$$P(Y=3|X) = \log\left(\frac{P(Y=1|X)+P(Y=2|X)+P(Y=3|X)}{P(Y=4|X)}\right)$$

write $\text{logit}(y) = \frac{1}{1-y}$

$$\boxed{\log\left[\frac{Pr(Y\le j\,|X)}{Pr(Y>j\,|X)}\right] = \alpha_j + \beta_j' x}$$

Proportional Odds Model

If in total 4 levels

$$P(Y=1|X) = \log\left(\frac{P(Y=1|X)}{P(Y=2|X)+P(Y=3|X)+P(Y=4|X)}\right)$$

$$= \hat\beta_0 + \hat\beta_1 X$$

$$P(Y=2|X) = \log\left[\frac{P(Y=1|X)+P(Y=2|X)}{P(Y=3|X)+P(Y=4|X)}\right]$$

$$= \hat\beta_0 + \hat\beta_2 X$$

3   2   4

$$P(Y \leq 3 \mid x) = \log\left[\frac{Pr(Y \leq 3 \mid x)}{P(Y > 3 \mid x)}\right]$$

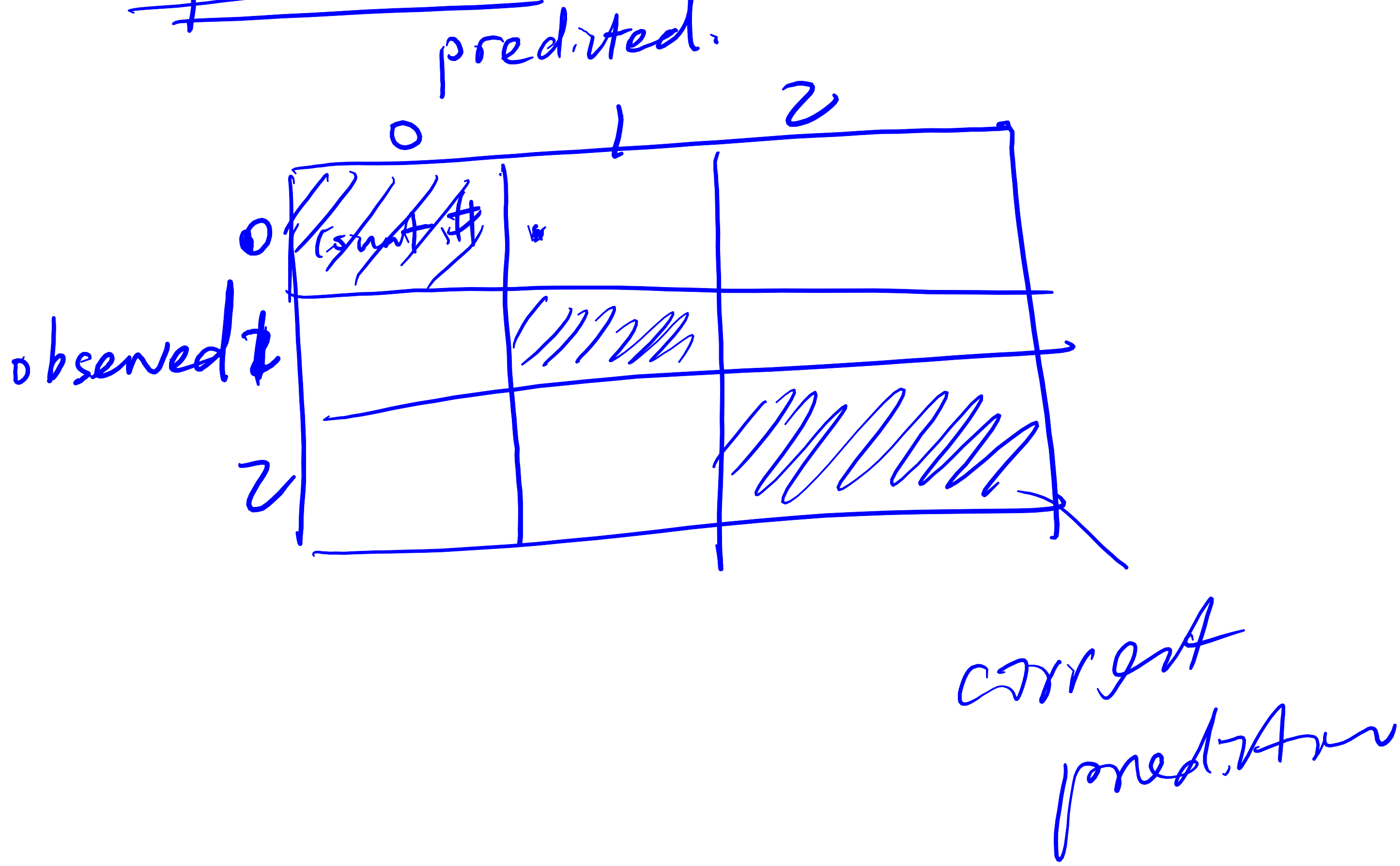
j

$$= \left[\log\left(\frac{Pr(1) + P(2) + P(3)}{P(4)}\right)\right]$$

j levels $\Rightarrow \boxed{j-1}$ equations

$$P(Y=4) = 1 - P(1) - P(2) - P(3)$$

Confusion Matrix



$$\hat{y} = X\beta + e$$

$$\hat{y}_1 \quad \hat{y}_2 \quad \cdots \quad \hat{y}_n.$$

Evaluate prediction

$$\frac{1}{n}\sum_{i=1}^{n}\left(\hat{y}_{i\,pred} - \hat{y}_{i\,observed}\right)^2$$

mean square error

Id	var	
1	Brown	1
2	C	0
3	D	0

appBrown	appC	appCl
10	5	2

ID	Vote
1.	Brown

ID	Vote	app
1 ~~vote~~	B.	10
1	C	5
1	Cl.	2