PRINCETON UNIVERSITY

ORF 474: High Frequency Trading
Spring 2020
Robert Almgren

Lecture 10a

April 13, 2020

Pairs trading

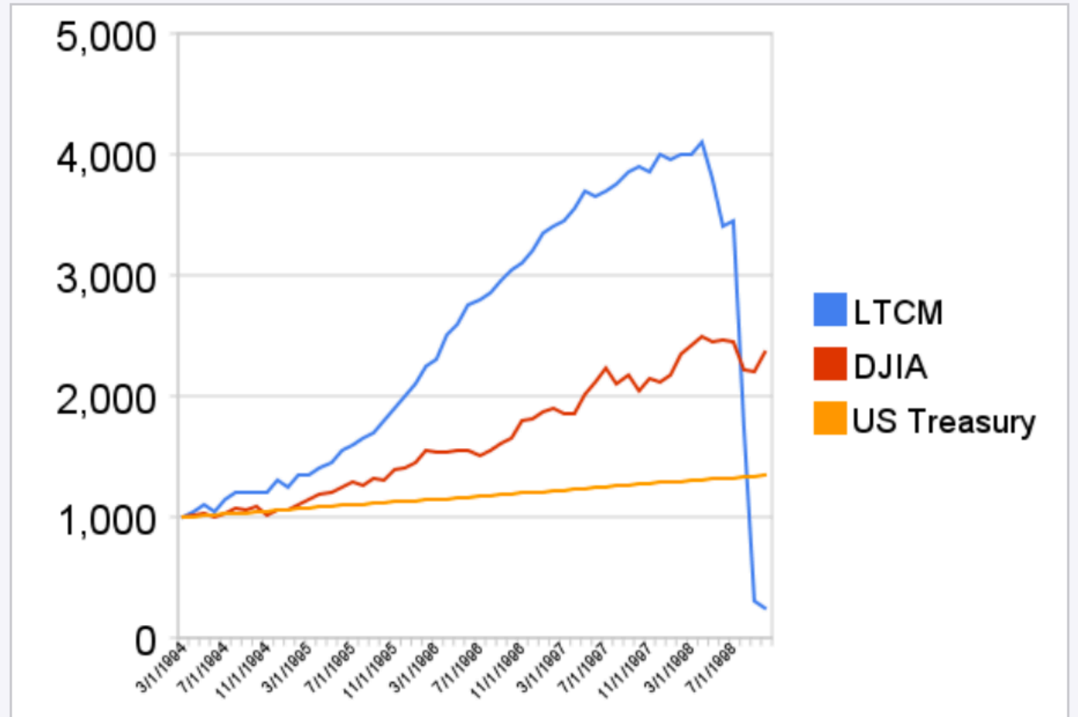
2 assets have linear relationship

- 2 stocks (BAC/JPM, UAL/DAL)
- 2 futures on related products (CL/HO)
- 2 futures on underlyings of different durations (ZF/ZN)
- 2 futures of different maturity on same underlying

- Should move together enough to revert
- Should move apart enough to give trade opportunities

Long-Term Capital Management

WIKIPEDIA
The Free Encyclopedia



The value of $1,000 invested in LTCM,[17] the Dow Jones Industrial Average and invested monthly in U.S. Treasuries at constant maturity.

Trading strategies [edit]

The core investment strategy of the company was then known as involving convergence trading: using quantitative models to exploit deviations from fair value in the relationships between liquid securities across nations and asset classes. In fixed income the company was involved in US Treasuries, Japanese Government Bonds, UK Gilts, Italian BTPs, and Latin American debt, although their activities were not confined to these markets or to government bonds.[13]

Bloomberg

How Long Term Capital Rocked Stocks, Too

It wasn't just the bond market that LTCM endangered

Leah Nathans Spiro
November 9, 1998, 12:00 AM EST

The hedge fund was deeply involved in three types of equity trading: "pairs" trading, risk arbitrage, and bets on overall market volatility. A major Wall Street firm says that LTCM's arbitrage positions in merger stocks alone, called risk arbitrage, reached $6.5 billion, and LTCM's positions in individual takeover stocks were 5 to 10 times as large as this Wall Street firm's own arbitrage positions. Some pairs-trading positions were even larger.

Of most concern was a massive $2.3 billion position in Royal Dutch Petroleum and Shell Transport, two closely related stocks. This position is called a pairs trade, or an arbitrage between two stocks, often in the same industry, that usually move closely together but sometimes diverge. Shell Transport owns 40% of Royal Dutch/Shell Group, while Royal Dutch Petroleum owns 60% of Royal Dutch/Shell Group. Both get their income from dividends from Royal Dutch/Shell Group.

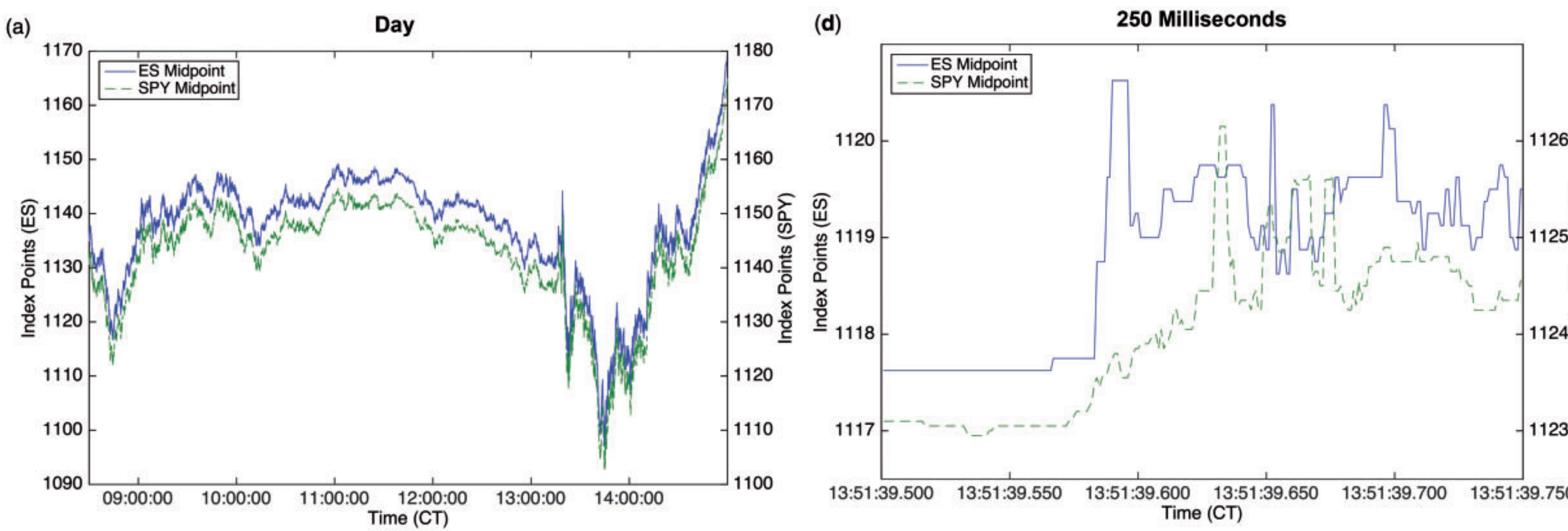
THE
QUARTERLY JOURNAL
OF ECONOMICS

Vol. 130 November 2015 Issue 4

THE HIGH-FREQUENCY TRADING ARMS RACE: FREQUENT BATCH AUCTIONS AS A MARKET DESIGN RESPONSE*

ERIC BUDISH
PETER CRAMTON
JOHN SHIM

This figure illustrates the time series of the E-mini S&P 500 future (ES) and SPDR S&P 500 ETF (SPY) bid-ask midpoints over the course of a trading day (August 9, 2011) at different time resolutions: the full day (a), an hour (b), a minute (c), and 250 milliseconds (d). SPY prices are multiplied by 10 to reflect that SPY tracks $\frac{1}{10}$ the S&P 500 Index. Note that there is a difference in levels between the two financial instruments due to differences in cost-of-carry, dividend exposure, and ETF tracking error; for details see Section V.B. For details regarding the data, see Section IV.



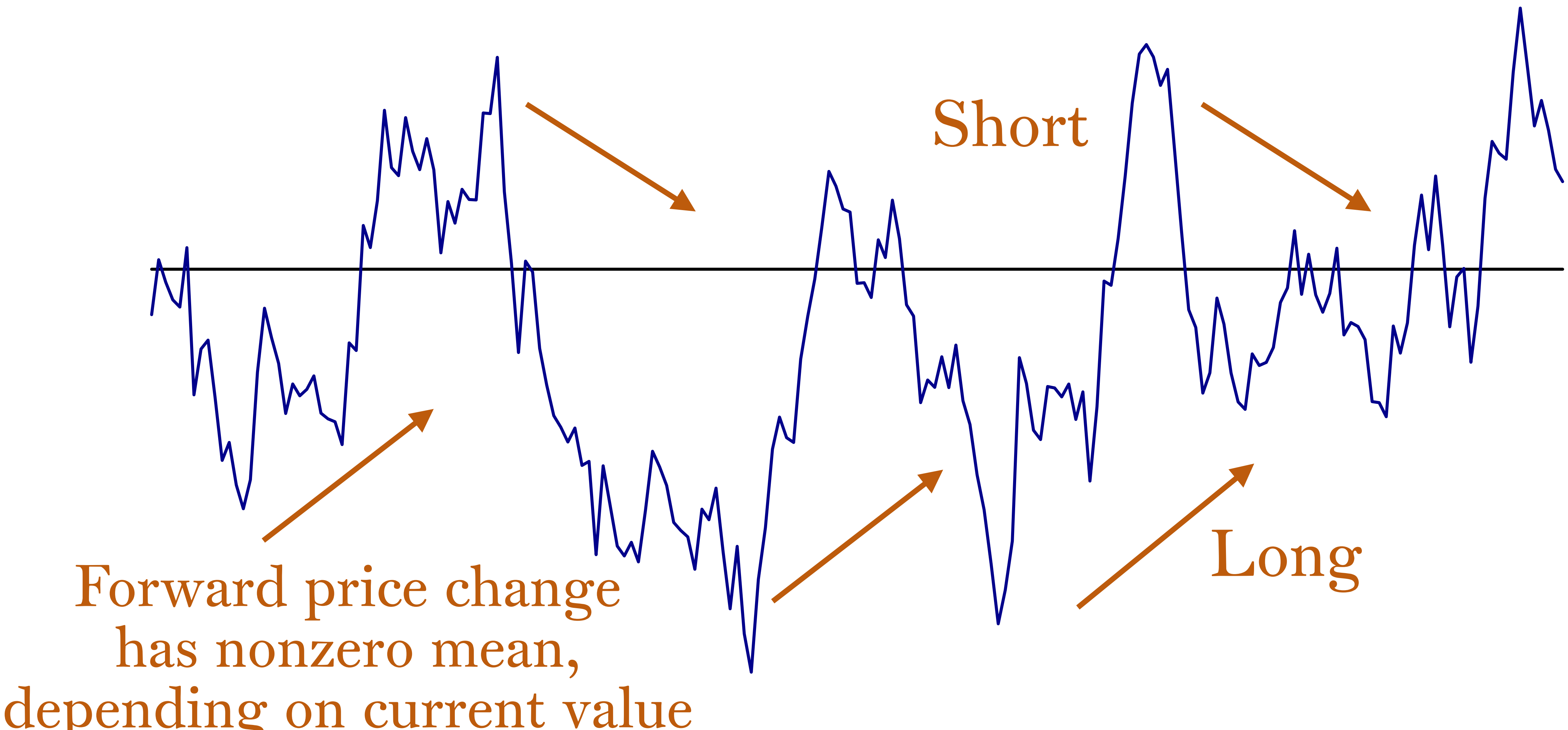
Main idea:
Draw line through past prices
regression or principal components
Assume that line is stable in future
deviations will revert to historical line

Problem: how to draw line

- Use fixed price offset (usually with arbitrage)
  - stock index futures / ETF
  - dual listed company
- Use relationship from previous day
- Use dynamic relationship updating intraday
  - How to do rolling regression?

General mean reversion

- Identify mean-reverting tradeable process
- Trade to take advantage of reversion



Can be more systematic about trading strategy

Journal of Economic Dynamics & Control 37 (2013) 1972–1981

Contents lists available at SciVerse ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Dynamic pairs trading using the stochastic control approach

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ABSTRACT

We propose a model for analyzing dynamic pairs trading strategies using the stochastic control approach. The model is explored in an optimal portfolio setting, where the portfolio consists of a bank account and two co-integrated stocks and the objective is to maximize for a fixed time horizon, the expected terminal utility of wealth. For the exponential utility function, we reduce the problem to a linear parabolic partial differential equation which can be solved in closed form. In particular, we exhibit the optimal positions in the two stocks.

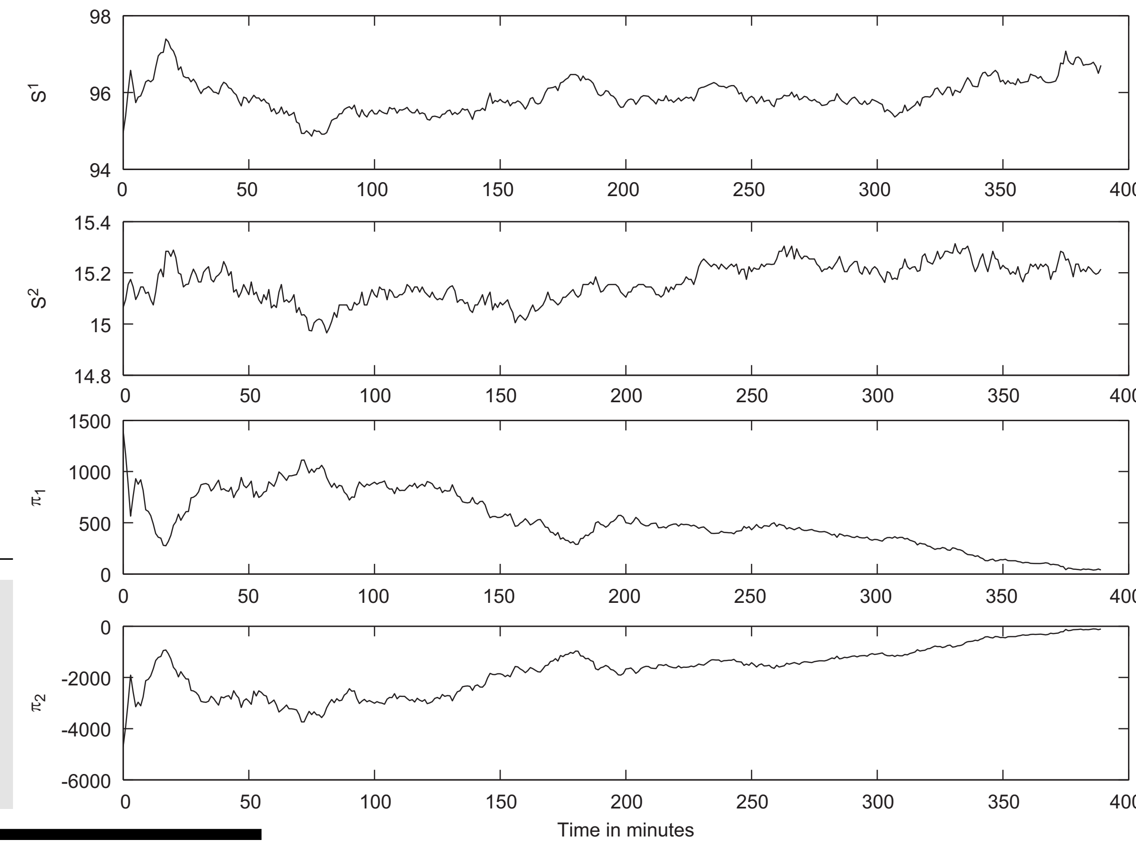
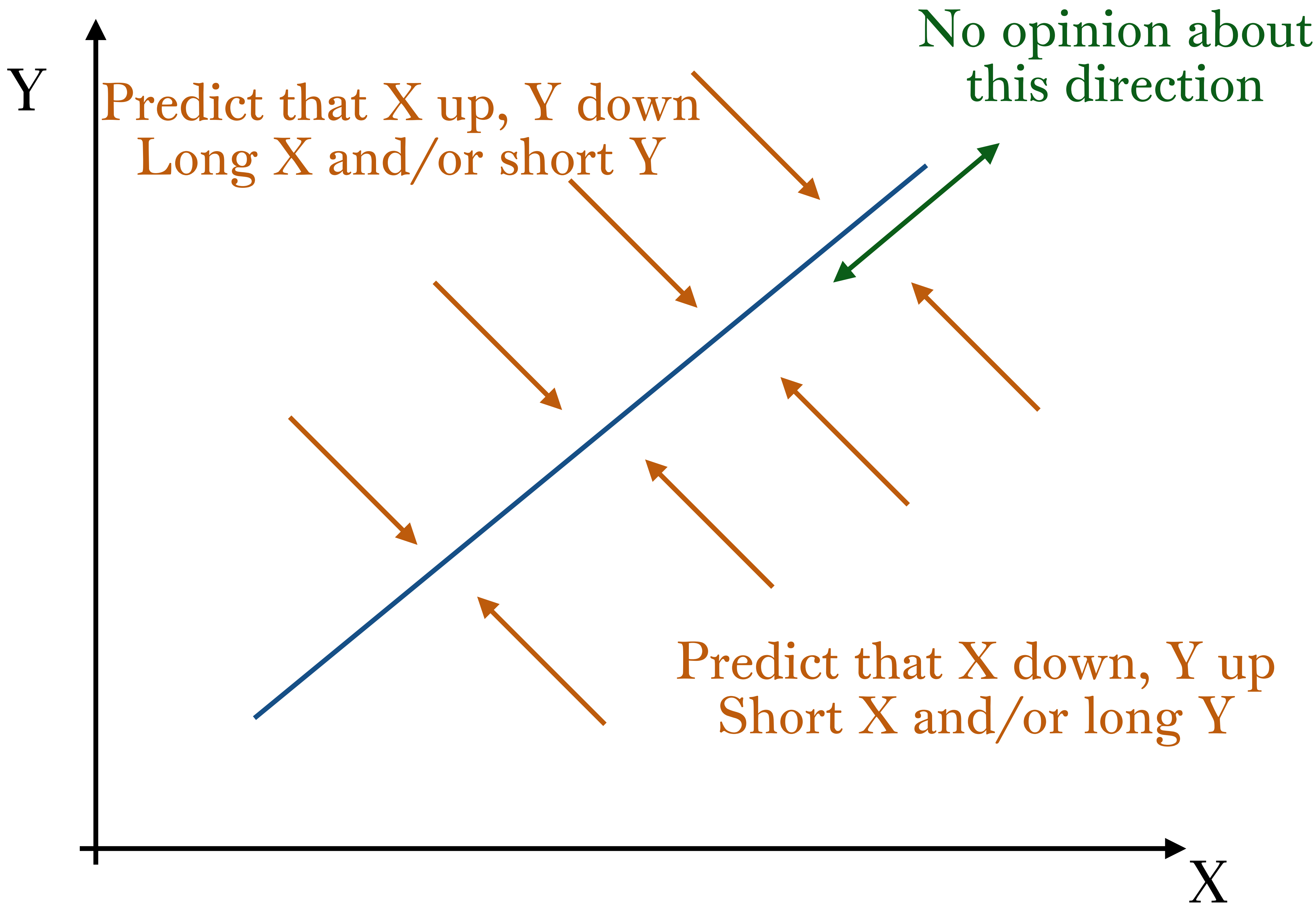


Fig. 1. Stocks and optimal policies.

Cointegration relationship with two assets



Application areas

- Assets that should have some relationship:
  - Treasury futures of different durations
  - Eurodollar futures (LIBOR at different dates)
  - Crude Oil vs Heating Oil or Gasoline
  - Commodities for different delivery dates
- Can be more than 2 dimensions total
  - Random walk directions: no opinion on change
    - interest rates go up or down
    - yield curve can tilt
  - Reverting directions: all other directions

Execution Strategies in Fixed Income Markets

Robert Almgren*

April 2, 2013

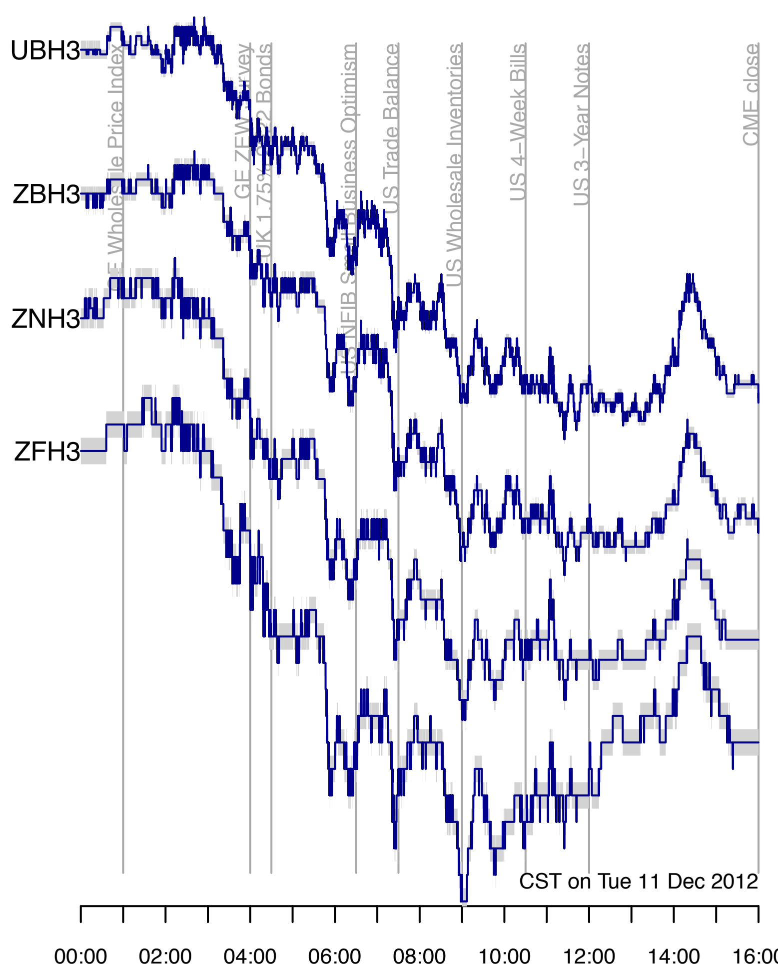
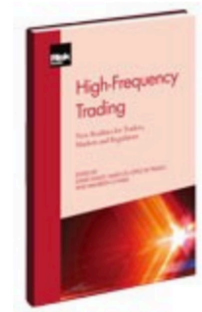


Figure 4: From bottom to top, the CME 5-year Treasury futures (ZF), 10-year (ZN), 30-year (ZB), and Ultra (UB), expiring in March 2013 (H3), from midnight to market close at 4 PM Chicago time, on Dec. 11, 2012. Price scale is arbitrary, to show relationships. Gray band is bid-ask spread; line is bid-ask midpoint. These contracts move very closely together; trading in ignorance of the relationships between them would give poor performance.



Risk books

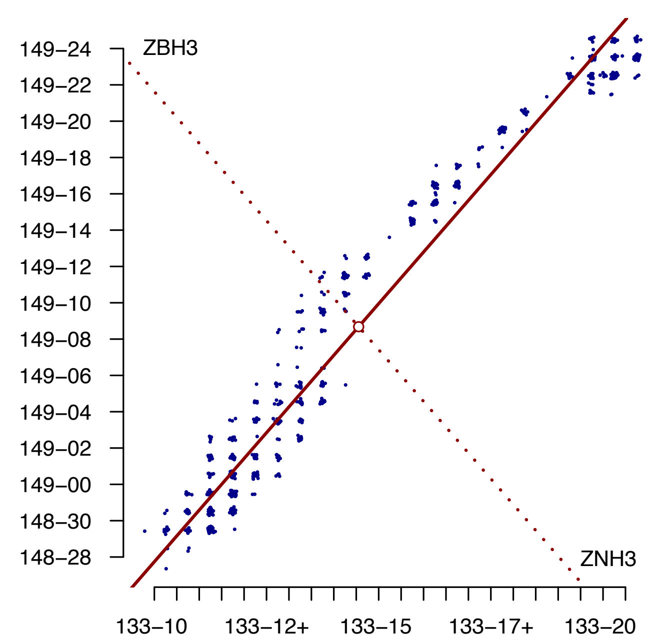
High-Frequency Trading - New Realities for Traders, Markets and Regulators

Edited By David Easley, Marcos López de Prado and Maureen O'Hara

1 Review(s) | Add Your Review

Overview

This is *the* survival guide for trading in a world where high-frequency trading predominates in markets, accounting for upwards of 60% of trading in equities and futures, and 40% in foreign exchange. High-frequency trading is the subject of extensive debate, particularly as to whether it is beneficial for traders and markets or instead allows some traders to benefit at others expense. This book provides you with an important overview and perspective on this area, with a particular focus on how low-frequency traders and asset managers can survive in the high frequency world.



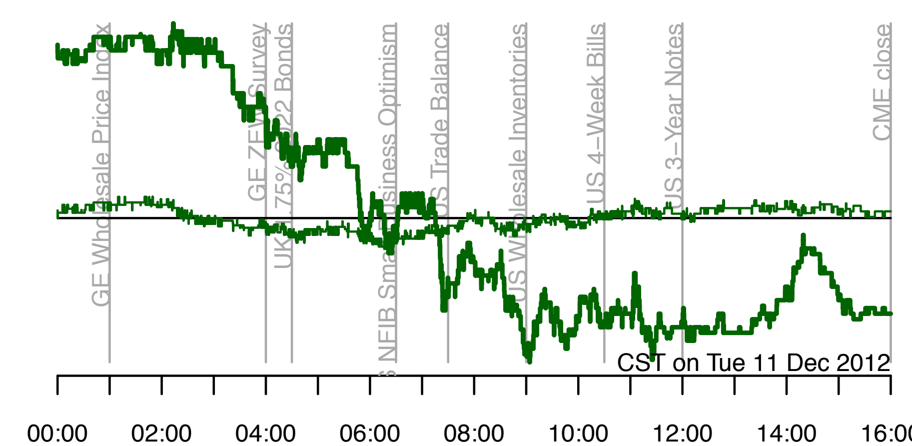
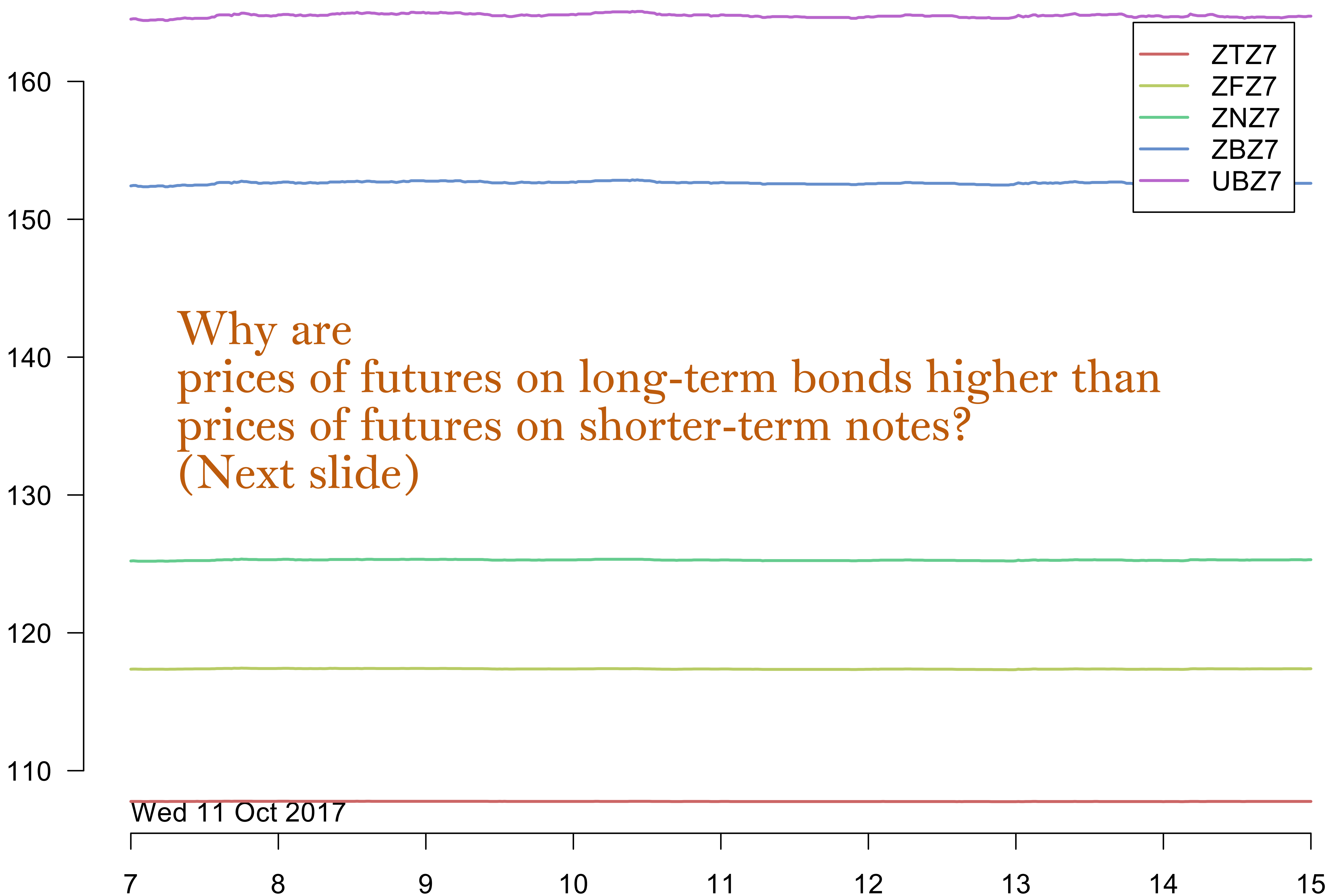


Figure 6: Projection of the price series in Figure 5 onto the principal axes of the correlation matrix. Vertical scale is arbitrary but is identical for the two components. The thick line is the projection onto the primary axis (solid line in Figure 5), reflecting the overall market motion (compare Figure 4) and is largely unpredictable. The thin line is the projection onto the secondary axis (dotted line in Figure 5) which shows mean reversion and is useful for prediction.

Treasury products



Why are
prices of futures on long-term bonds higher than
prices of futures on shorter-term notes?
(Next slide)

Why are prices of futures on long-term bonds higher than prices of futures on shorter-term notes?

Answer: Coupon payments

- Note = maturity 10 yrs or less
  Bond = maturity more than 10 years
- Futures price is very close to underlying price
- Note and bonds pay coupons:
  semi-annual payments to holder of instrument
- Coupon rate is (recently) generally higher than interest rate
- Long-term instrument has more time to collect payments

Issued notes and bonds

TreasuryDirect

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Home › Institutional › Announcements, Data & Results › Auction Query

Average yield
= effective interest rate

<

Interest rate
= fixed coupon payment

Financial Institutions

Treasury Marketable Securities

Announcements, Data & Results

- Announcement and Results Press Releases
- Record Setting Auction Data
- Auction Query
- TIPS/CPI Data
- FRN Daily Indexes
- Treasury Buybacks

Auction Query

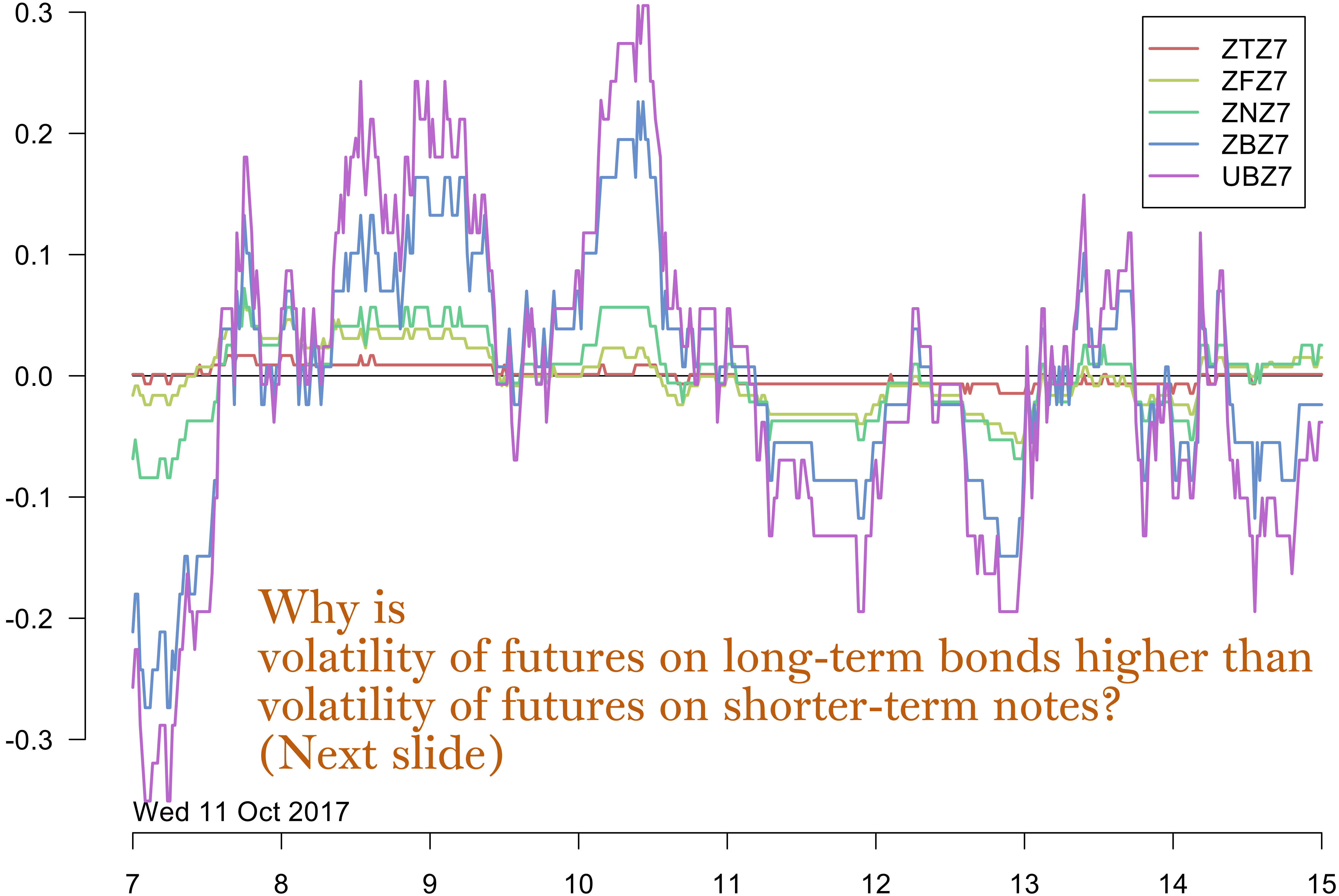
Download as: CSV | JSON | TSV | XML | HELP

Securi.	Security Term	Auction Dat	Maturity D...	Price per $...	Average / M...	Interest Rate
Select						
Bond	29-Year 10-Month	04/08/2020	02/15/2050	116.588638	1.270%	2.000%
Note	9-Year 10-Month	04/07/2020	02/15/2030	106.783664	0.724%	1.500%
Note	3-Year	04/06/2020	04/15/2023	99.707782	0.318%	0.250%
Note	7-Year	03/26/2020	03/31/2027	99.624642	0.620%	0.625%
Note	5-Year	03/25/2020	03/31/2025	99.827547	0.480%	0.500%
Note	1-Year 10-Month	03/25/2020	01/31/2022	99.876498		
Note	2-Year	03/24/2020	03/31/2022	99.954228	0.350%	0.375%
Note	9-Year 10-Month	03/19/2020	01/15/2030	94.992914	0.499%	0.125%
Bond	29-Year 11-Month	03/12/2020	02/15/2050	116.761508	1.220%	2.000%
Note	9-Year 11-Month	03/11/2020	02/15/2030	106.179092	0.750%	1.500%
Note	3-Year	03/10/2020	03/15/2023	99.813012	0.490%	0.500%
Note	7-Year	02/27/2020	02/28/2027	99.185208	1.189%	1.125%
Note	5-Year	02/26/2020	02/28/2025	99.878957	1.102%	1.125%

Go to page: 1 Show rows: 100 1-100 of 2087

https://www.treasurydirect.gov/instit/annceresult/annceresult_query.htm

Shift to all have mean zero



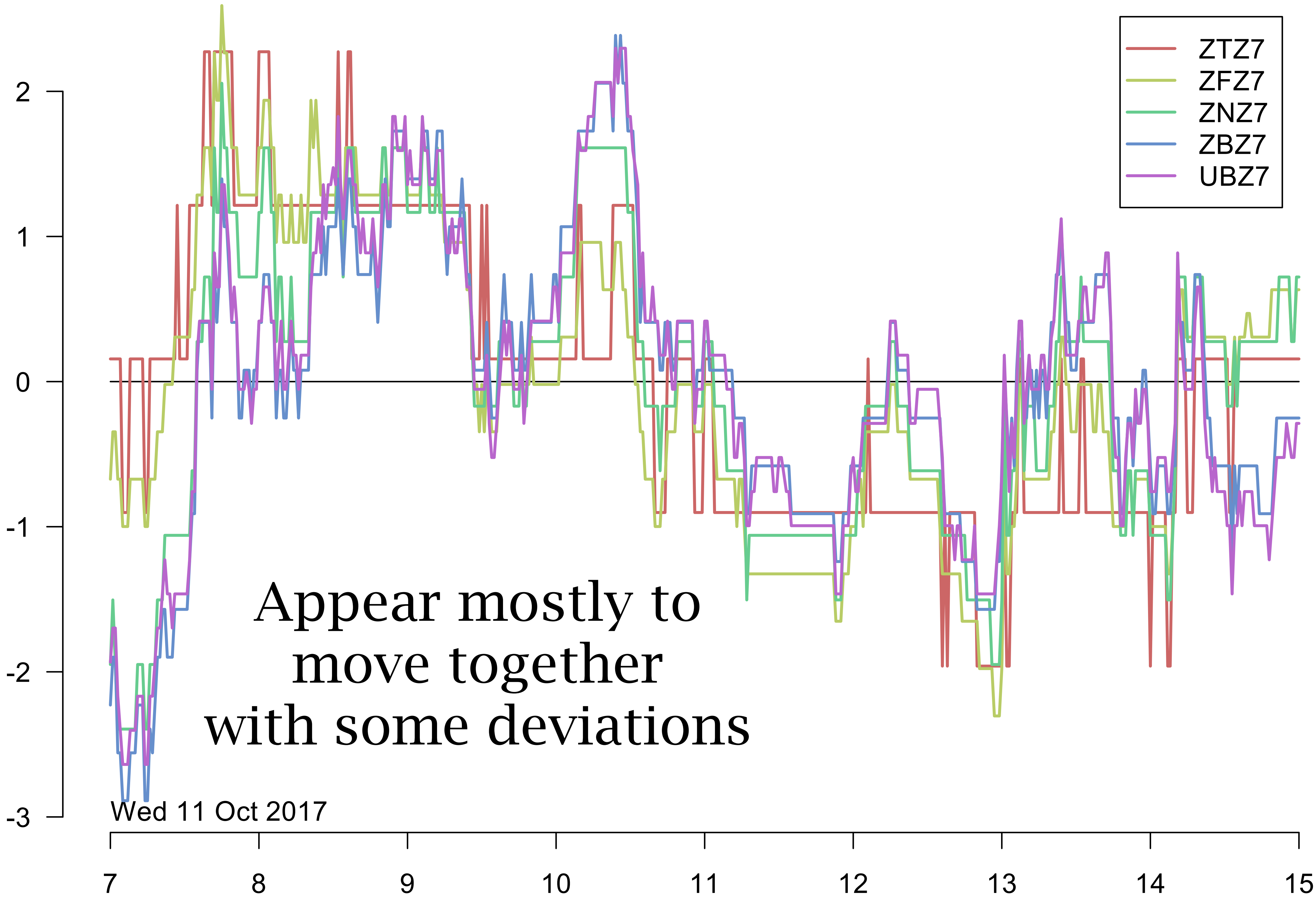
Why is
volatility of futures on long-term bonds higher than
volatility of futures on shorter-term notes?
(Next slide)

Why is volatility of futures on long-term bonds higher than volatility of futures on shorter-term notes?

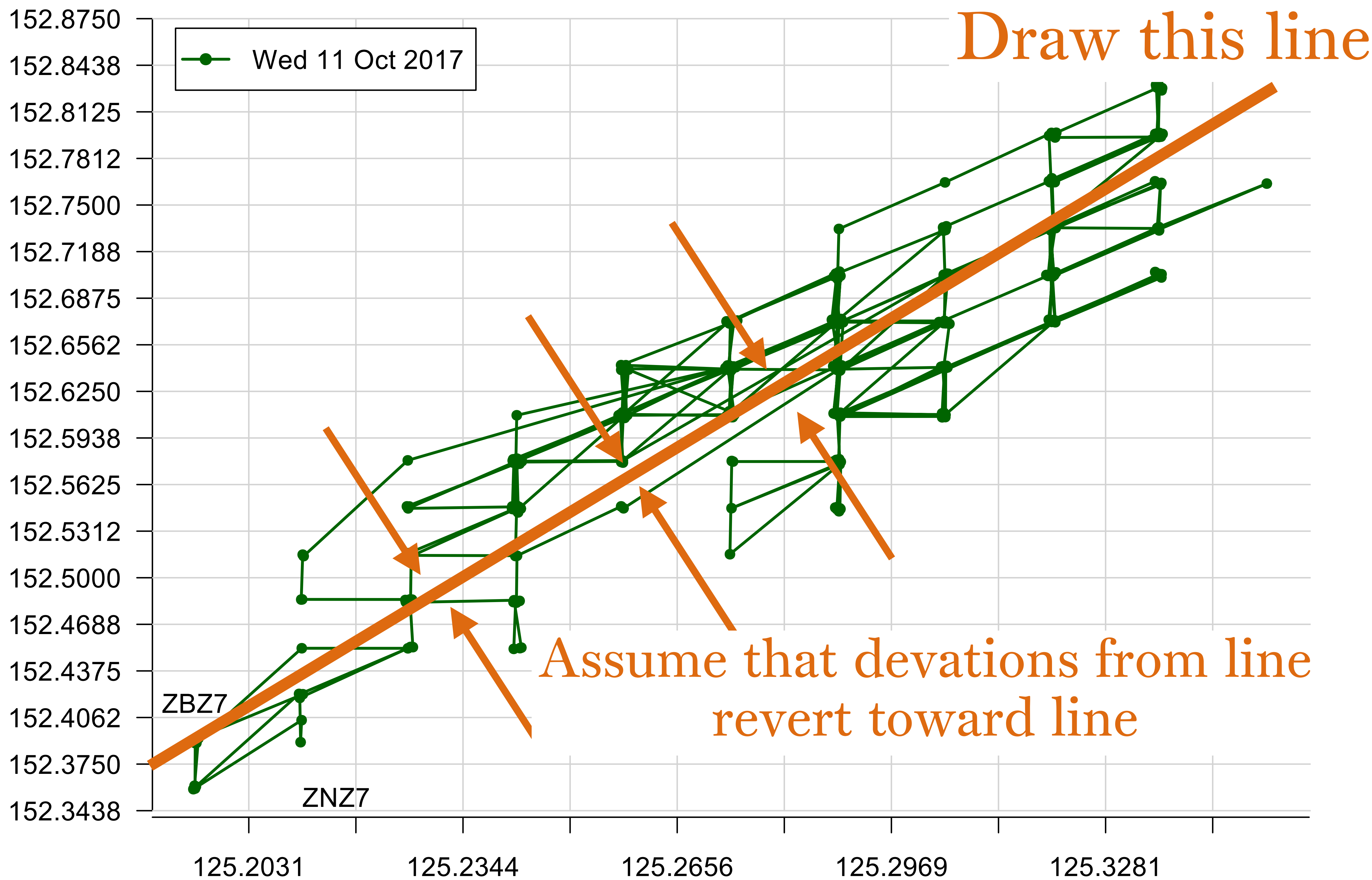
Answer: Duration

- Fixed payment $P(T)$ at future time T
- Suppose no intermediate coupon payments
- Constant interest rate r
- Current price $P(0) = P(T) \exp(-r T)$
- Duration $D = d \log(P(0)/P(T)) / d r$
- For zero-coupon instrument, $D = T$
- In general, D measures "effective time horizon" = sensitivity to underlying rate
- Longer duration has more time to pay interest
- Volatility of product = volatility of rate $\times$ duration

Shift and scale to all have mean zero and variance 1



Appear mostly to move together with some deviations


Draw this line
Wed 11 Oct 2017
152.8750
152.8438
152.8125
152.7812
152.7500
152.7188
152.6875
152.6562
152.6250
152.5938
152.5625
152.5312
152.5000
152.4688
152.4375
152.4062
152.3750
152.3438
ZBZ7
ZNZ7
125.2031
125.2344
125.2656
125.2969
125.3281
Assume that devations from line revert toward line

Forecasting algorithm

- Historical data points x_j, y_j
- Compute a rolling mean xmean, ymean
  exponential averaging in time of data points
- Compute correlation matrix of data points around mean
  exponential averaging in time of deviations
- Take principal components of correlation matrix
- Decide which components are random walk, which are mean reverting
- Forecast value is projection of reverting components

Principal component analysis

$$(x_j, y_j) \text{ for } j = 1, \ldots, n$$

Shift and scale: $\hat{x}_j = \dfrac{x_j - \bar{x}}{\sigma_x} \qquad \hat{y}_j = \dfrac{y_j - \bar{y}}{\sigma_y}$

$$\hat{\Sigma} = \begin{pmatrix} \langle \hat{x}^2 \rangle & \langle \hat{x}\hat{y} \rangle \\ \langle \hat{x}\hat{y} \rangle & \langle \hat{y}^2 \rangle \end{pmatrix} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

Principal vectors of $\hat{\Sigma}$ are $V = \begin{pmatrix} v_1 & v_2 \end{pmatrix}$

SVD computation

Eigenvalues of $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ are $\lambda = 1 \pm \rho$

Eigenvectors are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Matrices larger than 2x2:
get eigenvector for largest eigenvalue
very quickly by power method
(good for continuous updates)

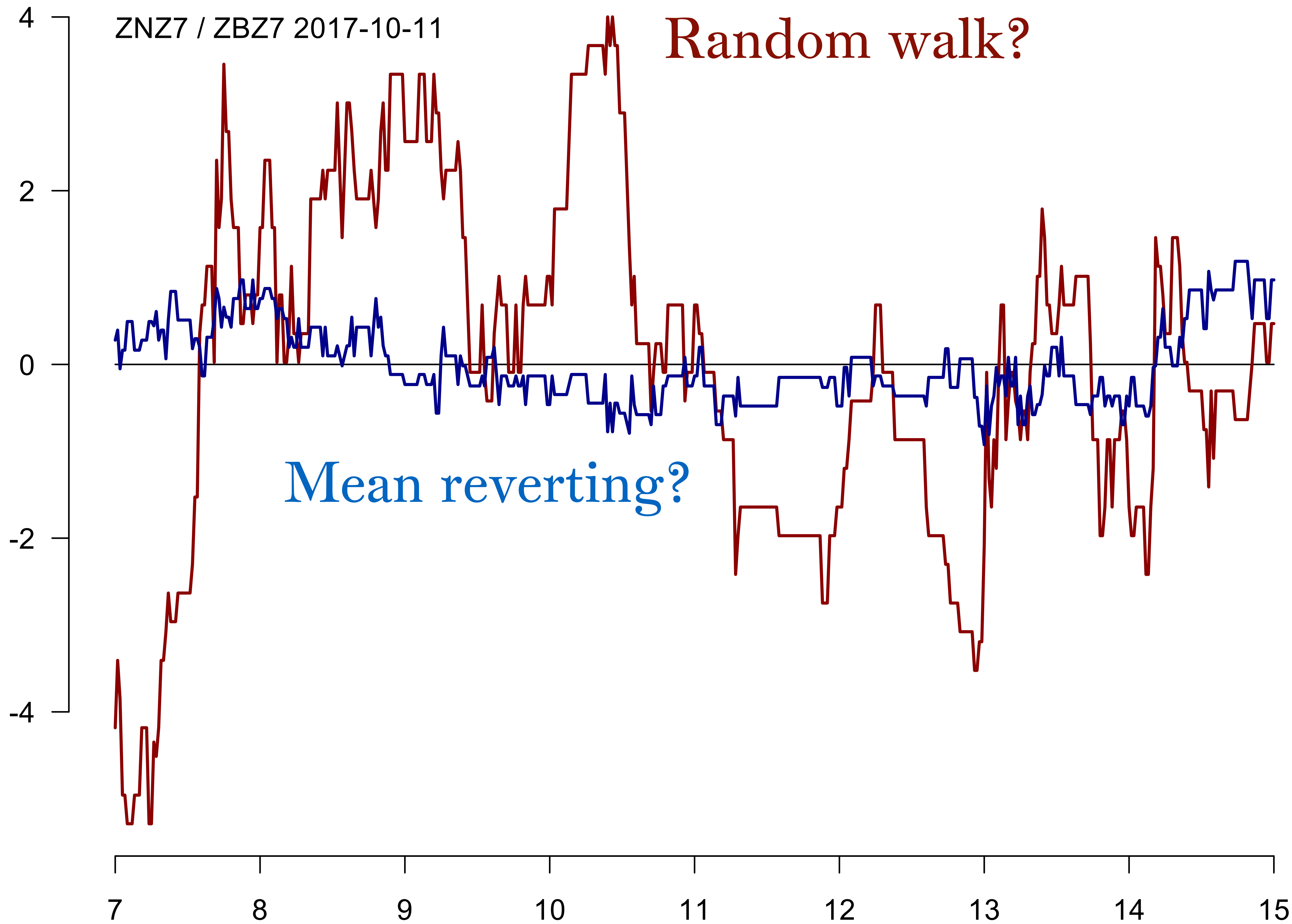
Coordinate transformation

$$(x, y) \longleftrightarrow (\xi, \eta)$$

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = V' \begin{pmatrix} (x - \bar{x})/\sigma_x \\ (y - \bar{y})/\sigma_y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sigma_x & \\ & \sigma_y \end{pmatrix} V \begin{pmatrix} \xi \\ \eta \end{pmatrix} + \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

Projection is η=0



ZNZ7 / ZBZ7 2017-10-11
Random walk?
Mean reverting?
4
2
0
-2
-4
7
8
9
10
11
12
13
14
15

Forecast algorithm for 2 assets

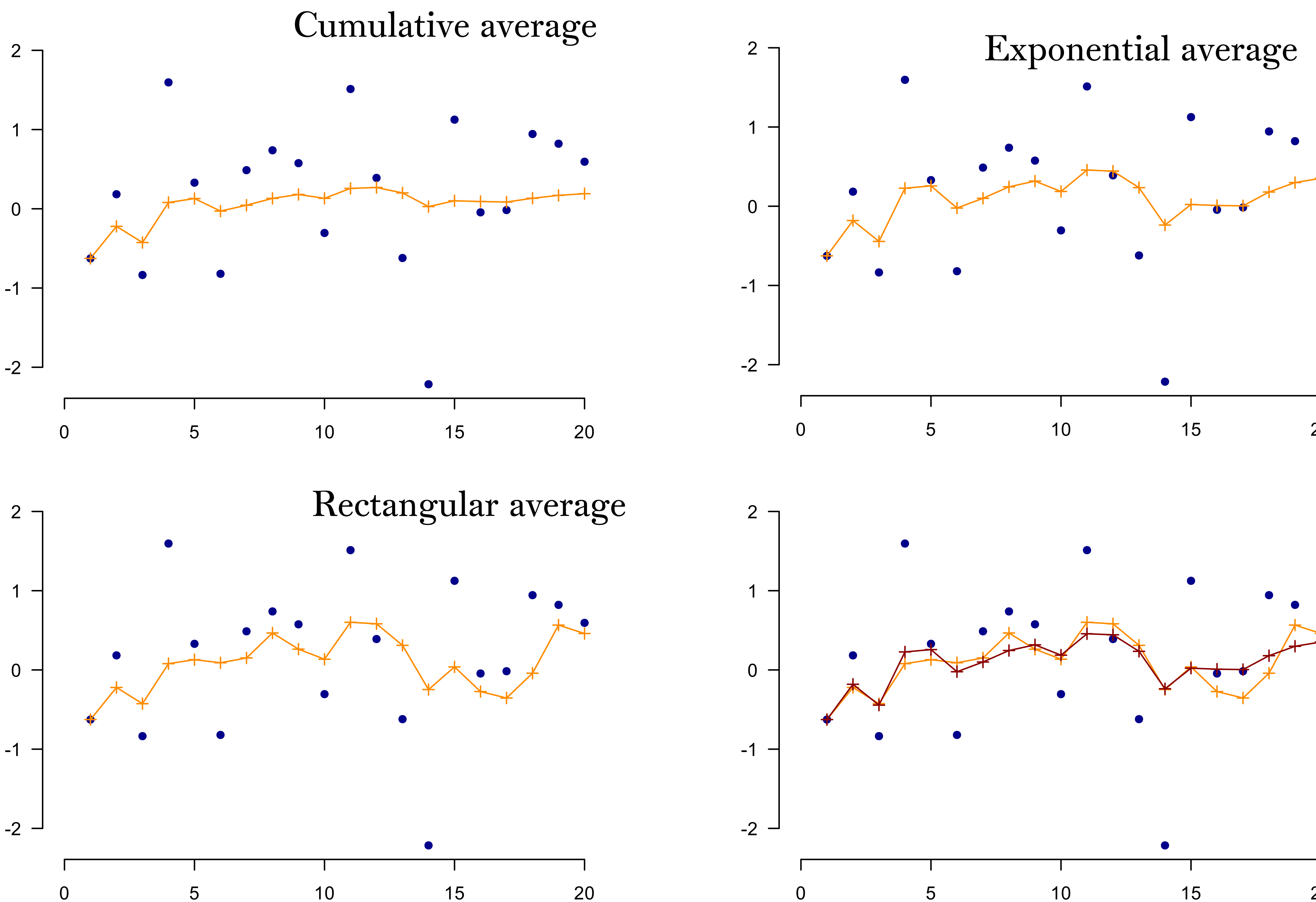
- Determine covariance model from data to date
- Transform (x,y) to (ξ,η)
- Project by setting $\eta = 0$
- Transform (ξ,0) back to (x,y)
- Result is forecast prices (x_*, y_*)
- Signal is $x - x_*$ for x, $y - y_*$ for y

Larger systems

- Have to determine "degrees of freedom"
- How many dimensions are "free"
- How many dimensions are mean-reverting?

Implementation challenges

- How to compute rolling means?
  - From start of day
  - With fixed window
  - Exponential means
  - Volume weighting
- How to compute covariance matrix?
  - need to subtract most recent value
- How to compute principal components?
  - SVD is slow, can use power method
- How to compute projection?
  - simple but can be confusing



Cumulative average
Exponential average
Rectangular average
2
1
0
-1
-2
0
5
10
15
20

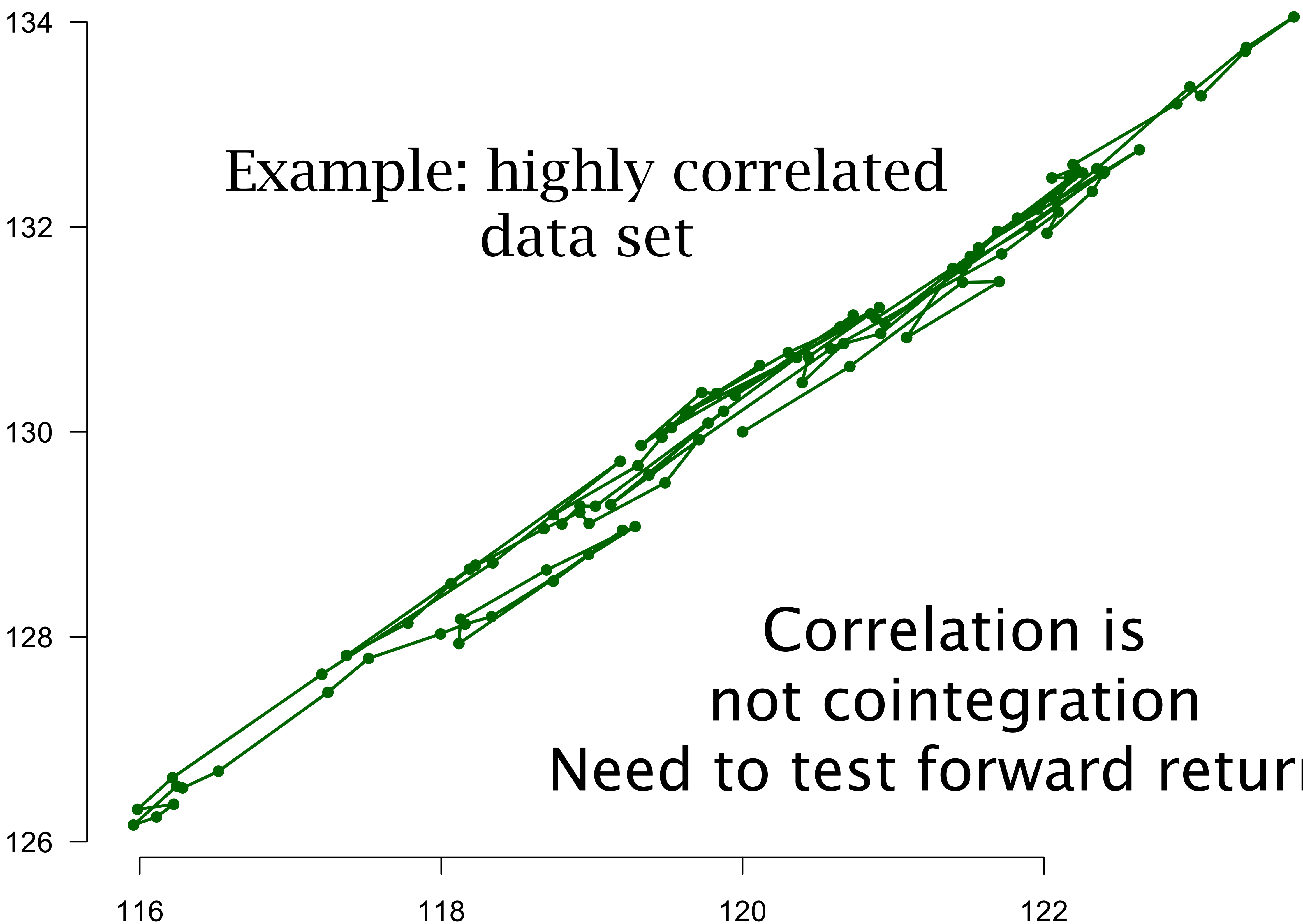
Need to determine intraday fits

- At time t during day, use only data $0 \leq s \leq t$
- Do this by restricting sums in regression and PCA
- Question: If data looks mean reverting,
  are points coming to line, or line to points

Example data set is random

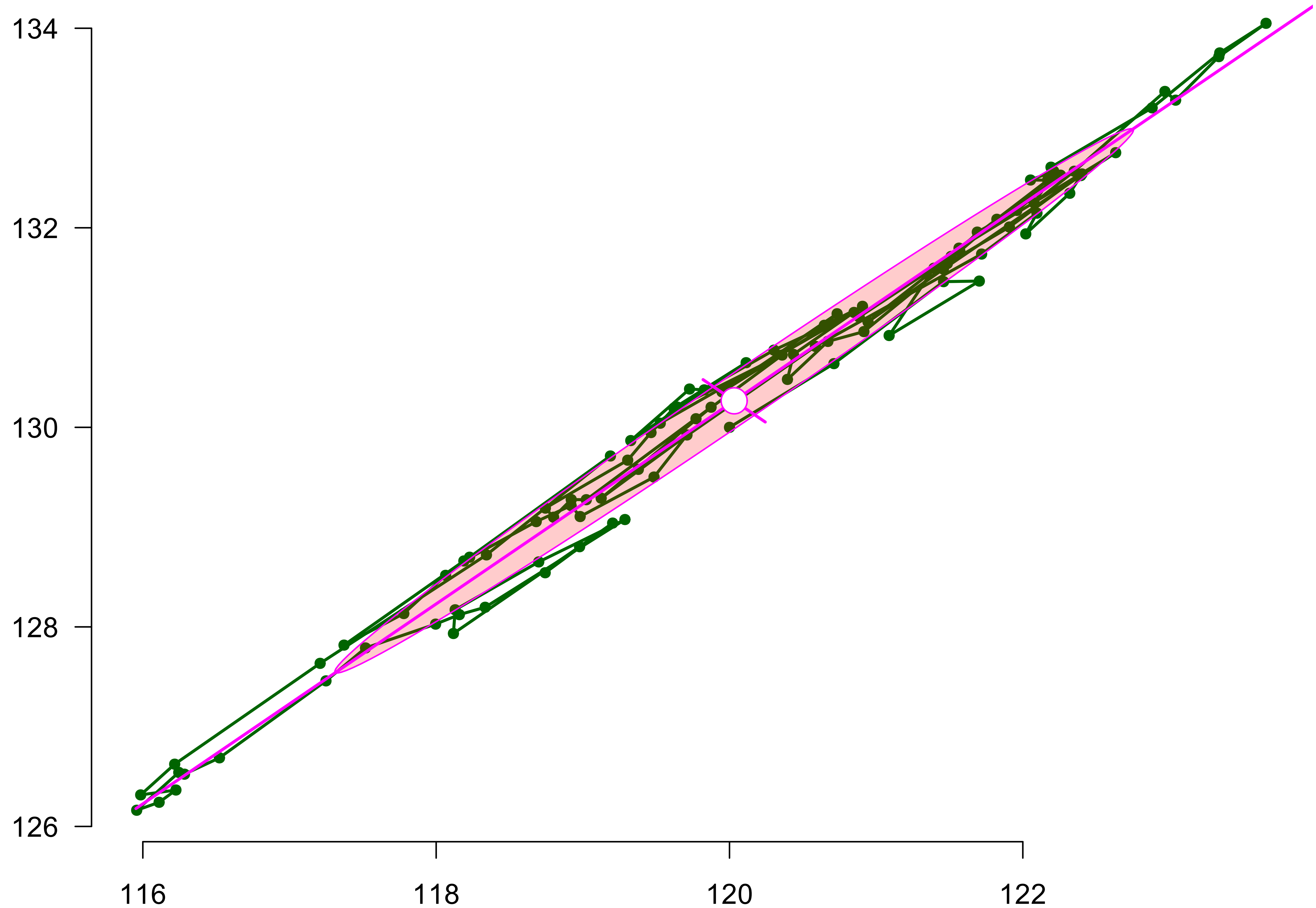
```
xm <- 120
ym <- 130

sigma1 <- 5
sigma2 <- .5

X <- data.frame( t=7 + (15-7)*(0:n)/n )
xi  <- sigma1 * sqrt(1/n) * c(0,cumsum(rnorm(n)))
eta <- sigma2 * sqrt(1/n) * c(0,cumsum(rnorm(n)))

X$p1 <- xm + xi + eta
X$p2 <- ym + xi - eta
```

Example: highly correlated data set

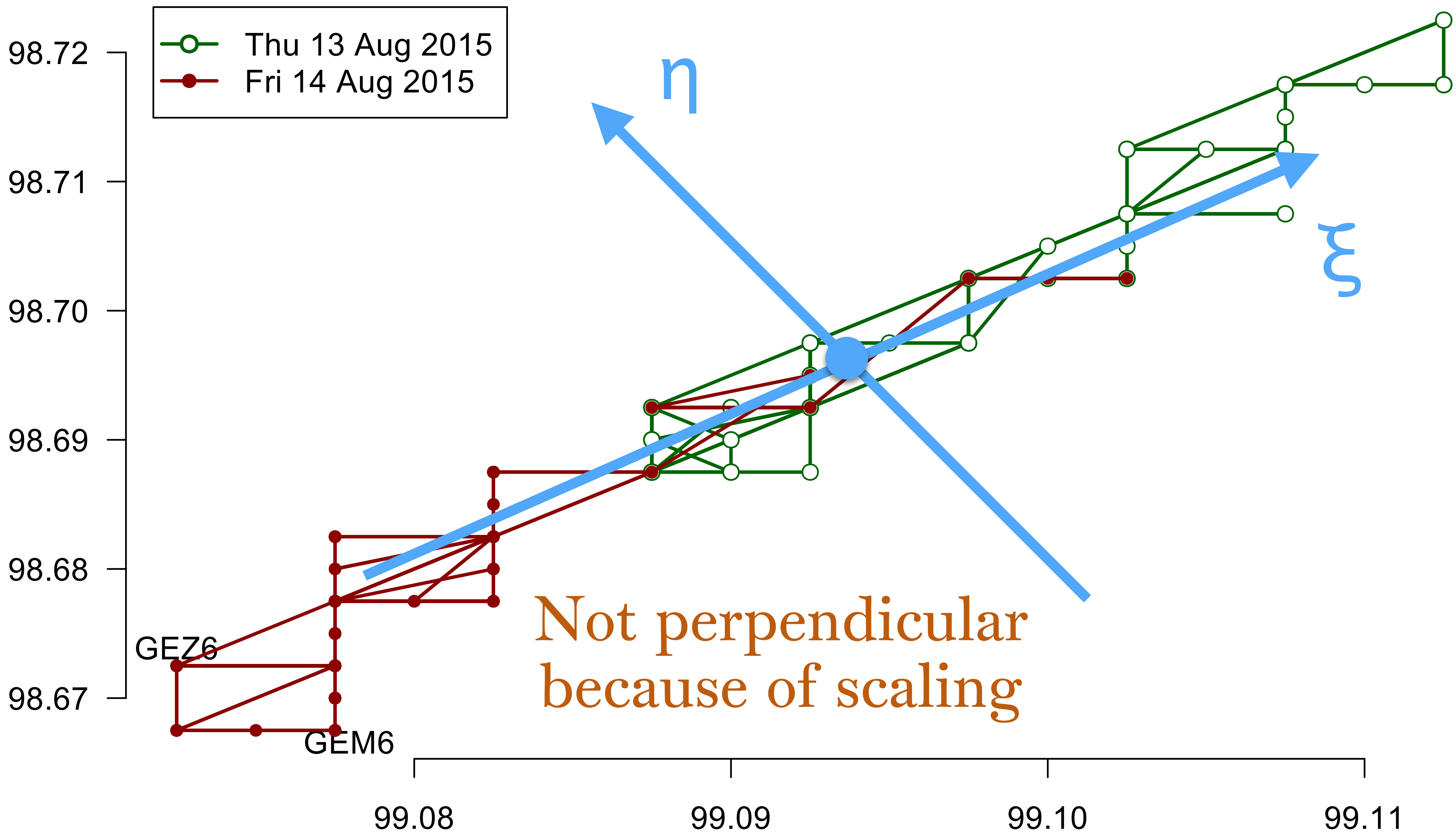


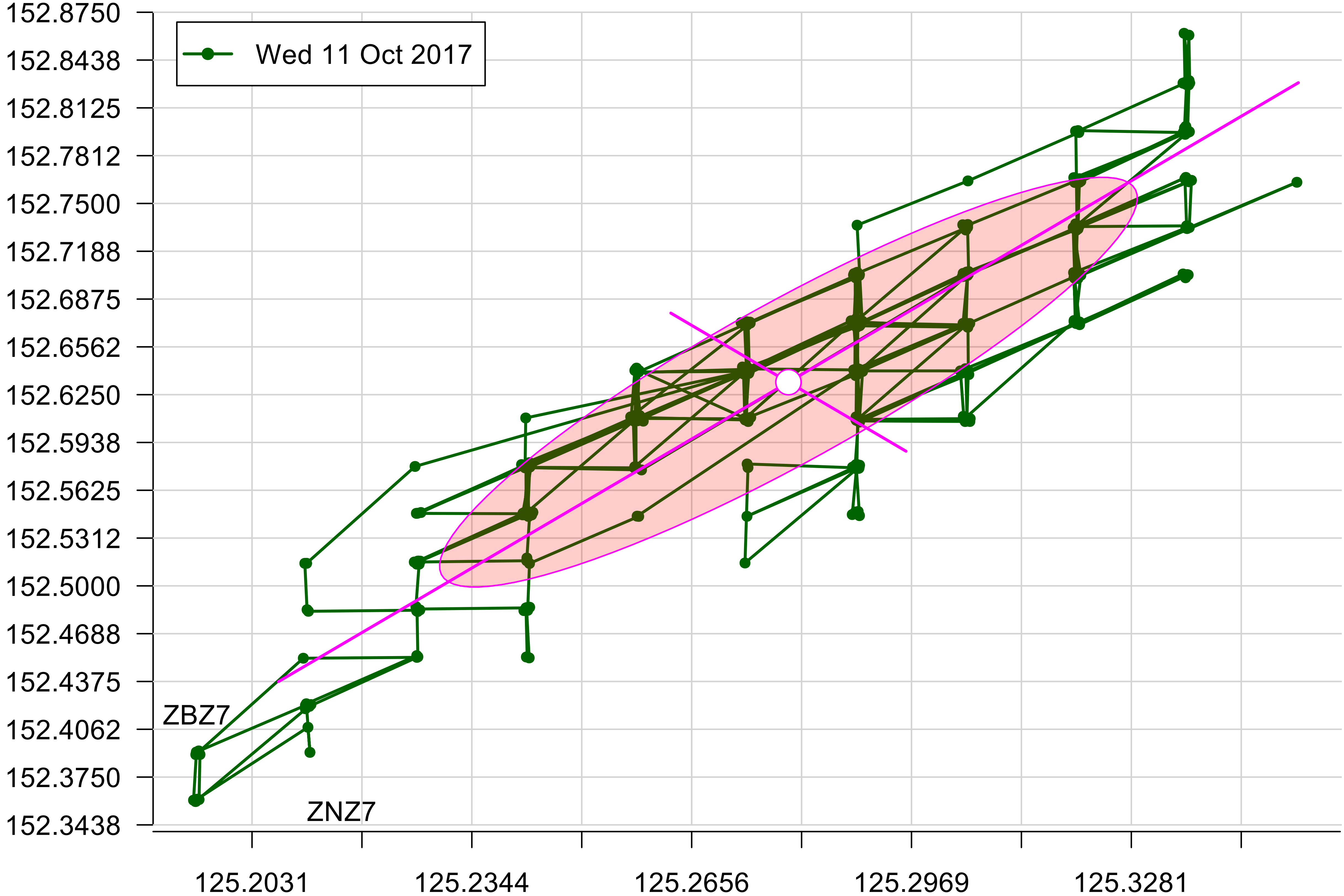
Correlation is not cointegration

Need to test forward returns


134
132
130
128
126
116
118
120
122

If offset is not constant, Can you use fit from previous day?

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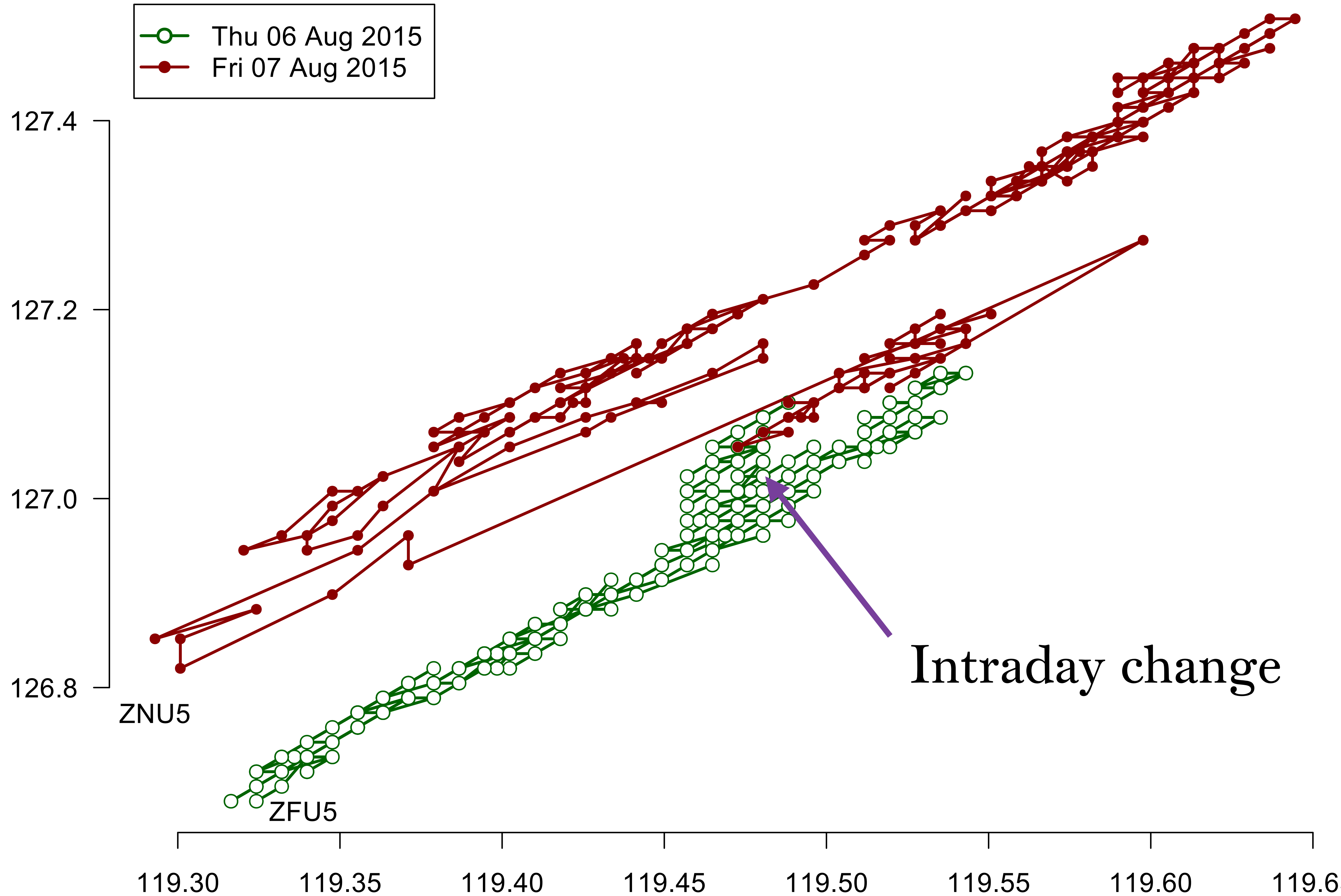


Thu 06 Aug 2015
Fri 07 Aug 2015
127.4
127.2
127.0
126.8
ZNU5
ZFU5
119.30
119.35
119.40
119.45
119.50
119.55
119.60
119.6
Intraday change

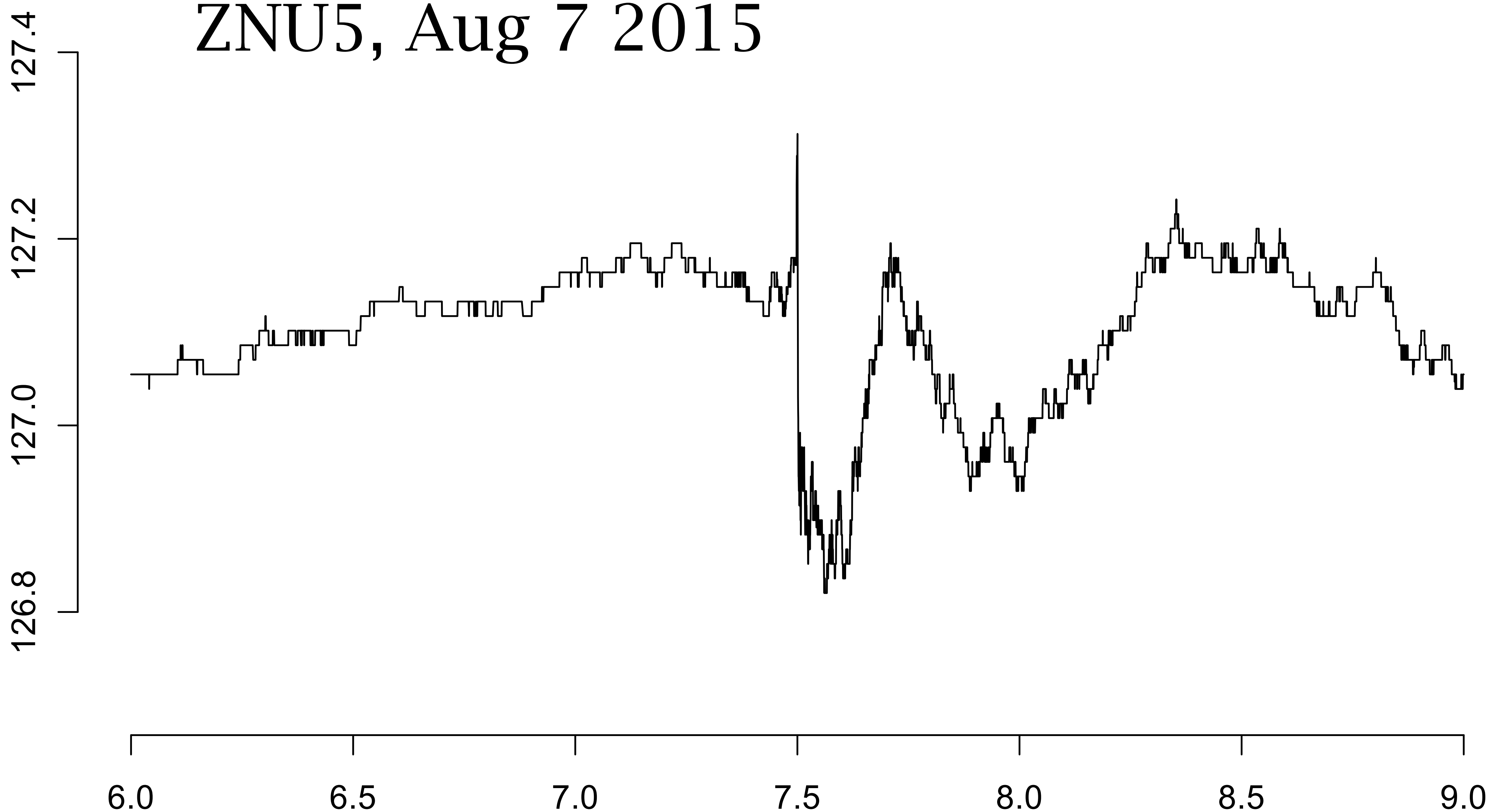
UNITED STATES DEPARTMENT OF LABOR

BUREAU OF LABOR STATISTICS

August, 2015

Date	Time	Release
Thursday, August 06, 2015	10:00 AM	**Productivity and Costs by Industry: Wholesale Trade, Retail Trade, and Food Services and Drinking Places, 2014** for Annual 2014
Friday, August 07, 2015	08:30 AM	**Employment Situation** for July 2015
Tuesday, August 11, 2015	08:30 AM	**Productivity and Costs (P)** for Second Quarter 2015
Wednesday, August 12, 2015	10:00 AM	**Job Openings and Labor Turnover Survey** for June 2015
Thursday, August 13, 2015	08:30 AM	**U.S. Import and Export Price Indexes** for July 2015
Friday, August 14, 2015	08:30 AM	**Producer Price Index** for July 2015
Tuesday, August 18, 2015	10:00 AM	**Summer Youth Labor Force** for 2015
Wednesday, August 19, 2015	08:30 AM	**Consumer Price Index** for July 2015
Wednesday, August 19, 2015	08:30 AM	**Real Earnings** for July 2015
Friday, August 21, 2015	10:00 AM	**Regional and State Employment and Unemployment (Monthly)** for July 2015

ZNU5, Aug 7 2015



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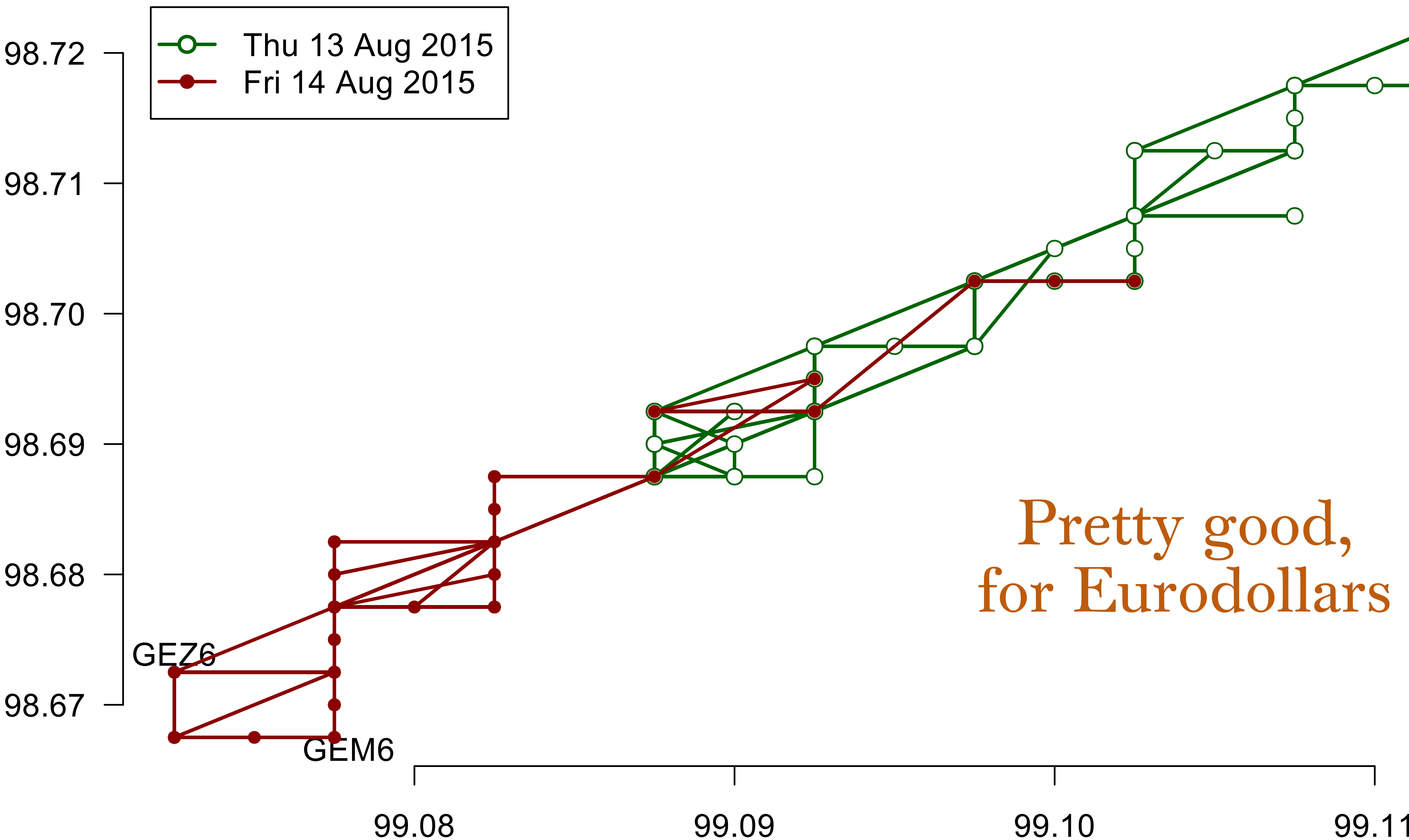
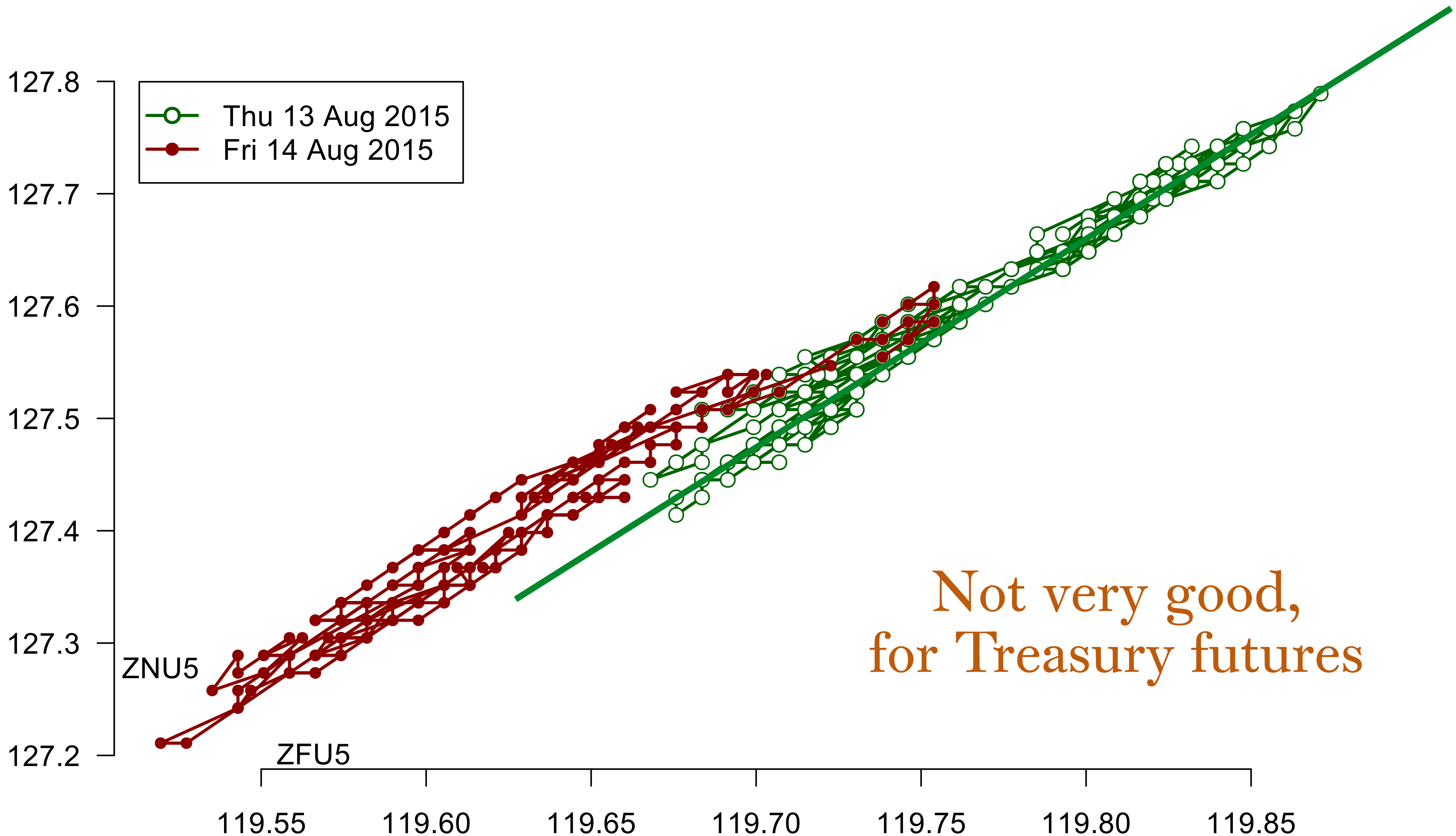
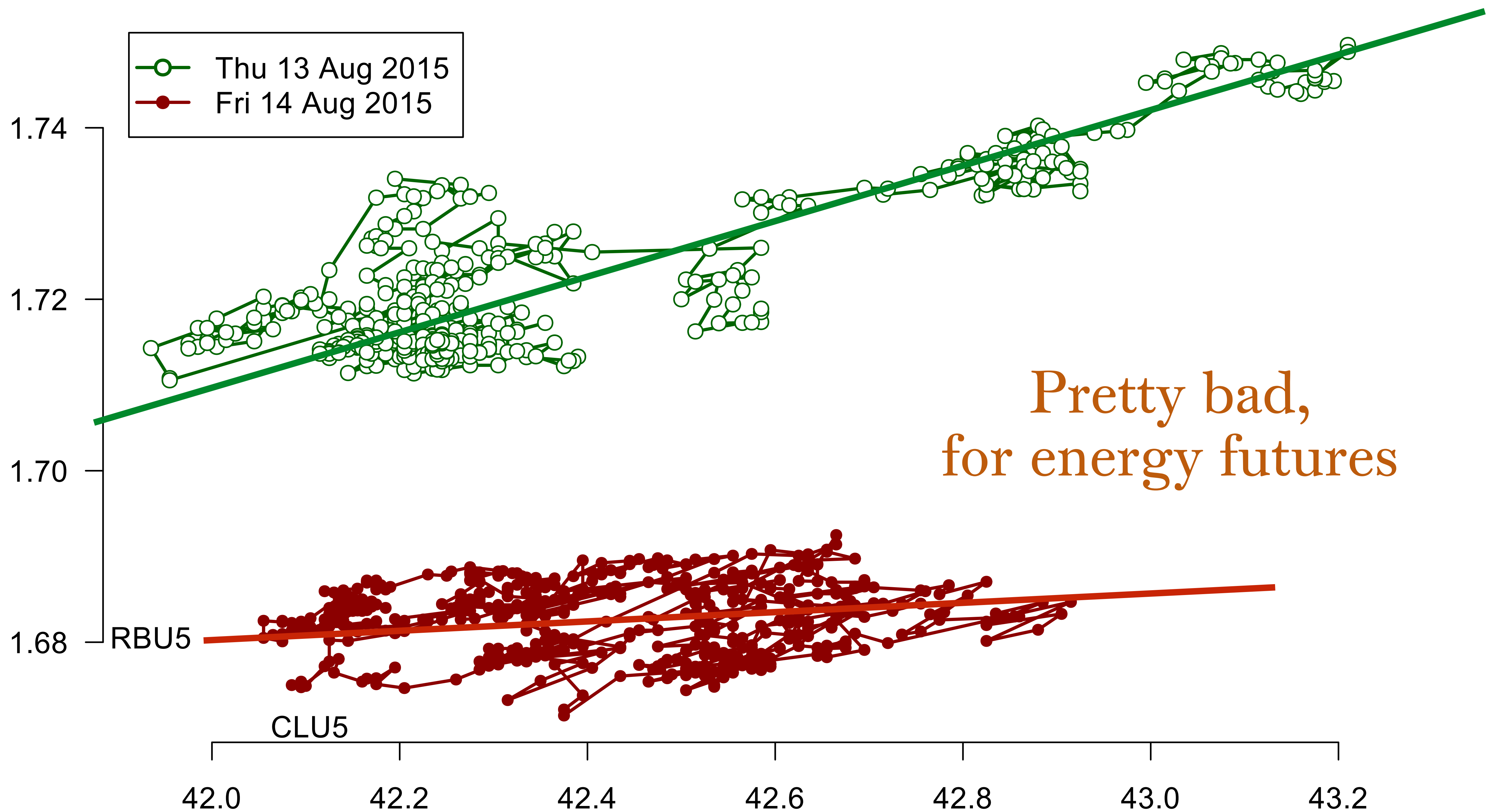


Pretty good,
for Eurodollars



Not very good,
for Treasury futures



Pretty bad,
for energy futures