1 Recursion

1.1 (Adapted from Fall 2013) Fill in the blanks in the implementation of `paths`, which takes as input two positive integers x and y. It returns a list of paths, where each path is a list containing steps to reach y from x by repeated incrementing or doubling. For instance, we can reach 9 from 3 by incrementing to 4, doubling to 8, then incrementing again to 9, so one path is [3, 4, 8, 9]

```python
def paths(x, y):
    """Return a list of ways to reach y from x by repeated
    incrementing or doubling.
    >>> paths(3, 5)
    [[3, 4, 5]]
    >>> sorted(paths(3, 6))
    [[3, 4, 5, 6], [3, 6]]
    >>> sorted(paths(3, 9))
    [[3, 4, 5, 6, 7, 8, 9], [3, 4, 8, 9], [3, 6, 7, 8, 9]]
    >>> paths(3, 3) # No calls is a valid path
    [[3]]
    """
    if _____:

        return _____

    elif _____:

        return _____

    else:

        a = _____

        b = _____

        return _____
```

1.2   We will now write one of the faster sorting algorithms commonly used, known as
merge sort. Merge sort works like this:

1. If there is only one (or zero) item(s) in the sequence, it is already sorted!

2. If there are more than one item, then we can split the sequence in half, sort each
   half recursively, then merge the results, using the `merge` procedure described
   below. The result will be a sorted sequence.

Using the algorithm described, write a function `mergesort(seq)` that takes an un-
sorted sequence and sorts it.

Recall the `merge` procedure is as follows:

```python
def merge(s1, s2):
    """ Merges two sorted lists """
    if len(s1) == 0:
        return s2
    elif len(s2) == 0:
        return s1
    elif s1[0] < s2[0]:
        return [s1[0]] + merge(s1[1:], s2)
    else:
        return [s2[0]] + merge(s1, s2[1:])


def mergesort(seq):
```

2 Trees

2.1 Implement long_paths, which returns a list of all *paths* in a tree with length at least
n. A path in a tree is a linked list of node values that starts with the root and ends
at a leaf. Each subsequent element must be from a child of the previous value's
node. The *length* of a path is the number of edges in the path (i.e. one less than
the number of nodes in the path). Paths are listed in order from left to right. See
the doctests for some examples.

```python
def long_paths(tree, n):
    """Return a list of all paths in tree with length at least n.

    >>> t = Tree(3, [Tree(4), Tree(4), Tree(5)])
    >>> left = Tree(1, [Tree(2), t])
    >>> mid = Tree(6, [Tree(7, [Tree(8)]), Tree(9)])
    >>> right = Tree(11, [Tree(12, [Tree(13, [Tree(14)])])])
    >>> whole = Tree(0, [left, Tree(13), mid, right])
    >>> for path in long_paths(whole, 2):
    ...     print(path)
    ...
    <0 1 2>
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 6 9>
    <0 11 12 13 14>
    >>> for path in long_paths(whole, 3):
    ...     print(path)
    ...
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 11 12 13 14>
    >>> long_paths(whole, 4)
    [Link(0, Link(11, Link(12, Link(13, Link(14)))))]
    """
```

2.2   Write a function that takes a `Tree` object and returns the elements at the depth with the most elements.

In this problem, you may find it helpful to use the second optional argument to `sum`, which provides a starting value. All items in the sequence to be summed will be concatenated to the starting value. By default, start will default to 0, which allows you to sum a sequence of numbers. We provide an example of sum starting with a list, which allows you to concatenate items in a list.

```python
def widest_level(t):
    """
    >>> sum([[1], [2]], [])
    [1, 2]
    >>> t = Tree(3, [Tree(1, [Tree(1), Tree(5)]),
    ...               Tree(4, [Tree(9, [Tree(2)])])])
    >>> widest_level(t)
    [1, 5, 9]
    """
    levels = []
    x = [t]

    while _____:

        _____

        _____ = sum(_____, [])

    return max(levels, key=_____)
```

3 Mutation

3.1 For each row below, fill in the blanks in the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and expressions may affect later expressions.

```
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
[1, 2, 23]

>>> dogs[1] = list(dogs)
>>> dogs[1]
[[1, 2, 23], None, [1, 2, 23]]

>>> dogs[0].append(2)
>>> cats
[1, 2, 23, 2]

>>> cats[1::2]
[2, 2]

>>> cats[:3]
[1, 2, 23]

>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
IndexError

>>> dogs
[[1, 2, 23, 2], [[1, 2, 23, 2], None, [1, 2, 23, 1, 3]], [1, 2, 23, 1, 3]]
```

4   OOP

4.1 (Summer 2015 Final) The TAs are building a social networking website called
CS61A+. The TAs plan to represent the network in a class called `Network` that
supports the following method:

- `add_friend(user1, user2)` adds `user1` and `user2` to each other's friends lists.
  If `user1` or `user2` are not in the `Network`, add them to the dictionary of friends.

Help the TAs implement these two methods to make their social networking website
popular!

```python
class Network:
    """
    >>> cs61a_plus = Network()
    >>> cs61a_plus.add_friend('Robert', 'Jeffrey')
    >>> cs61a_plus.friends['Robert']
    ['Jeffrey']
    >>> cs61a_plus.friends['Jeffrey']
    ['Robert']
    >>> cs61a_plus.add_friend('Jessica', 'Robert')
    >>> cs61a_plus.friends['Robert']
    ['Jeffrey', 'Jessica']
    """
    def __init__(self):
        self.friends = {}        # Maps users to a list of their friends

    def add_friend(self, user1, user2):


        if _____:


            _____


        if _____:


            _____


        _____


        _____
```

CS61A+ turns out to be unpopular. To attract more users, the TAs want to implement a feature that checks if two users have at most n degrees of separation. Consider the following CS61A+ Network:

```
self.friends = {
    'Robert':  ['Jeffrey', 'Jessica'],
    'Jeffrey': ['Robert', 'Jessica', 'Yulin'],
    'Jessica': ['Robert', 'Jeffrey', 'Yulin'],
    'Yulin':   ['Jeffrey', 'Jessica'],
    'Albert':  []
}
```

- There is 1 degree of separation between Robert and Jeffrey, because they are direct friends.

- There are 2 degrees of separation between Robert and Yulin (Robert $\rightarrow$ Jessica $\rightarrow$ Yulin)

- The degree of separation between Albert and anyone else is undefined, since Albert has no friends.

```
class Network:
    # Code from previous question

    def degrees(self, user1, user2, n):
        """In these doctests, assume cs61a_plus is a Network with the
        dictionary of friends described in the example.

        >>> cs61a_plus.degrees('Robert', 'Yulin', 2)    # Exactly 2 degrees
        True
        >>> cs61a_plus.degrees('Robert', 'Jessica', 2)  # Less than 2 degrees
        True
        >>> cs61a_plus.degrees('Yulin', 'Robert', 1)    # More than 1 degree
        False
        >>> cs61a_plus.degrees('Albert', 'Jessica', 10) # No friends!
        False
        """
        if _____:
            return True


        elif _____:
            return False


        for friend in _____:

            if _____:
                return True


        return _____
```

5 Mutable Linked Lists and Trees

5.1 Write a recursive function `flip_two` that takes as input a linked list `lnk` and mutates `lnk` so that every pair is flipped.

```python
def flip_two(lnk):
    """
    >>> one_lnk = Link(1)
    >>> flip_two(one_lnk)
    >>> one_lnk
    Link(1)
    >>> lnk = Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> flip_two(lnk)
    >>> lnk
    Link(2, Link(1, Link(4, Link(3, Link(5)))))
    """
```

6 Generators

6.1 Write a generator function that yields functions that are repeated applications of a one-argument function f. The first function yielded should apply f 0 times (the identity function), the second function yielded should apply f once, etc.

```
def repeated(f):
    """
    >>> double = lambda x: 2 * x
    >>> funcs = repeated(double)
    >>> identity = next(funcs)
    >>> double = next(funcs)
    >>> quad = next(funcs)
    >>> oct = next(funcs)
    >>> quad(1)
    4
    >>> oct(1)
    8
    >>> [g(1) for _, g in
    ...  zip(range(5), repeated(lambda x: 2 * x))]
    [1, 2, 4, 8, 16]
    """



    g = _____

    while True:


        _____


        _____
```

6.2 Ben Bitdiddle proposes the following alternate solution. Does it work?

```
def ben_repeated(f):
    g = lambda x: x
    while True:
        yield g
        g = lambda x: f(g(x))
```

6.3   Implement `accumulate`, which takes in an `iterable` and a function `f` and yields
each accumulated value from applying `f` to the running total and the next element.

```python
from operator import add, mul

def accumulate(iterable, f):
    """
    >>> list(accumulate([1, 2, 3, 4, 5], add))
    [1, 3, 6, 10, 15]
    >>> list(accumulate([1, 2, 3, 4, 5], mul))
    [1, 2, 6, 24, 120]
    """
    it = iter(iterable)
```

_____

_____

for _____:

_____

_____

7   Scheme

7.1   Write a function that takes a procedure and applies to every element in a given nested list.

The result should be a nested list with the same structure as the input list, but with each element replaced by the result of applying the procedure to that element.

Use the built-in `list?` procedure to detect whether a value is a list.

```
(define (deep-map fn lst)
```

```
scm> (deep-map (lambda (x) (* x x)) '(1 2 3))
(1 4 9)
scm> (deep-map (lambda (x) (* x x)) '(1 ((4) 5) 9))
(1 ((16) 25) 81)
```