CSC265 Fall 2021 Homework Assignment 7

due Wednesday, November 17, 2021

A *pennant* of height h is a tree consisting of 2^h nodes: a root with exactly one child, which is the root of a complete binary tree of the remaining $2^h - 1$ nodes. It also satisfies the max-heap property: the element at any node has priority at least as big as all the elements in the subtree rooted at that node. Note that a pennant of height 0 consists of a single node.

A *pennant forest* is a sequence of pennants $P_0, P_1, \ldots, P_m$ such that

- $\text{height}(P_{i-1}) \leq \text{height}(P_i)$, for $0 < i \leq m$.
- There are at most 2 pennants of any height.
- There are at least $i + 1$ pennants of height at most i, for $0 \leq i \leq m$.
- There are at most $i + 2$ pennants of height at most i, for $0 \leq i \leq m$.
- The element at the root of $P_{i-1} \leq$ the element at the root of P_i, for $0 < i \leq m$.

1. Prove that if $P_0, P_1, \ldots, P_m$ is a pennant forest, then it contains at least 2^m nodes and fewer than 2^{m+1} nodes.
2. Give an efficient algorithm that, given a max-heap of height h, constructs a pennant forest $P_0, \ldots, P_h$ with the same elements.
3. Give an efficient algorithm that, given a pennant forest $P_0, \ldots, P_m$, constructs a max-heap of height m with the same elements.
4. Give an efficient algorithm for splitting a pennant of height h into two pennants of height $h - 1$.
5. Give an efficient algorithm for merging two pennants of height $h - 1$ into a pennant of height h.
6. Give an algorithm that takes a sequence $P_0, \ldots, P_m$ of pennants that satisfies the first four properties of a pennant forest and constructs a pennant forest with the same nodes. Your algorithm should run in $O(m^2)$ time.

Explain why your algorithms are correct and run in the required times.