Macao Inter-school Math Competition Junior

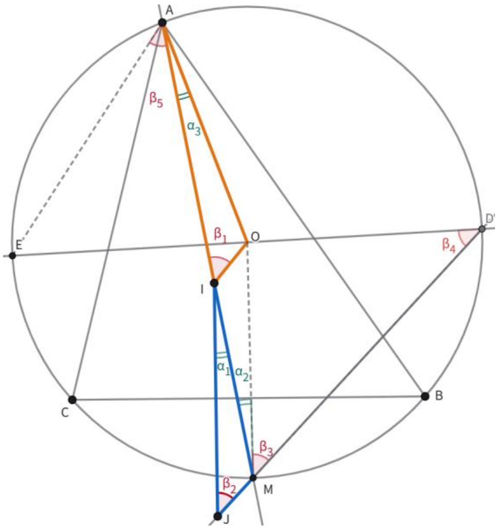
Problem 10:

Given ΔABC, its incenter I, circumcenter O, and circumcircle Ω.
AI intersecting Ω again at M, J is the reflection of I through segment BC.
JM intersecting Ω again at D, DO intersecting Ω again at E.

Prove that AE∥OI.

Solution:

We hope that $\angle AIO = \angle EAI$.
First let R denote the radius of Ω and r denote the radius of ΔABC' s incircle (why do I set up?).
We have $IJ \parallel OM$ *as* $IJ \perp BC$ *and* $OM \perp BC$*(By Fact 5 we know M is the midpoint of arc BC).*



Claim. $\Delta AIO \sim \Delta IJM$.
Proof: Since the definition of J, we have IJ = 2r.

By Euler's theorem we have $OI^2 - R^2 = -2Rr$, *and LHS is actually* $Pow_{\Omega}(I)$, *so* $AI \cdot IM = 2Rr$, *finally reduce to* $\frac{AI}{IJ} = \frac{AO}{IM}$.

And angle chase give the result:

$$\angle JIM = \angle IMO = \angle IAO$$

We're done.

By the following claim, quickly give that

$$\angle AIO = \angle IJM = \angle OMD = \angle ODM = \angle EAM = \angle EAI$$

as desired.

Remark:
J's definition ---- the reflection of I through BC is really strange(and I didn't experience before), but it implies that IJ = 2r and $IJ \parallel OM$. *we easily connect IJ = 2r with the Euler's theorem, and* $IJ \parallel OM$ *can use to do some angle chasing, which is useful after and during the main Claim.*