PRINCETON UNIVERSITY

ORF 474: High Frequency Trading
Spring 2020
Robert Almgren

Lecture 3a

Feb 17, 2020

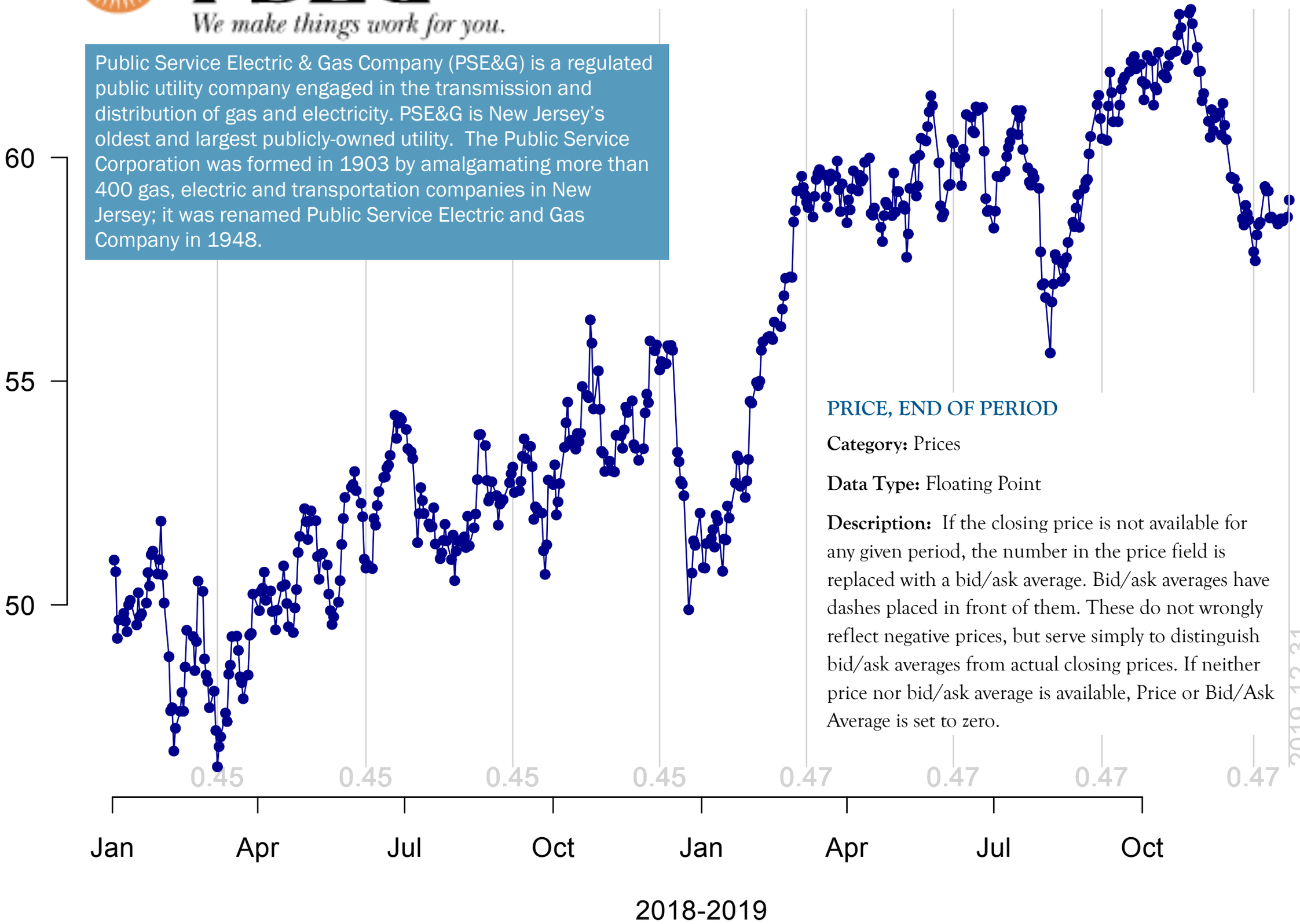
Today

- Daily volatility from CRSP data, and event info
- Market structure updates
- High frequency volatility from TAQ data
- Theoretical models for bid-ask spread

PSEG

We make things work for you.

Public Service Electric & Gas Company (PSE&G) is a regulated public utility company engaged in the transmission and distribution of gas and electricity. PSE&G is New Jersey's oldest and largest publicly-owned utility. The Public Service Corporation was formed in 1903 by amalgamating more than 400 gas, electric and transportation companies in New Jersey; it was renamed Public Service Electric and Gas Company in 1948.



PRICE, END OF PERIOD

Category: Prices

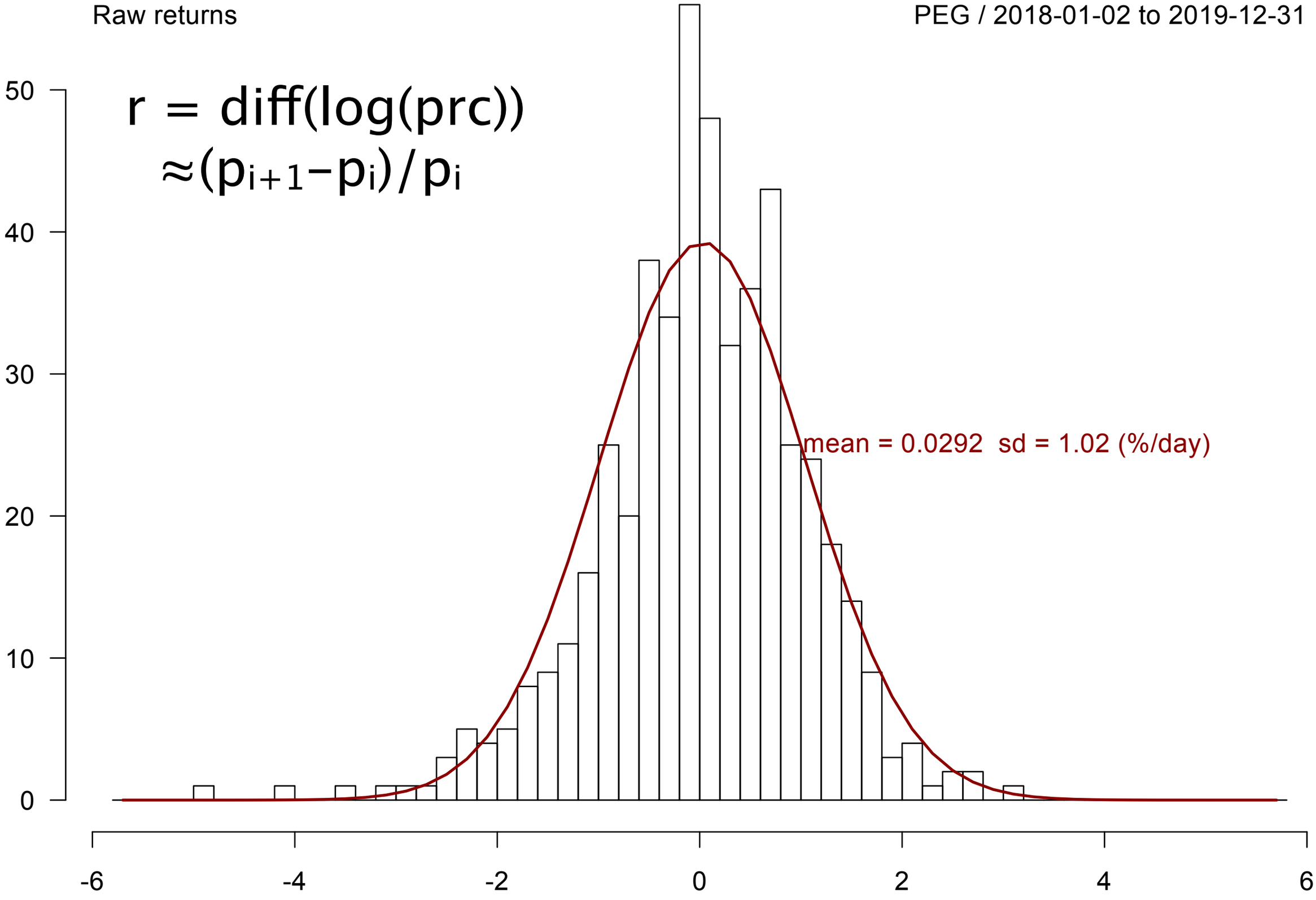
Data Type: Floating Point

Description: If the closing price is not available for any given period, the number in the price field is replaced with a bid/ask average. Bid/ask averages have dashes placed in front of them. These do not wrongly reflect negative prices, but serve simply to distinguish bid/ask averages from actual closing prices. If neither price nor bid/ask average is available, Price or Bid/Ask Average is set to zero.

Raw returns

PEG / 2018-01-02 to 2019-12-31

$$r = \text{diff}(\log(prc)) \approx (p_{i+1} - p_i)/p_i$$



CRSP dividend information

Stock / Events

For more about this dataset, see the Dataset List , Manuals and Overviews or FAQs .

Names | Distribution

Share Outstanding | Delist

Permno: 23712=PEG

	PERMNO	DISTCD	DIVAMT	FACPR	FACSHR	DCLRDT	EXDT	RCRDDT	PAYDT
1:	23712	1232	0.45	0	0	2018-02-20	2018-03-07	2018-03-08	2018-03-30
2:	23712	1232	0.45	0	0	2018-04-17	2018-06-07	2018-06-08	2018-06-29
3:	23712	1232	0.45	0	0	2018-07-17	2018-09-06	2018-09-07	2018-09-28
4:	23712	1232	0.45	0	0	2018-11-20	2018-12-06	2018-12-07	2018-12-31
5:	23712	1232	0.47	0	0	2019-02-19	2019-03-07	2019-03-08	2019-03-29
6:	23712	1232	0.47	0	0	2019-04-16	2019-06-06	2019-06-07	2019-06-28
7:	23712	1232	0.47	0	0	2019-07-16	2019-09-06	2019-09-09	2019-09-30
8:	23712	1232	0.47	0	0	2019-11-19	2019-12-09	2019-12-10	2019-12-31

DIVIDEND CASH AMOUNT

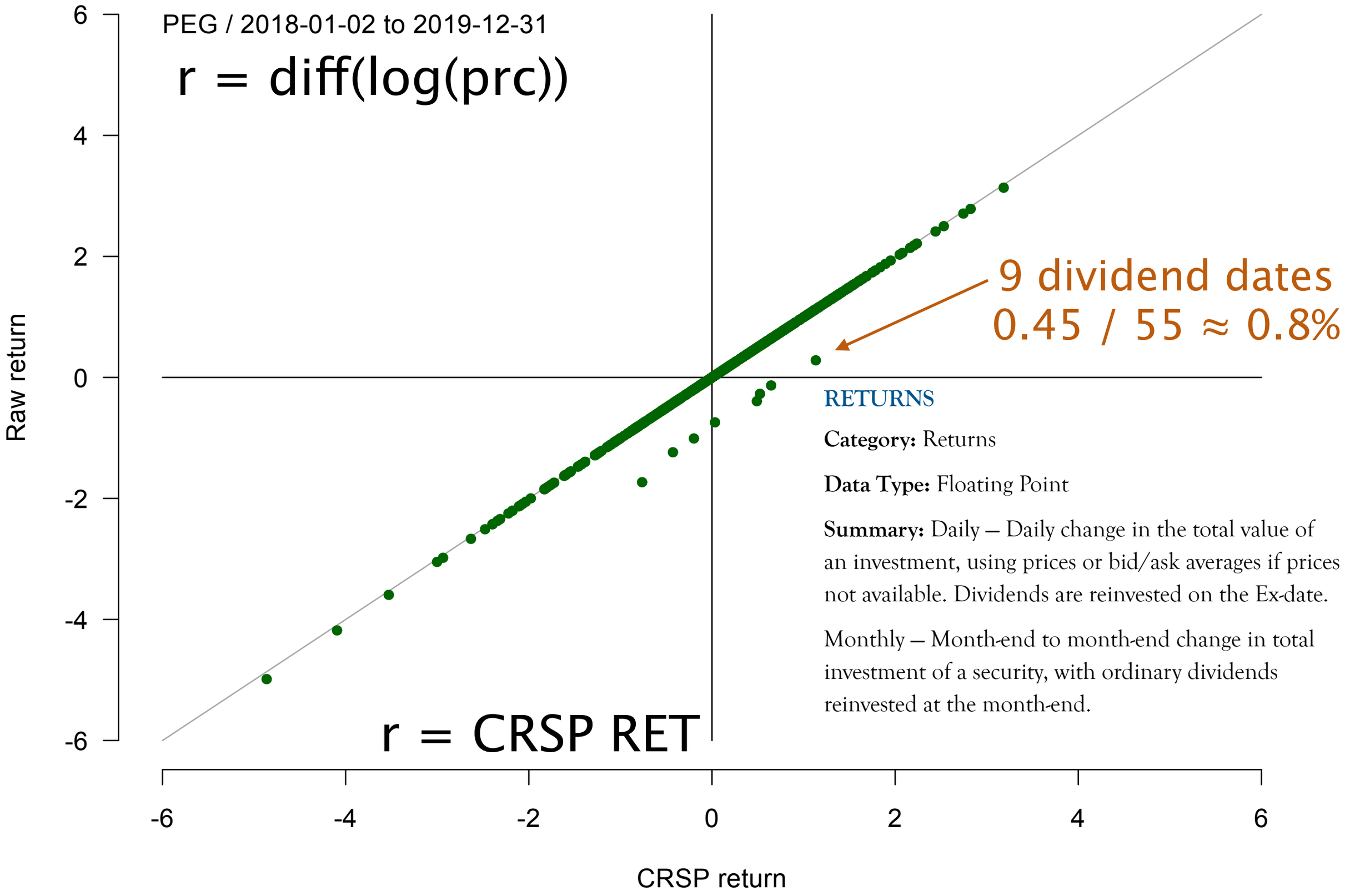
Dividend Cash Amount is the US dollar value per share of distributions resulting from cash dividends, spin-offs, mergers, exchanges, reorganizations, liquidations, and rights issues. Dividend Cash Amount includes the cash value of ordinary and non-ordinary (return of capital) dividends. When the distribution is paid in shares of a trading security, the Dividend Cash Amount is set to the exchange ratio times the price of the security at the close of the Ex-Distribution Date.

EX-DISTRIBUTION DATE

Ex-Distribution Date is the ex-dividend or ex-distribution date. It is the date on which the security is first traded without the right to receive the distribution.

RECORD DATE

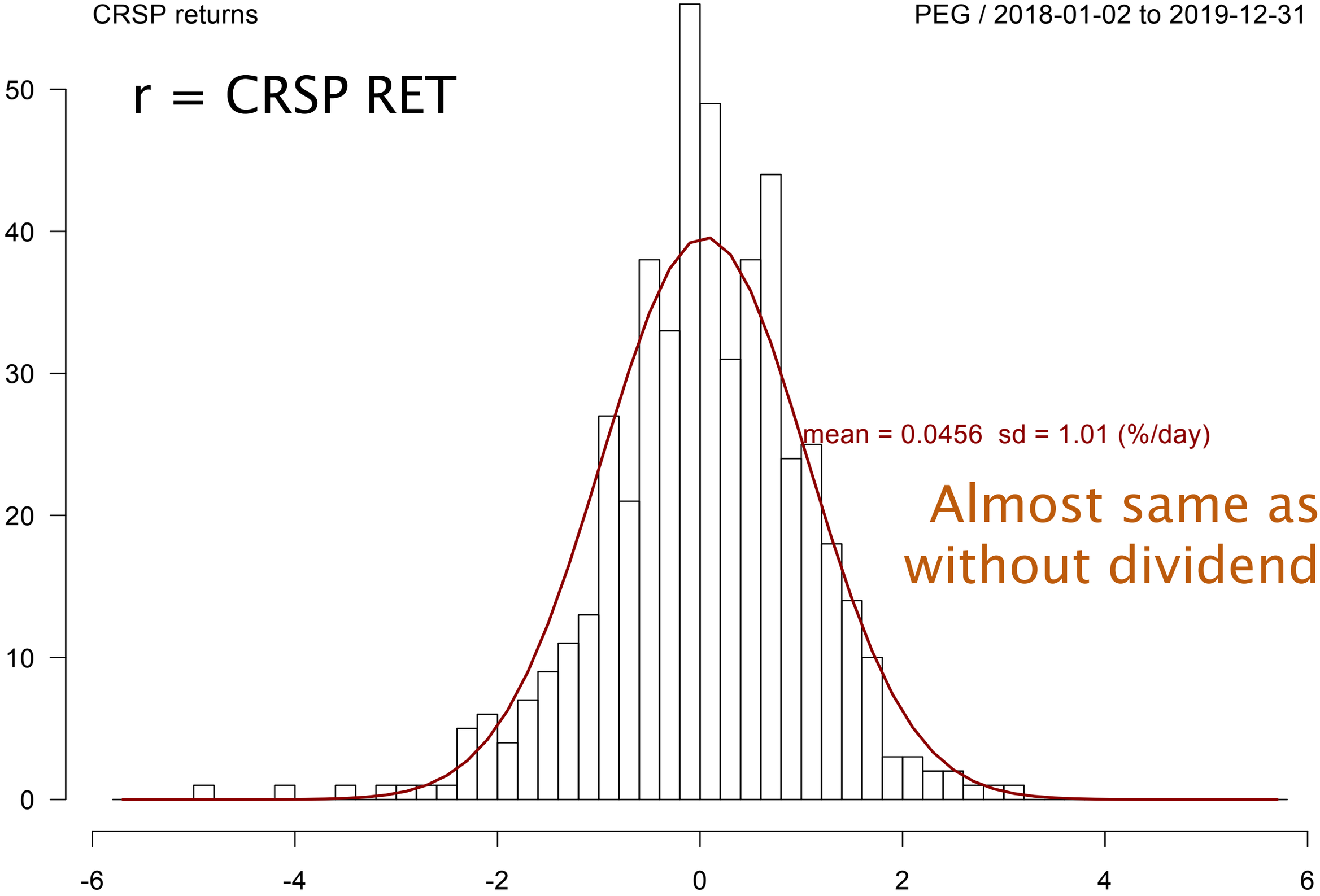
Record Date is the record date on which the stockholder must be registered as holder of record on the stock transfer records of the company in order to receive a particular distribution directly from the company.


PEG / 2018-01-02 to 2019-12-31
r = diff(log(prc))
r = CRSP RET
9 dividend dates
0.45 / 55 ≈ 0.8%
RETURNS
Category: Returns
Data Type: Floating Point
Summary: Daily — Daily change in the total value of an investment, using prices or bid/ask averages if prices not available. Dividends are reinvested on the Ex-date.
Monthly — Month-end to month-end change in total investment of a security, with ordinary dividends reinvested at the month-end.
Raw return
CRSP return
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

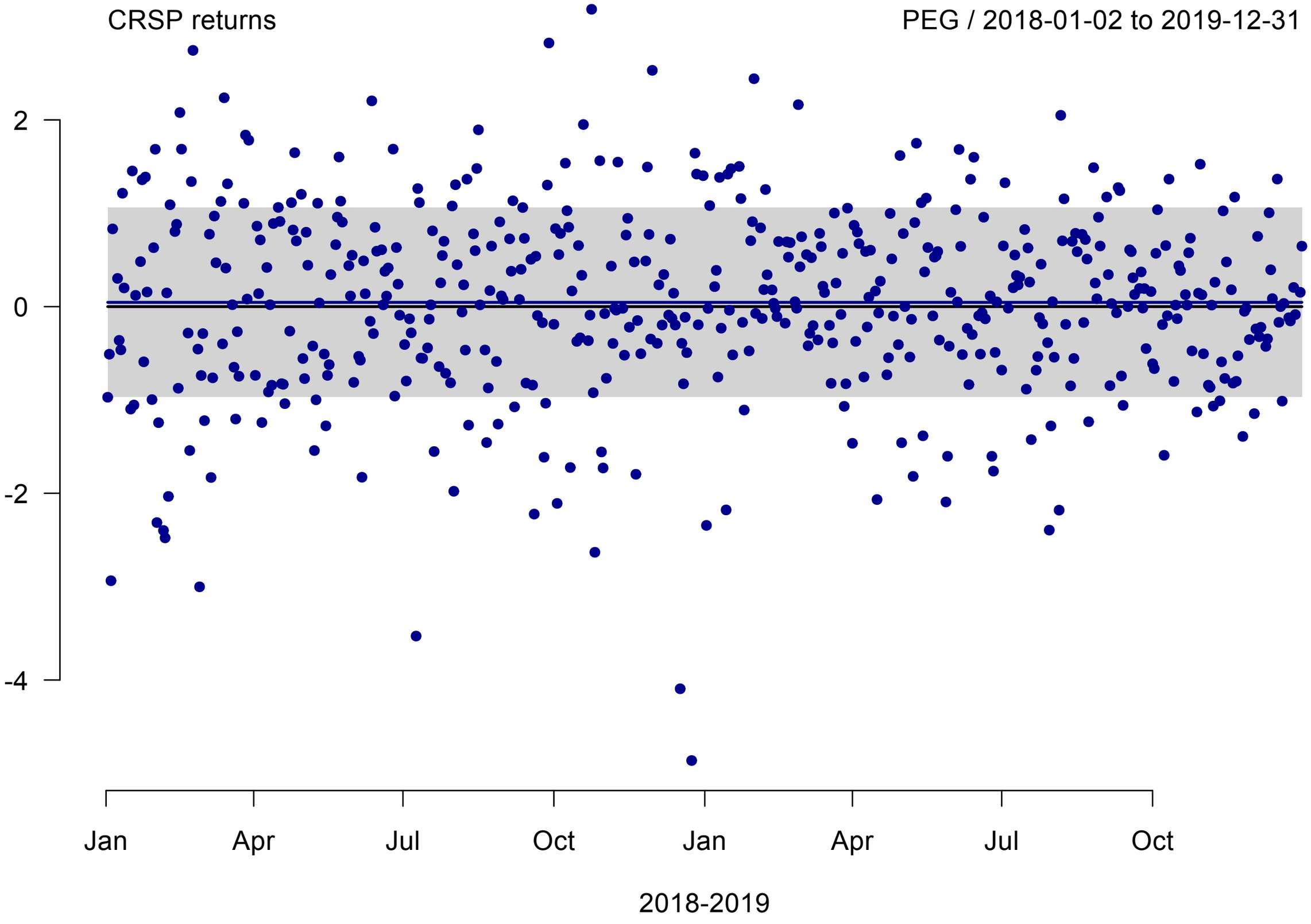
CRSP returns

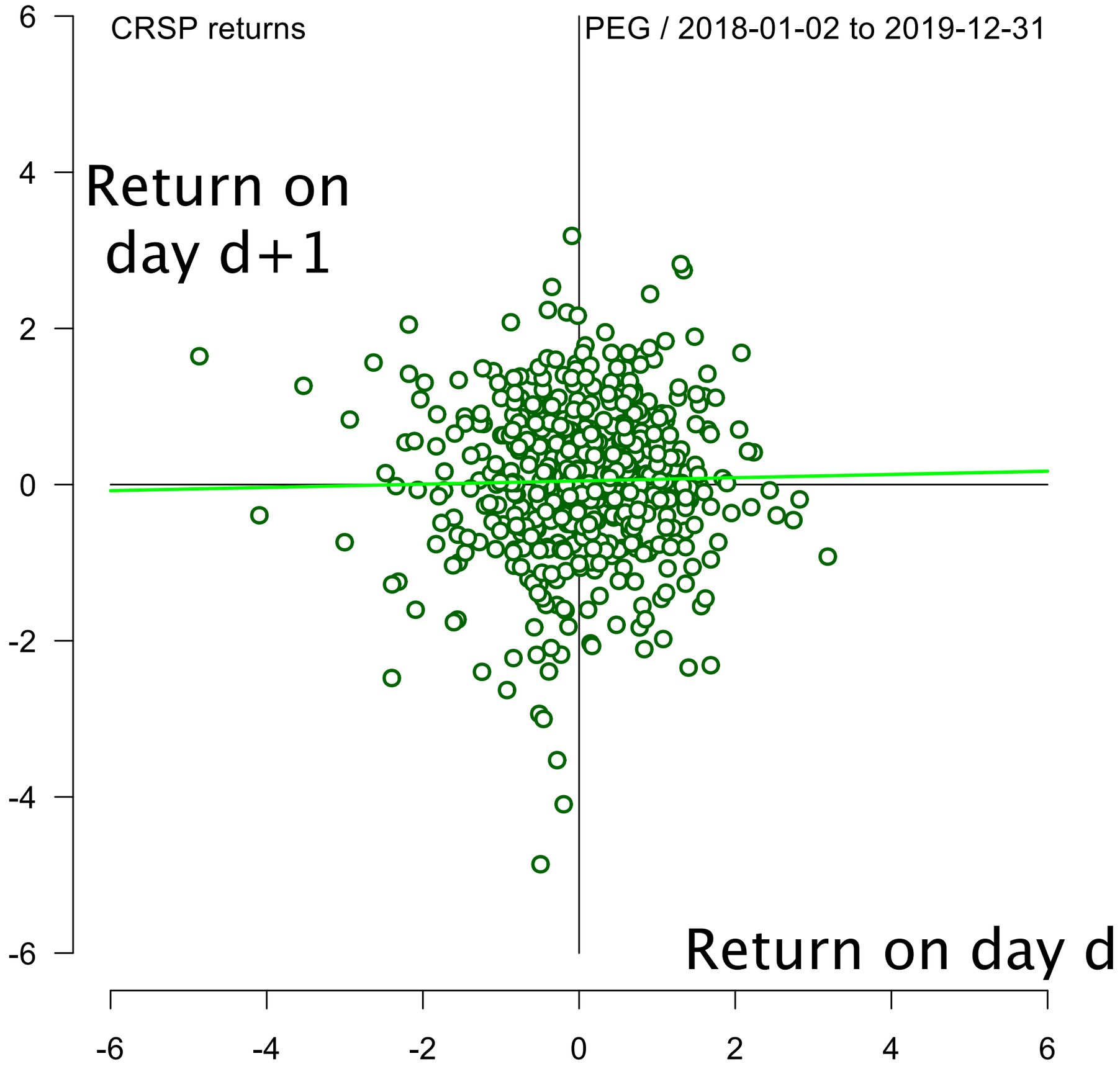
PEG / 2018-01-02 to 2019-12-31

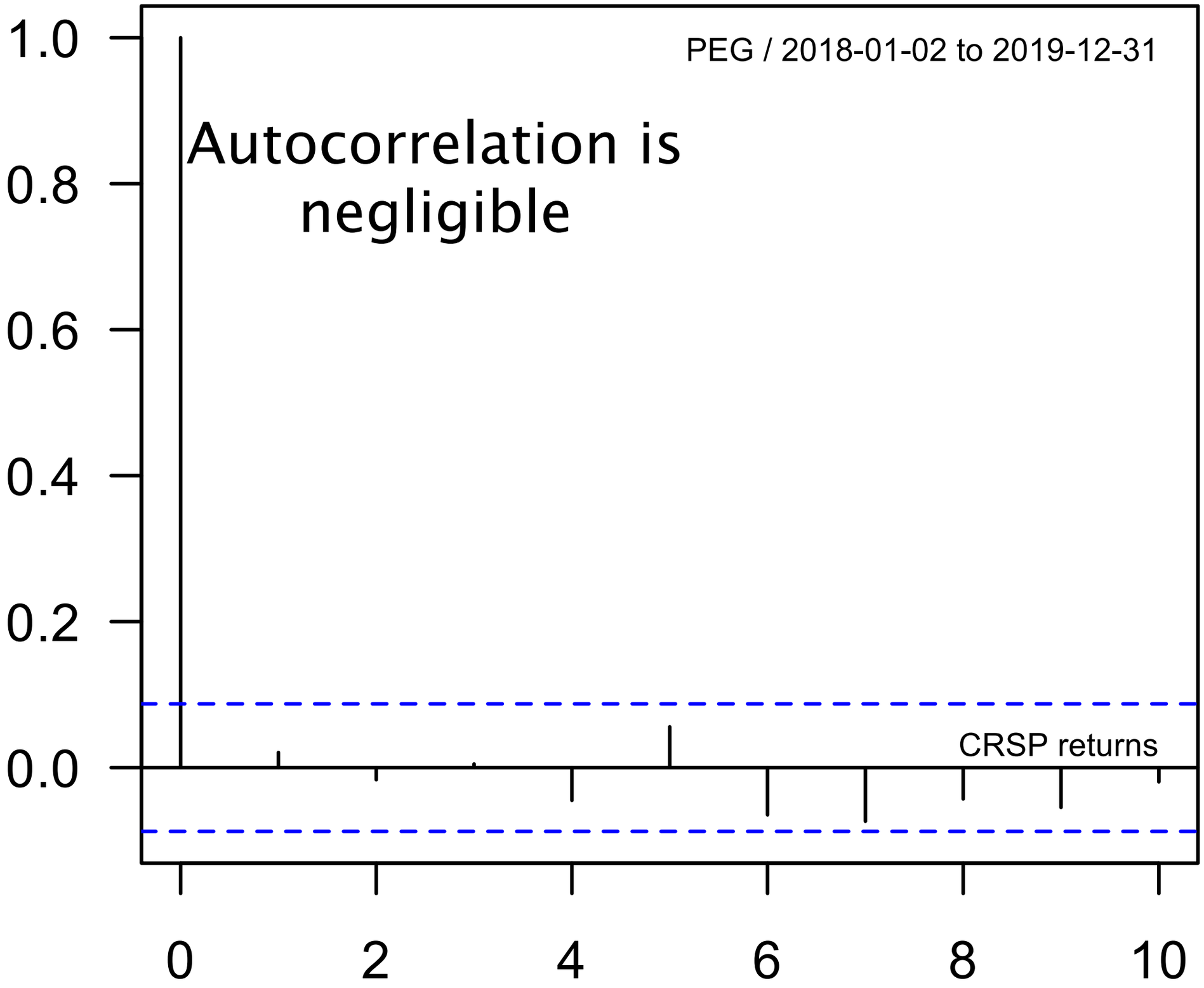
r = CRSP RET

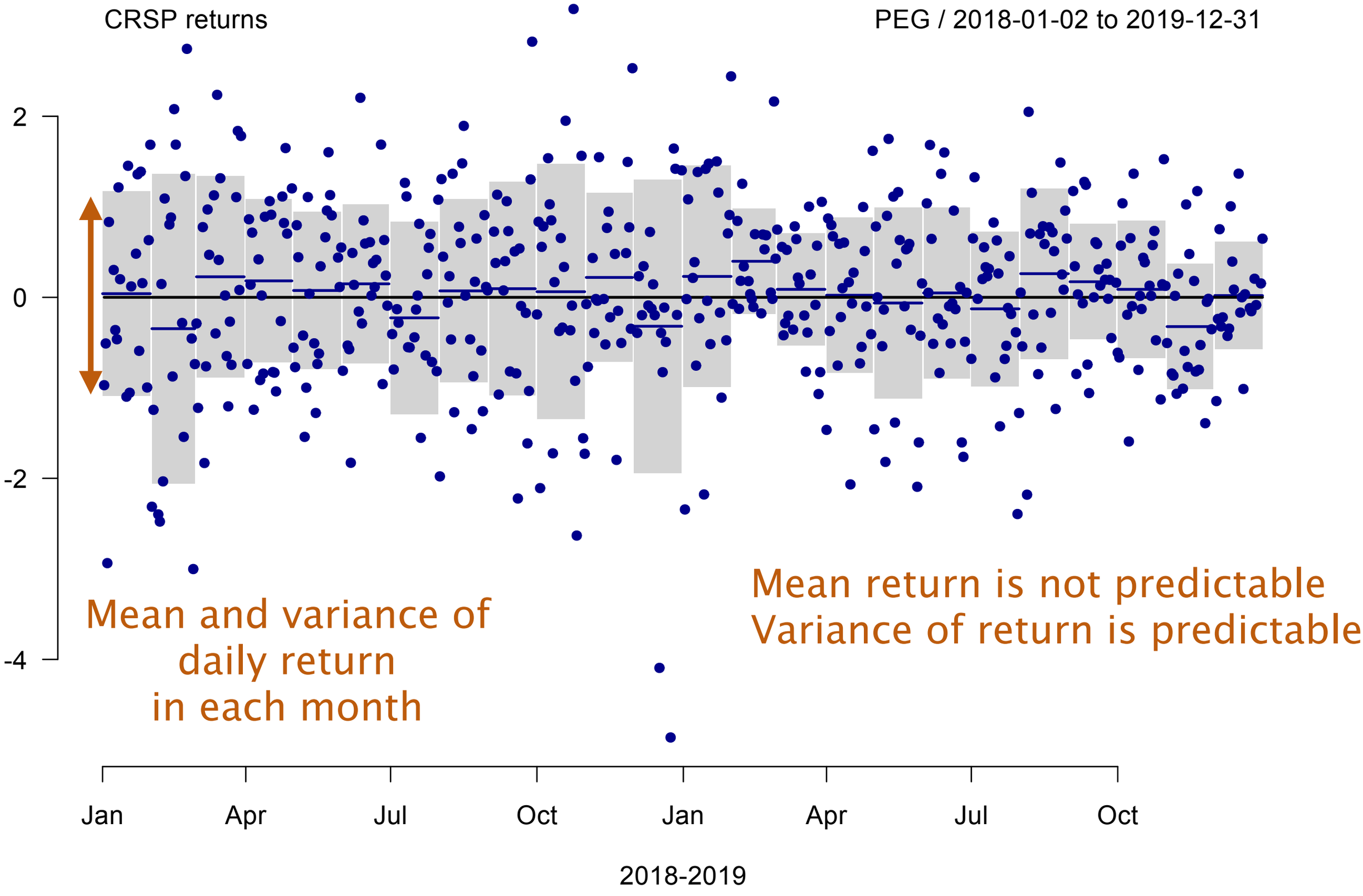


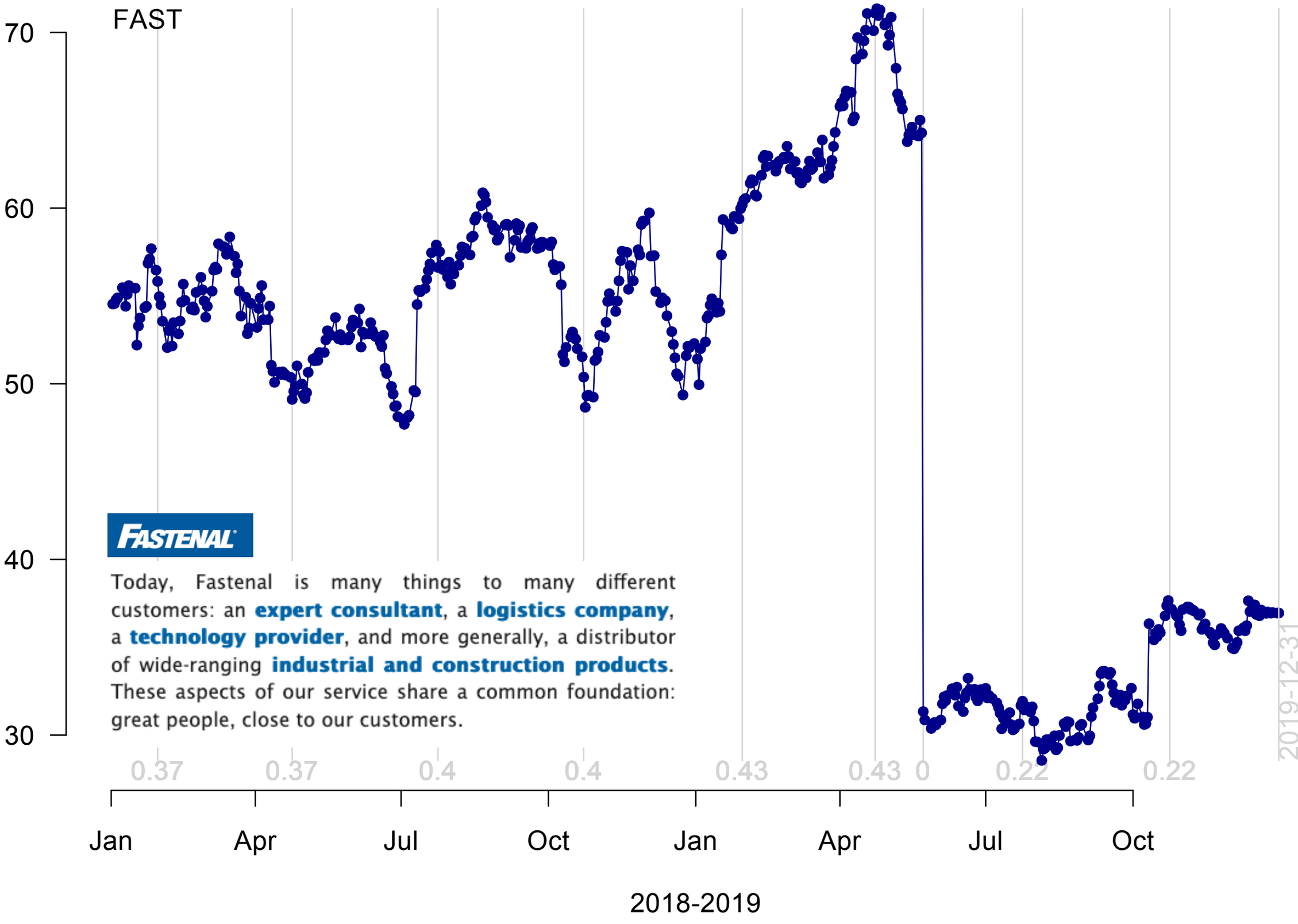
Almost same as without dividends

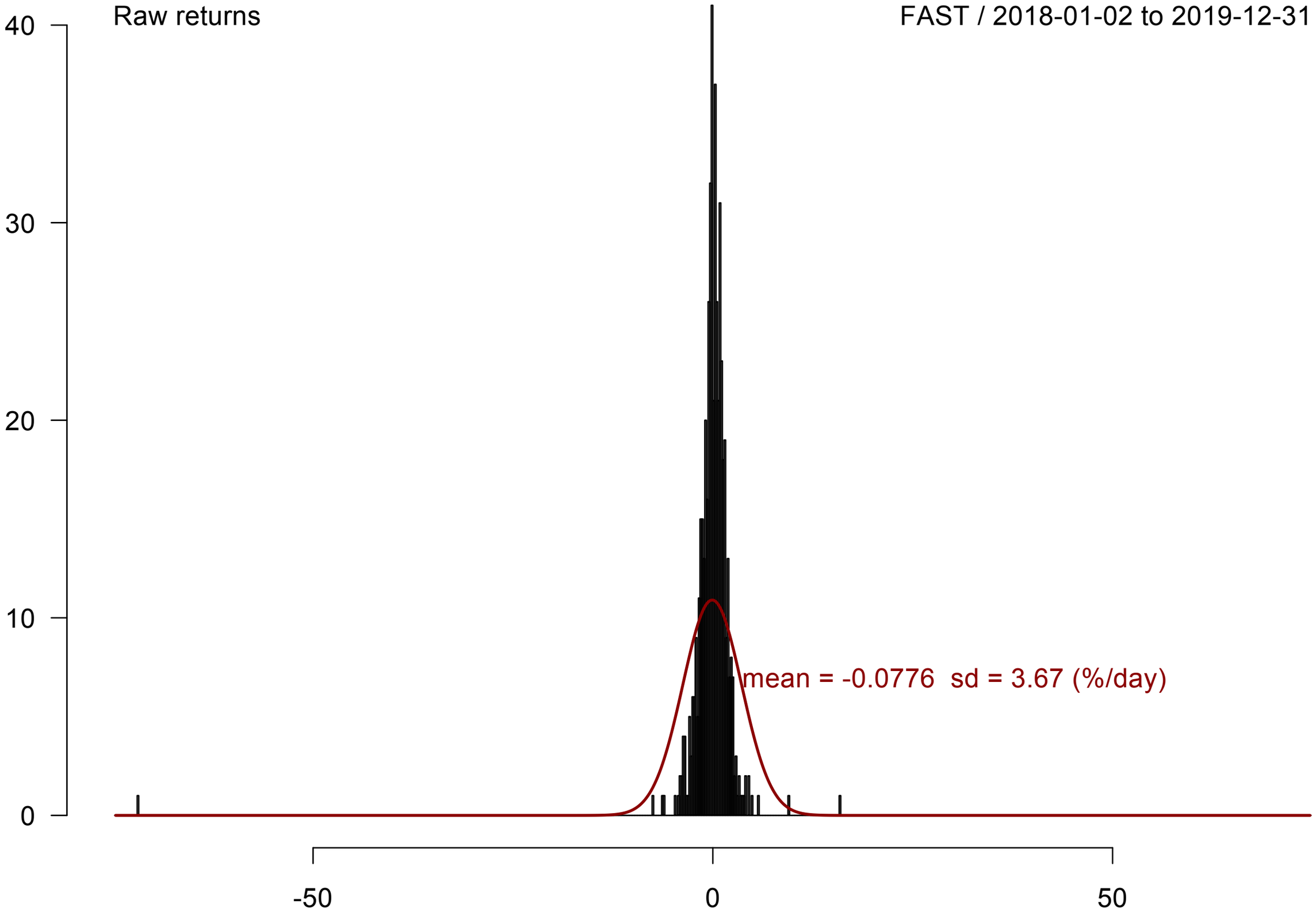

CRSP returns
PEG / 2018-01-02 to 2019-12-31
2
0
-2
-4
Jan
Apr
Jul
Oct
Jan
Apr
Jul
Oct
2018-2019

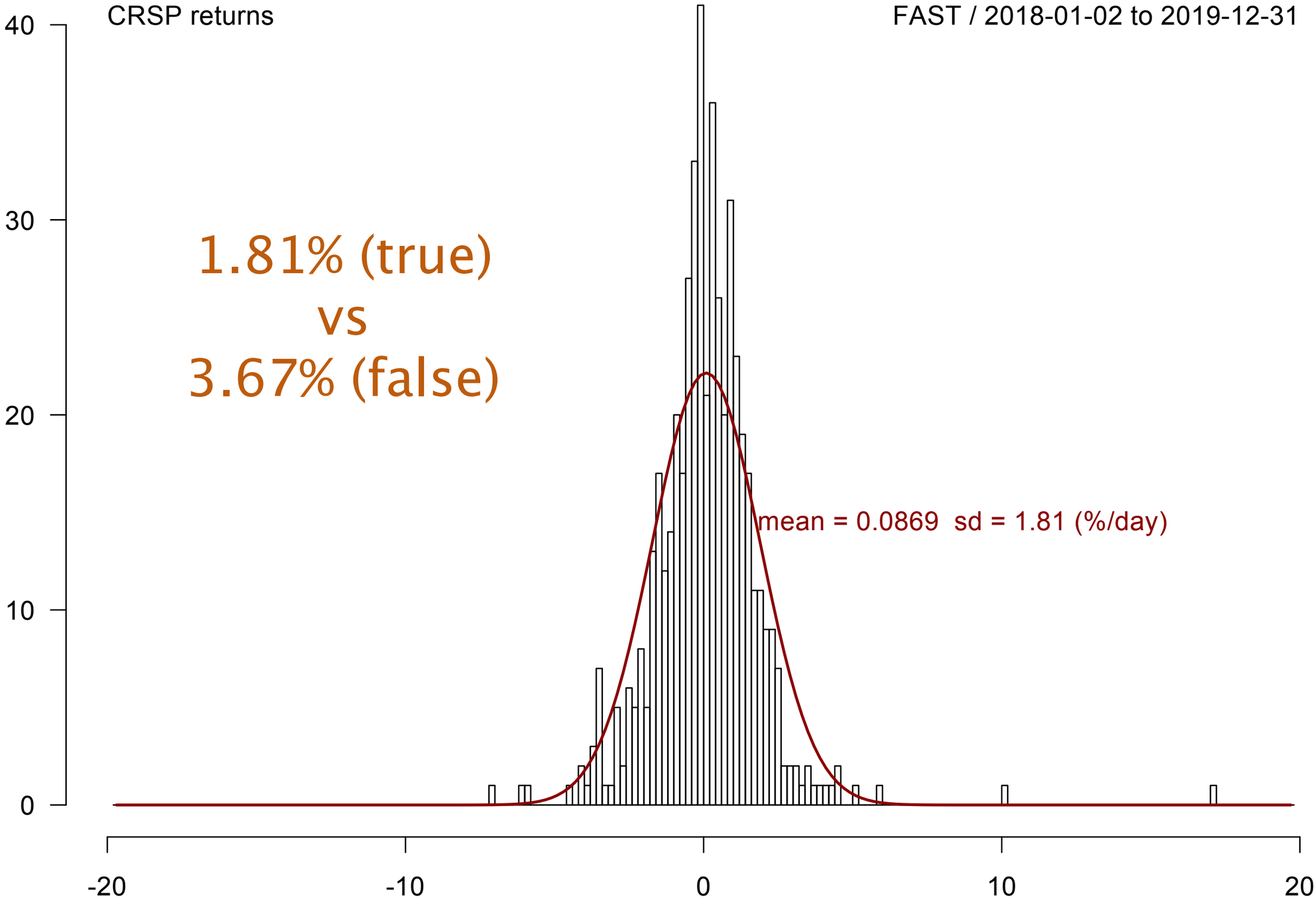

CRSP returns
PEG / 2018-01-02 to 2019-12-31
Return on day d+1
Return on day d
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6


PEG / 2018-01-02 to 2019-12-31
Autocorrelation is negligible
CRSP returns
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10


CRSP returns
PEG / 2018-01-02 to 2019-12-31
2
0
-2
-4
Mean and variance of
daily return
in each month
Mean return is not predictable
Variance of return is predictable
Jan
Apr
Jul
Oct
Jan
Apr
Jul
Oct
2018-2019


FAST
70
60
50
40
30
FASTENAL
Today, Fastenal is many things to many different customers: an expert consultant, a logistics company, a technology provider, and more generally, a distributor of wide-ranging industrial and construction products. These aspects of our service share a common foundation: great people, close to our customers.
0.37
0.37
0.4
0.4
0.43
0.43
0
0.22
0.22
2019-12-31
Jan
Apr
Jul
Oct
Jan
Apr
Jul
Oct
2018-2019


Raw returns
FAST / 2018-01-02 to 2019-12-31
40
30
20
10
0
mean = -0.0776 sd = 3.67 (%/day)
-50
0
50


CRSP returns
FAST / 2018-01-02 to 2019-12-31
1.81% (true)
vs
3.67% (false)
mean = 0.0869 sd = 1.81 (%/day)
40
30
20
10
0
-20
-10
0
10
20

Model for daily returns

Historical data is more or less consistent with model

$$
\begin{aligned}
r_i &= (p_i - p_{i-1})/p_{i-1} && \text{(price changes are small)} \\
&\approx \log p_i - \log p_{i-1} \\
&\sim \mathcal{N}(0, \sigma^2) && \text{(mean of daily price changes is much smaller than standard deviation)}
\end{aligned}
$$

or

$$
\begin{aligned}
\log p_i &= \log p_{i-1} + \sigma \xi_i \\
\sigma &= \text{volatility}
\end{aligned}
$$

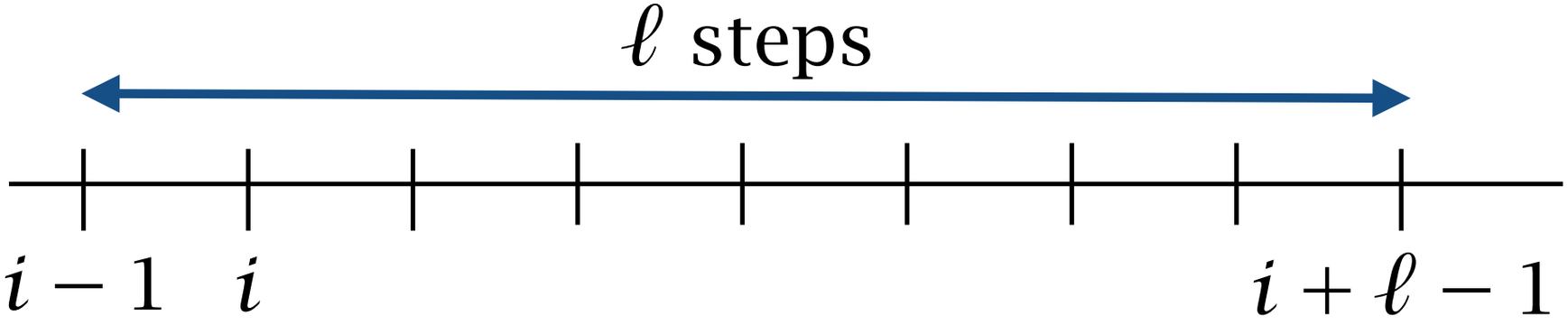
$$
\begin{aligned}
&\mathbb{E}(\xi_i) = 0 \\
&\text{Var}(\xi_i) = 1 \\
&\xi_i, \xi_j \text{ independent for } i \neq j \\
&\text{For example, } \xi_i \sim \mathcal{N}(0,1)
\end{aligned}
$$

Volatility is mean squared daily fractional price change.

Daily model also tells you statistics on longer intervals

$$\log p_i \;=\; \log p_{i-1} \;+\; \sigma\, \xi_i$$

$$\log p_{i+\ell-1} \;=\; \log p_{i-1} \;+\; \sigma \sum_{j=i}^{i+\ell-1} \xi_j$$



$$\sigma \sum_{j=i}^{i+\ell-1} \xi_j \;\sim\; \mathcal{N}\left(0,\, \ell\sigma^2\right)$$

because of serial independence, and for large lag

Volatility squared = variance per day

Units of σ are day$^{-1/2}$

Assumptions of model

- Log price does a Brownian motion
- Drift is negligible
- Volatility **σ** is constant:

  no random variation

  no cyclical structure

Then forecast volatility is historical mean

All these assumptions can be criticized

Summary of daily volatility

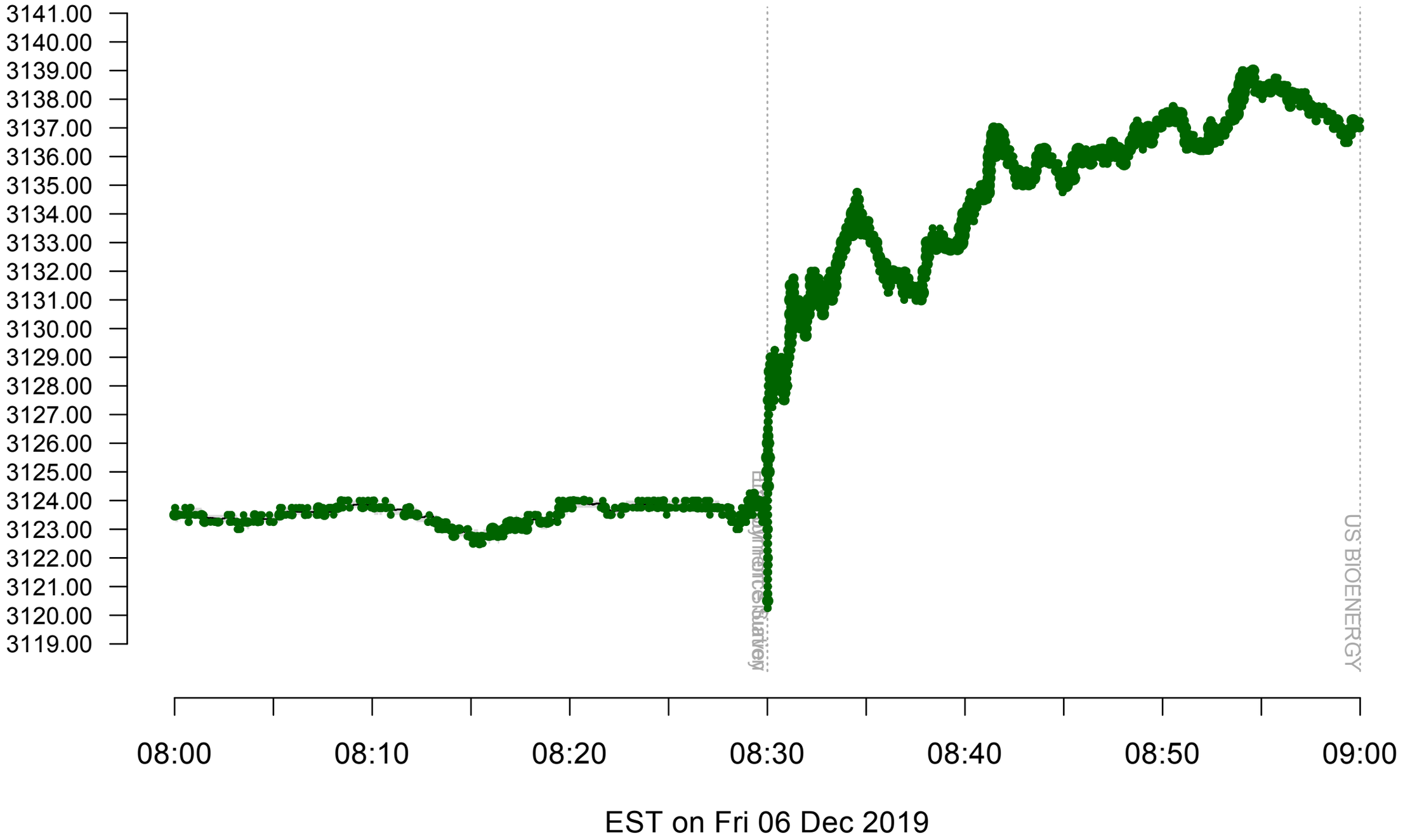
- Calibrated from historical data,
  suggests stochastic model for future evolution
  - approximately normal daily returns
  - no serial dependence in daily returns
- Volatility often taken to be constant in model,
  though in reality certainly varies randomly
  (predictable annual profile is less convincing)
- Daily data is the natural finest resolution
  can build up longer intervals by aggregating daily

Significance for HFT

- Intraday events are important for volatility
- This volatility is "real", unlike dividends and splits
- Decide whether you want volatility

  with jumps: important part of total daily volatility

  without jumps: purely continuous part


3141.00
3140.00
3139.00
3138.00
3137.00
3136.00
3135.00
3134.00
3133.00
3132.00
3131.00
3130.00
3129.00
3128.00
3127.00
3126.00
3125.00
3124.00
3123.00
3122.00
3121.00
3120.00
3119.00
Employment Survey
US BIOENERGY
08:00
08:10
08:20
08:30
08:40
08:50
09:00
EST on Fri 06 Dec 2019

Equity market structure -- changes in 2020

Cboe

January 2020

Cboe's Vision

Equity Market Structure Reform

Introduction

Cboe Global Markets (Cboe) is a multi-asset exchange holding company, well-versed in the art and science of defining markets – from equities, options, and futures, to global foreign exchange and exchange traded products. This holistic view of the markets motivates us to seek what is fair and beneficial to all participants. We listen to and work with our customers and broader industry participants to create, innovate and develop unique solutions that address their needs.

As a prominent equities exchange operator, Cboe provides robust and resilient markets that bring together buyers and sellers across numerous asset classes. Our four equities exchanges execute about 17% of the total average daily U.S. volume. Cboe's significant experience and insight into the functioning of the U.S. equities markets has focused our recommendations on proposals that will benefit investors, while continuing to foster competition and market efficiency.

Cboe's Recommendations

Cboe recommends four key modifications to the current equities market structure:

- **Round Lots and Odd Lots** - Reduce the standard round lot size from 100 shares to 10 shares or 1 share for high-priced securities, and broaden odd lot transparency by disseminating top-of-book odd lot quotation data through the Securities Information Processors (SIPs). These proposed changes will enhance price discovery, reduce spreads and increase the transparency of limit orders with fewer than 100 shares in high-priced stocks.
- **Distributed SIPs** - Implement SIPs in multiple locations in order to significantly reduce the geographic latency that slows the receipt of consolidated real-time quote and trade information.
- **Tick Size** - Cboe recommends a targeted reduction in tick sizes for securities priced above $1.
- **Sub-Penny Pricing** - Establish sub-penny pricing standards that permit fair and competitive price-improvement opportunities between exchanges and off-exchange venues.

http://www.cboe.com/aboutcboe/government-relations/pdf/cboes-vision-equity-market-structure-reform-2020.pdf

Reg NMS exempts "round lots"

§ 242.600 NMS security designation and definitions.

(a) The term ***national market system security*** as used in section 11A(a)(2) of the Act (15 U.S.C. 78k-1(a)(2)) shall mean any NMS security as defined in paragraph (b) of this section.

(b) For purposes of Regulation NMS (§§ 242.600 through 242.612), the following definitions shall apply:

(1) ***Actionable indication of interest*** means any indication of interest that explicitly or implicitly conveys all of the following information with respect to any order available at the venue sending the indication of interest:

(i) Symbol;

(ii) Side (buy or sell);

(iii) A price that is equal to or better than the national best bid for buy orders and the national best offer for sell orders; and

(iv) A size that is at least equal to one round lot.

(8) ***Best bid and best offer*** mean the highest priced bid and the lowest priced offer.

(9) ***Bid*** or *offer* means the bid price or the offer price communicated by a member of a national securities exchange or member of a national securities association to any broker or dealer, or to any customer, at which it is willing to buy or sell one or more round lots of an NMS security, as either principal or agent, but shall not include indications of interest.

(51) ***Odd-lot*** means an order for the purchase or sale of an NMS stock in an amount less than a round lot.

(84) ***Transaction report*** means a report containing the price and volume associated with a transaction involving the purchase or sale of one or more round lots of a security.

https://www.law.cornell.edu/cfr/text/17/242.600

"round lot" seems to be defined by exchanges (always 100 lots)

U.S. Securities and Exchange Commission

Division of Trading and Markets:

Responses to Frequently Asked Questions Concerning Rule 611 and Rule 610 of Regulation NMS

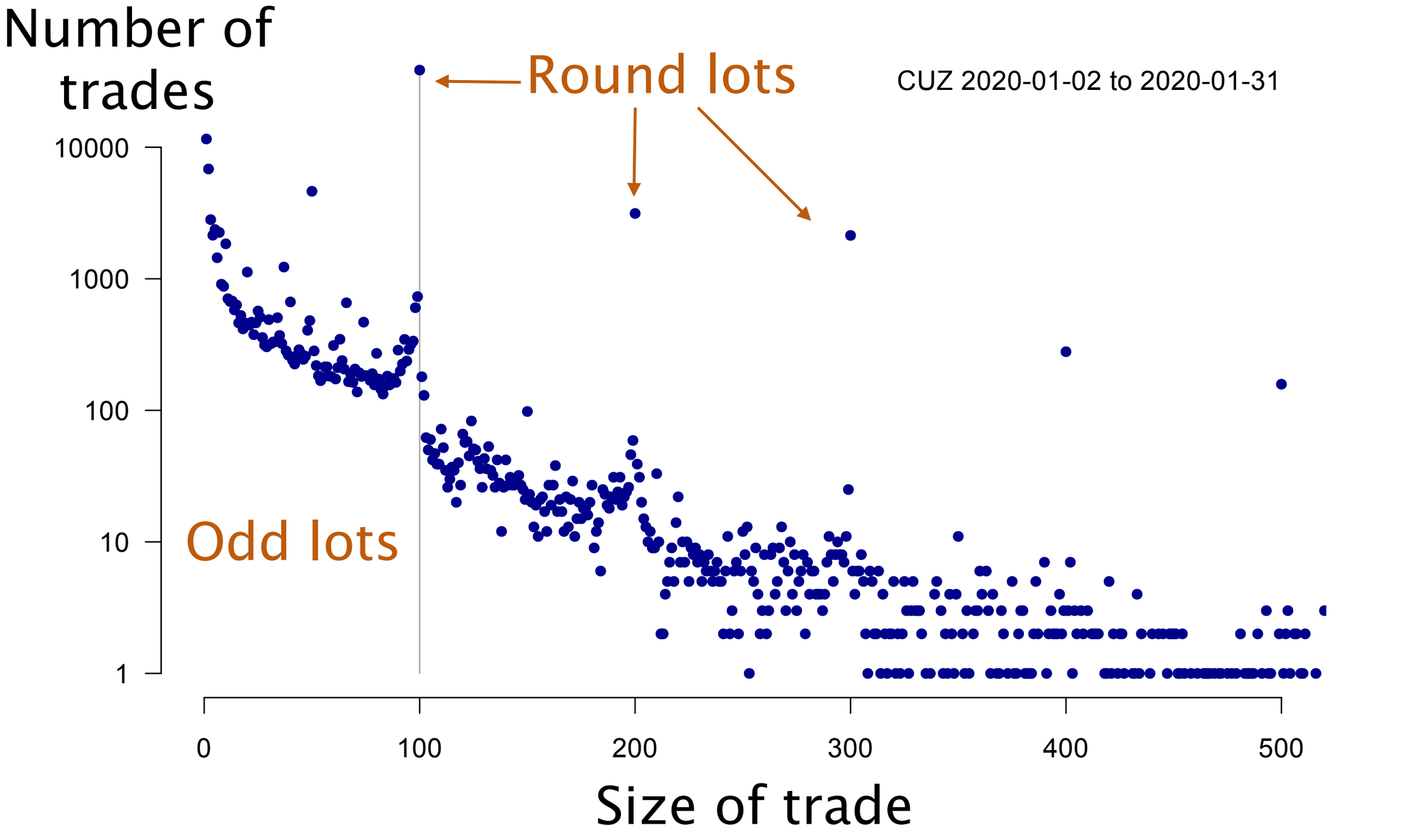
On April 6, 2005, the Commission adopted Regulation NMS, a series of initiatives designed to modernize and strengthen the national market system for equity securities. Regulation NMS was published in Securities Exchange Act Release No. 51808 (Jun. 9, 2005), 70 FR 37496 (Jun. 29, 2005) ("NMS Release"). It includes: (1) Rule 610, which addresses access to markets; (2) Rule 611, which provides intermarket price priority for displayed and accessible quotations; (3) Rule 612, which establishes minimum pricing increments; and (4) amendments to the joint-industry plans and rules governing the dissemination of market data.

Do Rule 610 or Rule 611 apply to odd-lot orders (those with sizes of less than one round lot in an NMS stock) or to the odd-lot portions of mixed-lot orders?

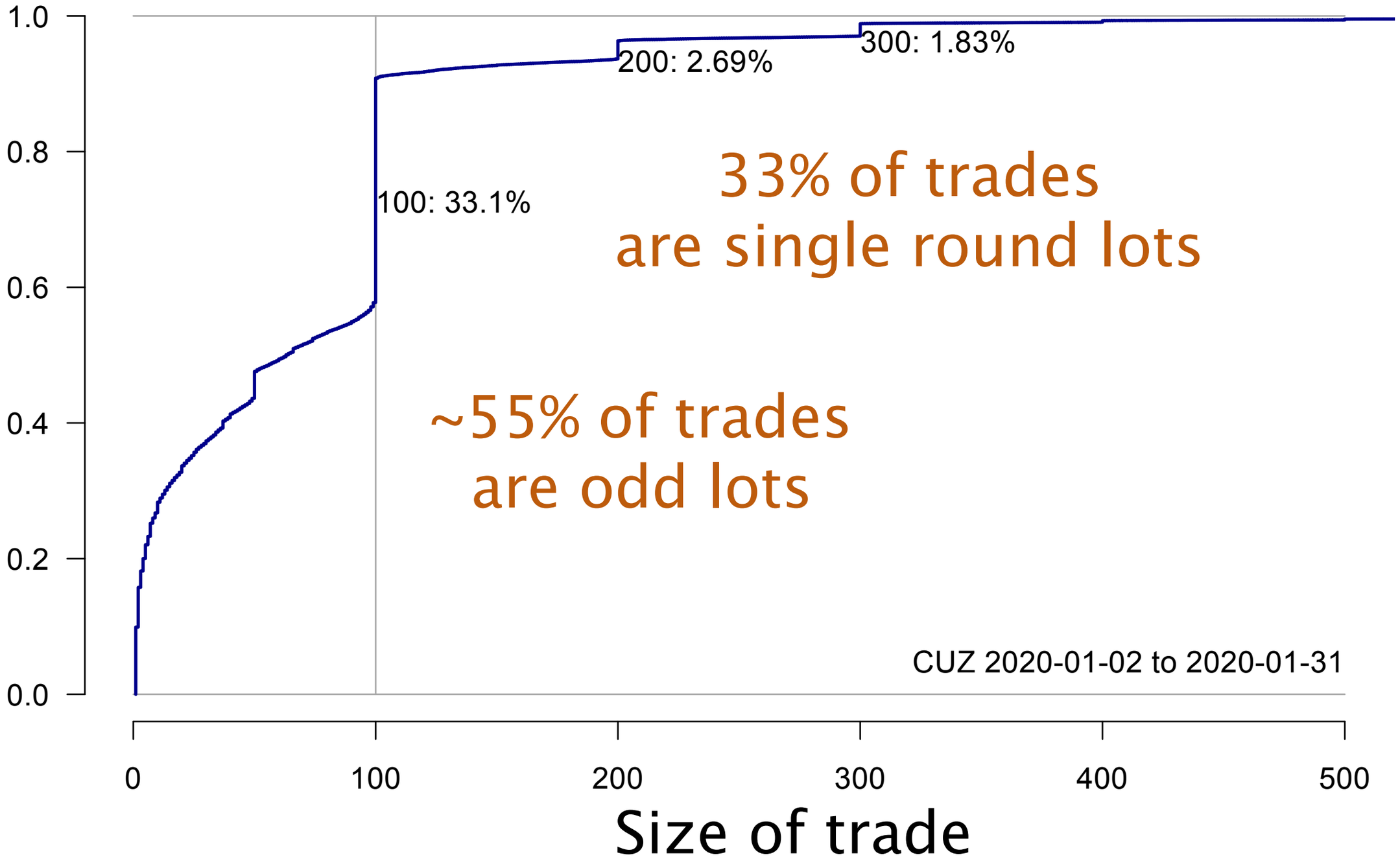
Answer: No, Rule 610 and Rule 611 do not apply to odd-lot orders or to the odd-lot portions of mixed-lot orders. Rule 600(b)(8) defines "bid" or "offer" as the bid price or offer price for one or more round lots of an NMS security. This definition is embedded in the definition of "quotation" in Rule 600(b)(62), as well as the definition of "protected bid" or "protected offer" in Rule 600(b)(57). Consequently, trading centers are permitted to establish their own rules for handling odd-lot orders and the odd-lot portions of mixed-lot orders. For example, although trading centers are not required to handle odd-lot orders or the odd-lot portions of mixed lot orders in accordance with the requirements for automated quotations set forth in Rule 600(b)(3), they are free to incorporate such requirements in their rules if they wish to do so.

https://www.sec.gov/divisions/marketreg/nmsfaq610-11.htm

Trade size distribution



Cumulative distribution



CBOE proposal

Round Lots and Odd Lots: The Increase in High-Priced Stocks

In recent years, the general rise of the stock market coupled with fewer declared stock splits have contributed to a major increase in the average stock price. Almost 600 listed securities, including ones with meaningful retail interest, are now trading above $100 per share, and the price of an S&P 500 stock now averages approximately $130 per share, more than tripling from the year 2000.

High-priced stocks, particularly those of considerable interest to individual investors, naturally cause a substantial increase in odd lot trading, as trading fewer shares is more affordable for the everyday person. In addition, limit orders initially entered as round lots often become odd lots as a result of contra-side odd lot order executions against them. All orders, regardless of whether their origination was as a round lot or an odd lot, can lose both displayed representation on the SIPs and the benefit of order protection when their remaining order size is reduced to a quantity below one round lot.

There are currently no established standards or guidelines to adjust round lot sizes when specific price thresholds are reached. As the number of high-priced securities grows, Cboe supports the development of a structured methodology through which round lot sizes would be adjusted downward. This would ensure that these orders are displayed to retail and other investors through the SIPs and extend order protection to a more meaningful percentage of orders in higher-priced securities.

Cboe recommends initially confining round lot adjustments to securities that trade at prices greater than $500 per share, applying a round lot size of 10 shares to securities priced between $500 and $1,000, and applying a round lot size of 1 share for securities priced above $1,000 per share. Today, this change would affect a total of approximately 25 securities. Adopting new round lot sizes, even for this small group of securities, would be a significant change for market participants. Current stocks with nonstandard round lot sizes are lightly traded with little retail interest. Changing round lot sizes for actively traded stocks will entail an educational process for brokers, vendors and customers. System changes may also be needed to ensure an understanding of quote sizes.[6]

Current stocks with nonstandard round lot sizes are lightly traded with little retail interest

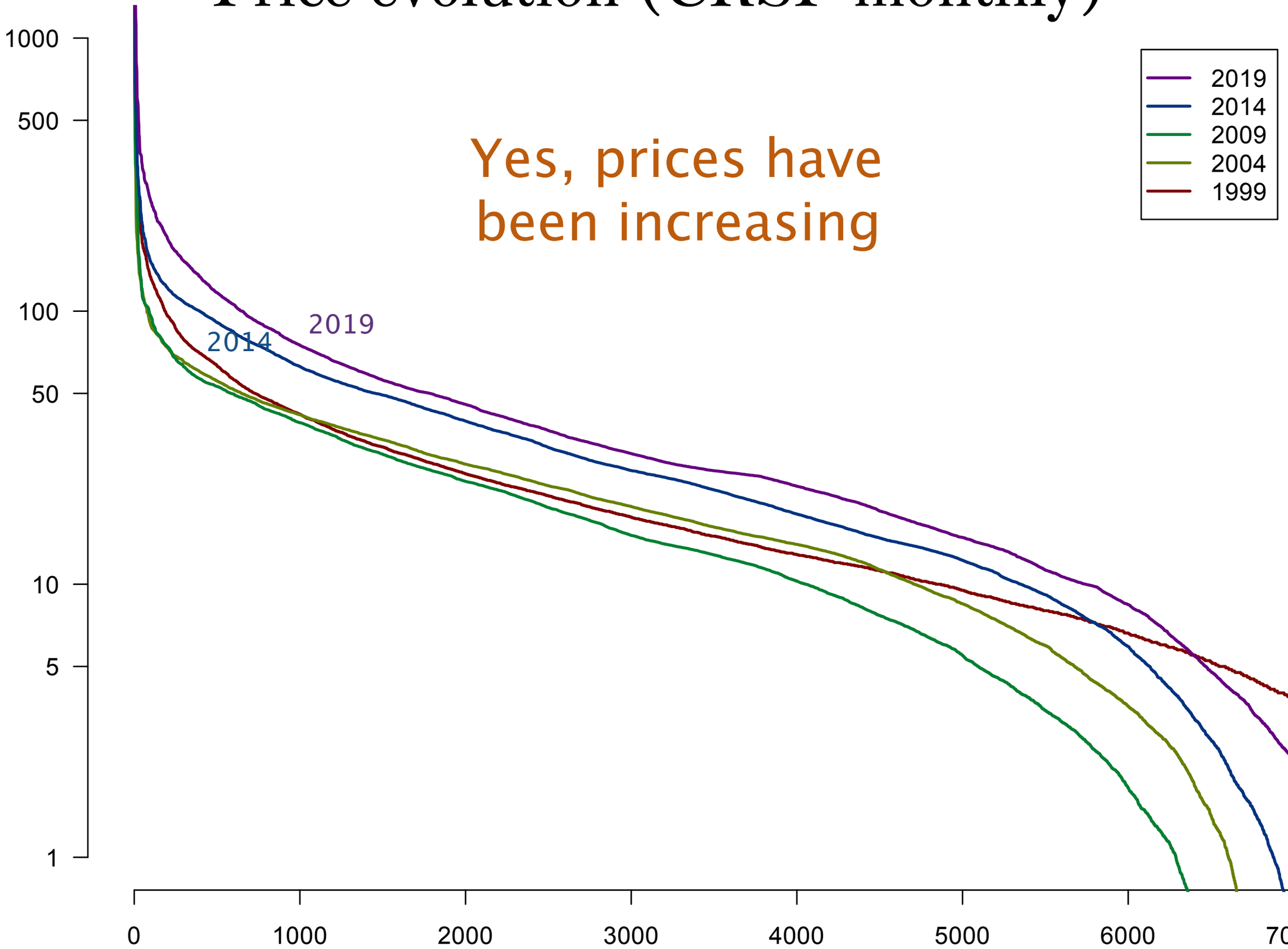
[6] Changing round lot sizes would not necessitate amending Reg NMS. However, the SIPs disseminate size using integer values to represent round lots, meaning that a security's round lot size must be known in order to understand the quote size representation.

Broaden Odd Lot Transparency

Cboe supports the addition of top-of-book odd lot quotation information to the SIPs.

Odd lot orders currently have no quote representation by the securities information processors (SIPs), meaning that resting odd lot orders have no opportunity to contribute directly to price discovery. Cboe strongly supports the CTA and UTP Operating Committees' proposal for the SIPs to disseminate top-of-book odd lot quotation data.[8] We believe this addition will broaden odd lot transparency and provide a way for investors to view better-priced orders that do not meet the Reg NMS round lot size requirement for order protection.

Price evolution (CRSP monthly)



CTA proposal

The Consolidated Tape Association (CTA) oversees the dissemination of real-time trade and quote information in New York Stock Exchange LLC (Network A) and Bats, NYSE Arca, NYSE American and other regional exchange (Network B) listed securities. Since the late 1970s, all SEC-registered exchanges and market centers that trade Network A or Network B securities send their trades and quotes to a central consolidator where the Consolidated Tape System (CTS) and Consolidated Quote System (CQS) data streams are produced and distributed worldwide.

Odd Lots

Securities Information Processor ("SIP") Operating Committees are considering a proposal for the SIPs to disseminate certain consolidated odd lot quotation data as ancillary information on the SIP data feeds. They are seeking feedback on the below draft proposal.

The Participants of the Nasdaq Unlisted Trading Privileges (UTP) Plan and the Consolidated Quotation (CQ) Plan operate the SIPs that are responsible for disseminating the national best bid and national best offer (NBBO) for quotations in NMS stocks. Pursuant to Regulation NMS, the NBBO as disseminated by the SIPs is calculated using bids and offers to buy or sell one or more round lots of an NMS stock[1]. Round lots are defined by the exchanges and generally refer to quotes to buy or sell 100 shares of a given security or a larger number of shares divisible by 100. Odd lots, or orders for fewer than 100 shares, are not included in the NBBO and are not currently distributed by the SIPs.

- Create separate fields in which top-of-book odd lot quotes can be represented. The fields will display odd lot information in a form that parallels, but is separate from, the NBBO fields for round lot quotes. Individual exchange odd lot bids and offers will be ranked by price, size, and time, in the same manner as for the NBBO.
    - Odd lot quotes will be ancillary information and will in no way affect how the NBBO and round lot quotes are represented.
- Each exchange will send its top-of-book odd lot quotes to the SIP in the same form in which they send their top-of-book round lot quotes to the SIP.
- The overall odd lot best bid and offer, while calculated in the same manner as the round lot NBBO, will have a unique characteristic:
    - Across exchanges, the highest odd lot bid or lowest odd lot offer will not be represented whenever it is worse than the NBBO.
- The Odd Lot Bid will have an associated Odd Lot Bid Size, and the Odd Lot Offer will have an associated Odd Lot Offer Size. Size will be represented in actual shares.

https://www.ctaplan.com/oddlots
https://www.ctaplan.com/publicdocs/CTA_Odd_Lots_Proposal.pdf

TAQ data

DAILY TAQ CLIENT SPECIFICATION

Version	Date
3.0d	May 31, 2019

2. Daily TAQ Master File

The Daily TAQ Master file contains static security information for all securities traded by participants of CTA and UTP (except Nextshares).

8	**Listed Exchange**	1	Text	Listing exchange of the security. Primary Listing Market codes and description. See Participant IDs for codes
9	**Tape**	1	Text	The tape of this symbol. Valid values: A, B, C
10	**Unit of Trade**	3	Numeric	Minimum incremental size of a trade, in shares. E.g. if Unit of Trade is 100, possible trade sizes are 100, 200, 300… but not 150. Valid values: 100,50,10,1
11	**Round Lot**	3	Numeric	The number of shares the security quotes at. Valid values: 100,50,10,1

3. Daily TAQ Trades File

Every trade reported to the consolidated tape by all CTA and UTP participants.

5	**Trade Volume**	10	Numeric	The number of shares traded
6	**Trade Price**	21	Numeric	The share price of this trade

4. Daily TAQ Quotes File

Every quote reported to the consolidated tape by all CTA and UTP participants.

5	**Bid Size**	10	Numeric	The maximum number of shares the highest bidder is willing to buy, in round lots.
7	**Offer Size**	10	Numeric	The maximum number of shares available at the offer price, in round lots.

WRDS seems not to have Master File

Trades are numbers of shares

Quote sizes are numbers of round lots

U.S. SECURITIES AND EXCHANGE COMMISSION

Press Release

SEC Proposes to Modernize Key Market Infrastructure Responsible for Collecting, Consolidating, and Disseminating Securities Market Data

Seeks to Introduce Competitive Forces to Core Components of the System for the First Time

Washington D.C., Feb. 14, 2020 — The Securities and Exchange Commission today proposed to modernize the infrastructure for the collection, consolidation, and dissemination of market data for exchange-listed national market system (NMS) stocks. The proposal would update and expand the content of NMS market data to better meet the diverse needs of investors in today's equity markets.

In 1975, one of Congress's principal objectives for the national market system was to assure the availability of information with respect to quotations and transactions in securities. Currently, the national securities exchanges and the Financial Industry Regulatory Authority (collectively, the SROs) act jointly under three NMS plans (the Equity Data Plans) to collect, consolidate and disseminate information for NMS stocks. For each NMS stock, the SROs are required to provide specified market data (the NMS market data) to exclusive securities information processors (SIPs). The SIPs then consolidate that information and make it available to the public. Today's proposal is designed to improve the NMS market data infrastructure by reducing the current disparity in content and latency between NMS market data and the proprietary data products that some of the individual exchanges sell directly to market participants. The proposal would replace the "exclusive SIP" model with a decentralized model of "competing consolidators."

https://www.sec.gov/news/press-release/2020-34

Round lot quotes to still be part of NBBO but not protected (?)

Content of Current vs. Proposed NMS Market Data		
	Current	**Proposed**
Last sale data/transaction reports	The price, size and exchange of the last sale of the NMS stock, including odd-lot transactions.	Same as current.
Best bid and best offer ("BBO")	BBOs for each SRO in current round lot sizes (e.g., 100 shares)	BBOs for each SRO in revised round lot sizes based on the proposed new "round lot" definition: • $50.00 or less per share: round lot = 100 shares; • $50.01 to $100.00 per share: round lot = 20 shares; • $100.01 to $500.00 per share: round lot = 10 shares; • $500.01 to $1,000.00 per share: round lot = 2 shares; and • $1,000.01 or more per share: round lot = 1 share.
National best bid and national best offer	NBBO is based on the current round lot size quotations.	NBBO would be based on the proposed new round lot size quotations.
Protected quotations	Protected quotations are in current round lots.	The definition of protected quotation is proposed to be amended to be 100 shares or more, regardless of round lot size. Protected quotations would be included in proposed NMS market data.

Distributed SIPs

SIPs and the National Market System

In 1975, Congress recognized that investors were best served by efficient access to the best market prices, and mandated the creation of the National Market System, which assured a single source for investors to gain real-time access to market data for all NMS stocks. The SIPs are the mechanism through which this vision is accomplished. Each SIP processes an exclusive set of securities, with the SEC requiring each national securities exchange to provide all of its trades and top-of-book quotations through two equities SIPs: CTA/CQ for Tape A and Tape B information, and UTP for Tape C information. Real-time consolidated trade and quote information is disseminated for NYSE-listed securities on Tape A, Nasdaq securities on Tape C, and listings from all other national securities exchanges on Tape B. In addition to quote and trade information, the SIPs also disseminate and calculate critical market information that includes the national best bid and offer (NBBO), Limit Up-Limit Down Price Bands, short sale restrictions and regulatory halts.

Over the past several years, technological advances have dramatically reduced SIP processor latency from several hundred microseconds to tens of microseconds to process both quotes and trades. SIP latency today is mostly caused by geographic latency, which is a function of the inbound distance from a specific market's location to the single SIP processor location. Additional latency is incurred from the distance outbound disseminated quote and trades travel to reach each recipient.

Each SIP is located in a single data center, with the CTA processor (Tapes A and B) in Mahwah, NJ, and the UTP processor (Tape C) in Carteret, NJ. Quote and trade data from each exchange must first be sent to the data center, processed and then disseminated to recipients. Cboe's data center in Secaucus, NJ, is roughly equidistant from both SIP data centers but all Cboe quotes and trades encounter significant geographic latency to reach them. Even greater geographic latencies are experienced by Nasdaq to send its quotes and trades in Tape A and B securities to Mahwah and by the NYSE to send its quotes and trades in Tape C securities to Carteret.

Cboe strongly supports the implementation of distributed SIPs, designed to introduce multiple instances of SIP consolidation in strategic data center locations, which will contribute to a major reduction in geographic latency. All SIP subscribers would be offered a choice of locations from which to receive the SIP market information and to achieve the fastest data delivery.

A More Level Playing Field for Exchange and Off-Exchange Markets

In 2001, as part of the conversion of equities market quotations from fractions to decimals, a minimum price variation (MPV) of $0.01 for quotations in equity securities was established for the marketplace.[9] Subsequently, the Reg NMS Sub-Penny Rule prohibited trading venues and broker-dealers from displaying, ranking or accepting orders in sub-pennies for all securities priced at $1 or more per share. Stocks under $1 per share can be quoted in increments of $0.0001 or larger.

Tick Sizes

Overall, Reg NMS has been highly successful in fostering competition, which has directly contributed to consistently narrow spreads for active securities, given the one-cent MPV restriction for all NMS securities priced at or above $1 per share. Cboe thinks the time has come to reduce the MPV for an appropriate subset of securities in which investors would often realize meaningful savings from executions conducted in an environment with narrower spreads. The criteria for establishing narrower quote increments should be based on a combination of a security's price, volume and liquidity.

The criteria for establishing narrower quote increments should be based on a combination of a security's price, volume and liquidity

Based on a recent Cboe analysis of listed securities, approximately 2,400 stocks currently have prices between $1 and $20 per share. More than 400 of those securities have an average daily consolidated volume in excess of one million shares, and almost 300 of the stocks in the above-mentioned price and volume ranges have an average spread of less than 1.1 cents. As a starting point, Cboe supports selecting 50 securities from this group to begin trading with a half-cent MPV rather than with today's required one cent MPV.

Sub-Penny Pricing

The Sub-Penny Rule restricts publicly displayed quotes to a minimum one-cent spread on even pennies. Although exchanges are permitted to accept midpoint-peg orders that may execute at the national best bid and offer (NBBO) midpoint, where contra-side orders are permitted to rest, exchanges are otherwise generally unable to execute at sub-penny prices. Since price-matching of buy and sell orders cannot occur at price points at which orders cannot rest on an exchange's order book, exchange executions are confined to either full penny or half-penny prices.

Currently, exchanges cannot accept, rank or display orders in sub-penny increments for stocks priced at or above $1 per share. As a result, executions are largely confined to either full penny or half-penny midpoint prices, while principal off-exchange venues may provide more granular execution prices. Cboe believes that permitting exchanges to accept orders and quote in sub-penny prices would enhance executions in the public markets and lead to better price discovery and formation.

Finance

Here's One Tax Every Candidate Ought to Back

A tiny tax on financial transactions could raise big money for health care, education and infrastructure without impeding the efficient allocation of capital.

By Antonio Weiss
February 14, 2020, 3:00 PM UTC

An FTT is a small tax that would apply to trading in stocks, bonds, and derivatives. For example, an FTT of 10 basis points (0.10%) would result in a $10 tax for every $10,000 in stock sold.

A 10 basis point FTT would likely result in a more than 50% reduction in trading volume. If you think high-frequency trading and other trading innovations have been helpful to the efficient allocation of capital, you might not like that result. On the other hand, if you think a lot of that activity is essentially rent-seeking behavior that doesn't particularly enhance fundamental price discovery or the allocation of capital in the economy, you might be perfectly content.

SIFMA Insights

Ramifications of an FTT

A Financial Transaction Tax Will Harm US Capital Markets & Individual Investors

October 2019

Key Takeaways

In this report, we use case studies from across the globe to assess the potential ramifications of FTTs. FTTs fail to reach objectives, as they:

- Increase costs and lower returns for individual investors;
- Typically, and often significantly, miss revenue generation projections, as the taxable base declines with volume migration;
- Not only do they not curb volatility but instead increase it as trading volumes decline, harming capital markets;
- Increase financing costs for municipalities, the federal government and corporations;
- Increase prices for consumer goods; and
- Generally damage economic growth by decreasing revenues and jobs in the U.S. as volumes migrate.

The original concept behind modern financial transaction taxes (FTT) was the Tobin Tax. This proposed, but never enacted, currency transaction tax was meant to eliminate exchange rate differentials among countries across the globe.

The original Tobin proposal was meant to maintain the benefits capital markets bring to investors and economies. It was not meant to impact long term investments, nor was it meant to be a revenue generator for governments with ballooning deficits. Tobin himself disavowed this tax as a means of revenue raising for social purposes and eventually backed off his own proposal.

Despite the fact that the proposed benefits of a Tobin Tax have always been, and remain today, controversial, many countries have tried versions of this tax, now commonly known as FTTs. The primary driver behind FTTs is to raise revenue and curb volatility. **The results have failed to meet these objectives. FTTs have been shown to harm individual investors, and the harm generally outweighs any benefits.**

As shown in this report, FTTs fail to reach objectives on the following accounts: (a) they increase costs and lower returns for individual investors; (b) they typically, and often significantly, miss revenue generation projections, as the taxable base declines with volumes; (c) not only do they not curb volatility but instead increase it as trading volumes decline, harming capital markets; (d) they increase financing costs for municipalities, the federal government and corporations; (e) they increase prices for consumer goods; and (f) they generally damage economic growth by decreasing revenues and jobs in the U.S. as volumes migrate.

About SIFMA

SIFMA is the leading trade association for broker-dealers, investment banks and asset managers operating in the U.S. and global capital markets. On behalf of our industry's nearly 1 million employees, we advocate for legislation, regulation and business policy, affecting retail and institutional investors, equity and fixed income markets and related products and services. We serve as an industry coordinating body to promote fair and orderly markets, informed regulatory compliance, and efficient market operations and resiliency. We also provide a forum for industry policy and professional development. SIFMA, with offices in New York and Washington, D.C., is the U.S. regional member of the Global Financial Markets Association (GFMA).

https://www.sifma.org/wp-content/uploads/2019/10/SIFMA-Insights-Ramifications-of-an-FTT.pdf

High-frequency volatility compared to daily

- There is no natural finest time scale
  - data arrives in continuous time
- It is not obvious what "the price" is
  - last trade price? bid? ask? midpoint?
- Serial independence is violated on short time scales
  - require special statistical techniques to use fine data
- Intraday profiles cannot be neglected

Model for time evolution

(drop the logarithms -- not important on intraday time scales)

- Discrete time (daily data): Typically, constant volatility

difference eqn: $$p_i - p_{i-1} = \sigma \xi_i, \qquad \xi_i \sim \mathcal{N}(0,1)$$

summed: $$p_i = p_0 + \sigma \sum_{j=1}^{i} \xi_j$$

- Continuous time (intraday data):

Possibly time-varying volatility

stochastic differential eqn: $$dp(t) = \sigma(t)\, dB(t)$$

$B(t) =$ Brownian process

integrated: $$p(t) = p(0) + \int_0^t \sigma(s)\, dB(s)$$

Variance across time intervals

- Discrete time:

$$p_{i+\ell} - p_i \quad \text{has variance} \quad \sigma^2 \ell$$

$$\sigma^2 \quad \text{is variance per step}$$

- Continuous time:

$$p(T) - p(0) \quad \text{has variance} \quad \int_0^T \sigma(t)^2 \, dt$$

$$\sigma(t)^2 \quad \text{is variance per unit time}$$

Estimation from historical data

- Discrete time:

$$N\sigma^2 \approx \sum_{j=1}^{N} (p_j - p_{j-1})^2$$

- Continuous time:

"quadratic variation"

$$\int_0^T \sigma(t)^2\, ds \approx \sum_{j=1}^{N} \Big(p(t_j) - p(t_{j-1})\Big)^2$$

any points $\{t_j\}_{j=0}^{N}$ with

$$0 = t_0 < t_1 < \cdots < t_{N-1} < t_N = T$$

What prices to use?

- Possible prices:
  - last trade price
  - last quote update whether bid or ask (really bad choice)
  - bid-ask midpoint
- Choose the one that gives smallest serial dependence on reasonable time scales
  - trade prices have strong serial covariance (Roll model)

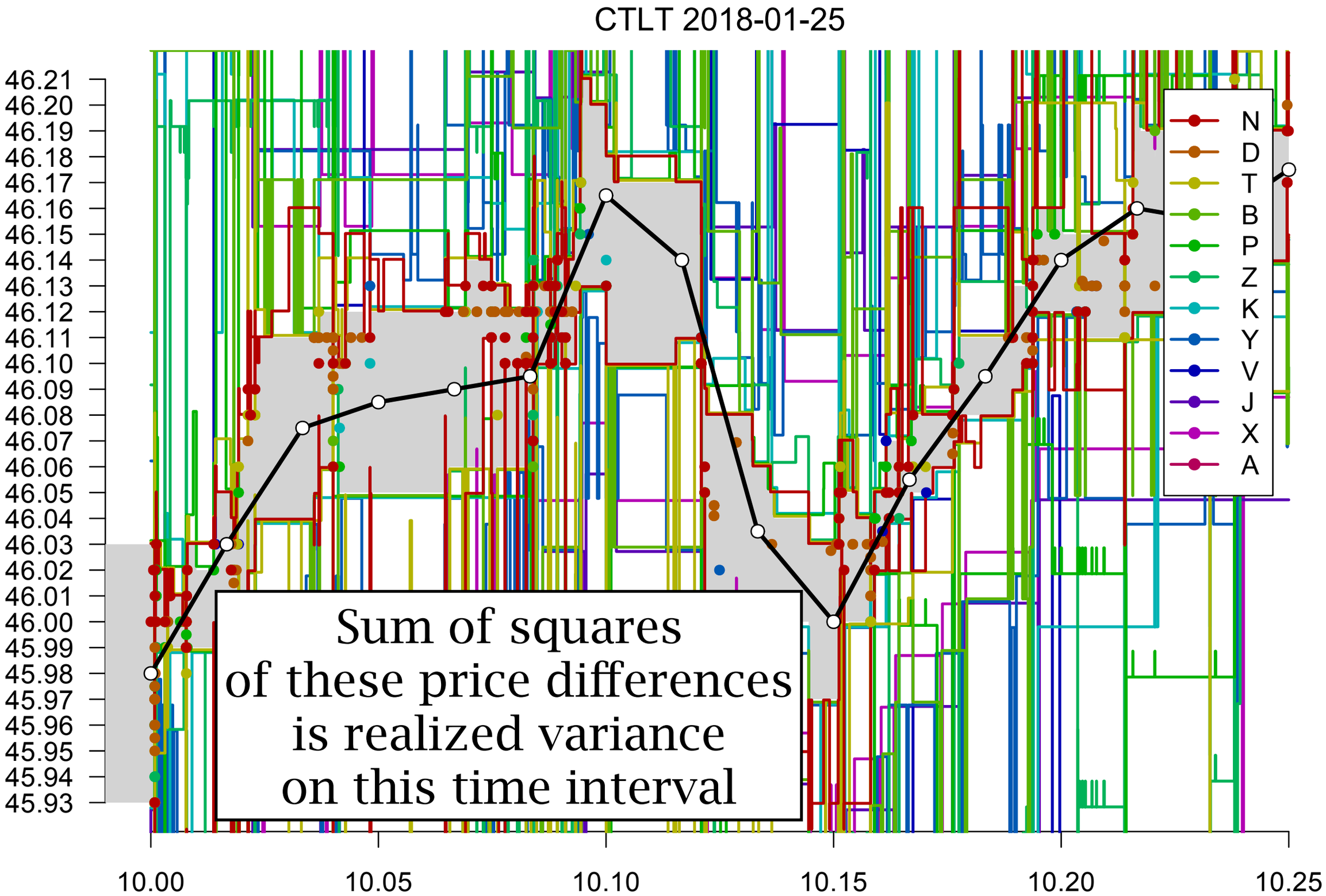
$\Longrightarrow$ Use bid-ask midpoint as "the price"

What times to use for sample?

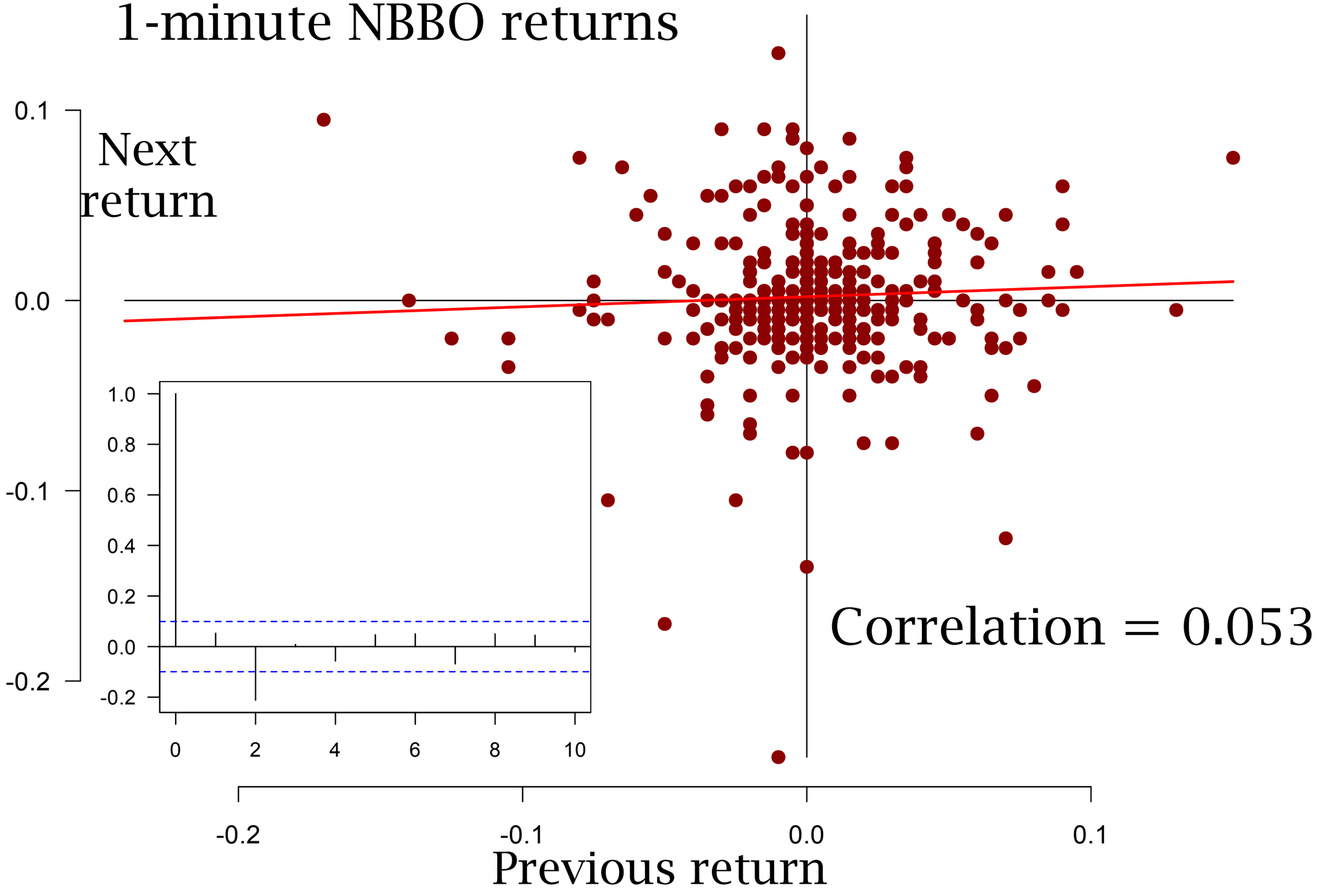
- Want to use fine times, to have lots of observations, and get good statistical precision
- Serial correlation is nontrivial at fine time scales

$\Longrightarrow$ 1-minute sampling is OK to start with

NBBO midpoint sampling at 1-minute intervals

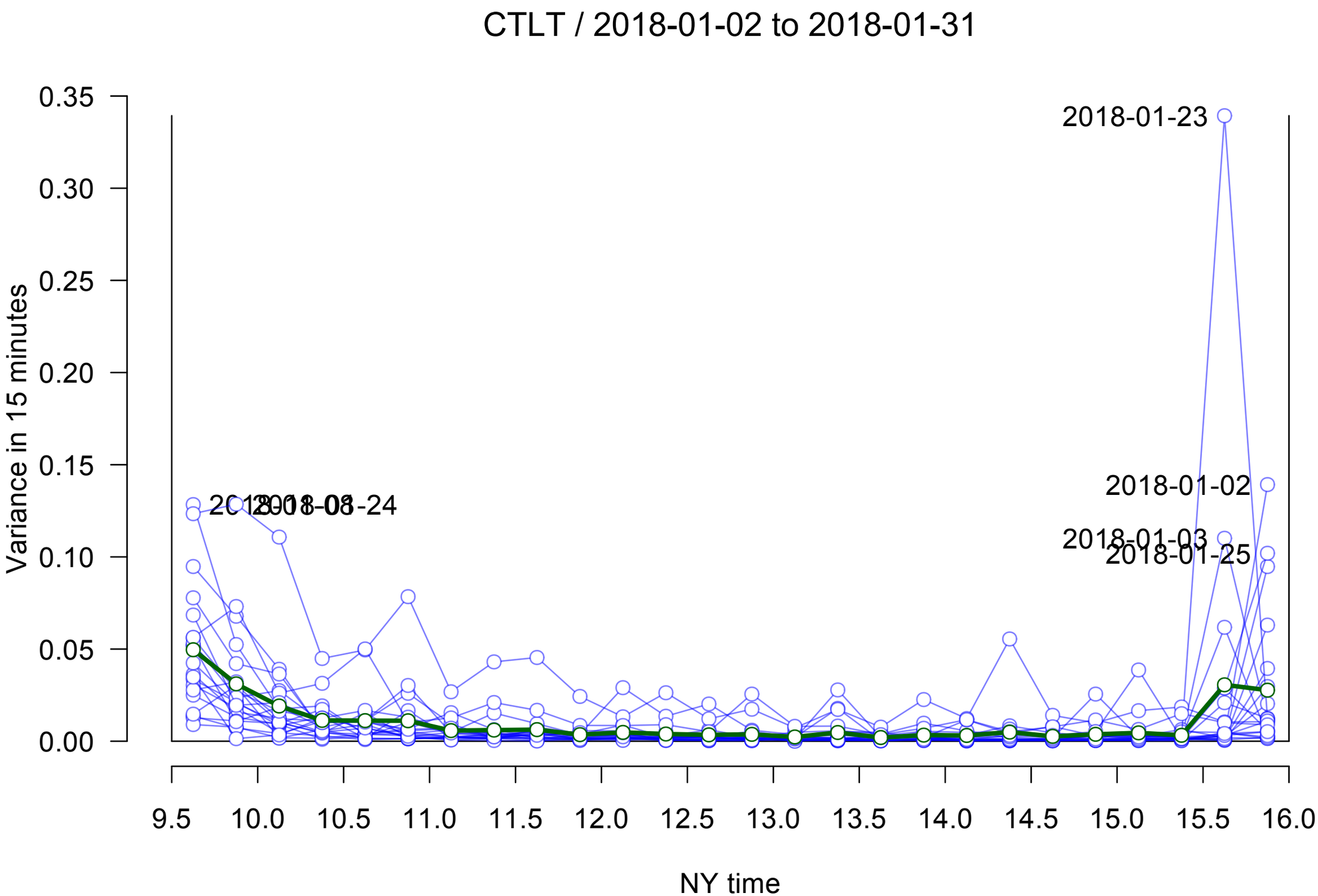


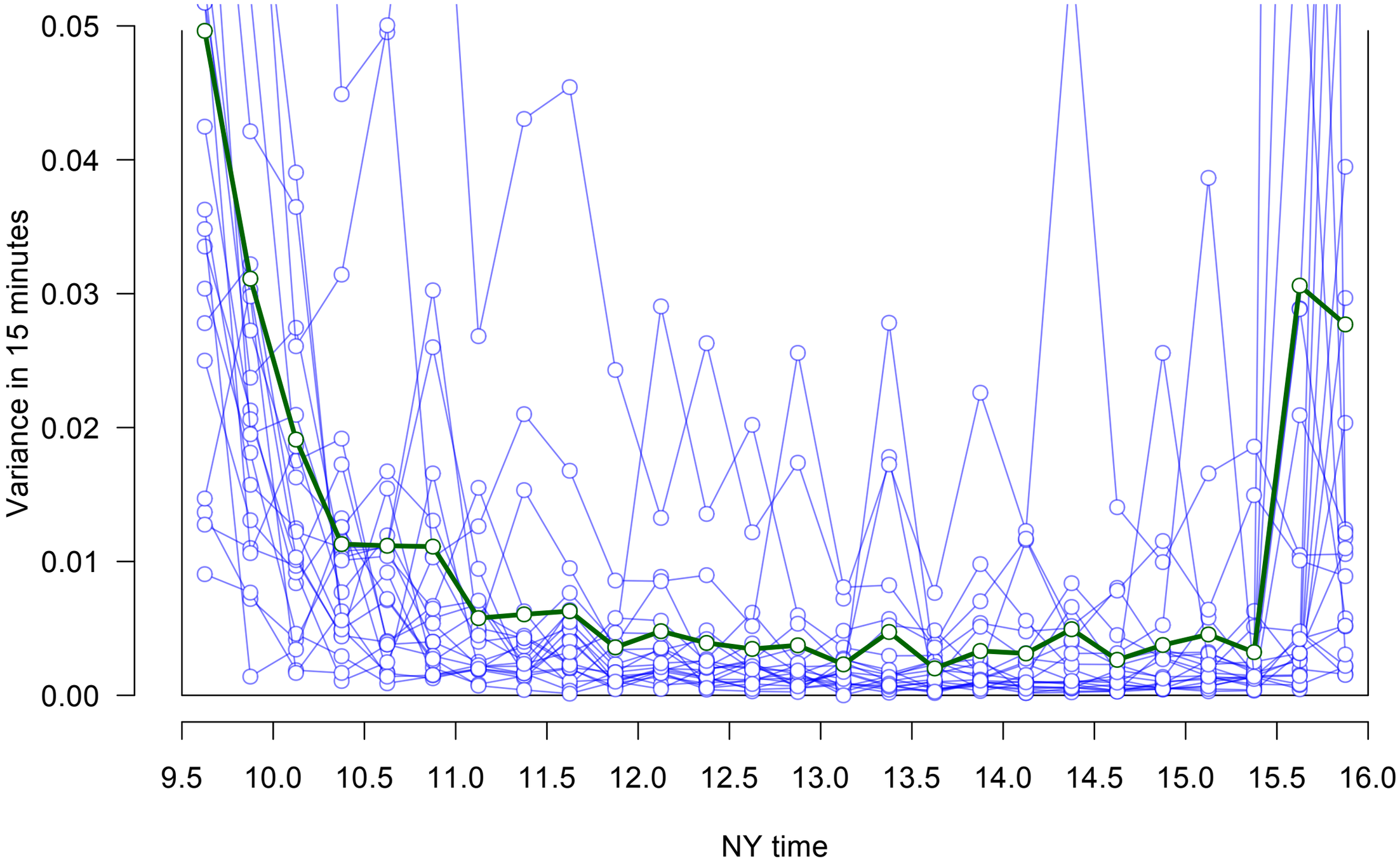
CTLT 2018-01-25
1-minute NBBO returns

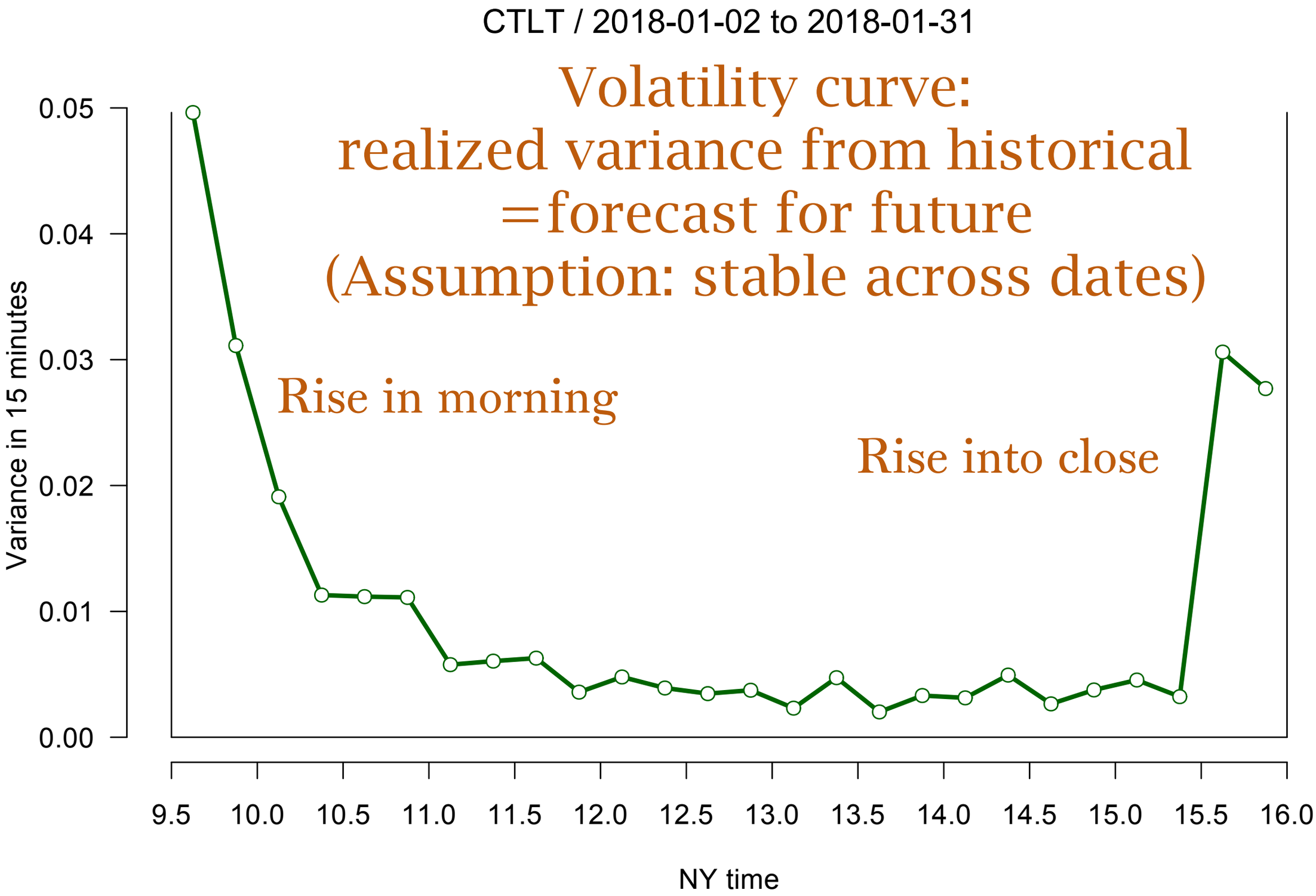


Summary of procedure for realized variance

- Compute NBBO bid and ask at 1-minute time points
  compute bid and ask from each exchange at each time
  NBBO bid is max of bids, NBBO ask is min of asks
- Take differences of successive prices
- Take sum of squares of differences in each 15 min
- Compute intraday volatility curves
  by averaging each bin across many days


CTLT / 2018-01-02 to 2018-01-31
Variance in 15 minutes
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2018-01-23
2018-01-02
2018-01-03
2018-01-25
9.5
10.0
10.5
11.0
11.5
12.0
12.5
13.0
13.5
14.0
14.5
15.0
15.5
16.0
NY time


CTLT / 2018-01-02 to 2018-01-31
Variance in 15 minutes
0.05
0.04
0.03
0.02
0.01
0.00
9.5
10.0
10.5
11.0
11.5
12.0
12.5
13.0
13.5
14.0
14.5
15.0
15.5
16.0
NY time


CTLT / 2018-01-02 to 2018-01-31
Volatility curve:
realized variance from historical
=forecast for future
(Assumption: stable across dates)
Rise in morning
Rise into close
Variance in 15 minutes
0.05
0.04
0.03
0.02
0.01
0.00
9.5
10.0
10.5
11.0
11.5
12.0
12.5
13.0
13.5
14.0
14.5
15.0
15.5
16.0
NY time

Subtleties to consider later

- Can we use finer time resolution,
  for example to get 1-minute variance?

  yes, but need to account for serial dependence
- Can we detect real-time changes in volatility?

  yes, though not instantly
- Is it stable across months
  in addition to intraday variation?

  more complicated, need product model

Trade direction

- Every trade has a buyer and a seller
- On an exchange, the limit order must be first
- The *aggressive* side is the market order
- The *passive* side is the limit order

Lee-Ready algorithm: direction from trades only

THE JOURNAL OF FINANCE • VOL. XLVI, NO. 2 • JUNE 1991

Inferring Trade Direction from Intraday Data

CHARLES M. C. LEE and MARK J. READY*

ABSTRACT

This paper evaluates alternative methods for classifying individual trades as market buy or market sell orders using intraday trade and quote data. We document two potential problems with quote-based methods of trade classification: quotes may be recorded ahead of trades that triggered them, and trades inside the spread are not readily classifiable. These problems are analyzed in the context of the interaction between exchange floor agents. We then propose and test relatively simple procedures for improving trade classifications.

The tick test is a technique which infers the direction of a trade by comparing its price to the price of the *preceding* trade(s). The test classifies each trade into four categories: an uptick, a downtick, a zero-uptick, and a zero-downtick. A trade is an uptick (downtick) if the price is higher (lower) than the price of the previous trade. When the price is the same as the previous trade (a zero tick), if the last price change was an uptick, then the trade is a zero-uptick. Similarly, if the last price change was a downtick, then the trade is a zero-downtick. A trade is classified as a buy if it occurs on an uptick or a zero-uptick; otherwise it is classified as a sell.

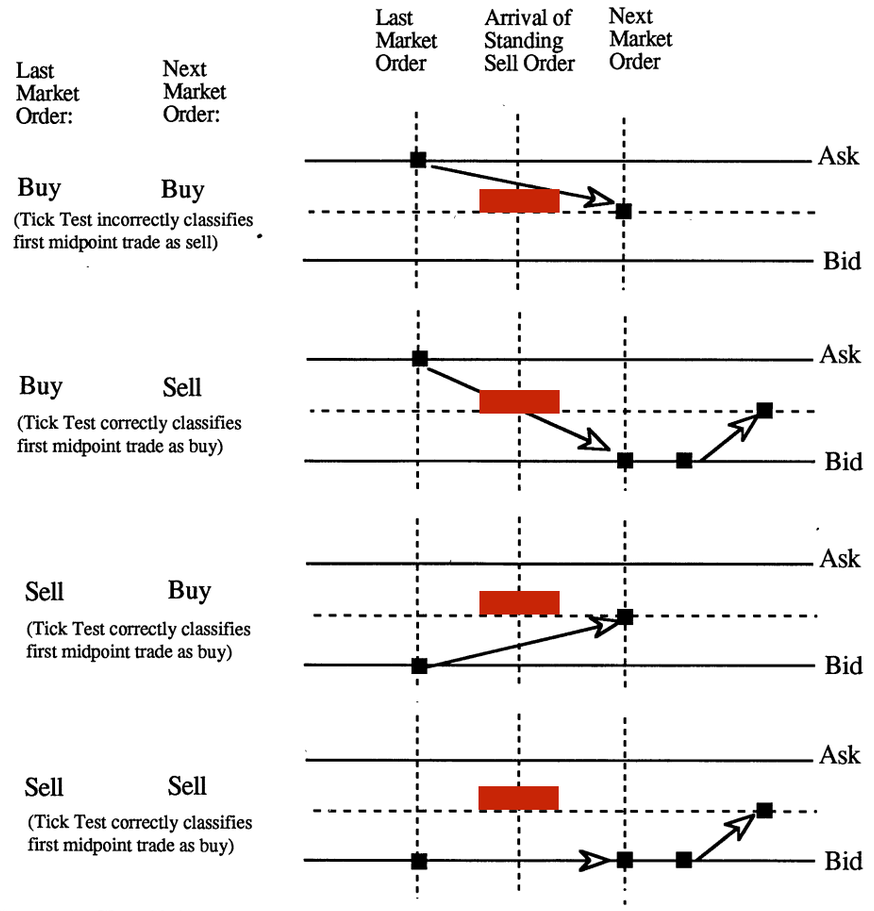
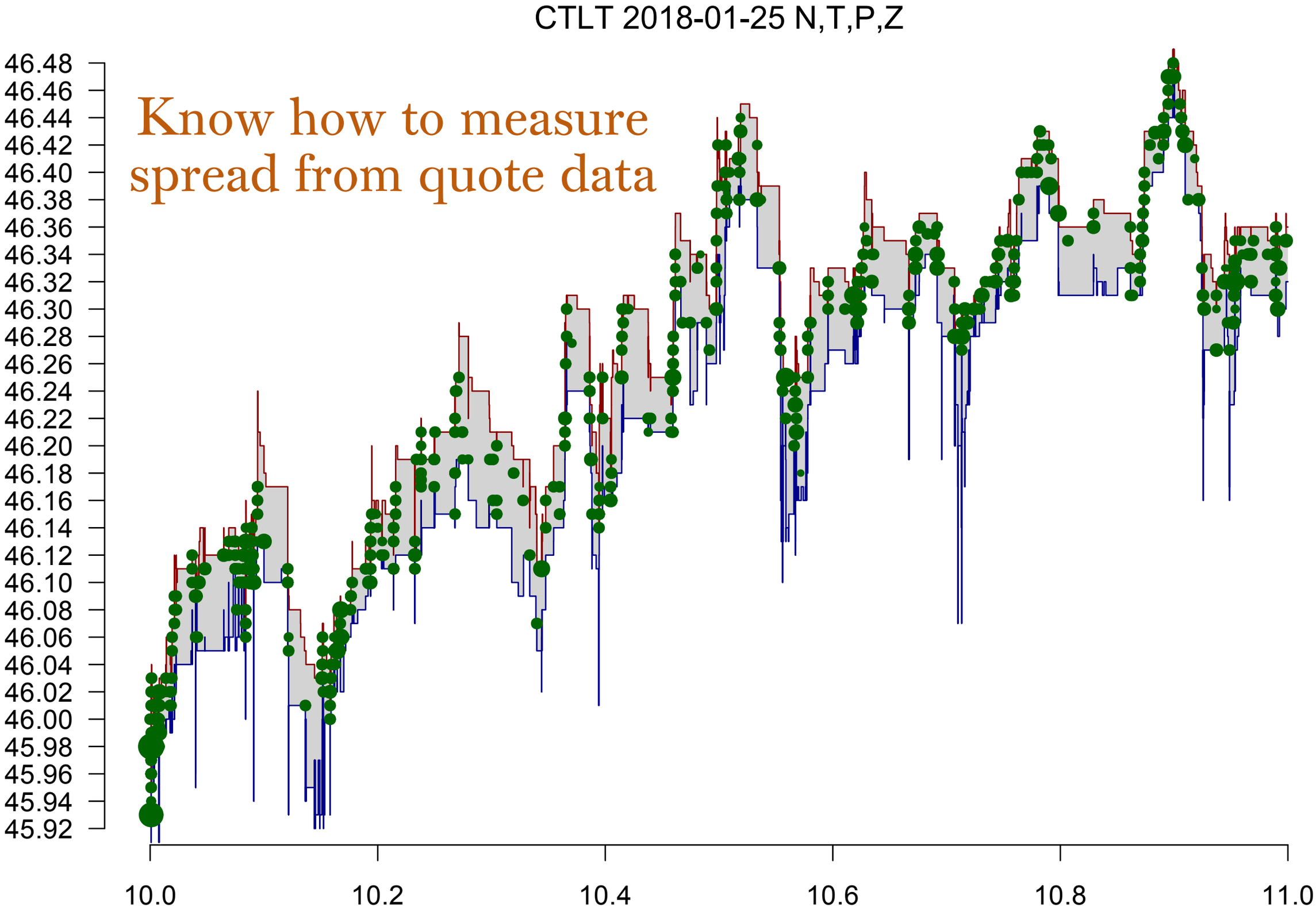
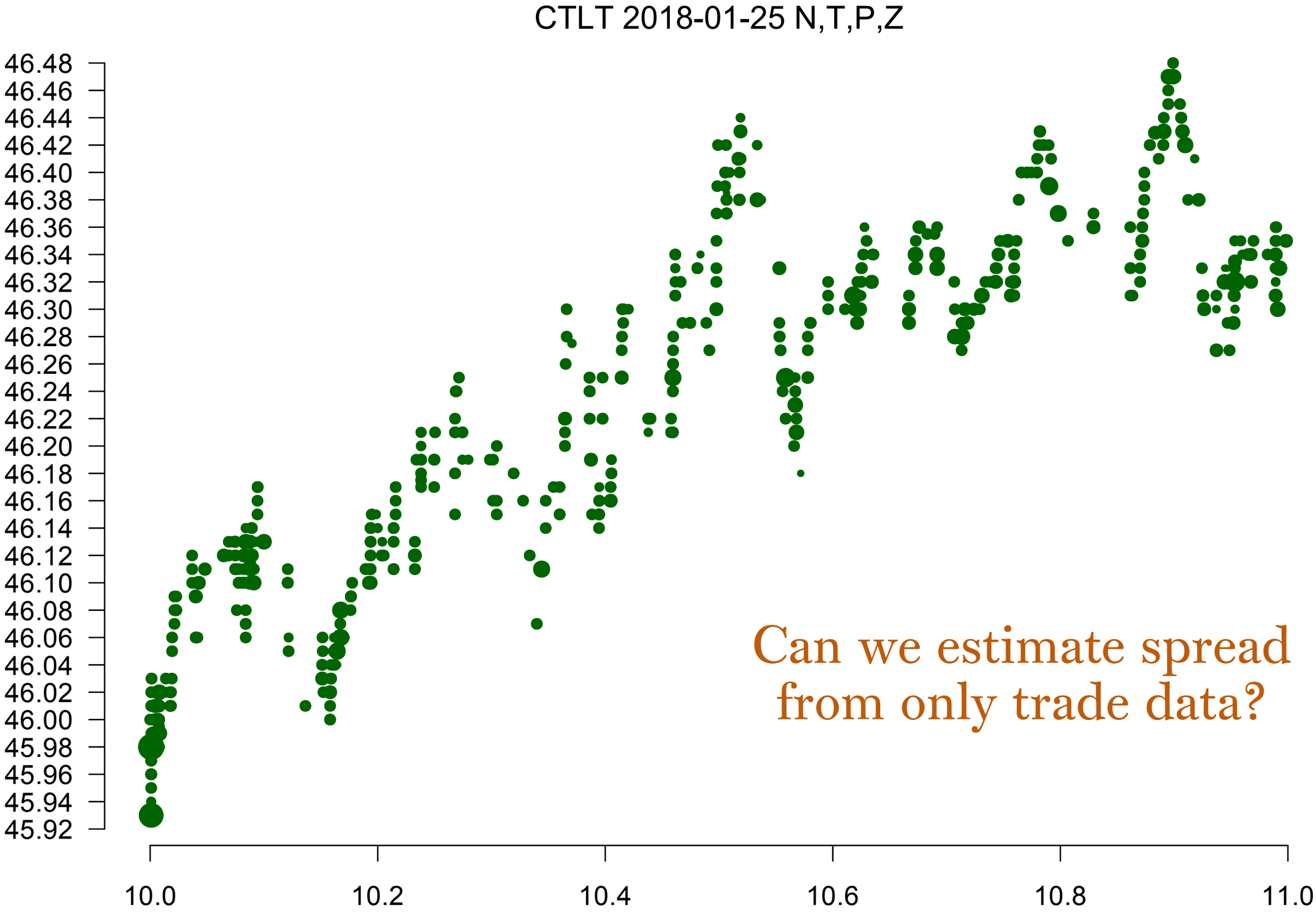


Figure 4. Classifying the first midpoint trade after the arrival of a standing sell order. Assume constant bid and ask prices and a 1/4 spread. Further assume market buy and sell orders follow independent Poisson processes with the same arrival rate. Standing buy and sell orders also follow independent Poisson processes with a lower average arrival rate than market orders. The four equally likely patterns of market orders immediately before and after the arrival of a standing order are depicted above (buy/buy, buy/sell, sell/buy, and sell/sell). In three of the four cases, the tick test classification for the first midpoint trade is correct.


CTLT 2018-01-25 N,T,P,Z
Know how to measure spread from quote data
46.48
46.46
46.44
46.42
46.40
46.38
46.36
46.34
46.32
46.30
46.28
46.26
46.24
46.22
46.20
46.18
46.16
46.14
46.12
46.10
46.08
46.06
46.04
46.02
46.00
45.98
45.96
45.94
45.92
10.0
10.2
10.4
10.6
10.8
11.0


CTLT 2018-01-25 N,T,P,Z
46.48
46.46
46.44
46.42
46.40
46.38
46.36
46.34
46.32
46.30
46.28
46.26
46.24
46.22
46.20
46.18
46.16
46.14
46.12
46.10
46.08
46.06
46.04
46.02
46.00
45.98
45.96
45.94
45.92
10.0
10.2
10.4
10.6
10.8
11.0
Can we estimate spread from only trade data?

Roll model (1984)

reversion of trade prices gives effective spread

THE JOURNAL OF FINANCE • VOL. XXXIX, NO. 4 • SEPTEMBER 1984

A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market

RICHARD ROLL*

ABSTRACT

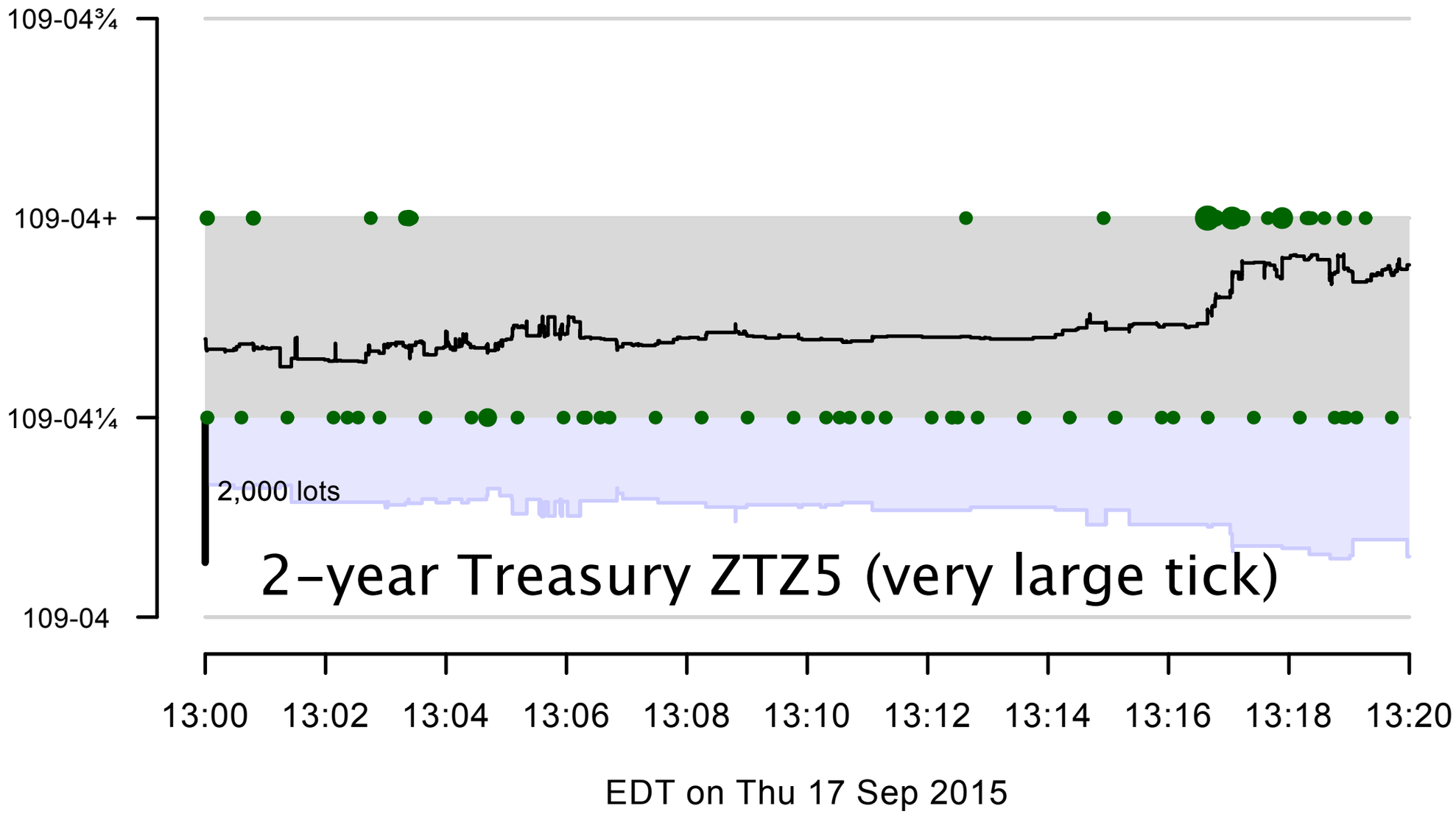
In an efficient market, the fundamental value of a security fluctuates randomly. However, trading costs induce negative serial dependence in successive observed market price changes. In fact, given market efficiency, the effective bid-ask spread can be measured by

$$\text{Spread} = 2\sqrt{-\text{cov}}$$

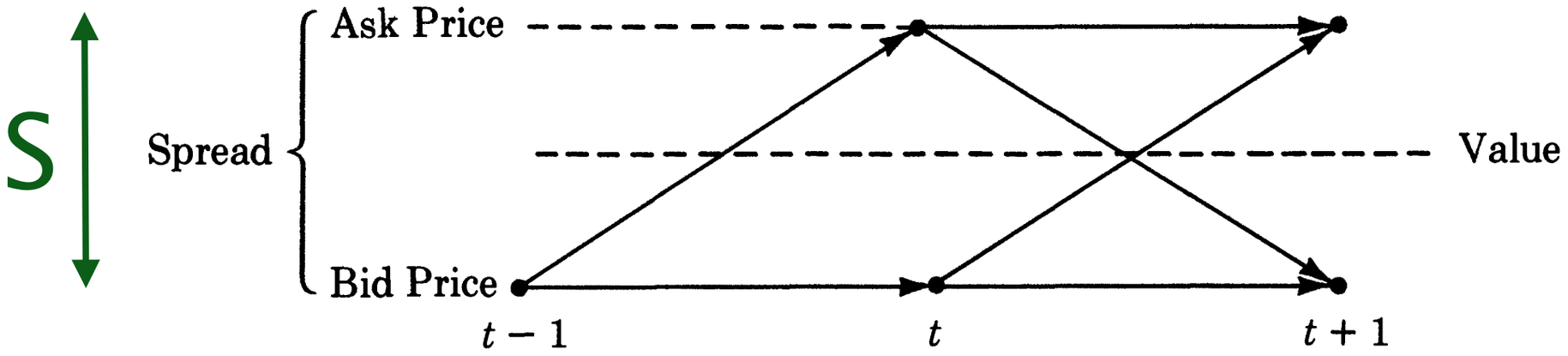
where "cov" is the first-order serial covariance of price changes. This implicit measure of the bid-ask spread is derived formally and is shown empirically to be closely related to firm size.

Spread estimation using only trade data

trades happen at bid and ask



The schematic below illustrates possible paths of observed market price between successive time periods, given that the price at time $t-1$ was a sale to the market maker, at his bid, *and* given that no new information arrives in the market.



Each path is equally likely. There is a similar but opposite asymmetric pattern if the price at $t-1$ happened to be a purchase from the market maker, at his ask price.

4 price sequences, equally likely:
(0,0), (0,+S), (+S,0), (+S,–S)

Covariance $C = \langle \Delta p_i \cdot \Delta p_{i+1} \rangle = -\frac{1}{4} S^2$

Roll formula

$$\text{Effective spread} \quad S = 2\sqrt{-C}$$

- Not affected by midpoint motion if uncorrelated to bid-ask bounce

The following are proofs that: (A) the covariance between successive price changes cannot be due to new information if markets are informationally efficient, (B) the implicit spread measure is independent of the observation interval if markets are efficient, and (C) even if the spread changes in reaction to news, the serial covariance will still be $-s^2/4$ where s^2 is the average squared spread in the sample.

- Independent of observation interval

  Roll used daily data
- Multiple fills from one market order?

  group by time? by second?
- Useful when cannot observe actual quotes (Hasbrouck 2003 futures pit data)

Roll model verification

Second, the *expected value* of the spread-induced serial covariance is independent of the time interval chosen for collecting successive prices.[4] This is implied by the fact that the serial covariance depends only on whether successive sampled transactions are at the bid or the ask, not on whether any news arrives between the sample observations. Of course, in the interest of efficient estimation, the more frequent the observations the better—because nonstationarity is less likely to affect the results and because the larger sample size means that the spread will be buried in relatively less noise.

more data is better

To verify directly that the resulting estimates of spreads are valid, it would be necessary to collect bid-ask spreads from market data (a costly procedure we are attempting to avoid). But the results can be validated indirectly by relating the measured implicit spread to firm size.

no quote data
clever test

Let $\hat{c}_{j,t}$ be the estimated serial covariance of returns of stock j in year t; then, according to our previous analysis

$$\hat{s}_{j,t} = 200\sqrt{-\hat{c}_{j,t}} \qquad (2)$$

is an estimate[7] of the percentage bid-ask spread for the stock. (The constant 200 instead of 2.0 converts the units to percent). Two estimates of serial covariance were made for each stock, one estimate using daily returns and one estimate using weekly returns. A "sufficient number of observations" was arbitrarily chosen to be one month (21 trading days) for calculations with daily returns and 21 weeks for calculations with weekly returns.

daily and weekly returns from CRSP close prices

Problem: probabilities are not equal

- buys follow buys, sells follow sells
- Reasons:
  - trade splitting on single exchange because of match rules
  - bunched trades across different exchanges
  - large order splitting (algo execution)

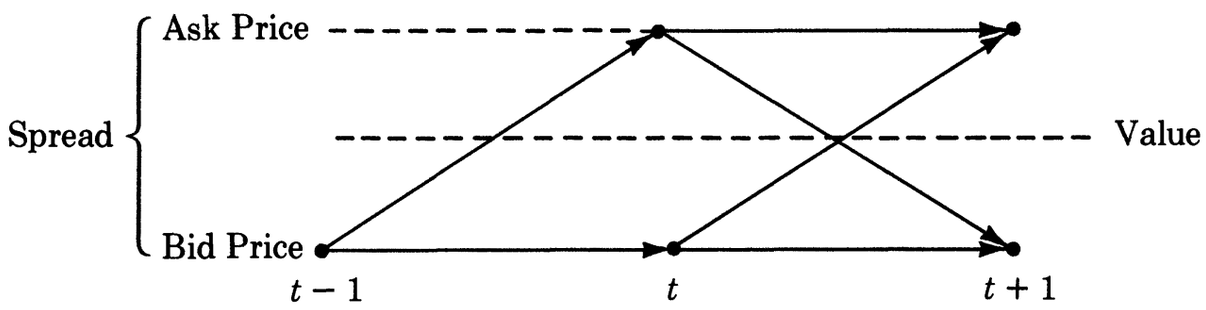
Roll model with 1-period serial correlation in trade sign

$$\mathbb{P}(\, P_{i+1} = \text{ask} \,|\, P_i = \text{ask}\,) \;=\; \frac{1}{2}(1+\rho)$$

$$\mathbb{P}(\, P_{i+1} = \text{bid} \,|\, P_i = \text{ask}\,) \;=\; \frac{1}{2}(1-\rho)$$

Correlation ρ typically 80% or more

Roll model with serial correlation

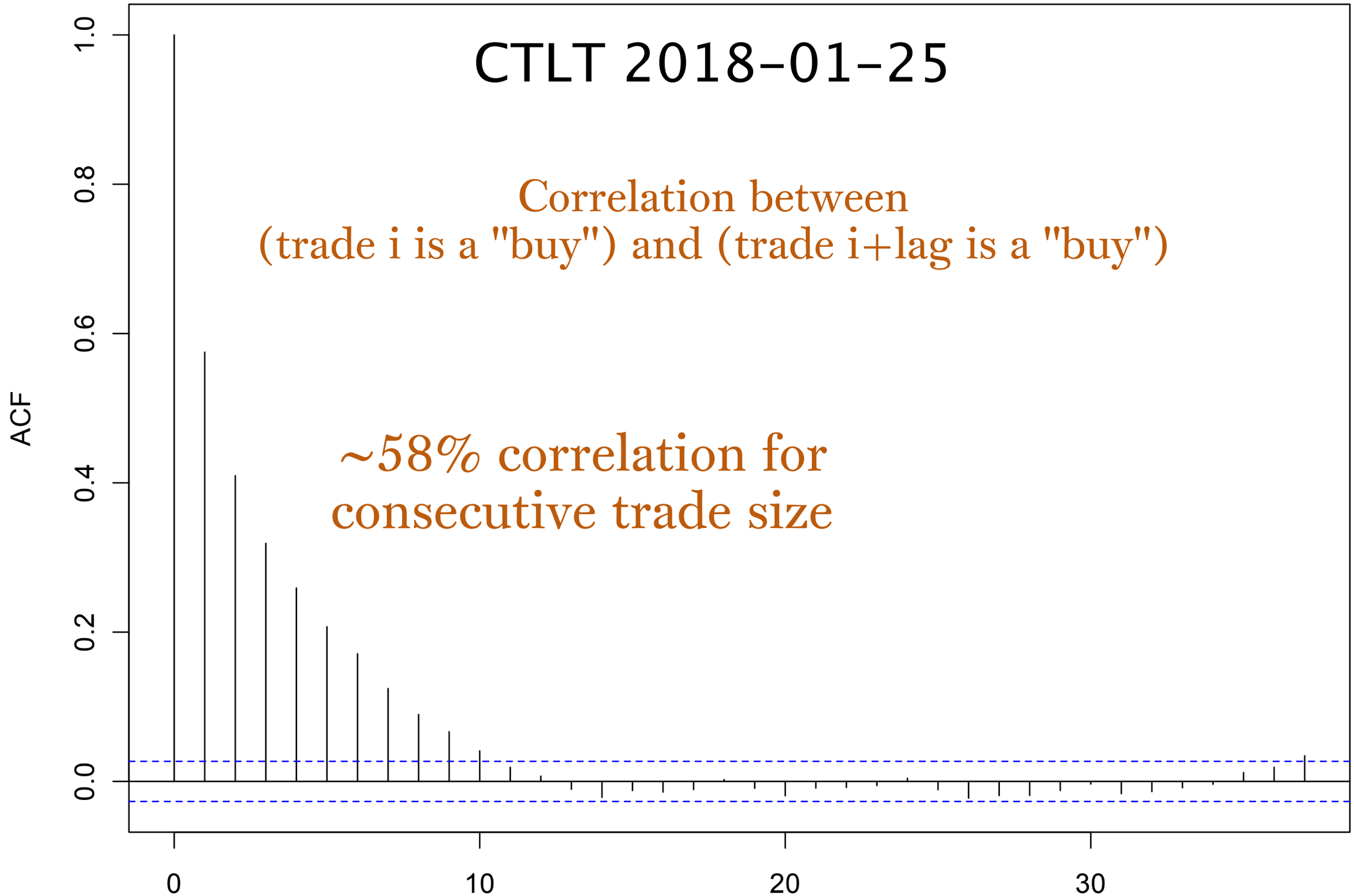


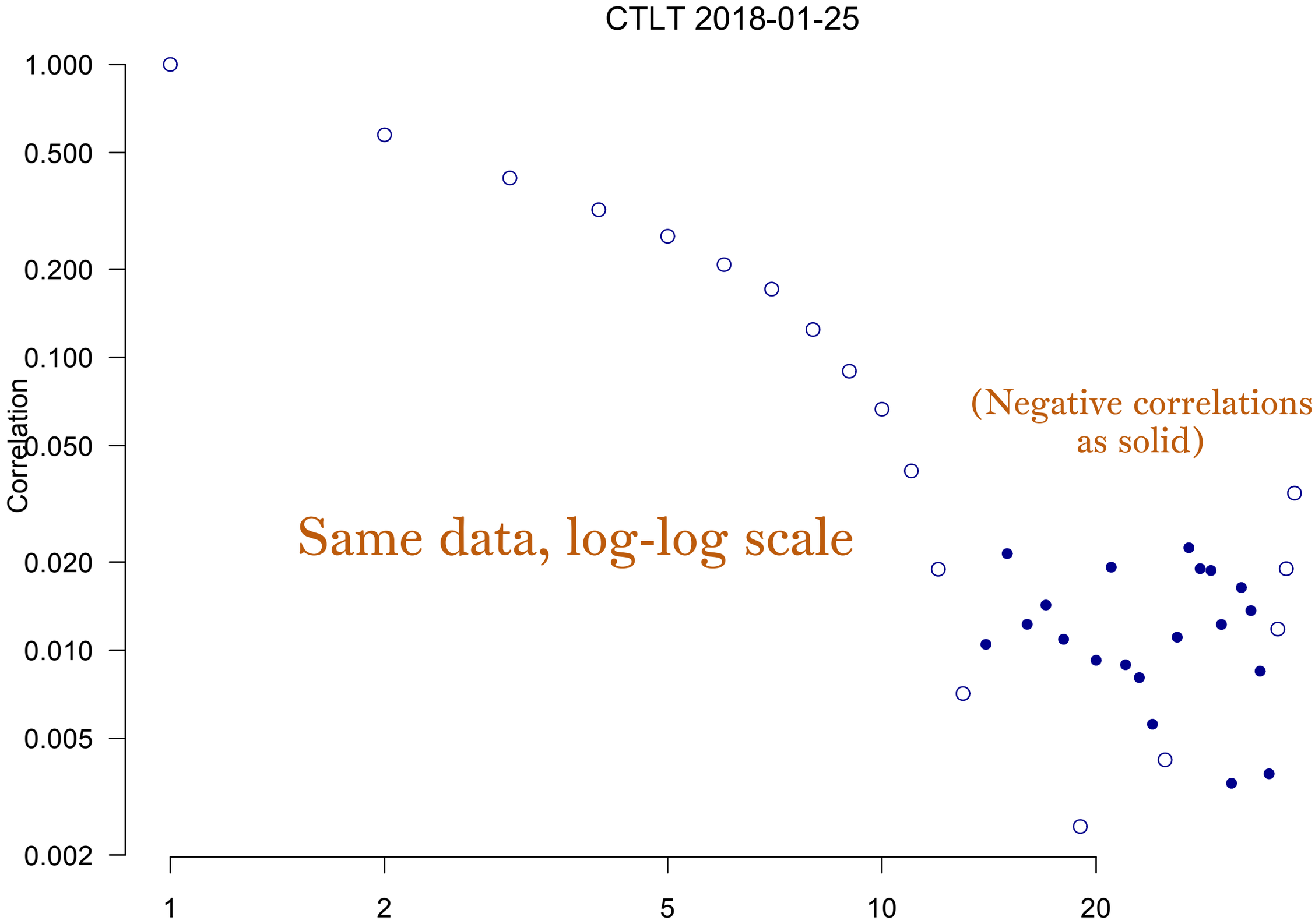
Price changes	Probability
$(0,0)$	$\frac{1}{4}(1+\rho)^2$
$(0,+S)$	$\frac{1}{4}(1+\rho)(1-\rho)$
$(+S,0)$	$\frac{1}{4}(1-\rho)(1+\rho)$
$(+S,-S)$	$\frac{1}{4}(1-\rho)^2$

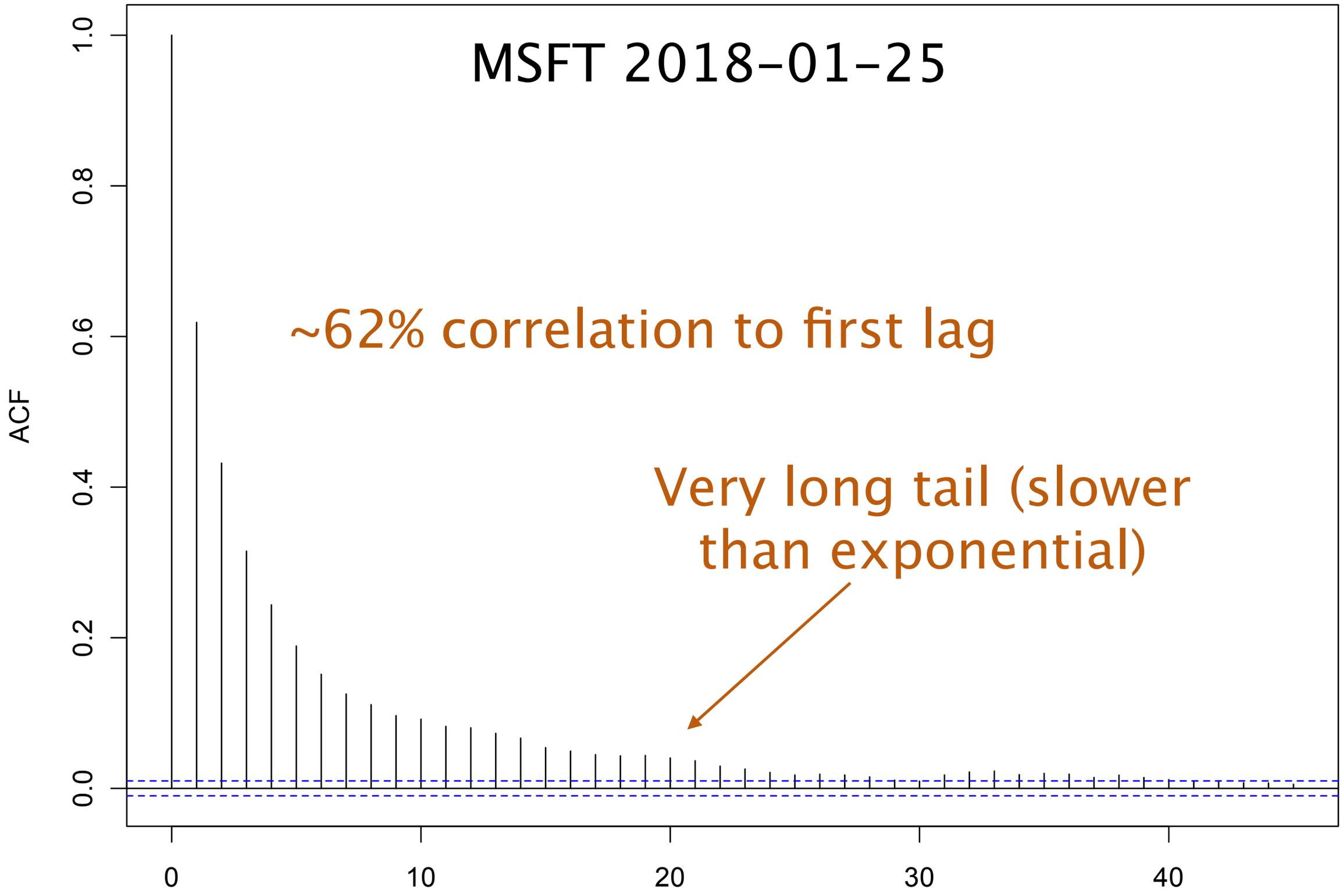
$$C = \langle \Delta p_i \cdot \Delta p_{i+1} \rangle = -\frac{1}{4}(1-\rho)^2 S^2$$

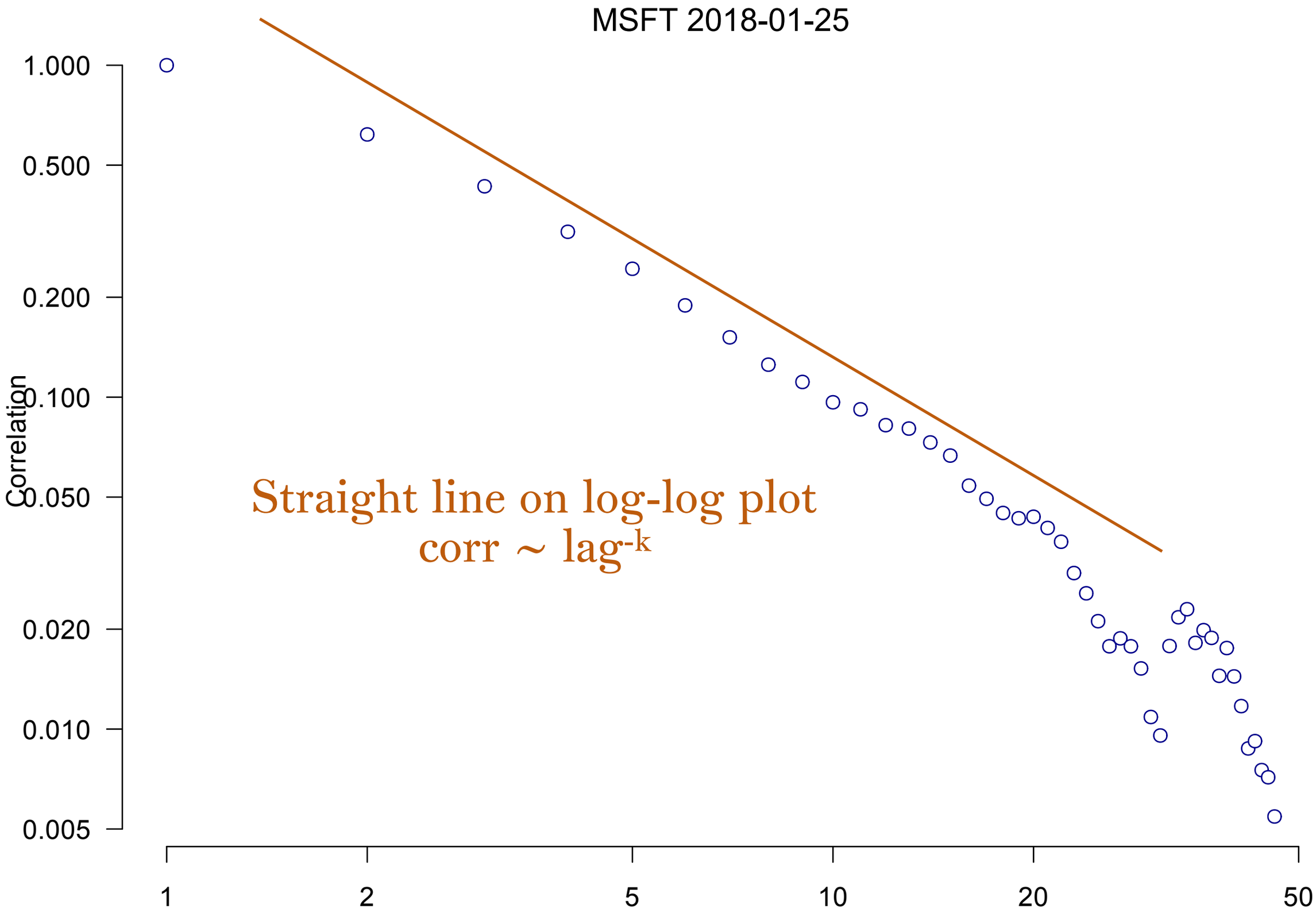
$$S = \frac{2\sqrt{-C}}{1-\rho}$$

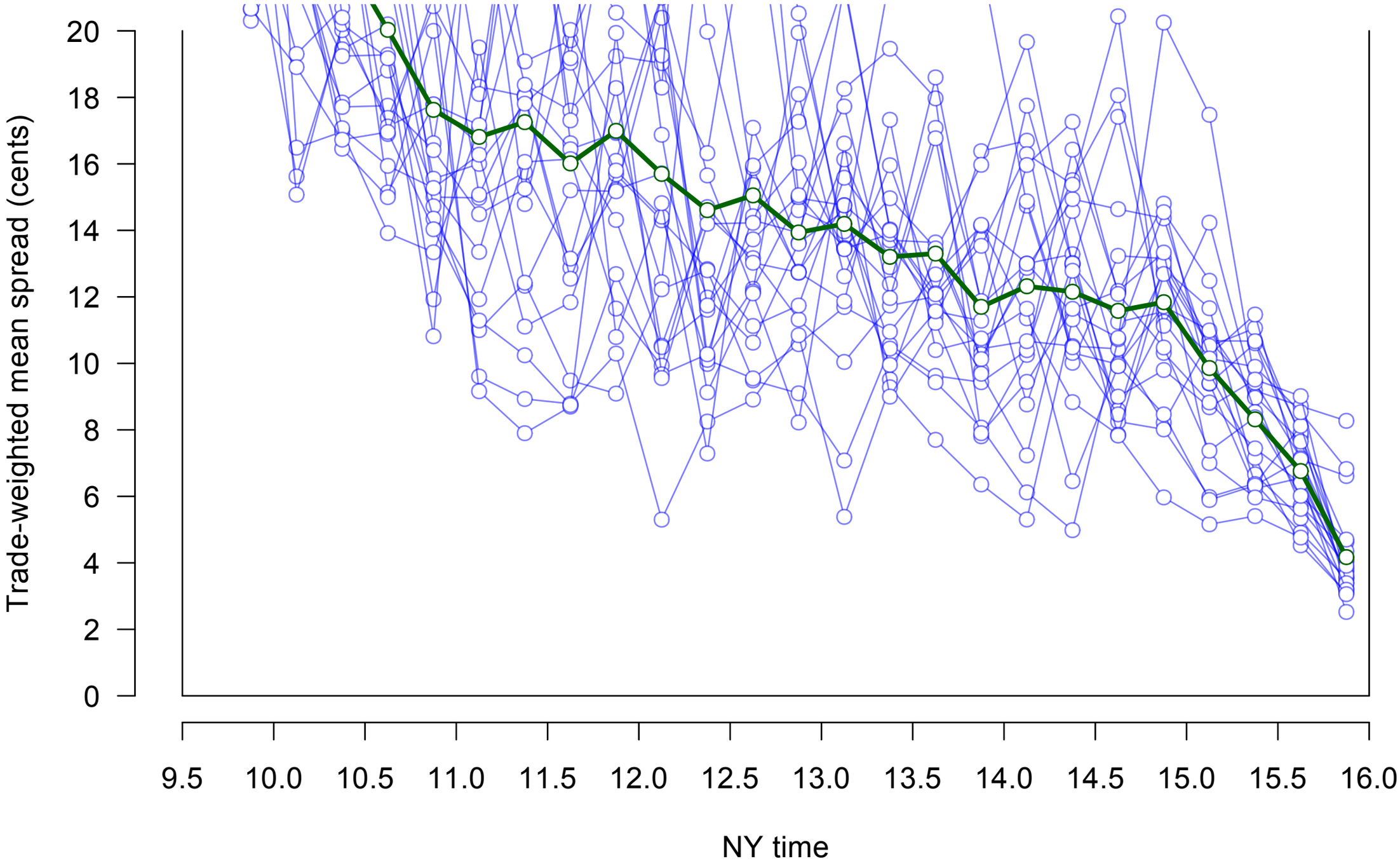
Problem: need independent estimate of ρ (need quotes)

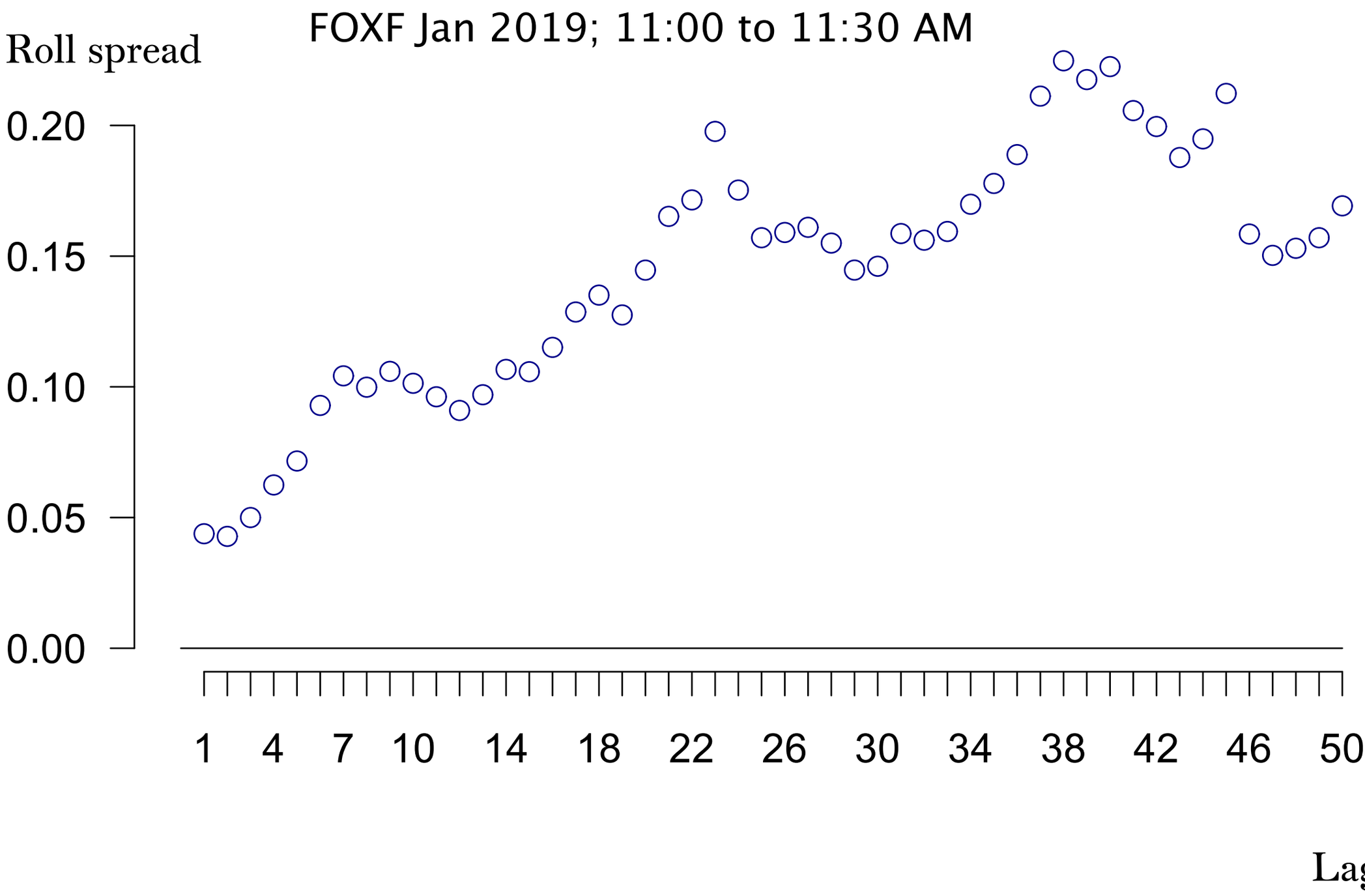

CTLT 2018-01-25
Correlation between
(trade i is a "buy") and (trade i+lag is a "buy")
~58% correlation for
consecutive trade size
ACF
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30


CTLT 2018-01-25
Correlation
1.000
0.500
0.200
0.100
0.050
0.020
0.010
0.005
0.002
1
2
5
10
20
(Negative correlations as solid)
Same data, log-log scale


MSFT 2018-01-25
~62% correlation to first lag
Very long tail (slower than exponential)
ACF
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40


MSFT 2018-01-25
Corr
1.000
0.500
0.200
0.100
0.050
0.020
0.010
0.005
1
2
5
10
20
50
Straight line on log-log plot
corr ~ lag^-k


FOXF / 2019-01-02 to 2019-01-31
Trade-weighted mean spread (cents)
0
2
4
6
8
10
12
14
16
18
20
9.5
10.0
10.5
11.0
11.5
12.0
12.5
13.0
13.5
14.0
14.5
15.0
15.5
16.0
NY time


FOXF Jan 2019; 11:00 to 11:30 AM
Roll spread
0.20
0.15
0.10
0.05
0.00
1
4
7
10
14
18
22
26
30
34
38
42
46
50
Lag

Economics of spread

- Why is bid-ask spread the size it is?
- Market makers want to capture profit
- But compete with each other for order flow
- Spread is a result of this competition
- Theoretical models next week

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20 May 2016

Jack L. Treynor and the Birth of the Quants

By Mark Harrison, CFA

Imagine for a moment an investment world *without* the accomplishments of Jack L. Treynor.

Treynor was a key member of a tiny group of theorists from which the efficient markets hypothesis (EMH), the capital asset pricing model (CAPM), and the random walk hypothesis emerged in the 1960s. In this imaginary Treynor-less world, mass casino psychology and fund manager guru-worship might rule, unchallenged by any metrics other than crude popularity and marketing spend.

In another *FAJ* piece, "The Only Game in Town," written under the pseudonym Walter Bagehot, Treynor evaluated the economics of market making, asking, Why do investors persist in trading despite their dismal long-run trading record and why isn't trading against the public consistently profitable?

THE MARKET CONSENSUS

It is well known that market makers of all kinds make surprisingly little use of fundamental information. Instead they observe the relative pressure of buy and sell orders and attempt to find a price that equilibrates these pressures. The resulting market price at any point in time is not merely a consensus of the transactors in the market place, it is also a consensus of their mistakes.

On the other hand the market maker always gains in his transactions with liquidity-motivated transactors. The essence of market-making, viewed as a business, is that in order for the market maker to survive and prosper, his gains from liquidity-motivated transactors must exceed his losses to information-motivated transactors. To the market maker, the two kinds of transactors are largely indistinguishable. The spread he sets between his bid and asked price affects both: the larger the spread, the less money he loses to information-motivated transactors and the more he makes from liquidity-motivated transactors (assuming that a wider spread doesn't discourage the latter transactions).

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BID, ASK AND TRANSACTION PRICES IN A SPECIALIST MARKET WITH HETEROGENEOUSLY INFORMED TRADERS*

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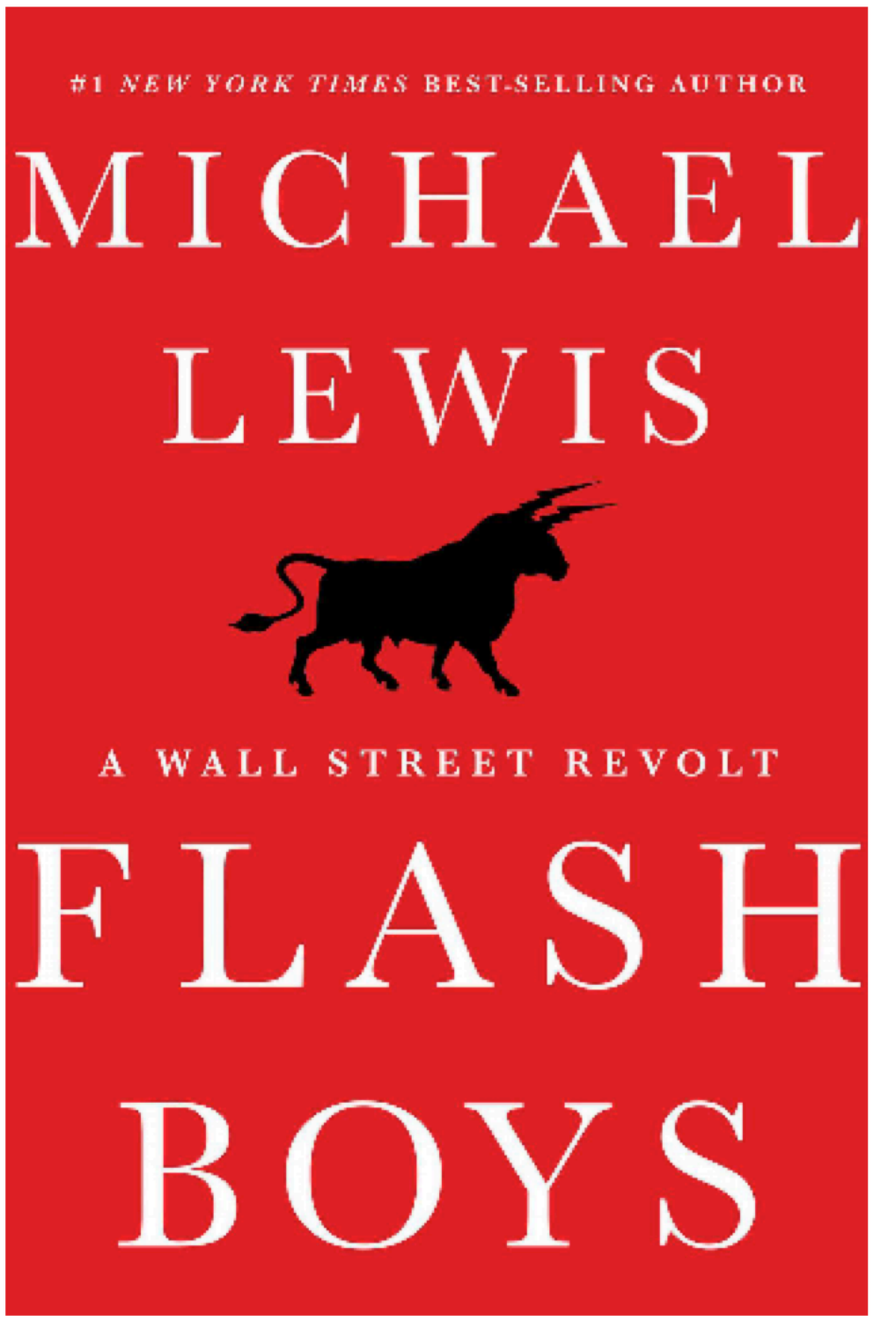
The presence of traders with superior information leads to a positive bid–ask spread even when the specialist is risk-neutral and makes zero expected profits. The resulting transaction prices convey information, and the expectation of the average spread squared times volume is bounded by a number that is independent of insider activity. The serial correlation of transaction price differences is a function of the proportion of the spread due to adverse selection. A bid–ask spread implies a divergence between observed returns and realizable returns. Observed returns are approximately realizable returns plus what the uninformed anticipate losing to the insiders.

The approach taken in this paper is based on the idea that a bid–ask spread can be a purely informational phenomenon, occurring even when all the specialist's fixed and variable transactions costs (including his time, inventory costs, etc.) are zero and when competition forces the specialist's profit to zero. The core idea is that the specialist faces an adverse selection problem, since a customer agreeing to trade at the specialist's ask or bid price may be trading because he knows something that the specialist does not. In effect, then, the specialist must recoup the losses suffered in trades with the well informed by gains in trades with liquidity traders. These gains are achieved by setting a spread. This informational source of the spread has also been suggested by Bagehot (1971) and formally analyzed by Copeland and Galai (1983).



An electronics company in Singapore called Flextronics announced its intention to buy a smaller rival, Solectron, for a bit less than $4 a share. A big investor called Brad and said he wanted to sell 5 million shares of Solectron. The public stock markets—the New York Stock Exchange (NYSE) and Nasdaq—showed the current market. Say it was 3.70–3.75, which is to say you could sell Solectron for $3.70 a share or buy it for $3.75. The problem was that, at those prices, only a million shares were bid for and offered. The big investor who wished to sell 5 million shares of Solectron called Brad because he wanted Brad to take the risk on the other 4 million shares. And so Brad bought the shares at $3.65, slightly below the price quoted in the public markets. But when he turned to the public markets—the markets on his trading screens—the share price instantly moved.

Almost as if the market had read his mind. Instead of selling a million shares at $3.70, as he'd assumed he could do, he sold a few hundred thousand and trigged a minicollapse in the price of Solectron. It was as if someone knew what he was trying to do and was reacting to his desire to sell before he had fully expressed it. By the time he was done selling all 5 million shares, at prices far below $3.70, he had lost a small fortune.