PRINCETON UNIVERSITY

ORF 474: High Frequency Trading
Spring 2020
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Lecture 5b

March 4, 2020

What does a model add to description?

- More information about how effects work
- Quantitatively testable predictions
- Try to measure some of these from real data

Regression models

- Roll model

  without trade size **γ**

  (with trade size)

- Glosten-Milgrom model

  ("classic" without trade size)

  with trade size for market impact **λ**

- Inventory model

  trade sign autocorrelation to eliminate degeneracy **β, φ**

Goals of testing regression models

- What are the coefficients?
  - how do they compare to each other
  - what are the main determinants of spread/liquidity
- How do they vary
  - across stocks and other assets like futures
  - are values correlated with other asset properties
    - market cap, tick size, volatility, etc
  - across time -- intraday profiles and event response
- Are the models reasonable and correct?
  - is one version better than another
  - do we need the extra terms to fit data?

Model structure

- Inputs to model

  Δp = change in successive trade price

  d = trade sign

  q = signed trade size

- Trade-centric -- why trades, not quotes?

  Trades are real economic events: money changes hands

  Quote data not always available or reliable

    not all markets are limit order books

    dealer markets: corporate bonds, US Treasuries somewhat

  Posted quotes may not be effective quotes

    narrow spread from hidden orders

    wider spread for orders of realistic size

    shifts within spread (midpoint not reliable)

Suggestions for TAQ data

- Include only 10:00 - 15:30 (avoid open and close)
- Null or zero values do not participate in NBBO

  set bid = -Inf where bid = 0

  set ask = +Inf where ask = 0
- Group trades at same price with no quote between

  I have not done this -- is hard for equities
- Or use futures data from single exchange

Description of how to use TAQ

A Comparison of Execution Quality across U.S. Stock Exchanges

Elaine Wah, Stan Feldman, Francis Chung, Allison Bishop, and Daniel Aisen

Investors Exchange*

April 19, 2017

The complex and fragmented nature of the U.S. equities exchange landscape has given rise to structural inefficiencies that have created the potential for inherent conflicts of interest between market participants. The introduction of the Investors Exchange, or IEX, offers the unique opportunity to evaluate market quality across exchanges with varying design characteristics and fee structures. As execution quality on a stock exchange cannot be examined independently of equity market structure, we take a holistic approach: we study four dimensions of market quality—liquidity, execution costs, price discovery, and market stability—and within each category we examine structural mechanics responsible for observed disparities in execution quality. We observe a consistent trend in venue stratification by fee structure across a number of market quality metrics: maker-taker exchanges dominate the U.S. equities trading landscape in market share despite greater adverse selection, less stability around executions, significantly longer queues at the inside, and a lower probability of execution. This suggests that access fees and rebates perpetuate economic incentives misaligned with the tenets of best execution, and may promote activity detrimental to market quality. We employ a publicly available dataset (Daily TAQ) in an effort to facilitate replication of our metrics, via which market participants can independently evaluate venue performance.

We use Daily TAQ data, in which quotes and trades are timestamped to the microsecond. Our dataset includes 8,522 symbols, of which 1,773 are exchange-traded funds (ETFs) and 6,749 are corporate equities. The time period of the dataset is the first quarter of 2017 (January 3, 2017 through March 31, 2017). We exclude quotes and trades outside of regular market hours (9:30 AM to 4:00 PM Eastern Time) and during the opening and closing auctions. We omit quotes and trades during locked and crossed markets unless otherwise specified, as measurements of market quality are not meaningful during these "economically nonsensical states" (Holden & Jacobsen, 2014). To filter out abnormal quotes for metrics benchmarked to the NBBO, we only include quotations for which the NBO is within the range $\left[\frac{1}{3}NBB, 3\,NBB\right]$, where NBB is the national best bid and NBO is the national best offer. We also omit instances where either the bid or offer price is 0 or missing.

Trade-signing algorithms serve to classify individual trades as initiated by a marketable buy or sell order, which facilitates identification of the resting order. When the initiating direction of an execution is necessary, we sign trades by determining whether the trade occurred at the quote (either the bid or the offer) or inside the NBBO spread. For trades inside the quote, we avoid making assumptions about the side of midpoint trades, but otherwise sign based on whichever side is closer (e.g., an execution inside the spread and strictly less than the midpoint would be signed as a resting buy order that executed against a liquidity-removing seller). We do not compute statistics that require assumptions about which side is adding liquidity in a midpoint trade, but instead develop alternative metrics to assess the execution quality of midpoint executions.

Regression models

- Roll

$$\Delta p_t \;=\; \gamma\,\Delta d_t \;+\; \epsilon_t .$$

- Glosten-Milgrom

$$\Delta p_t \;=\; \lambda\, d_t \;+\; \gamma\,\Delta d_t \;+\; \epsilon_t$$

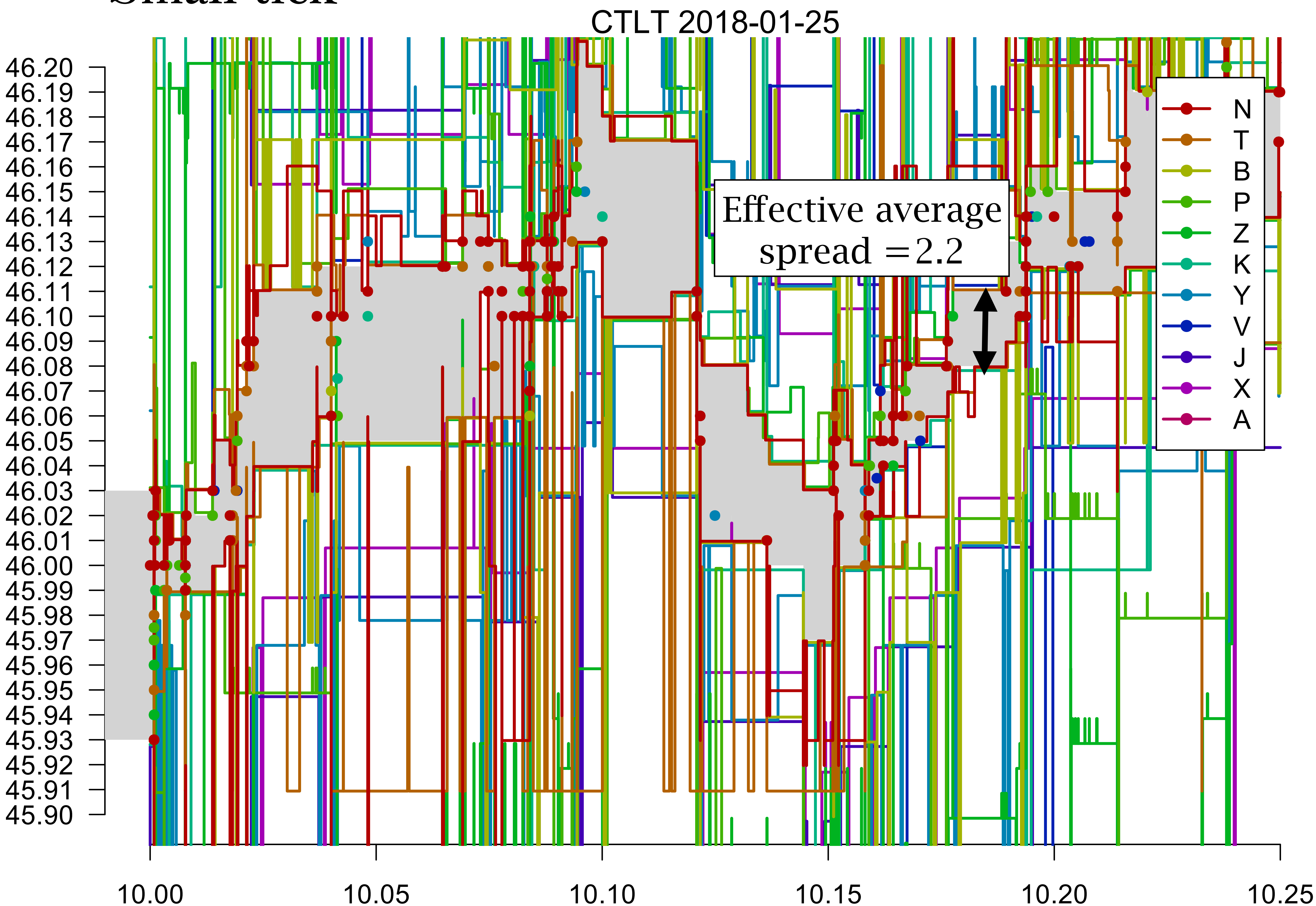
$$\Delta p_t \;=\; \lambda\, q_t \;+\; \gamma\,\Delta d_t \;+\; \epsilon_t$$

- Inventory risk

$$\Delta p_t \;=\; (\lambda+\beta) q_t \;-\; \lambda\phi\, q_{t-1} \;+\; \gamma\,\Delta d_t \;+\; \epsilon_t .$$

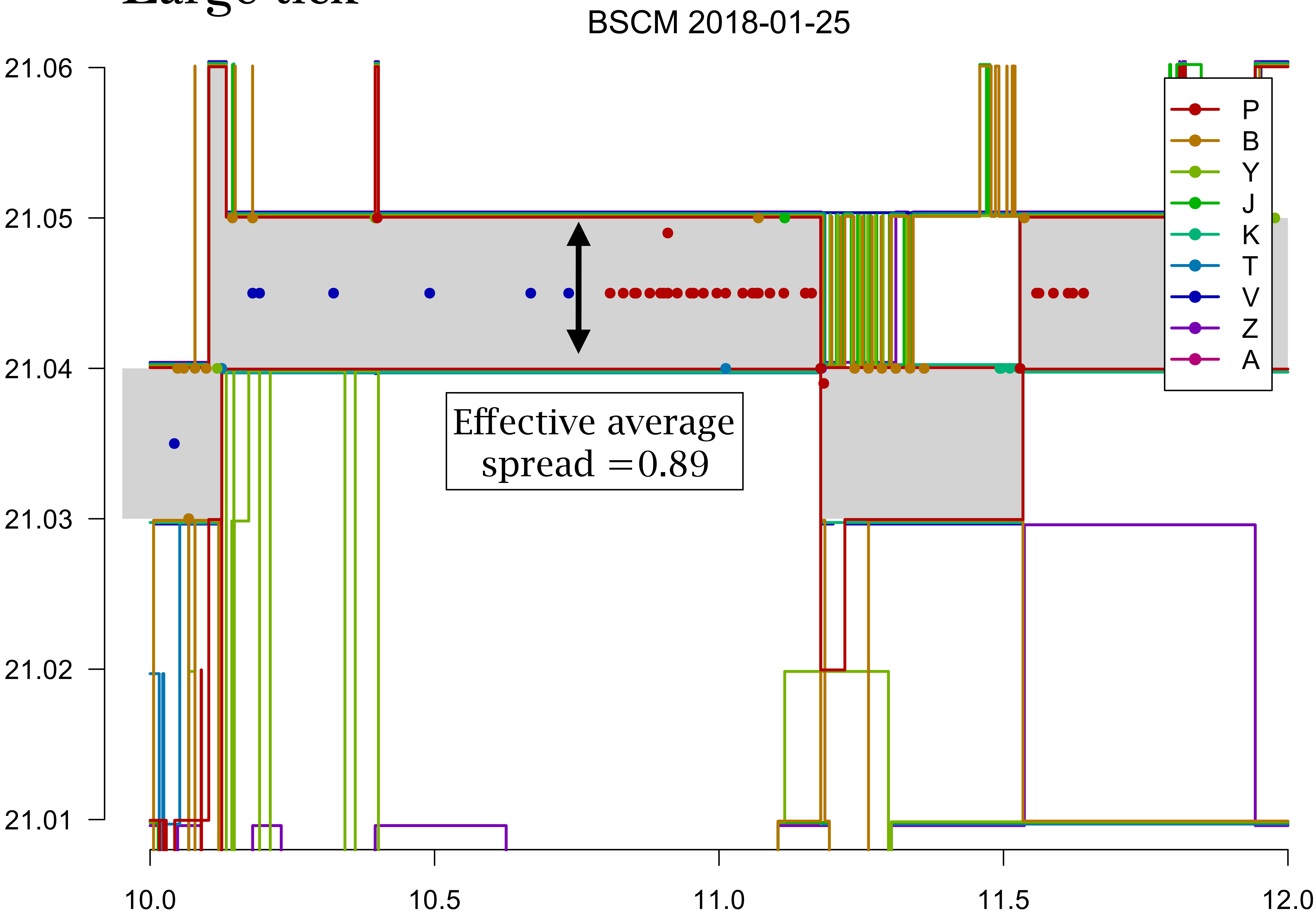
$$q_t \;=\; \phi\, q_{t-1} \;+\; \eta_t$$

Small tick

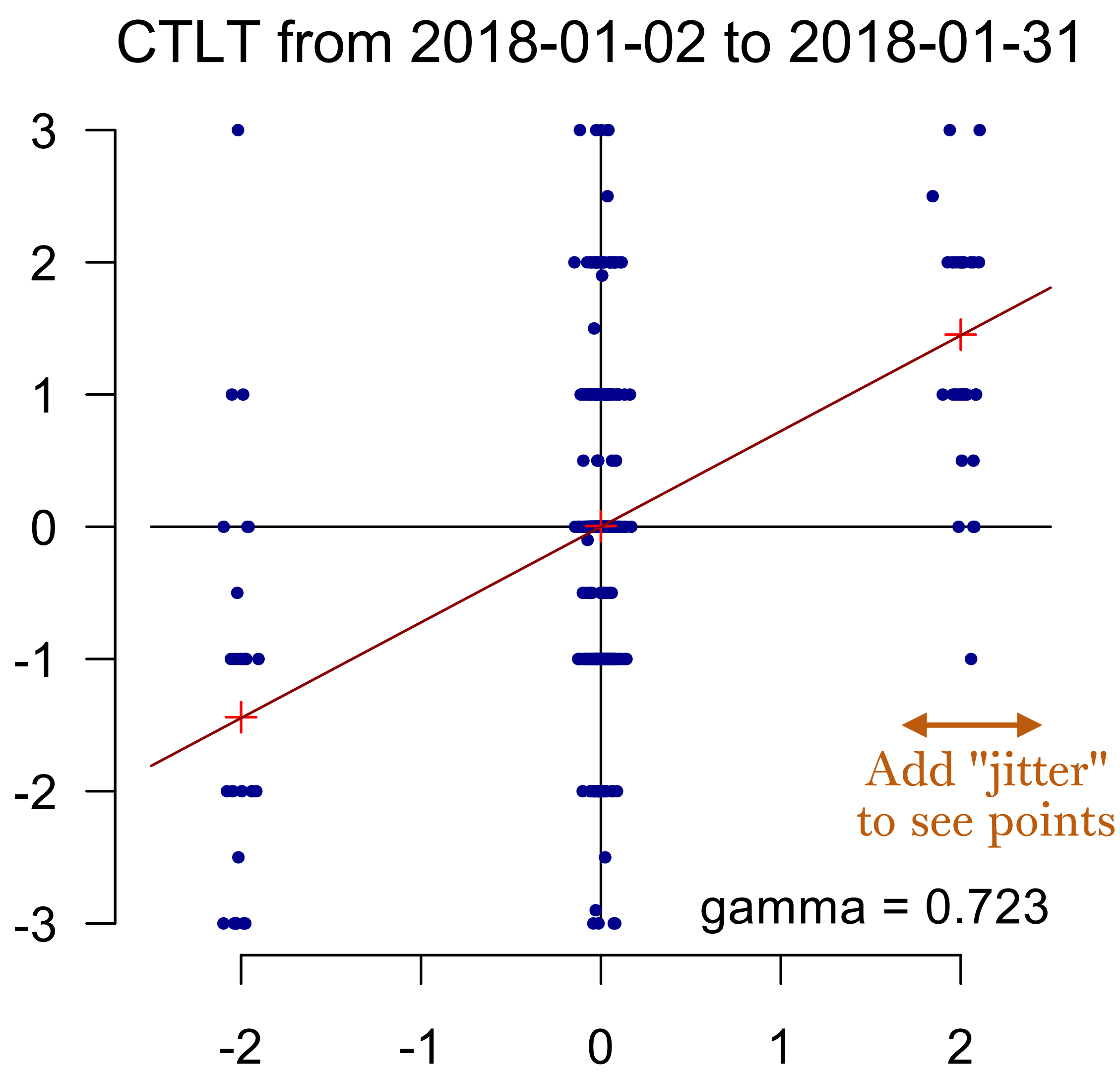


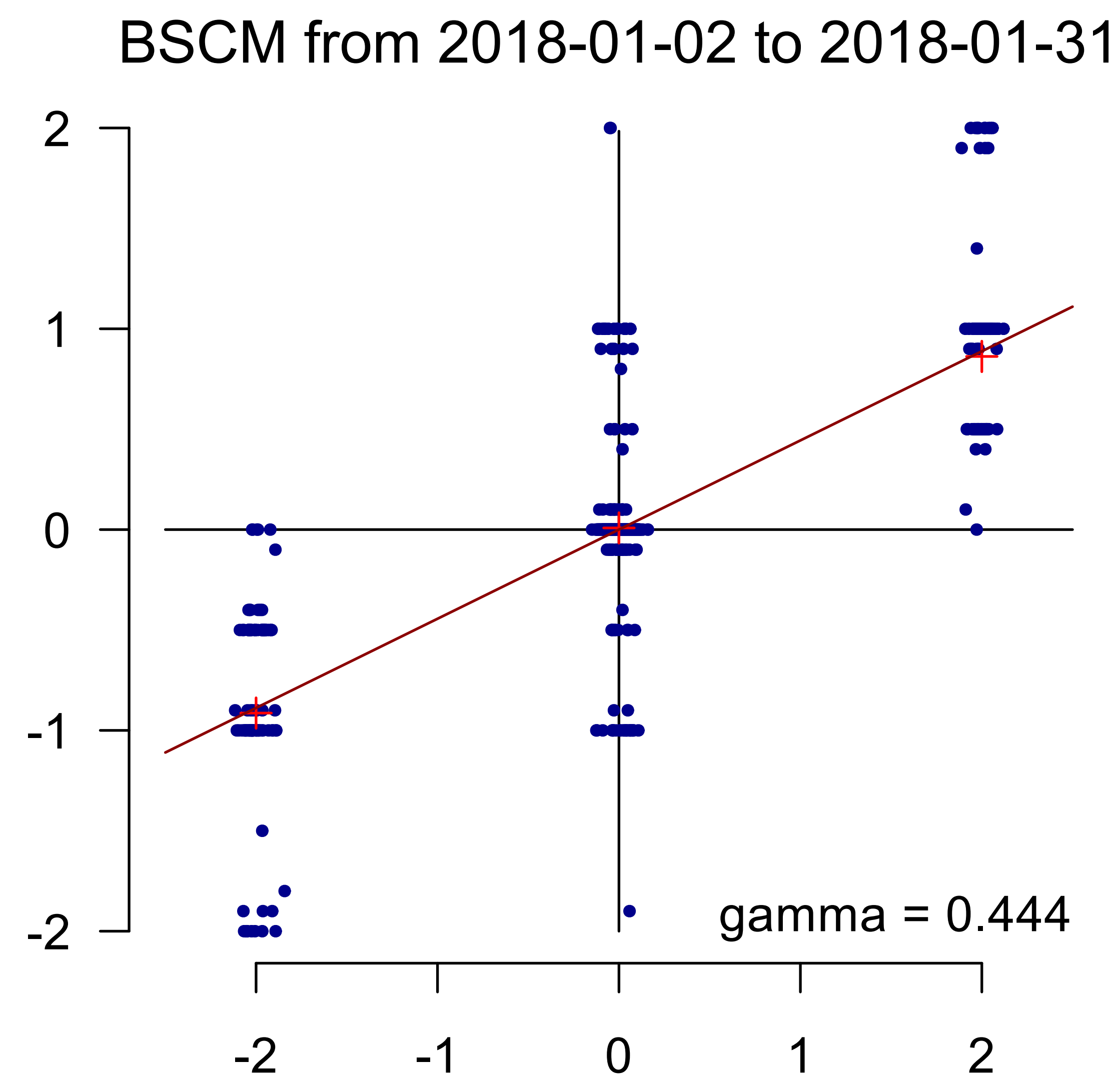
CTLT 2018-01-25
46.20
46.19
46.18
46.17
46.16
46.15
46.14
46.13
46.12
46.11
46.10
46.09
46.08
46.07
46.06
46.05
46.04
46.03
46.02
46.01
46.00
45.99
45.98
45.97
45.96
45.95
45.94
45.93
45.92
45.91
45.90
10.00
10.05
10.10
10.15
10.20
10.25
Effective average spread = 2.2
N
T
B
P
Z
K
Y
V
J
X
A

Large tick


BSCM 2018-01-25
21.06
21.05
21.04
21.03
21.02
21.01
10.0
10.5
11.0
11.5
12.0
P
B
Y
J
K
T
V
Z
A
Effective average spread $=0.89$

Roll regression





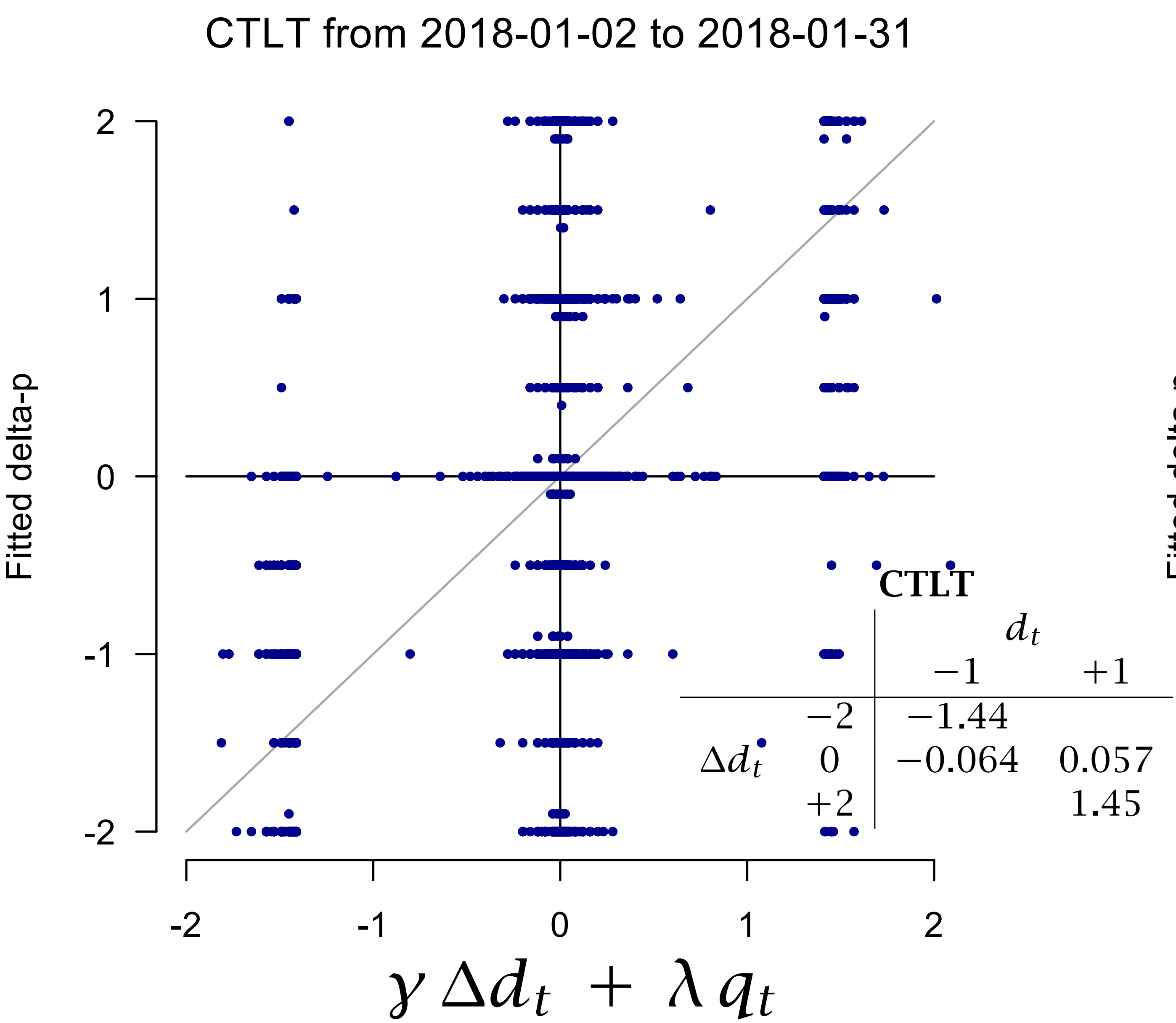
CTLT = small-tick

effective spread = 1.4¢

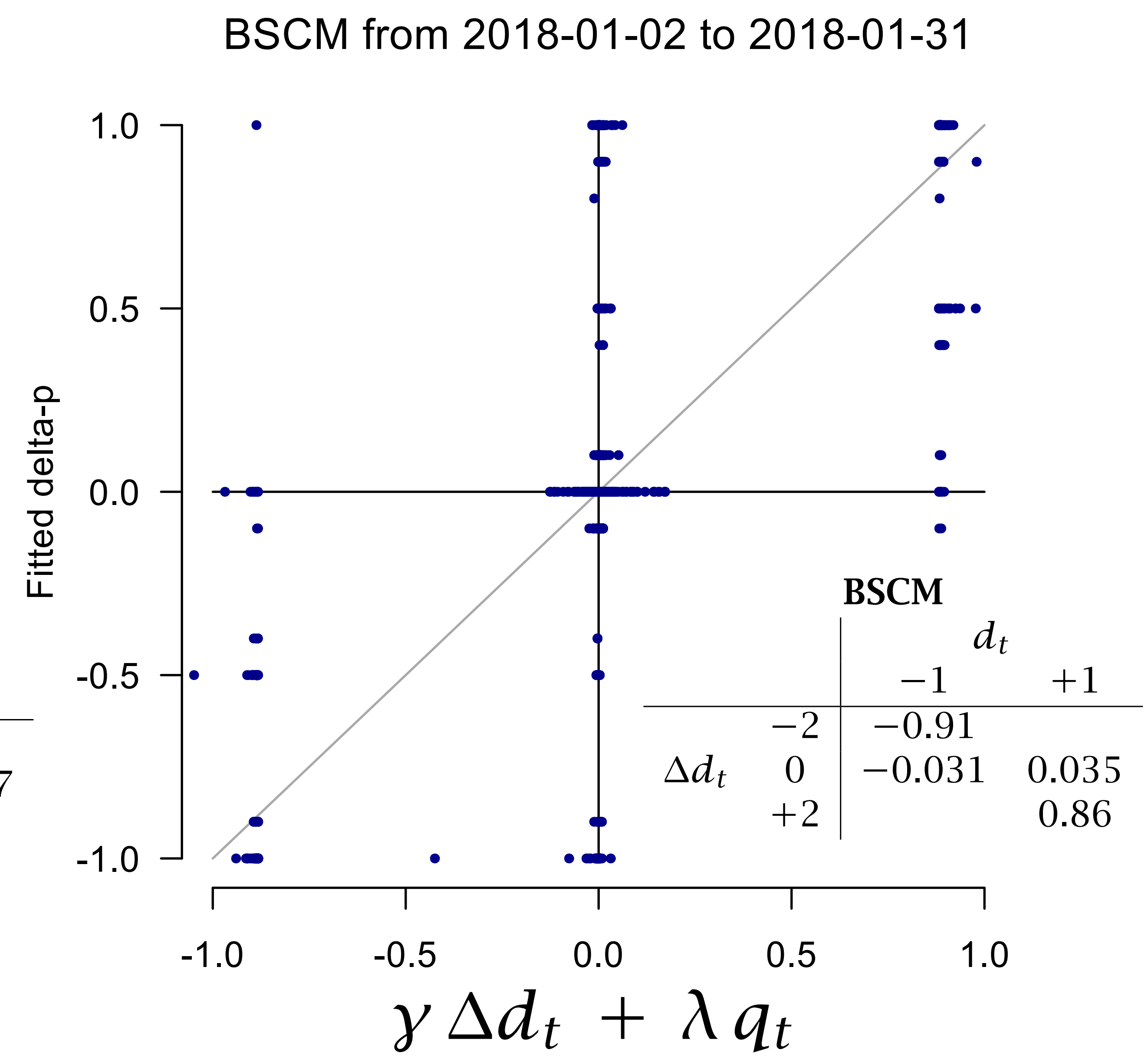
BSCM = large-tick

effective spread = 0.89¢

Glosten–Milgrom regression
Sign is more important than size



$\gamma = 0.705,\ \lambda = 0.0401$



$\gamma = 0.441,\ \lambda = 0.0029$

Large-tick vs small tick

GM regression

- CTLT: small-tick

  Υ = 0.705 cents

  λ = 0.04 cents per 100 lots

  Υ/λ = 17.6 * 100 lots

- BSCM: large-tick

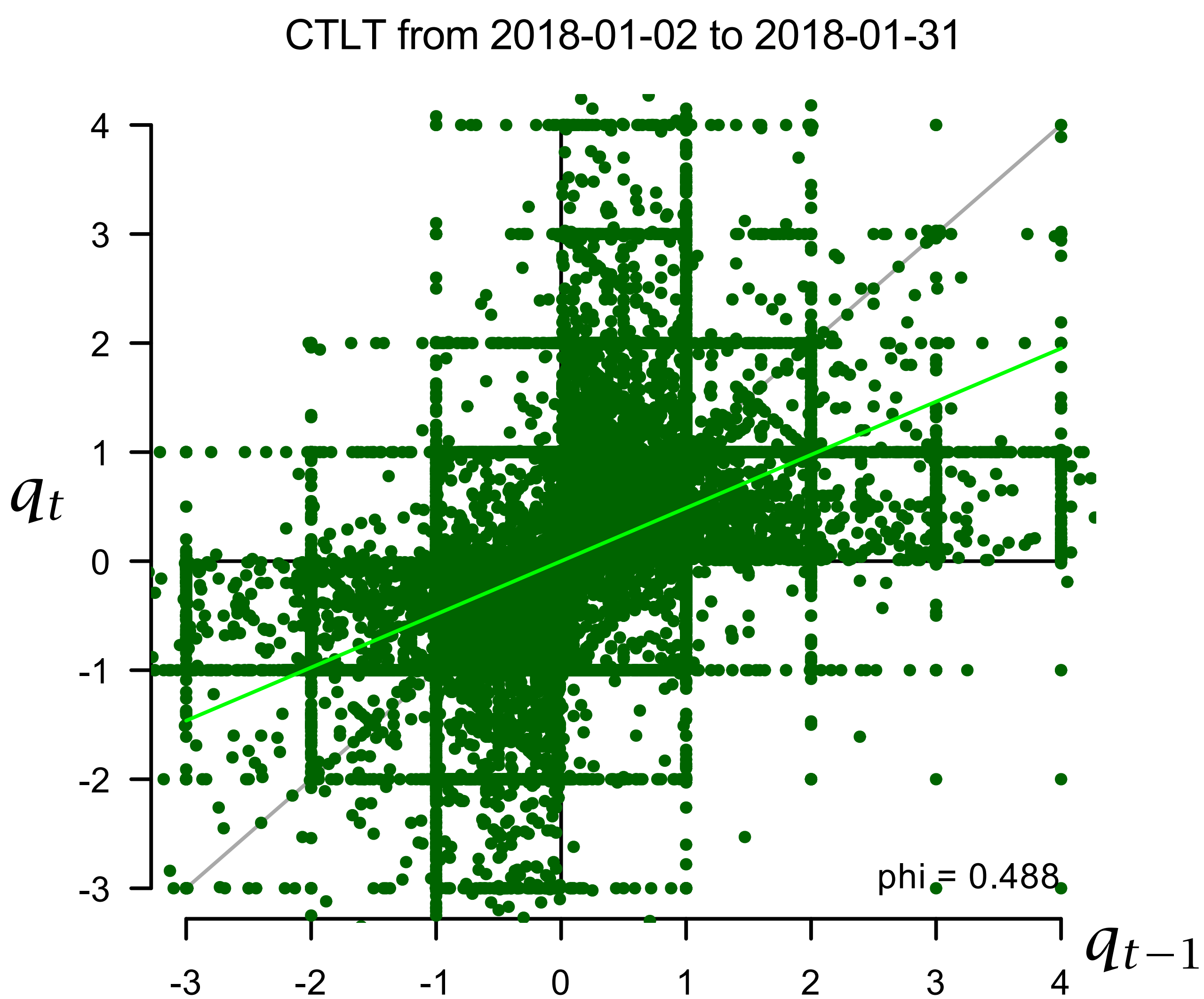
  Υ = 0.44 cents

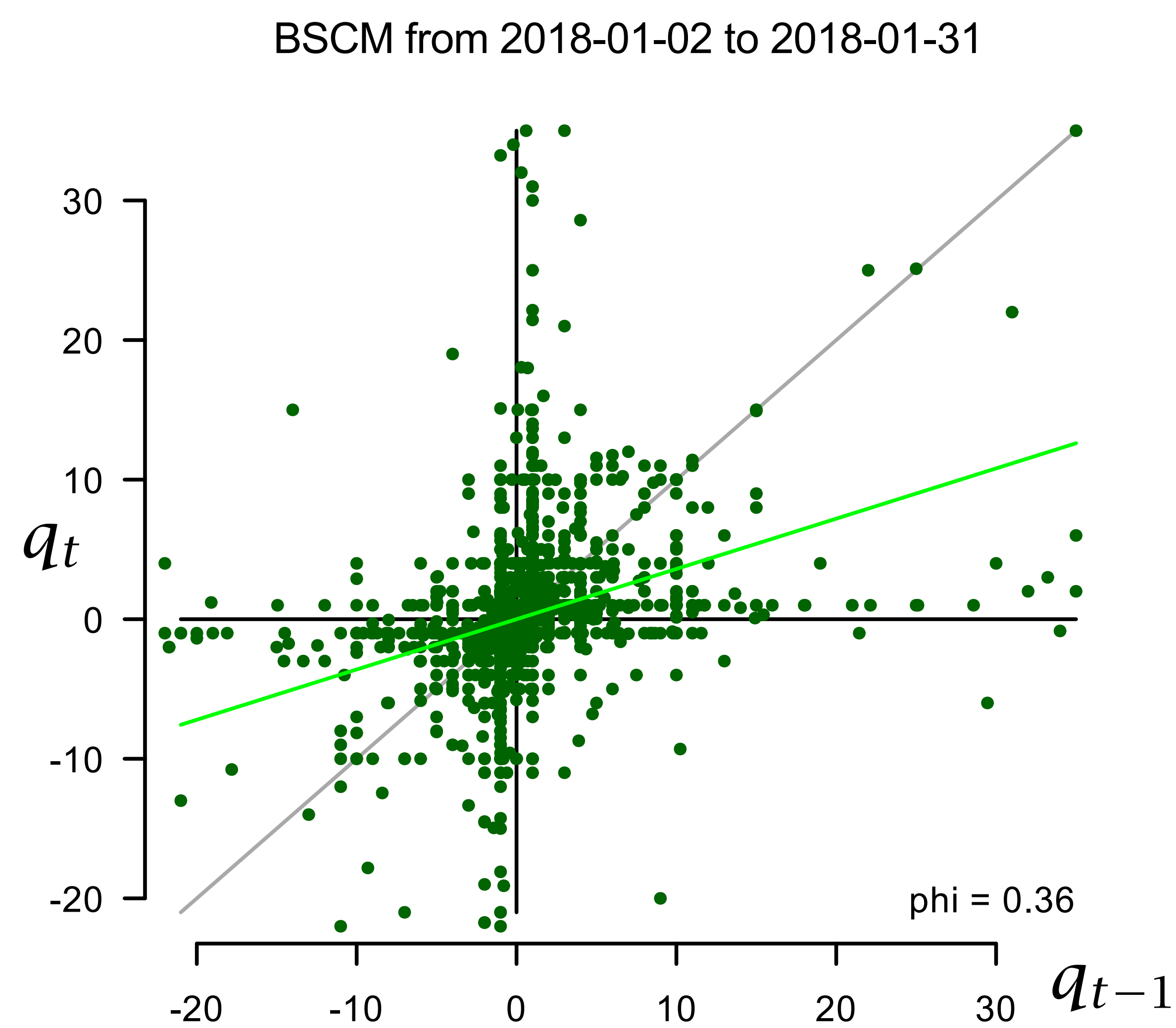
  λ = 0.0029 cents per 100 lots

  Υ/λ = 154 * 100 lots

Necessary trade size to exhaust liquidity is much larger for BSCM

Sequential size correlation for inventory model





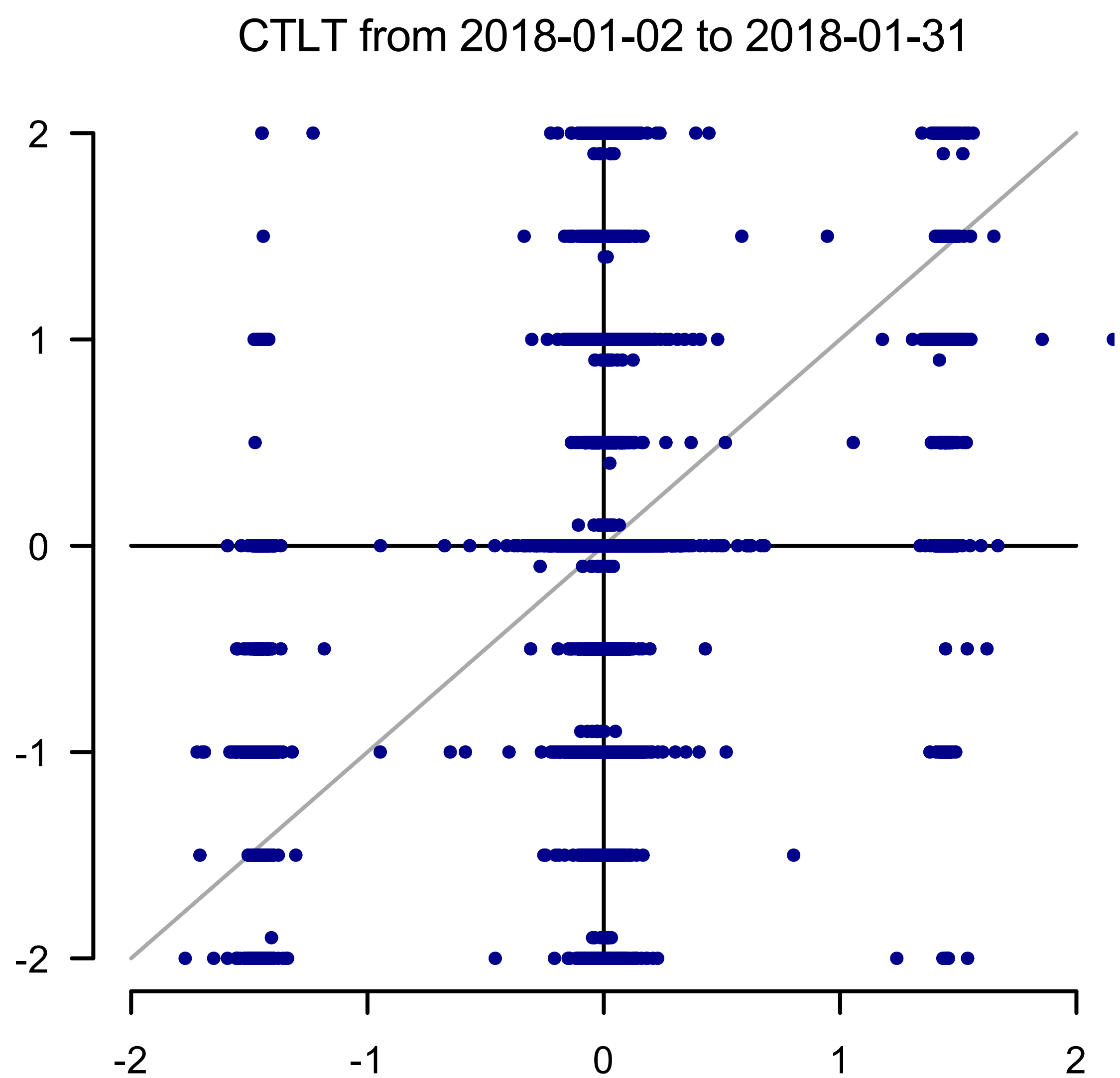
$\mathbf{\Phi}_0 = 0.86$ (sign only)

$\mathbf{\Phi} = 0.49$ (with size)

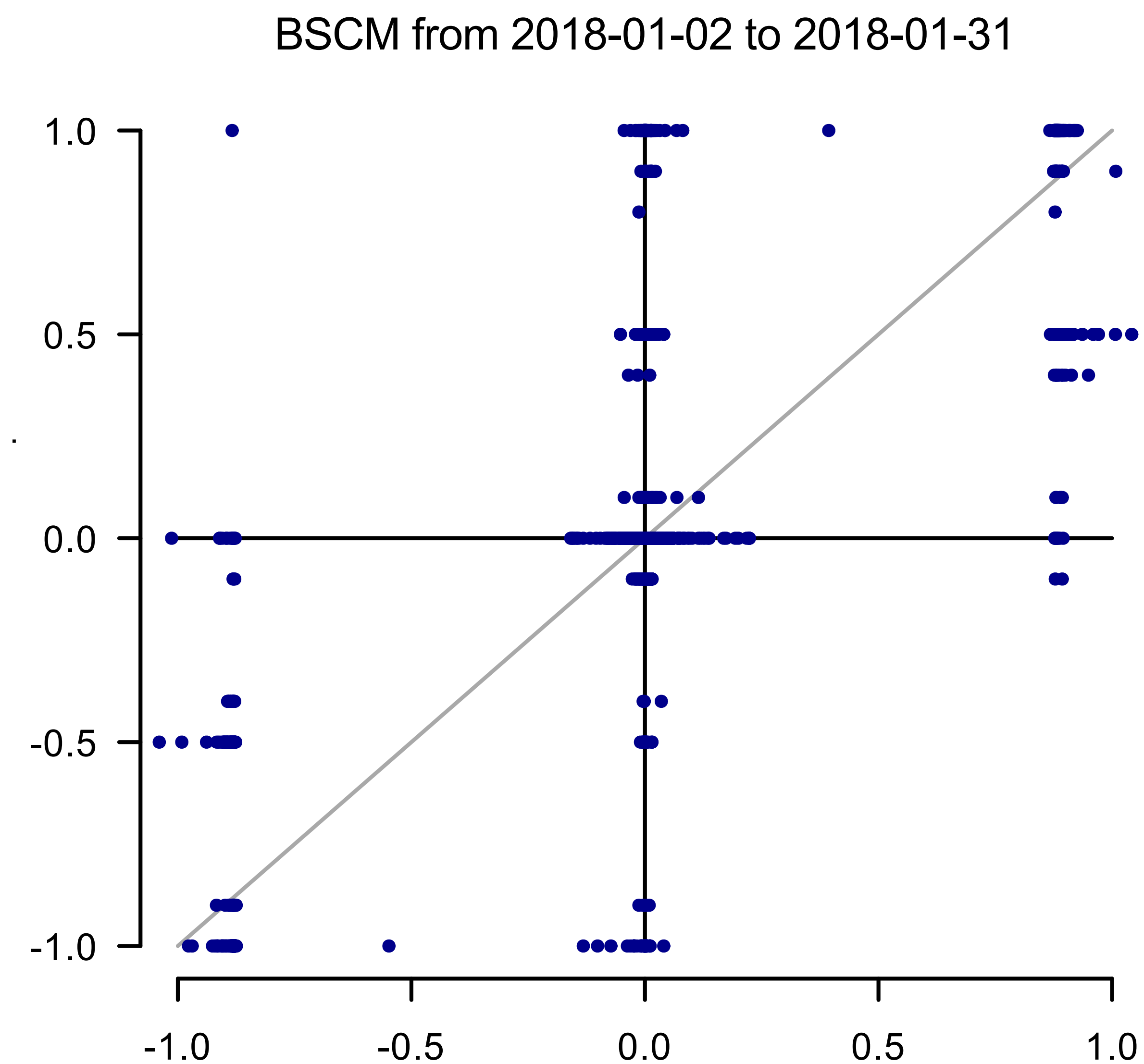
$\mathbf{\Phi}_0 = 0.64$ (sign only)

$\mathbf{\Phi} = 0.36$ (with size)

Inventory model



$$(\lambda + \beta) q_t \; - \; \lambda \phi \, q_{t-1} \; + \; \gamma \, \Delta d_t$$



$$(\lambda + \beta) q_t \; - \; \lambda \phi \, q_{t-1} \; + \; \gamma \, \Delta d_t$$

Inventory model (on size not just sisde)

- CTLT

  γ = 0.72 cents

  $\lambda + \beta$ = 0.0029

  $-\lambda\,\Phi$ = -0.020

  λ = -0.04 cents/100 lots

  β = 0.07 cents/100 lots

- BSCM

  γ = 0.44 cents

  $\lambda + \beta$ = 0.0039

  $-\lambda\,\Phi$ = -0.0027

  λ = 0.0075 cents/100 lots

  β = -0.0036 cents/100 lots

Critique of inventory model

- Depends on trade correlation
  unstable coefficient
  crude model (large tick have long dependence)
- Coefficient on $\boldsymbol{\lambda}+\boldsymbol{\beta}$ is very small (unstable)
- Suggests that market impact is smaller than spread
  discrete grid plays an essential role
- No useful insight for these markets
- Possibly in less public dealer markets

How to evaluate a model

- Draw picture, somehow
- Do coefficients make sense?
- Do you get some insight from results?
- Financial data has lots of noise

  R^2 always terrible

  p-values can be decent if lots of data

  error distribution is never normal

Conclusions from regression models

- Simple is better
- Inventory effects are not central in equities
  - may work better in dealer markets
- Decent results from Roll, G-M models
- Some insight into spread vs liquidity
  - different effects for large tick / small tick
  - intraday profiles