Week 13

Time Series and Forecasting

Learning Objectives

- ❑ Describe the basic components of time series data
- ❑ Use smoothing technique to extrapolate trend
- ❑ Forecast future value using smoothed data
- ❑ Evaluate the forecasting performance of a predictive regression model

Time Series Data

Time Series Data:

- a sequence of observations over regular intervals of time
  - E.g. no. of newly registered company per year, monthly sales of a firm, daily share price, etc.

- A time series data can be plotted against time and show different patterns

Time Series Data

Time series are examined for at least two reasons:

- to understand the past
- to predict the future

By analyzing a time series, we can identify patterns and tendencies that help explain historical variations in the variable of interest. This understanding contributes to our ability to forecast future values of the variable.

Time Series Data

Any time series can be broke into four components:

- **Trend** component (T)
- **Seasonal** component (S)
- **Cyclical** component (C)
- **Irregular** component (I)

Additive:

$$y = T + S + C + I$$

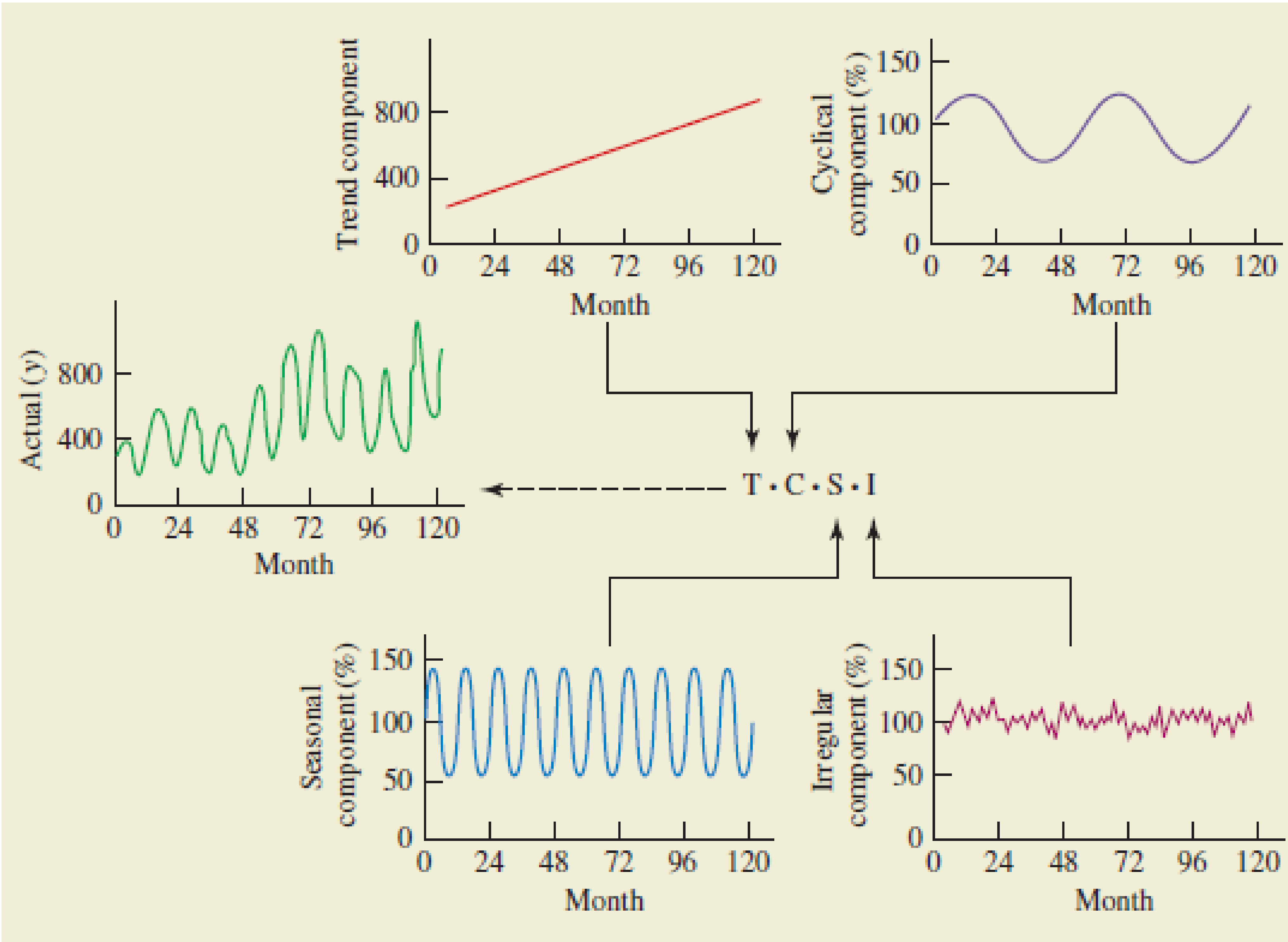
Multiplicative:

$$y = T \cdot S \cdot C \cdot I$$

Time Series Components

Time Series

Trend	Seasonal	Cyclical	Irregular
Overall, persistent, long-term movement	Regular periodic fluctuations within a year	Repeating movement over period of more than a year	Erratic, residual variations



Trend component
800
400
0
0 24 48 72 96 120
Month
Cyclical component (%)
150
100
50
0
0 24 48 72 96 120
Month
Actual (y)
800
400
0
0 24 48 72 96 120
Month
T • C • S • I
Seasonal component (%)
150
100
50
0
0 24 48 72 96 120
Month
Irregular component (%)
150
100
50
0
0 24 48 72 96 120
Month

Smoothing Technique

Usually, the most important component of time series is **trend**, as it is the only long term component that fit for forecasting.

We can get the trend from the time series by 'smoothing' it using some techniques:

- Moving Average
- Exponential Smoothing

Moving Average

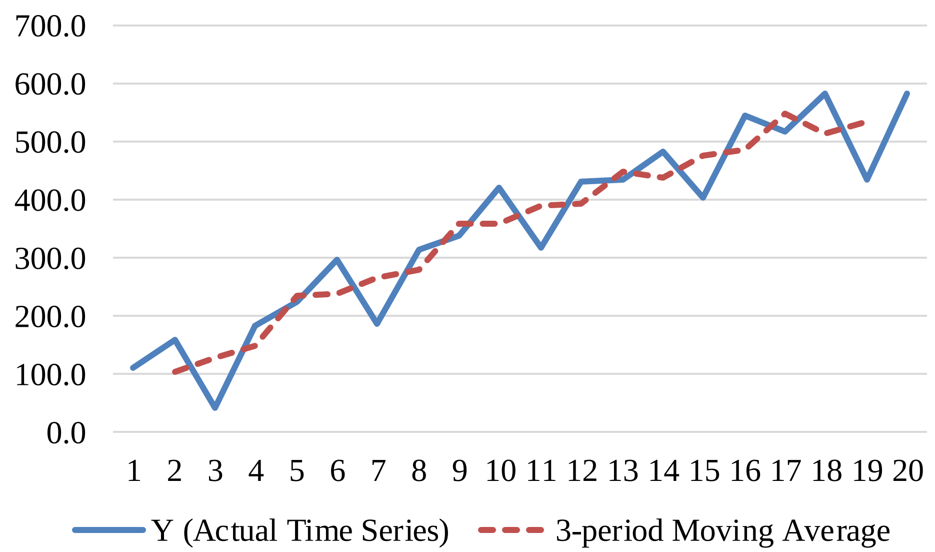
- The **moving average** replaces the original time series with another series, each point of which is the center of and the average of *N* points from the original series

- A.k.a. *centered* moving average (CMA)

- The purpose of the moving average is to take away the short-term seasonal and irregular variation, leaving behind a combined trend and cyclical movement

Moving Average: Example

Quarter	Y (Actual Time Series)	3-period Moving Total	3-period Moving Average
1	112.0		
2	159.0	313.0	104.3
3	42.0	383.0	$\boxed{127.7}$
4	182.0	448.3	149.4
5	224.3		



Moving Average on 3rd quarter:

$$127.7 = \frac{159.0 + 42.0 + 182.0}{3}$$

Exponential Smoothing

- Each exponentially smoothed (E) value is a weighted average of (1) the actual y value for that time period and (2) the exponentially smoothed value calculated for the previous time period.

- $$E_t = (\alpha)y_t + (1-\alpha)E_{t-1}, \quad 0 \leq \alpha \leq 1$$

- The larger the smoothing constant (α), the more importance is given to the actual y value for the time period.

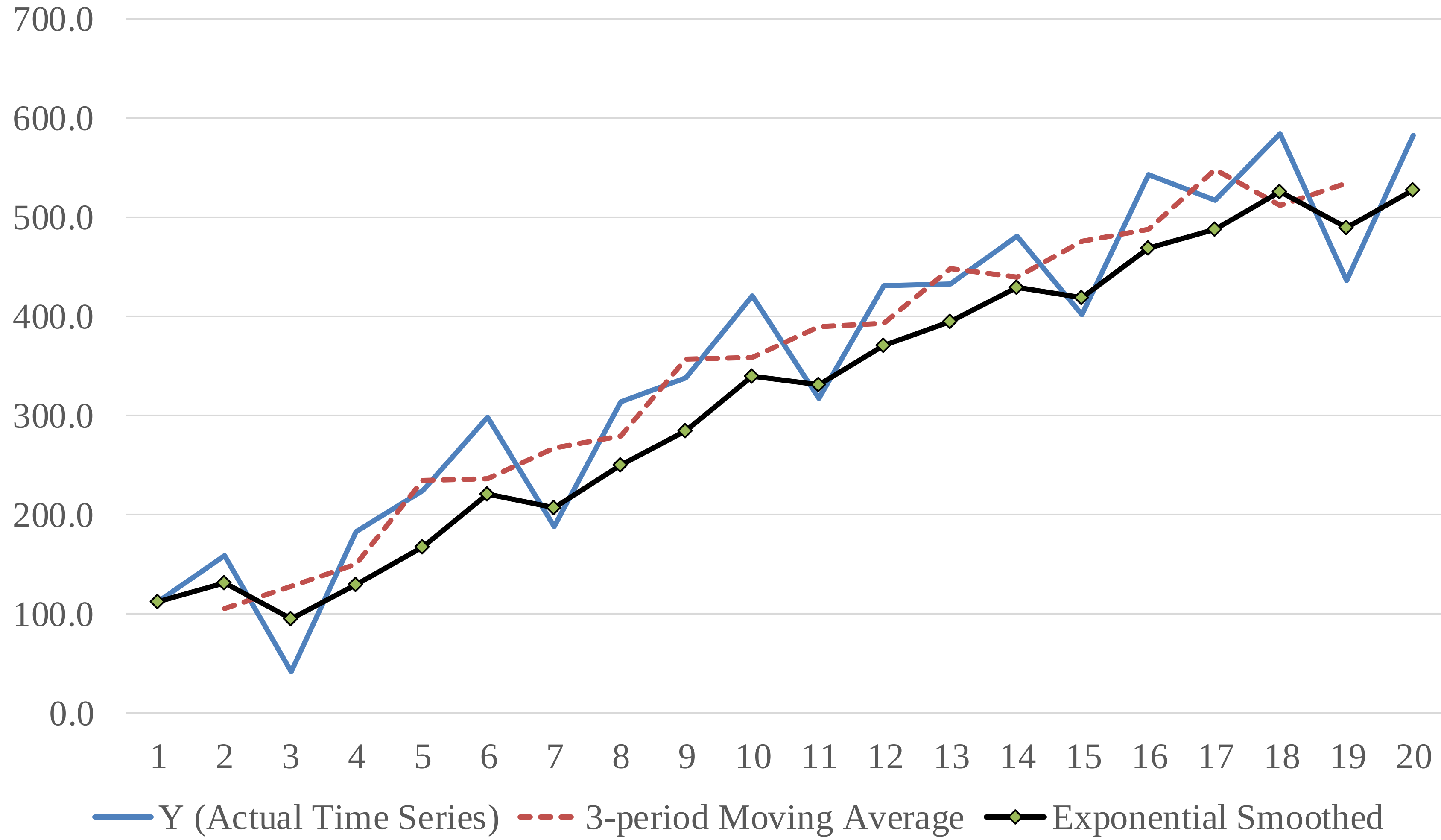
Exponential Smoothing: Example

Using the same data:

- Assume that the smoothing constant α is 0.40
- Let the initial smoothed value = the initial actual value, that is $E_1 = y_1$

Quarter	Y (Actual Time Series)	Exponential Smoothed (α = 0.4)	
1	112.0	112.0	$\rightarrow E_1 = y_1$
2	159.0	130.8	
3	42.0	95.3	$\rightarrow = 0.4(42) + 0.6(130.8)$
4	182.0	130.0	
5	224.3	167.7	
6	298.3	219.9	

Moving Average vs Exponential Smoothing



Forecasting

In forecasting, we use data from the past in predicting the future value of the variable of interest

There are at least 2 ways that we can use time series for forecasting:

- Trend equation
- Exponential Smoothing

Forecast using the Trend Equation

This technique requires a trend equation:

- Linear trend equation:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

- Quadratic trend equation:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2$$

where $\hat{y}$ is the trend line estimate of y
x is the time period

Forecast using the Trend Equation: Example

Once we've fitted a trend equation to the data, a forecast can be made by simply substituting a future value for the time period variable.

- For example, consider the trend equation for annual sales:

$$\hat{y} = 54.068 - 45.418x + 11.377x^2$$

where $\hat{y}$ is the trend line estimate of annual sales ('000)

x is the year code with 2000 = 1

- *Question*: What is the forecast value of annual sales in year 2020?

Forecasting with Exponential Smoothing

- Exponential smoothing can also be used in making a forecast for one time period into the future.

$$F_{t+1} = (\alpha)y_t + (1 - \alpha)F_t, \qquad 0 \leq \alpha \leq 1$$

where F_t is the forecast value at period t

Forecasting with Exponential Smoothing: Example

Quarter	Y (Actual Time Series)	Forecast ($\alpha = 0.5$)
1	112.0	
2	**159.0**	112.0
3	**42.0**	135.5
4	**182.0**	88.8
5	**224.3**	135.4
6	**298.3**	179.8
7	**187.3**	239.1
8	**314.3**	213.2
9	**337.0**	263.7
10	**420.0**	300.4

Evaluating Forecast Model

But how to evaluate the performance of a forecast model?

- Idea: estimate the model on a fraction of the available data and use the remaining part to evaluate out-of-sample forecast performance

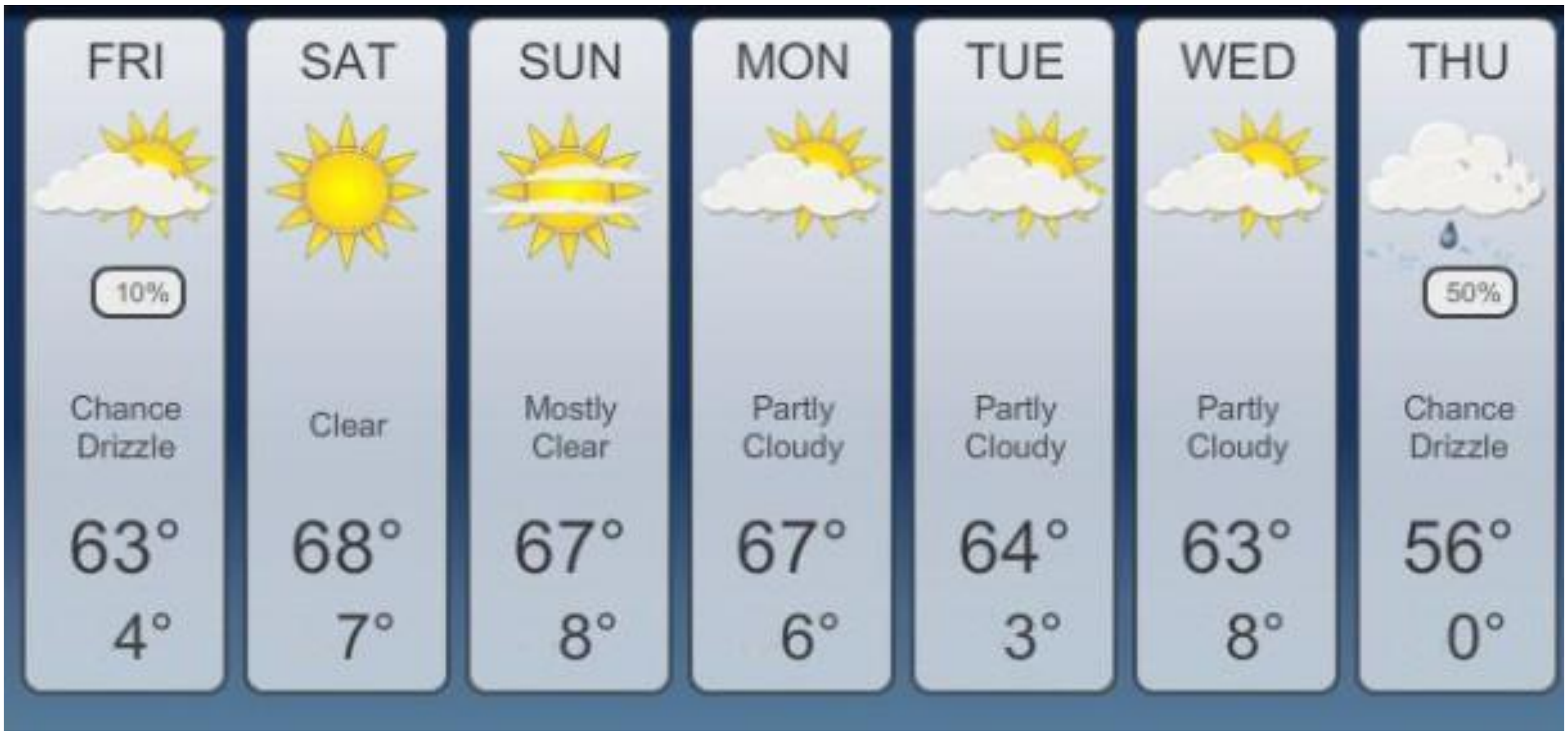
- Consider the following Example 1 and 2

Evaluating Forecast Model

Example 1:
21 Dec 2012 (Friday) - the end of a long cycle according to the Mayan calendar

Evaluating Forecast Model

Example 2:



Evaluating Forecast Model

Is either a good forecast model?

- Example 1:
  - It is very precise, but far from the actual observation
  - Low uncertainty but large bias
- Example 2
  - Might be unbiased, but high uncertainty

Evaluating Alternative Models: MAD & MSE

When two or more models are fitted to the same time series, it's useful to have one or more criteria on which to compare them (low uncertainty, close to average).

Two criterions:

- Mean Absolute Deviation (*MAD*)
- Mean Squared Error (*MSE*)

Mean Absolute Deviation (MAD)

From a given set of models or estimation equations fit to the same time series data, the model or equation that best fits the time series is the one with the **lowest value** of

$$MAD = \frac{\sum |y_t - \widehat{y_t}|}{n}$$

where

y_t = actual value of y

$\widehat{y_t}$ = forecast value of y

n = the number of time periods

Mean Squared Error (MSE)

From a given set of models or estimation equations fit to the same time series data, the model or equation that best fits the time series is the one with the **lowest value** of

$$MSE = \frac{\sum(y_t - \widehat{y_t})^2}{n}$$

where

y_t = actual value of y

$\widehat{y_t}$ = forecast value of y

n = the number of time periods

Evaluating Alternative Models: Example

Model 1: Linear Trend Model

Quarter	Y (Actual Time Series)	$\hat{Y}$ (Forecast Value)	Error $Y-\hat{Y}$	Absolute Error $\lvert Y-\hat{Y}\rvert$	Squared Error $(Y-\hat{Y})^2$
1	112.0	111.6	0.4	0.4	0.2
2	159.0	136.7	22.3	22.3	498.7
3	42.0	161.8	-119.8	119.8	14347.0
4	182.0	186.9	-4.9	4.9	23.9
5	224.3	212.0	12.3	12.3	150.9
6	298.3	237.1	61.2	61.2	3741.9
			Total:	**220.9**	**18762.7**

$$MAD = \frac{\sum|y_t - \hat{y}_t|}{n} = \frac{220.9}{6} = \mathbf{36.8}$$

$$MSE = \frac{\sum(y_t - \hat{y}_t)^2}{n} = \frac{18762.7}{6} = \mathbf{3127.1}$$

Evaluating Alternative Models: Example

Model 2: Exponential Forecast ($\alpha = 0.5$)

Quarter	Y (Actual Time Series)	$\hat{Y}$ (Forecast Value)	Error $Y - \hat{Y}$	Absolute Error $\lvert Y - \hat{Y} \rvert$	Squared Error $(Y - \hat{Y})^2$
1	112.0	112.0	0.0	0.0	0.0
2	159.0	135.5	23.5	23.5	552.3
3	42.0	88.8	-46.8	46.8	2185.6
4	182.0	135.4	46.6	46.6	2173.9
5	224.3	179.8	44.5	44.5	1976.2
6	298.3	239.1	59.2	59.2	3507.9
			Total:	**220.6**	**10395.8**

$$MAD = \frac{\sum \lvert y_t - \hat{y_t} \rvert}{n} = \frac{220.6}{6} = \mathbf{36.8}$$

$$MSE = \frac{\sum (y_t - \hat{y_t})^2}{n} = \frac{10395.8}{6} = \mathbf{1732.6}$$

Evaluating Alternative Models: MAD & MSE

- Model 2 performs better as it has the lower MSE and a similar MAD as compared to Model 1

- Compared to the MAD criterion, the MSE approach places a greater penalty on estimates for which there is a large error.
  - This is because MSE is the mean of the squared errors instead of simply the mean of the absolute values.
  - Therefore the MSE criterion is to be preferred whenever the cost of an error in estimation or forecasting increases in more than a direct proportion to the amount of the error.

Summary

- Time series and its components
- Smoothing techniques
- Forecasting using trend equation and exponential technique
- Evaluating forecast performance using MAD and MSE