Homework 2, Math 450, Spring 2017
Scaling & Dimensional analysis

due February 1, 2017

Instruction: Answer each of the following questions, showing and explaining your work as you go. Partial credit will be awarded based on how well I can follow your work and how far you get, so please use sentences, description, diagrams, and clear definitions to communicate your results as best you can. All diagrams and plots should be labelled.

1. Table 2, Page 45 of J. Huxley's 1932 book Problems of Relative Growth provides a table of red deer antler statistics relative to body mass. Use python (or a language of your choice) to recalculated a scaling relationship between the deer mass and the antler mass from this data. In your answer, present the data in a new table, your parameterization for the scaling law, your fitted parameter values, and a plot comparing the data to the fitted scaling law.

2. In a 2009 article (link), Head and other authors use a single fossil vertebrae to deduce that about 60 million years ago, there existed a neotropical snake that grew to 13 meters in length and weighted more than 1,000 kilograms (42 feet and more than 2,000 pounds). They called this snake *Titanoboa cerrejonensis*. This discovery and others inspired the PBS Secrets of the Dead episode Graveyard of the giant beasts in 2016. (youtube)

   (a) What were the two regression lines the team obtained for the relationship between between vertabra width and total body length using the extreme values of 60% and 65% for vertebral position?

   (b) Create a plot of width vs total body length using known 60% and 65% position data. Add to this plot the regression lines from part (a). Make sure your axes scales are large enough to show the predicted body length of *Titanoboa*, given that the discovered vertabra was 12 cm wide.

   (c) Do you believe their results? Defend your opinion.

3. In *Principia Mathematica*, Isaac Newton discusses the resistance a fluid poses to an object moving through it. These ideas were subsequently applied to the calculation of the lift force created by an inclined plate moving through the air. The lift force F depends on the density of the fluid ρ, the surface area of the plate S, the velocity of the plate through the fluid v and the angle of attack of the plate θ.

   (a) Use dimensional analysis to derive an equation for the attack angle θ as a function of a dimensionless product of the other variables.

   (b) Solve your equation for the lift force F.

   (c) Newton believed that when the angle of attack was small, the lift force scaled like the square of the angle. In 1804, George Cayley tested Newton's idea and found it in poor agreement with experimental data. Subsequent experiments found that lift was in fact proportional to the angle of attack for small angles. Why might this mistake have had a big discouraging effect on the practical development of flight?

4. A bicyclist traversing a turn on a road leans into the turn to keep her bicycle stable.

   (a) Find 3 dimensional variables that together should determine the angle at which she should lean.

   (b) Determine the units of each variable, in terms of the fundamental units of length, time, mass, charge, and temperature.

   (c) According to the Buckingham Π theorem, how many dimensionless groups can be constructed from your 3 dimensional variables.

(d) Find a general formula for a functional relationship between your variables and the angle of lean, expressed in terms of dimensionless groups.

5. When you cook a roast, the cooking time (T) is defined as the time needed for the center of the roast to reach a pre-defined temperature. The cooking time actually depends on the thermal conductivity (k) of the roast, its density (ρ), the radius of the roast (R), and the specific-heat capacity at constant pressure (s_p).

   (a) Find a dimensionless group relating these five variables.

   (b) If we halve the size of the roast, how should the cooking time be changed, assuming everything else stays the same?

6. (from Giordano, Fox, Horton, & Weir) A windmill is rotated by air flow to produce power to pump water. It is desired to find the power output P of the windmill. Assume that P is a function of the density of the air ρ, viscosity of the air μ, diameter of the windmill d, wind speed v, and the rotational speed of the windmill ω (radians per second). Thus,

$$P = f(\rho, \mu, d, v, \omega)$$

   (a) Using dimensional analysis, find a relationship for P.

   (b) Do your results make common sense? Explain,

   (c) Discuss how you would design an experiment to determine the nature of your function.