澳門學界數學比賽解答
Macao Inter-school Contest Junior 2026/10 Solution

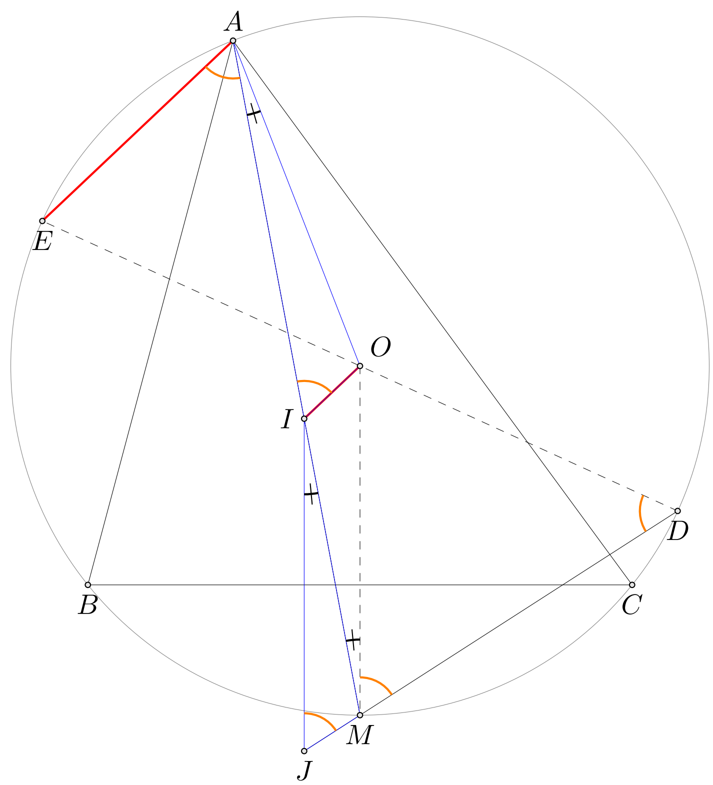
HaoJiang Tsai

2026 年 2 月 27 日

1 Problem

Given Acute ΔABC, its incentre I, circumcentre O and circumcircle Ω. $\overline{AI}$ intersecting Ω again at M, J is the reflection of I through $\overline{BC}$. $\overline{JM}$ intersecting Ω again at D, DO intersecting Ω again at E.

Prove that $AE \parallel OI$.



2 Solution

First let R denote the radius of Ω and r denote the radius of ΔABC's incircle (why do I set up?).

We have $IJ \parallel OM$ as $IJ \perp BC$ and $OM \perp BC$(Fact 5 said M is the midpoint of arc BC).

Claim: $\Delta AIO \sim \Delta IJM$.

Proof:

Since the definition of J, we have $IJ = 2r$.

By Euler’ s theorem we have $OI^2 - R^2 = -2Rr$, and LHS is actually $Pow_{\Omega}(I)$, so $AI \cdot IM = 2Rr$, finally reduce to $\frac{AI}{IJ} = \frac{AO}{IJ}$.

And angle chase: $\angle JIM = \angle IMO = \angle IAO$, we’ re done. $\square$

By the following claim, quickly give that

$$\angle AIO = \angle IJM = \angle OMD = \angle ODM = \angle EAM = \angle EAI$$

as desired.