Project 1 Report

Haoliang (Jack) Hu, Javier Padilla, Boyuan Lu
McCombs School of Business, University of Texas at Austin
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Dr. Daniel Mitchell
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Questions 1 - 3

As a data scientist in the marketing department, our team has been tasked by the chief marketing officer to recommend how to allocate a $10M budget across different marketing mediums. An external consulting firm has assessed the ROI of each medium. To start, we first read the data in the CSV files provided to us by our boss, which can be seen in the code below:

Setup

```
!pip install gurobipy
```

```
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Requirement already satisfied: gurobipy in /usr/local/lib/python3.8/dist-packages (10.0.1)
```

```
[ ] import numpy as np
    import gurobipy as gp
    import pandas as pd
```

```
[ ] #Read in the files
    df1 = pd.read_csv('ROI_data.csv')
    df2 = pd.read_csv('roi_mat.csv')
    df3 = pd.read_csv('index_data.csv')
```

To find the optimal allocation using the first ROI data given from the first consulting firm, we first constructed a model containing a variable for each type of marketing medium. Then, we applied all three of the constraints given by our boss such that the amount of money allocated to print and TV is more than the amount allocated towards Facebook and email, the total amount used for social media is at least twice SEO and AdWords, and each of the mediums is allocated no more than $3M (our upper bound constraint).

```
Mod=gp.Model()
ModX = Mod.addMVar(10, ub = 3) # tell the model how many variables there are
ROI1 = np.array(df1.loc[0])
for index in range(len(ROI1)):
  ROI1[index] = ROI1[index]+1


# add the constraints to the model
conlist=[0]*3
conlist[0] = Mod.addConstr(ModX[0] + ModX[1] + ModX[2] + ModX[3] + ModX[4] + ModX[5] + ModX[6] + ModX[7] + ModX[8] + ModX[9] <= 10)
conlist[1] = Mod.addConstr(1*ModX[0] + 1*ModX[1] - 1*ModX[4] - 1* ModX[9] <= 0)
conlist[2] = Mod.addConstr(-2*ModX[2] + -2*ModX[3] + 1* ModX[4] + 1*ModX[5] + 1*ModX[6] + 1*ModX[7] + 1*ModX[8] >= 0)
Mod.setObjective(ROI1 @ ModX, sense=gp.GRB.MAXIMIZE)


Mod.Params.OutputFlag = 0 # tell gurobi to shut up!!
Mod.Params.TimeLimit = 3600 # stop if taking too long
Mod.optimize() # solve the LP, will take the most time
print(Mod.objVal) # optimal ROI
print(Mod.x) # how much to invest in each investment
```

```
10.456
[0.0, 3.0, 0.0, 1.0, 0.0, 0.0, 3.0, 0.0, 0.0, 3.0]
```

Using Gurobi, we were able to find our **optimal income to be $10.456M and that we should allocate $3M, $1M, $3M, and $3M to TV, AdWords, Instagram, and Email, respectively, with no amount of money going towards the other marketing mediums**. This information is shown in the table below:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	0	3	0	1	0	0	3	0	0	3

Questions 4 - 5

According to the second projected ROI for each platform, we have decided that the following investment amount for each platform will be the optimal investment strategy for the company:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	3	0	0	1	3	3	0	0	0	0

We used the same model as we did with the last dataset, except that we changed the corresponding ROI data to fit with the second prediction.

If the company decides to use the allocation derived from the second model with the second ROI, and if the first ROI is true, then the objective will be **$0.204 million lower** than the optimal income calculated with the first ROI and allocation (10.252M - 10.456M = -0.204M).

If the company decides to use the allocation from the first model with the first ROI, and if the second ROI is true, then the objective will be **$0.192 million lower** than the optimal income calculated with the second ROI and allocation (10.264M - 10.456M = -0.192M).

Although the optimal allocation changed, the **optimal income stayed the same** ($10.465M) both with and without the third constraint (for each platform, the investment should be no more than $3M). Therefore, **we can conclude the third constraint is not useful**.

Question 6

After doing sensitivity analysis on the model with the first ROI data, we get the following results:
Objective function's upper bound: array([1.049, 1.062, 1.039, 1.046, 1.029, 1.039, inf, 1.039, 1.039, inf])
Objective function's lower bound: array([-inf, 1.039, -inf, 1.033, -inf, -inf, 1.039, -inf, -inf, 1.029])

From the results we can see, if we want the allocation to stay the same using the first ROI:

- The ROI for print can go up to 4.9% and go down to negative infinity.
- The ROI for TV can go up to 6.2% and go down to 3.9%.
- The ROI for SEO can go up to 3.9% and go down to negative infinity.
- The ROI for AdWords can go up to 4.6% and go down to 3.3.
- The ROI for Facebook can go up to 2.9% and go down to negative infinity.
- The ROI for LinkedIn can go up to 3.9% and go down to negative infinity.
- The ROI for Instagram can go up to infinity and go down to 3.9%.
- The ROI for Snapchat can go up to 3.9% and go down to negative infinity.
- The ROI for Twitter can go up to 3.9% and go down to negative infinity.
- The ROI for Email can go up to infinity and go down to 2.9%.

After doing sensitivity analysis on the model with the second ROI data, we get the following results:
Objective function's upper bound: array([1.052, 1.049, 1.039, 1.046, inf, inf, 1.039, 1.039, 1.039, 1.029])
Objective function's lower bound: array([1.039, -inf, -inf, 1.0375, 1.029, 1.039, -inf, -inf, -inf, -inf])

From the results, we can see if we want the allocation to stay the same using the second ROI:

- The ROI for print can go up to 5.2% and go down to 3.9%.
- The ROI for TV can go up to 4.9% and go down to negative infinity.
- The ROI for SEO can go up to 3.9% and go down to negative infinity.
- The ROI for AdWords can go up to 4.6% and go down to 3.75%.
- The ROI for Facebook can go up to infinity and go down to 2.9%.
- The ROI for LinkedIn can go up to infinity and go down to 3.9.
- The ROI for Instagram can go up to 3.9 and go down to negative infinity.
- The ROI for Snapchat can go up to 3.9% and go down to negative infinity.
- The ROI for Twitter can go up to 3.9% and go down to negative infinity.
- The ROI for Email can go up to 2.9 and go down to negative infinity.

Question 7

The optimal allocation for each month is shown as the following:

January:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	3	0	0	1.333	0	0	2.667	0	0	3

February:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	3	0	0	2.395	3	0	0	0	1.791	0

March:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	0	0	0	3	0	3	1.203	0	3	0

April:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	0	0	0	3	0	3	3	0	1.204	0

May:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	1.200	0	0	0	0	0	3	0	3	3

June:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	3	0	0	0	0	0	3	0	1.206	3

July:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	0.606	0	0	3	0.606	0	3	0	3	0

August:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	2.714	0	0	1.5	0	0	0	0	3	3

September:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	0.609	0	0	3	0	3	0	0	3	0.609

October:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	0	0	0	3	0	3	3	0	0	1.201

November:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	3	0	0	1.186	0	0	3	0	0	3

December:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Allocation (Millions of Dollars)	3	2.110	0	0	3	0	0	0	0	2.110

Question 8

Given that a stable budget could be defined as a budget allocation where each of the amounts of money allocated to each medium does not change by more than 1M between months, we found that this budget allocation **would not be considered stable**. This is because the difference between the amounts of money allocated varies by more than $1M for several of the marketing mediums that we are trying to utilize. We found this by first creating a matrix for the objective function of each month. Then, we created a for loop that would find the optimal budget allocation for each month based on the monthly ROI while taking into account the constraints of the total amount of money we can invest. The total amount of money we can invest each month was calculated using the formula 10+0.5*(last month's return). These steps can be seen in detail below:

```
#Create a matrix for the objective function of each month
A = np.zeros((12,10))
for index1 in range(len(A)):
  ROI3 = np.array(df2.loc[index1])
  for index2 in range(len(ROI3)):
      ROI3[index2] = ROI3[index2]/100+1
  A[index1] = ROI3


#Create a for loop to find the optimal allocation based on each month's ROI
#Define the constrint regarding the total amount we can invest (based on 'money' variable) for the first month
money = 10
conlist3[2] = Mod3.addConstr(ModX3[0] + ModX3[1] + ModX3[2] + ModX3[3] + ModX3[4] + ModX3[5] + ModX3[6] + ModX3[7] +ModX3[8] + ModX3[9] <= money)
monthCount = 1
for month in A:
  #Update the constraint regarding the total amount we can invest for the current month
  conlist3[2].rhs = money
  Mod3.setObjective(month @ ModX3, sense=gp.GRB.MAXIMIZE)
  Mod3.Params.OutputFlag = 0 # tell gurobi to shut up!!
  Mod3.optimize()
  print(f"Month {monthCount}'s allocation is {Mod3.X}")
  print(f"Month {monthCount}'s income is {Mod3.objVal}")
  print(f"Month {monthCount +1}'s lowest money to invest for allocation to stay the same: {Mod3.SAObjLow}")
  print(f"Month {monthCount +1}'s highest money to invest for allocation to stay the same: {Mod3.SAObjUp}")
  #Find the amount of money we can invest for the next month
  # money = money + (Mod3.objVal-money)*0.5
  money = 10 + (Mod3.objVal-money)*0.5
  print(money)
  monthCount += 1
```

After running this for loop, we can see that just for the first two months many of the platforms being used do not stay within the $1M rate of change (highlighted below):

	Allocation (Millions of Dollars)										
Month		Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
	Month 1	3	0	0	1.333	0	0	3	0	0	3
	Month 2	3	0	0	2.395	3	0	0	0	1.791	0

In order to make allocation stable, the ROI of each platform should have very minimum changes between different months. The less the ROI changes, the less the allocation will change. We can **look for a threshold for ROI**. If the ROI changes within that threshold, then the change in allocation will be less than $1M. To apply this threshold to our model, we can simply **add some constraints** to limit the changes in ROI.

The following tables show the sensitivity analyses we performed for the first two months:

	Maximum ROI (%) for Allocation to Stay the Same										
Month		Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
	Month 2	inf.	3.9	3.9	4.1	3.4	3.6	3.7	3.6	3.6	3.7
	Month 3	inf.	4.0	3.8	4.3	inf.	3.7	3.7	3.7	3.8	3.6

	Minimum ROI (%) for Allocation to Stay the Same										
Month		Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
	Month 2	3.9	-inf	-inf	3.6	-inf	-inf	3.5	-inf	-inf	3.4
	Month 3	3.9	3.7	-inf.	3.7	3.5	-inf.	-inf.	-inf.	3.7	-inf.

Given the sensitivity analyses, we know that if the ROI of each platform **changes within these upper and lower bounds, then the allocation will be the same, and investment will be stable**. Therefore, if the monthly ROI changes are very close to these boundaries, then we would have a stable investment. However, since we cannot control the market and ROI of each platform, it would be difficult to achieve an extremely stable investment.