

CSC265 Fall 2021 Homework Assignment 8

due Wednesday November 24, 2021

1. Consider a comparison tree using 3-way comparisons that determines whether every input sequence x_1, x_2, \dots, x_n of n numbers contains a repetition.
 - (a) Prove that on the path from the root to a leaf taken by an input sequence of n distinct numbers, there must be a comparison between each pair of adjacent elements in the sorted order.
 - (b) Prove that the comparison tree has height at least $\log_2(n!)$.
2. A *mode* of a sequence of numbers is a number that occurs the most times in the sequence. Prove that the time complexity of determining a mode of a sequence of n numbers is in $\Theta(n \log n)$.
3. Suppose you are given two Boolean arrays $A[1..n]$ and $B[1..n]$ each of which is sorted (i.e. in each array, all occurrences of 0 occur before all occurrences of 1). The problem is to determine whether both arrays have the same number of occurrences of 1 and, if not, which array has more occurrences.
 - (a) Prove that, when $n = 1$, both bits must be read in the worst case and, when $n = 2$, all four bits must be read in the worst case.
 - (b) Determine the worst case *probe complexity* of this problem to within an additive constant (i.e., for some function $f(n)$, prove that any algorithm that solves this problem must read at least $f(n)$ bits in the worst case and give an algorithm solving this problem that reads at most $f(n) + O(1)$ bits).

If you cannot do this, you will get part marks for proving upper and lower bounds that match to within a constant factor.