CSC265 Fall 2021 Homework Assignment 4

due Wednesday October 13, 2021

Consider the abstract datatype SEQ whose objects are sequences of elements and which supports two operations:

- PREPEND(x, S), which inserts element x at the beginning of the sequence S and
- ACCESS(S, i), which returns the i'th element in the sequence.

Suppose that we represent S by a singly linked list. Then PREPEND(x, S) takes 1 step and ACCESS(S, i) takes i steps, provided S has at least i elements.

Suppose that S initially has exactly one element. A sequence of n operations are performed. Each operation in the sequence is (independently) chosen to be an ACCESS with probability q and a PREPEND with probability $1 - q$, where $0 < q < 1$. For each ACCESS operation, the value of the parameter i is chosen uniformly from $[1..|S|]$.

1. Let $0 \leq k \leq n$. Determine the expected length of the linked list after k operations have been performed.
2. Let $0 \leq k \leq n$. Determine the expected number of steps taken to perform the k'th operation.
3. Determine the expected number of steps taken to perform all n operations.

Justify your answers. Remember to begin by describing the relevant probability space(s) and defining appropriate random variables.