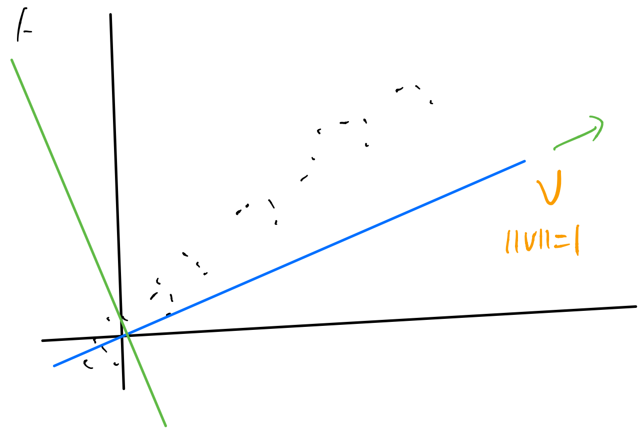
Week 6 - Lec 2

1.



PCA 找 var 最大的 v

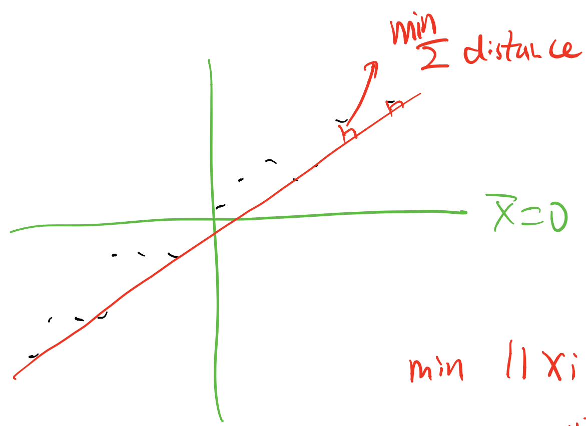
$V^T C V \to$ variance of V (C: covariance matrix; new co)

$tr(V^T C V) \to$ how much variance

① ?

换句话说

2.



$\|V\|=1$

$\sum_{i=1}^{N} \|x_i - v v^T x_i\|^2$

how far from original point

$\min \|x_i - v v^T x_i\|^2$

$= \|x_i\|^2 - 2\langle x_i, v v^T x_i\rangle + \|v v^T x_i\|^2$

$= \|x_i\|^2 - (v^T x_i)^2$

$\Downarrow$

$\max \sum_{i=1}^{N} (v^T x_i)^2 = \sum (v^T x - \overline{v^T x})^2$

2. $\bar{X}=0$ $\quad Cv = \lambda^* v \quad Xv$ eigen vector

$\frac{1}{N} X^T X = C \quad X^T X$

$\downarrow$

covariance matrix

$X^T y - X = \lambda \, Xv$

Kernerlized PCA ?

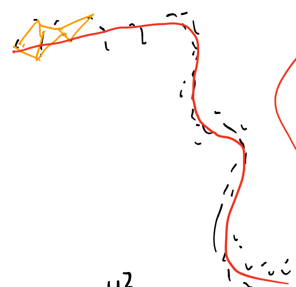
3. Laplacian eigen map

$x_i \in R^2$ $y_i \in R^1$

which points are neighbours of other points

$\min \sum_{ij} w_{ij} (y_i - y_j)^2$

If points are neigh, in y low 1D also close



$$\sum_{ij} w_{ij} y_i^2 + w_{ij} y_j^2 - 2 w_{ij} y_i y_j$$

$$= y^T (D - W) y$$

all pairs of weight

$$D_{ii} = \sum_j w_{ij}$$

$$= y^T L y$$

Laplacian graph

↳ has 0 eigen value and component

semidefinite

$$W_{ij} = e^{-\frac{\|x_i - x_j\|^2}{\sigma^2}}$$

If neighbours, w_{ij} ↑

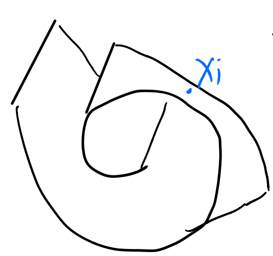
x_1 x_2 x_3 x_4

y_1, y_2 y_3 y_4

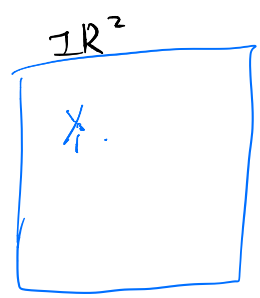
check

w ↑ want $y_i - y_j$ small

4. ISOMAP



3D →



① build 2D graph

MDS scaling

D D'

$(D_{ij} - D'_{ij})^2$

5、LLE

6、ICA: x_i original

independent component analysis

actural measure signal back

$x_i = M y_i$ — same D

y_i statistically independent

$p(y) = \prod_{i=1}^{d} p(y_i)$

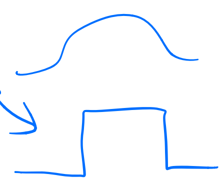
look for M

more guassian behave

more independent

$y_i = M^{-1} x_i$

want signal



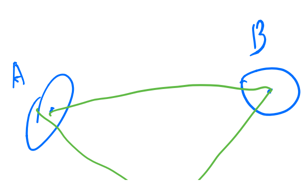
measure distance between ~~distance~~ distributions

$KL(p \| \prod p_i)$

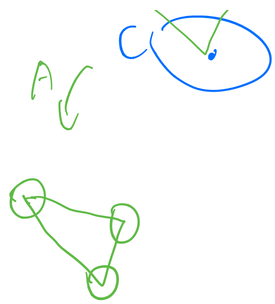
$\int p(x) \log \frac{p(x)}{\prod p_i(x_i)} dx > ?$

7、supervised method

LDA / fisher mapping.



$S_w = I$ within



$S_B =$

$S_B' = A^T S_B A$

$S_B S_W^{-1}$