PRINCETON UNIVERSITY

ORF 474: High Frequency Trading
Spring 2020

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Lecture 9b

April 8, 2020

Today

1. Microprice (imbalance) code/example
2. Measuring forward correlation
3. Pairs trading

Quote imbalance and trade imbalance

- Quote imbalance for large-tick contracts
- Trade direction probability
- Different ways to do historical averaging
- Trade direction imbalance as indicator

Quote imbalance

- Useful for large-tick products

  bid-ask spread is minimum price increment

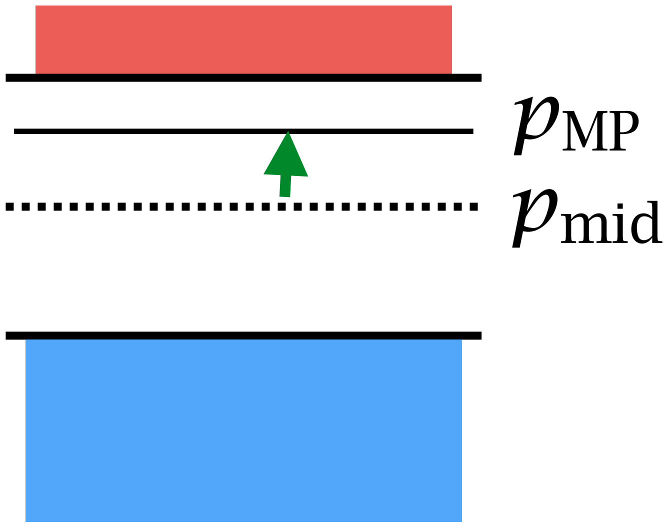
  price moves rarely

- Significant for predicting next one-tick move

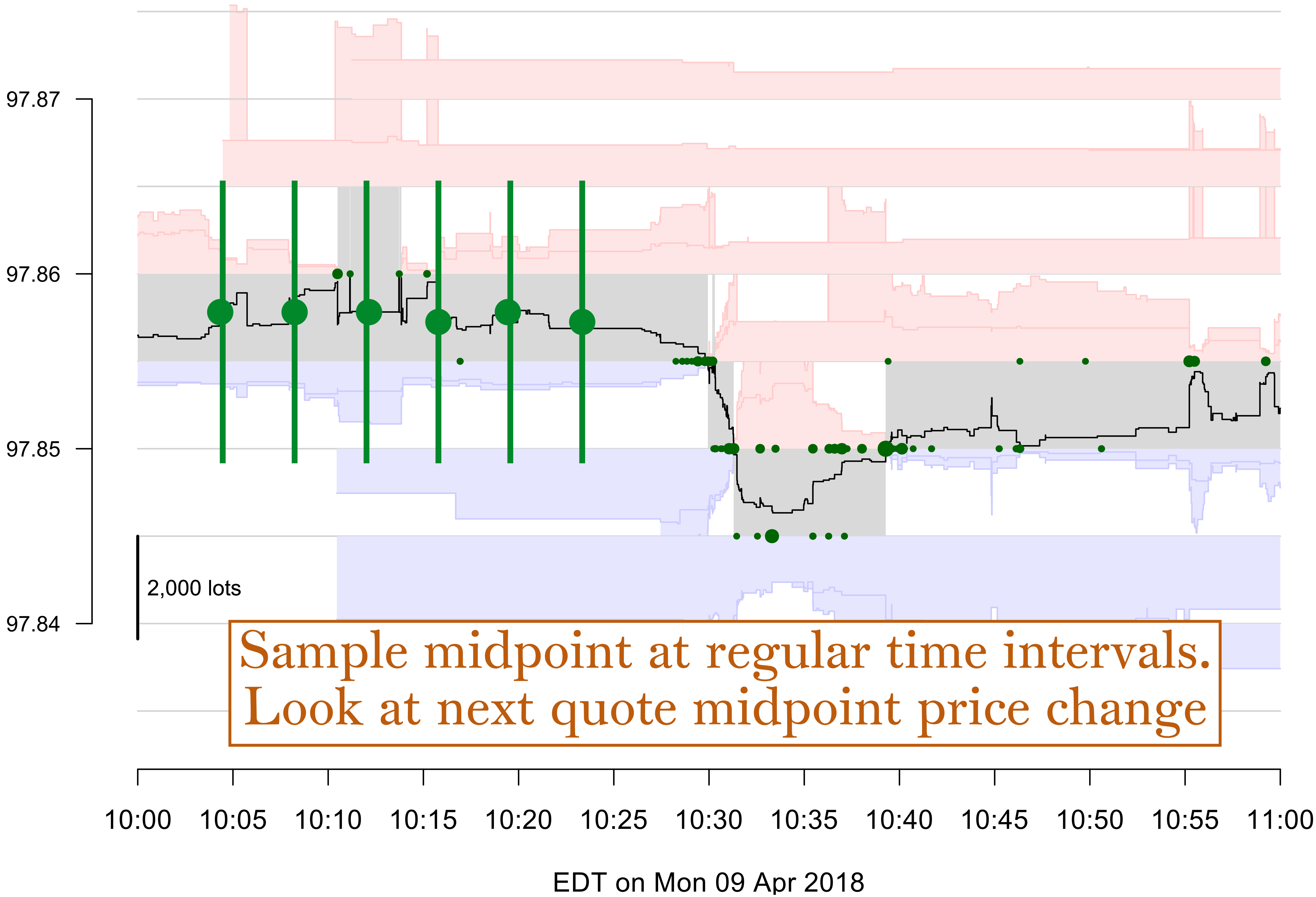
  predicting longer-term moves is much harder

- One-tick moves can be important

$$S = 2\,\frac{p_{\mathrm{MP}} - p_{\mathrm{mid}}}{p_{\mathrm{A}} - p_{\mathrm{B}}} = \frac{q_{\mathrm{B}} - q_{\mathrm{A}}}{q_{\mathrm{B}} + q_{\mathrm{A}}}$$



BAXH8: Mar 2018 Canadian Banker's Acceptance (a short term interest rate, like Eurodollar)



Sample midpoint at regular time intervals.
Look at next quote midpoint price change

Questions in microprice evaluation

- When to sample microprice?

  every quote update?

  trade times?

  regular time?

- When to sample forward price change?

  next sample point?

  next quote update?

  next quote midpoint change?

Determine direction of next midpoint change at each time on a regular grid, conditional on imbalance value at each time.

Regular time grid

```
q)t:([] t: 0D07:30:00 + 1000000000*til 8*60*60 )

q)5 # t
t
--------------------
0D07:30:00.000000000
0D07:30:01.000000000
0D07:30:02.000000000
0D07:30:03.000000000
0D07:30:04.000000000

q)-5 # t
t
--------------------
0D15:29:55.000000000
0D15:29:56.000000000
0D15:29:57.000000000
0D15:29:58.000000000
0D15:29:59.000000000
```

8 hours from 07:30,
at 1-second intervals

Imbalance on regular time grid

```
q)h"{[d;s] update S:S % min S where S>0 from
           update S:ask-bid,I:(bsiz-asiz)%bsiz+asiz from
        aj[ `t;
            ([] t: 0D07:30:00 + 1000000000*til 8*60*60 );
            select last bid,last ask,last bsiz,last asiz by t:time from quote where date=d,sym=s ]
    }[2017.10.25;`ZNZ7]"

t                    bid        ask        bsiz asiz S I
----------------------------------------------------------------------
0D07:30:00.000000000 124.296875 124.3125   41   35   1 0.0789473684211
0D07:30:01.000000000 124.296875 124.328125 179  179  2 0
0D07:30:02.000000000 124.3125   124.328125 16   100  1 -0.724137931034
0D07:30:03.000000000 124.296875 124.3125   55   79   1 -0.179104477612
0D07:30:04.000000000 124.296875 124.3125   60   271  1 -0.63746223565
0D07:30:05.000000000 124.28125  124.296875 286  55   1 0.677419354839
0D07:30:06.000000000 124.28125  124.296875 336  90   1 0.577464788732
0D07:30:07.000000000 124.28125  124.296875 343  117  1 0.491304347826
0D07:30:08.000000000 124.28125  124.296875 336  251  1 0.144804088586
0D07:30:09.000000000 124.28125  124.296875 333  282  1 0.0829268292683
0D07:30:10.000000000 124.28125  124.296875 376  294  1 0.122388059701
0D07:30:11.000000000 124.28125  124.296875 236  510  1 -0.367292225201
0D07:30:12.000000000 124.28125  124.296875 226  354  1 -0.220689655172
0D07:30:13.000000000 124.28125  124.296875 112  754  1 -0.741339491917
0D07:30:14.000000000 124.28125  124.296875 238  753  1 -0.519677093845
0D07:30:15.000000000 124.28125  124.296875 420  145  1 0.486725663717
0D07:30:16.000000000 124.296875 124.3125   47   991  1 -0.909441233141
0D07:30:17.000000000 124.28125  124.296875 502  77   1 0.73402417962
0D07:30:18.000000000 124.28125  124.296875 1277 106  1 0.846710050615
0D07:30:19.000000000 124.28125  124.296875 1259 154  1 0.782024062279
0D07:30:20.000000000 124.28125  124.296875 1451 91   1 0.881971465629
...
```

Next midpoint change

quotes with midpoint price change

```
q)h"{[d;s] update nextt:next t, nextdp:next dp from
      select from (
           update dp:deltas pm from
           select pm:0.5*(last bid)+last ask by t:time from quote where date=d,sym=s
       ) where 0<>dp
    }[2017.10.25;`ZNZ7]"

t                    | pm          dp         nextt                nextdp
---------------------| ------------------------------------------------------
-0D07:10:59.979331679| 124.625     124.625    -0D07:04:59.300944223 0.015625
-0D07:04:59.300944223| 124.640625  0.015625   -0D06:59:59.999340244 -0.0078125
-0D06:59:59.999340244| 124.6328125 -0.0078125 -0D06:56:32.208616637 -0.015625
-0D06:56:32.208616637| 124.6171875 -0.015625  -0D06:55:09.788880505 0.015625
-0D06:55:09.788880505| 124.6328125 0.015625   -0D06:52:38.990543529 0.015625
-0D06:52:38.990543529| 124.6484375 0.015625   -0D06:46:00.814645893 -0.0078125
-0D06:46:00.814645893| 124.640625  -0.0078125 -0D06:46:00.814531929 -0.0078125
-0D06:46:00.814531929| 124.6328125 -0.0078125 -0D06:44:14.193361505 -0.0078125
-0D06:44:14.193361505| 124.625     -0.0078125 -0D06:44:14.192657203 -0.0078125
-0D06:44:14.192657203| 124.6171875 -0.0078125 -0D06:44:14.184332959 0.0078125
-0D06:44:14.184332959| 124.625     0.0078125  -0D06:44:12.613319651 0.0078125
-0D06:44:12.613319651| 124.6328125 0.0078125  -0D06:40:49.203769349 -0.0078125
-0D06:40:49.203769349| 124.625     -0.0078125 -0D06:40:49.203746967 -0.0078125
-0D06:40:49.203746967| 124.6171875 -0.0078125 -0D06:39:20.038342961 0.0078125
-0D06:39:20.038342961| 124.625     0.0078125  -0D06:39:20.038320765 -0.0078125
-0D06:39:20.038320765| 124.6171875 -0.0078125 -0D06:39:20.038307235 0.015625
-0D06:39:20.038307235| 124.6328125 0.015625   -0D06:29:47.924877315 -0.015625
-0D06:29:47.924877315| 124.6171875 -0.015625  -0D06:23:54.565852725 0.015625
```

Put them together

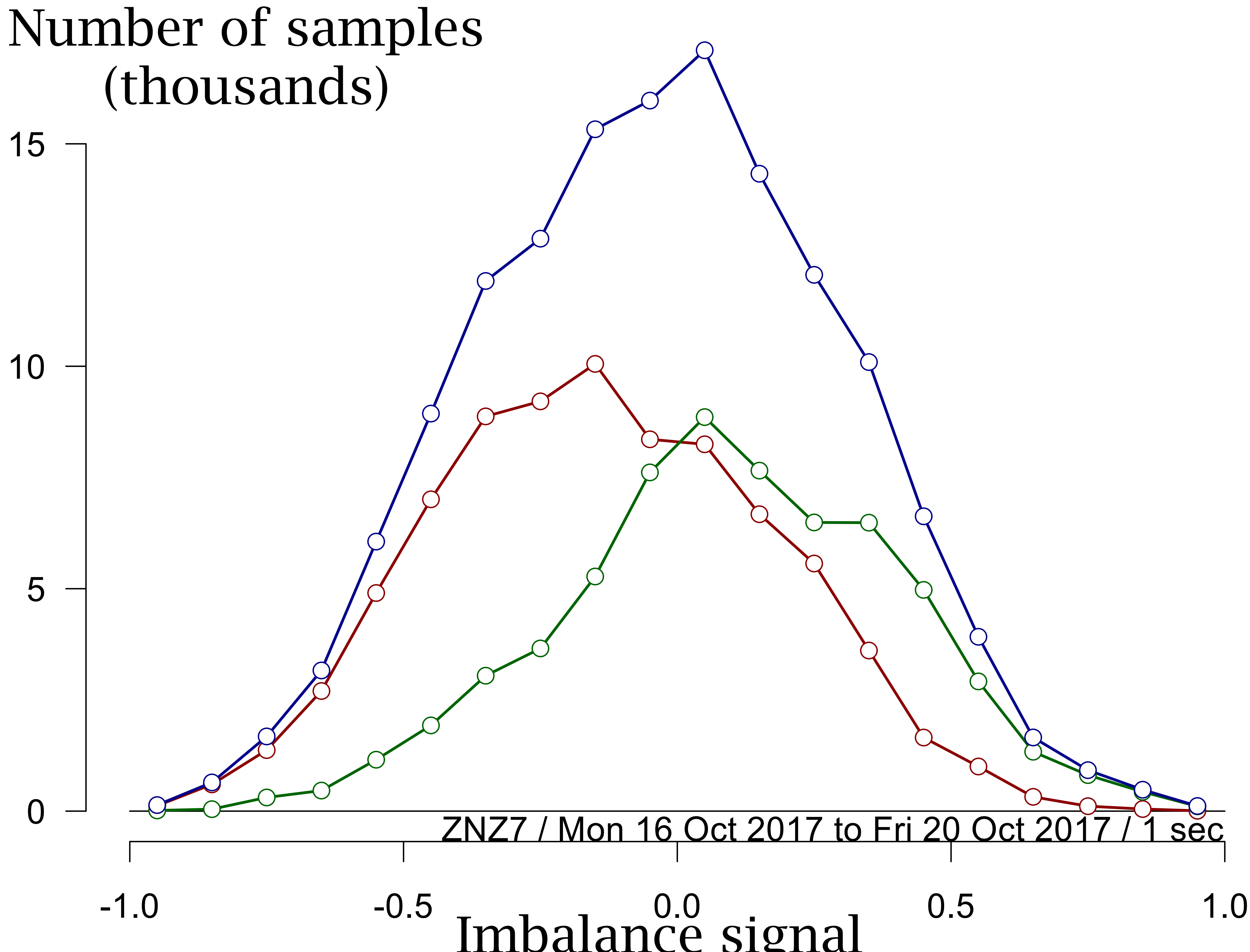
```
q)h"{[d;s] update dt:nextt-t from
     aj[ `t;
         select t,S,I from update S:S % min S from
            update S:ask-bid,I:(bsiz-asiz)%bsiz+asiz from
               aj[ `t; ([] t: 0D07:30:00 + 1000000000*til 8*60*60 );
                       select last bid,last ask,last bsiz,last asiz by t:time from quote where date=d,sym=s ];
         update nextt:next t,nextdp:next dp from
            select from (
                 update dp:deltas pm from select pm:0.5*(last bid)+last ask by t:time from quote where date=d,sym=s
            ) where 0<>dp]
    }[2017.10.25;`ZNZ7]"

t                    S I               pm          dp         nextt                nextdp     dt
---------------------------------------------------------------------------------------------------------------------
0D07:30:00.000000000 1 0.0789473684211 124.3046875 0.0078125  0D07:30:00.018402629 -0.0078125 0D00:00:00.018402629
0D07:30:01.000000000 2 0               124.3125    -0.0078125 0D07:30:01.185693301 0.0078125  0D00:00:00.185693301
0D07:30:02.000000000 1 -0.724137931034 124.3203125 0.0078125  0D07:30:02.028393247 -0.015625  0D00:00:00.028393247
0D07:30:03.000000000 1 -0.179104477612 124.3046875 -0.0078125 0D07:30:04.103676453 -0.0078125 0D00:00:01.103676453
0D07:30:04.000000000 1 -0.63746223565  124.3046875 -0.0078125 0D07:30:04.103676453 -0.0078125 0D00:00:00.103676453
0D07:30:05.000000000 1 0.677419354839  124.2890625 -0.015625  0D07:30:12.549772315 -0.0078125 0D00:00:07.549772315
0D07:30:06.000000000 1 0.577464788732  124.2890625 -0.015625  0D07:30:12.549772315 -0.0078125 0D00:00:06.549772315
0D07:30:07.000000000 1 0.491304347826  124.2890625 -0.015625  0D07:30:12.549772315 -0.0078125 0D00:00:05.549772315
0D07:30:08.000000000 1 0.144804088586  124.2890625 -0.015625  0D07:30:12.549772315 -0.0078125 0D00:00:04.549772315
0D07:30:09.000000000 1 0.0829268292683 124.2890625 -0.015625  0D07:30:12.549772315 -0.0078125 0D00:00:03.549772315
0D07:30:10.000000000 1 0.122388059701  124.2890625 -0.015625  0D07:30:12.549772315 -0.0078125 0D00:00:02.549772315
...
```

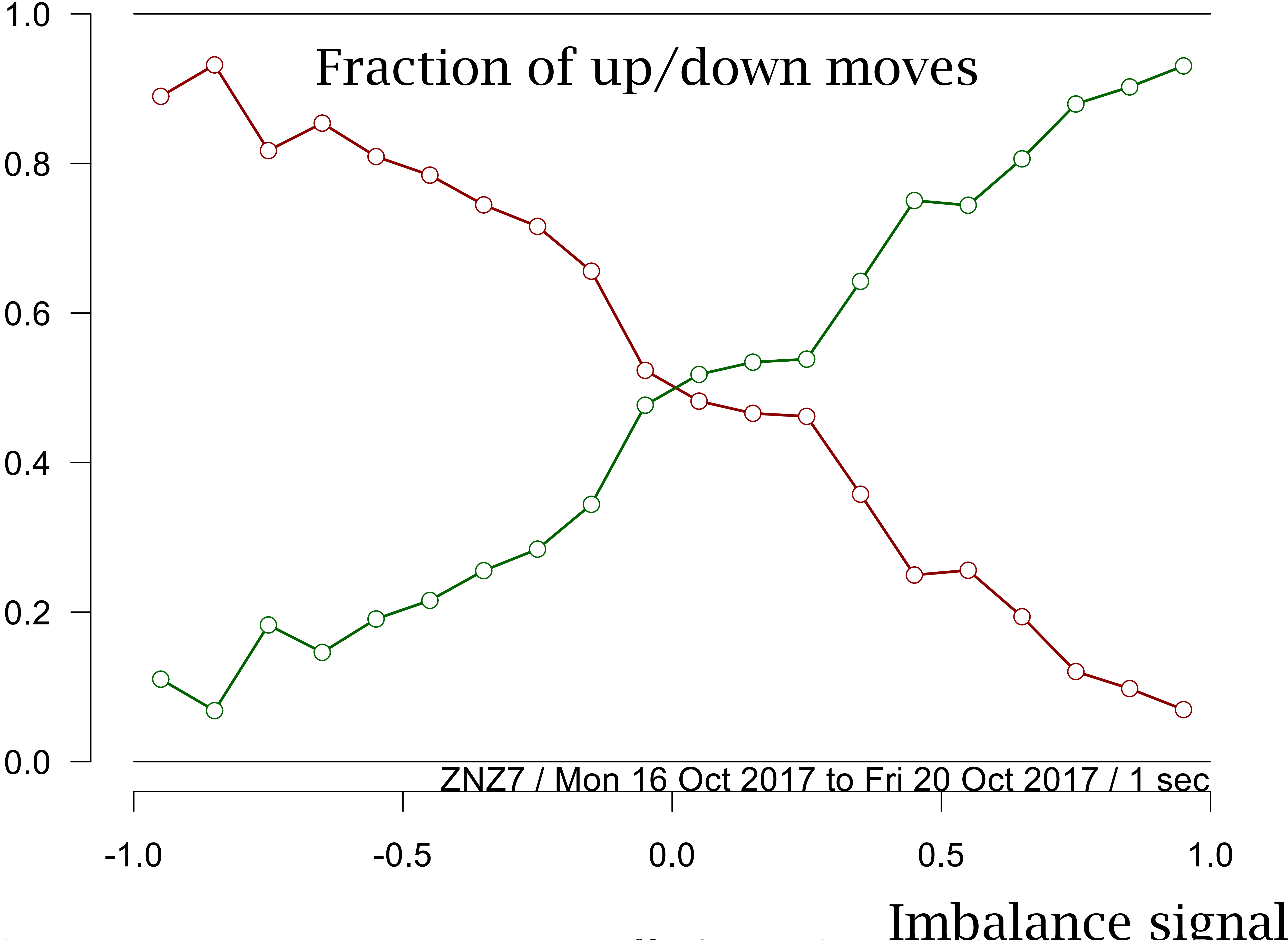
How do do this in R?

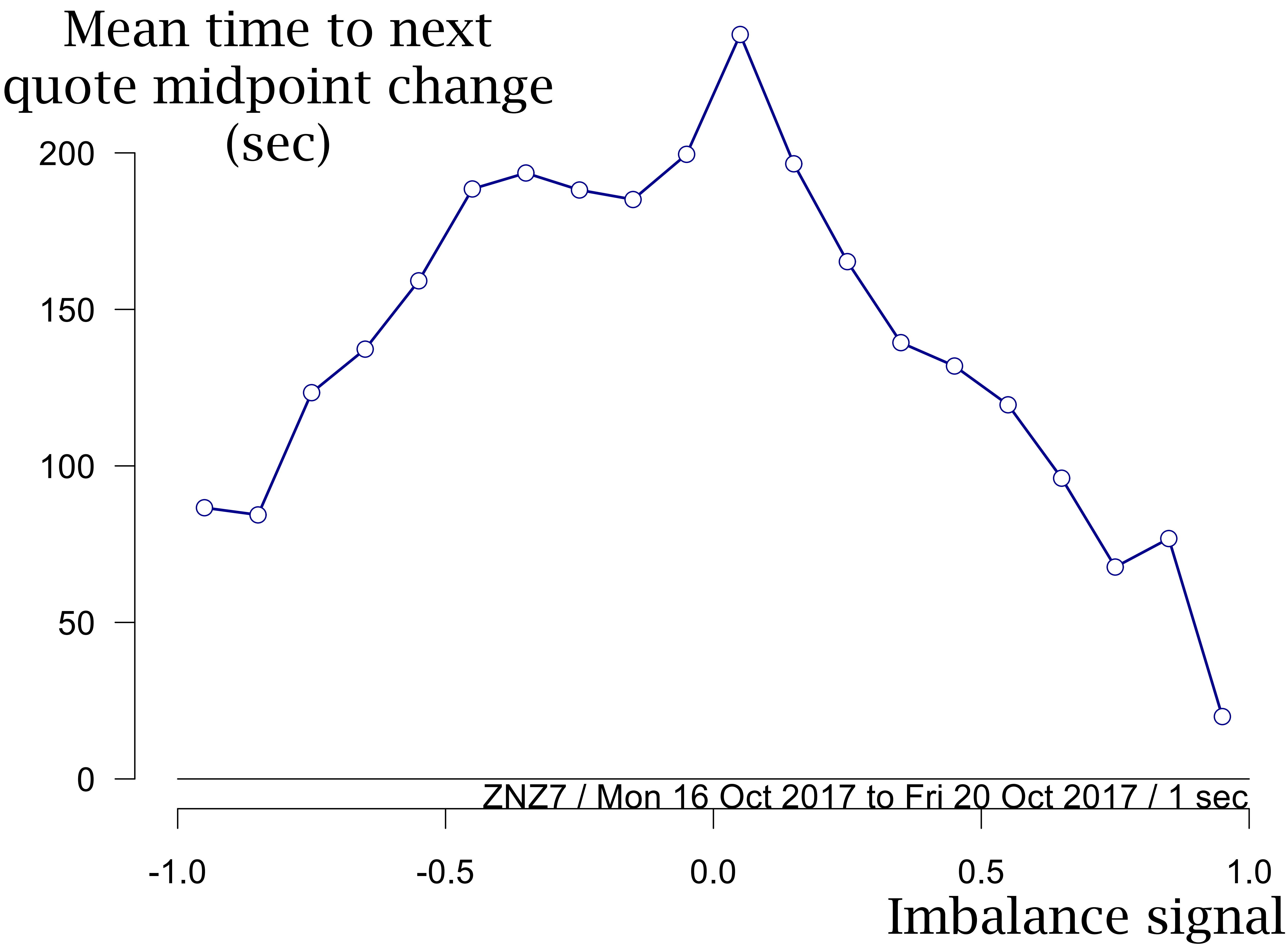
```
##########################################################################
#  Query for regular times

nt <- round( (tmax - tmin)*60*60 / dt )
print(paste('nt =',nt),quote=FALSE)

qT <- paste0('([] t:',tmsfmt(round(60*tmin)),'+',dt,'*1000000000*til ',nt,')')
if (verbose) print(qT)

qQ <- qappend('select last bid,last ask,last bsiz,last asiz by t:time from quote','date=d',scls)
if (verbose) print(qQ)

q1 <- paste0('aj[`t;',qT,';',qQ,']')
q1 <- paste('update S:ask-bid,I:(bsiz-asiz)%bsiz+asiz from',q1)
q1 <- paste('update S:S % min S where S>0 from',q1)
if (verbose) print(q1)

##########################################################################
#  Query for next midpoint change

q2 <- 'select pm:0.5*(last bid)+last ask by t:time from quote where date=d,sym=s'
q2 <- paste('update dp:deltas pm from',q2)
q2 <- paste0('select from (',q2,') where 0<>dp')
q2 <- paste('update nextt:next t, nextdp:next dp from',q2)
if (verbose) print(q2)

##########################################################################
#  Combine them together
q <- paste0('aj[`t;',q1,';',q2,']')
q <- paste('update dt:nextt-t from',q)
```

Piece together using string functions

"qappend" is a utility I have


Number of samples
(thousands)
15
10
5
0
ZNZ7 / Mon 16 Oct 2017 to Fri 20 Oct 2017 / 1 sec
-1.0
-0.5
0.0
0.5
1.0
Imbalance signal


Fraction of up/down moves
1.0
0.8
0.6
0.4
0.2
0.0
ZNZ7 / Mon 16 Oct 2017 to Fri 20 Oct 2017 / 1 sec
-1.0
-0.5
0.0
0.5
1.0
Imbalance signal



Mean time to next
quote midpoint change
(sec)
200
150
100
50
0
ZNZ7 / Mon 16 Oct 2017 to Fri 20 Oct 2017 / 1 sec
-1.0
-0.5
0.0
0.5
1.0
Imbalance signal

Trade pressure

Trades

Times	$t_1, t_2, \ldots$
Sizes	$v_1, v_2, \ldots$
Prices	$p_1, p_2, \ldots$

Want to determine partition into

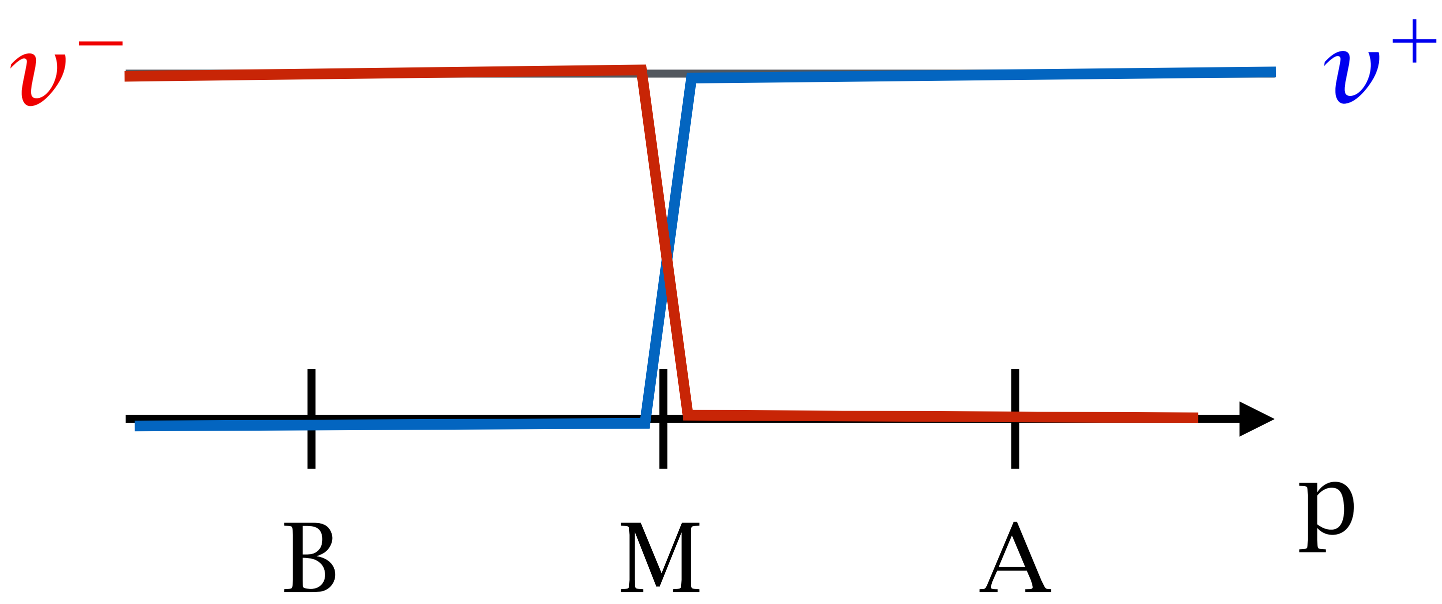
Buy volumes	$v_1^+, v_2^+ \ldots$
Sell volumes	$v_1^-, v_2^- \ldots$

$$v_j^+ + v_j^- = v_j$$

Not necessarily only for large-tick contracts

One possible partition

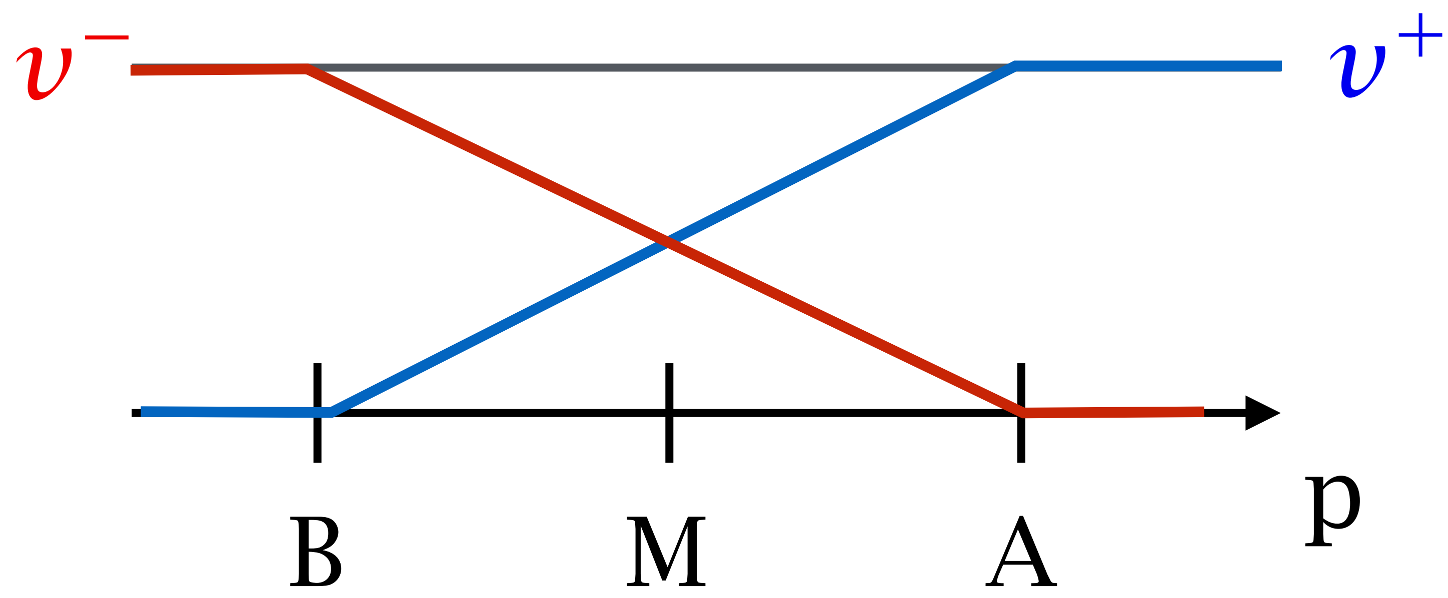
$p < p_{\text{mid}}$	$p = p_{\text{mid}}$	$p > p_{\text{mid}}$
$v_j^- = v_j$	$v_j^- = \dfrac{1}{2} v_j$	$v_j^- = 0$
$v_j^+ = 0$	$v_j^+ = \dfrac{1}{2} v_j$	$v_j^+ = v_j$



Another possible partition

$$v_j^+ \;=\; v_j \cdot \max\left\{ 0,\; \min\left\{ 1,\; \frac{p-B}{A-B} \right\} \right\}$$

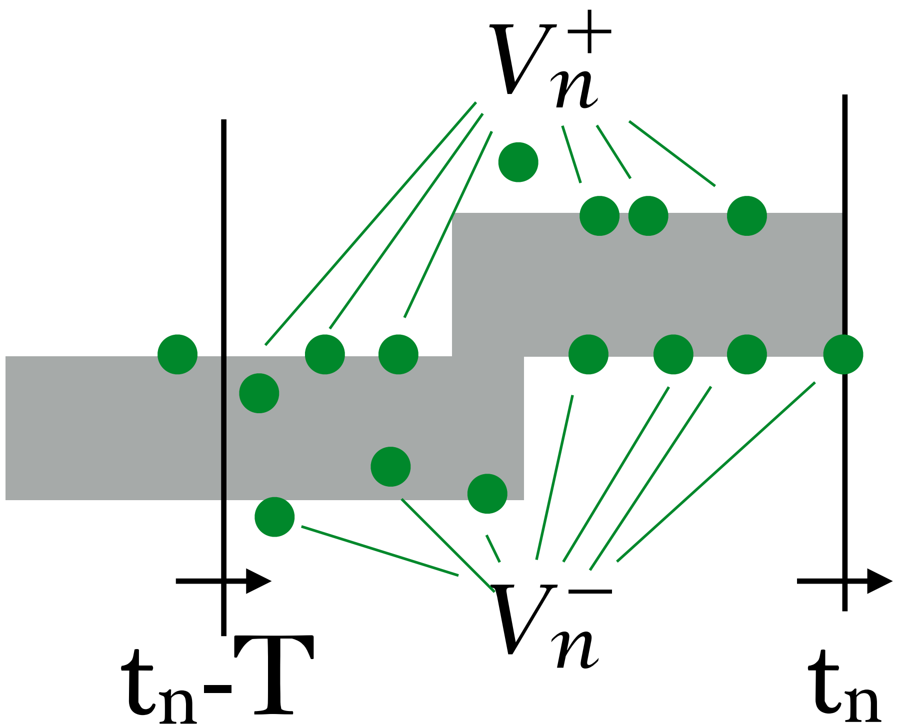
$$v_j^- \;=\; v_j \cdot \max\left\{ 0,\; \min\left\{ 1,\; \frac{A-p}{A-B} \right\} \right\}$$



Cumulative trade volumes

$$V_n^+ = \text{total buy volume in } t_n - T < t \le t_n$$
$$V_n^- = \text{total sell volume in } t_n - T < t \le t_n$$

$$V_n^+ = \sum_{t_n - T < t_j \le t_n} v_j^+$$
$$V_n^- = \sum_{t_n - T < t_j \le t_n} v_j^-$$



Can restrict to duration of this quote level but do not need to

Replace window sum by exponential

$$V_n^+ = \sum_{j=1}^{n} e^{-(t_n - t_j)/T} v_j^+$$

$$V_n^- = \sum_{j=1}^{n} e^{-(t_n - t_j)/T} v_j^-$$

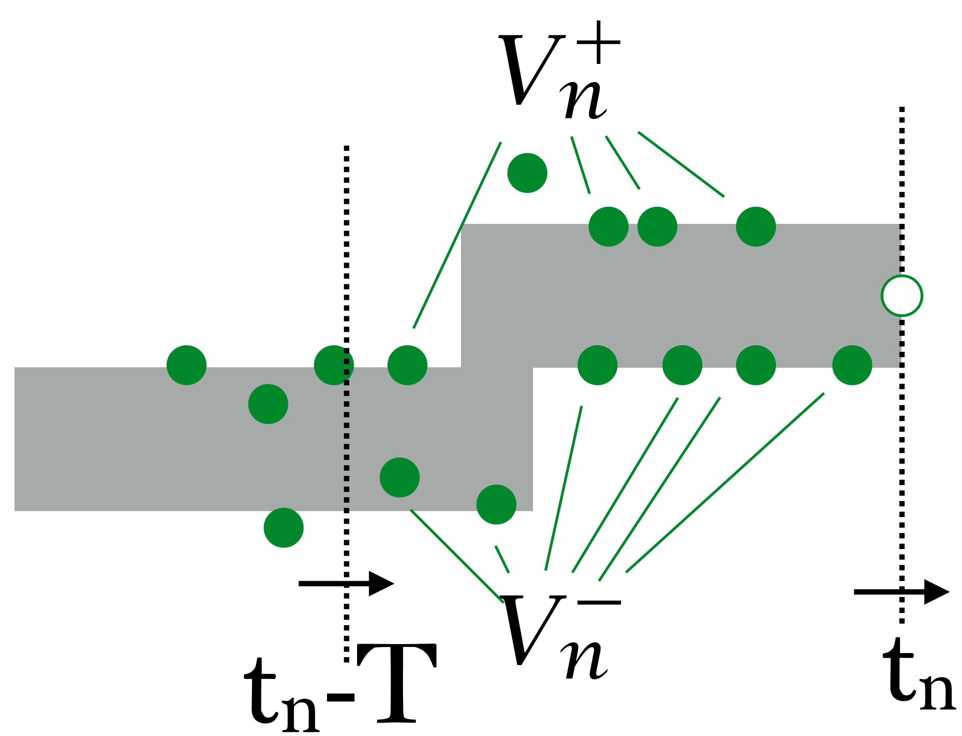
$$V_n^+ = e^{-(t_n - t_{n-1})/T} V_{n-1}^+ + v_n^+$$

$$V_n^- = e^{-(t_n - t_{n-1})/T} V_{n-1}^- + v_n^-$$

Easy to code and efficient

Cumulative volume at non-trade times t

- For example, to match quote imbalance at regular grid of (non-trade) times
- Add "phantom" trades with $v = 0$ (price does not matter)



Applications for cumulative volume

- Each volume independently indicates trade interest on that side

  Liquidity measure if you are trading against it

  Compare cumulative volume to historical averages

- Ratio indicates imbalance

  Can use to forecast price motions,
  maybe in combination with quote imbalance

Signed liquidity measurement

Order Execution Using Trading Signals

Michael G Sotiropoulos

Algorithmic Trading Quantitative Research
Bank of America Merrill Lynch

Princeton Quant Trading Conference
31-Mar-2012

Contents

Specific Examples: Trade Sign Autocorrelation (III)

Almgren R., “A New EMA Indicator”, Banc of America Securities Technical Report (2006)

Calculation of the signal (Almgren 2006)

1. For each trade define its “askness” a and “bidness” b as the distance of the transaction price from the bid (res. ask) in units of spread

$$a = \min\left(\left(\frac{P - P_b}{P_a - P_b}\right)^+, 1\right); \quad b = \min\left(\left(\frac{P_a - P}{P_a - P_b}\right)^+, 1\right). \tag{8}$$

   By construction, $a + b = 1$. A trade that hits the ask side has $a = 1, \ b = 0$.

2. At each trade time t_n compute the moving average of askness and bidness over a window of size $\tilde{\tau}_w$ as

$$A_n = \frac{1}{\tilde{\tau}_w} a_n + w_n A_{n-1}; \quad B_n = \frac{1}{\tilde{\tau}_w} b_n + w_n B_{n-1}, \tag{9}$$

   with exponentially decaying weights $w_n = e^{-\left(t_n - t_{n-1}\right)/\tilde{\tau}_w}$.

3. Normalize the moving averages by half the average trading speed

$$\bar{A}_n = \frac{2A_n}{N_{trd}/T_{day}}, \quad \bar{B}_n = \frac{2B_n}{N_{trd}/T_{day}} \tag{10}$$

An algorithm that tries to minimize impact cost will use the signal as follows:

- For a BUY (SELL) order trade faster when $\bar{B}_n$ ($\bar{A}_n$) is higher.

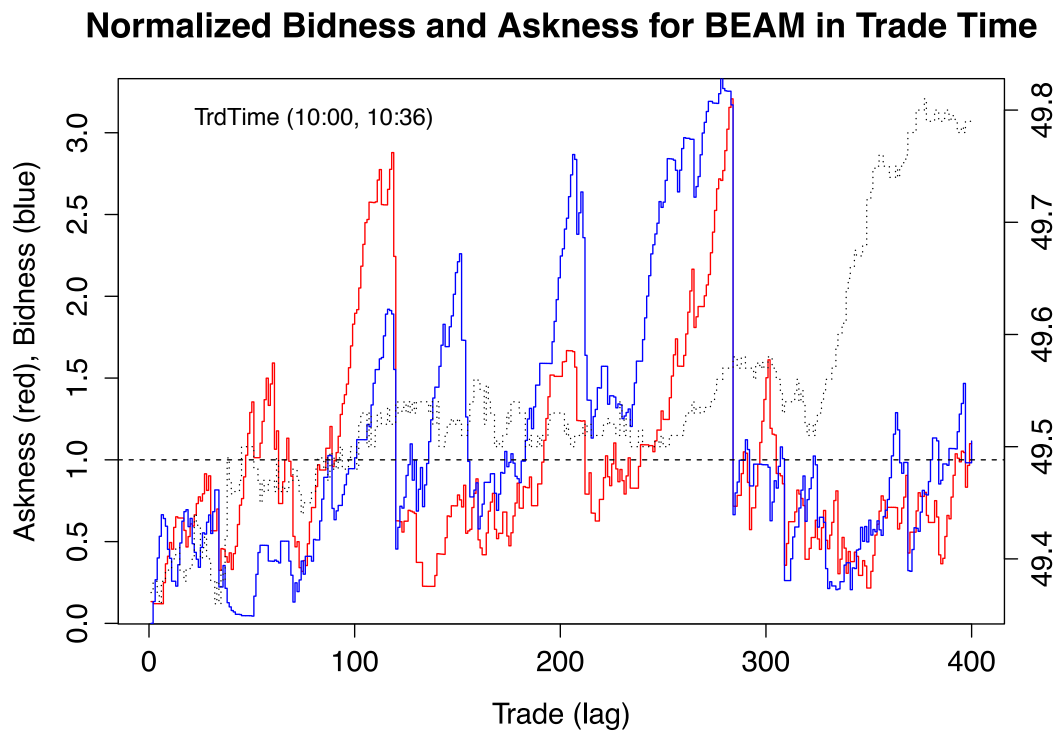
Michael G Sotiropoulos

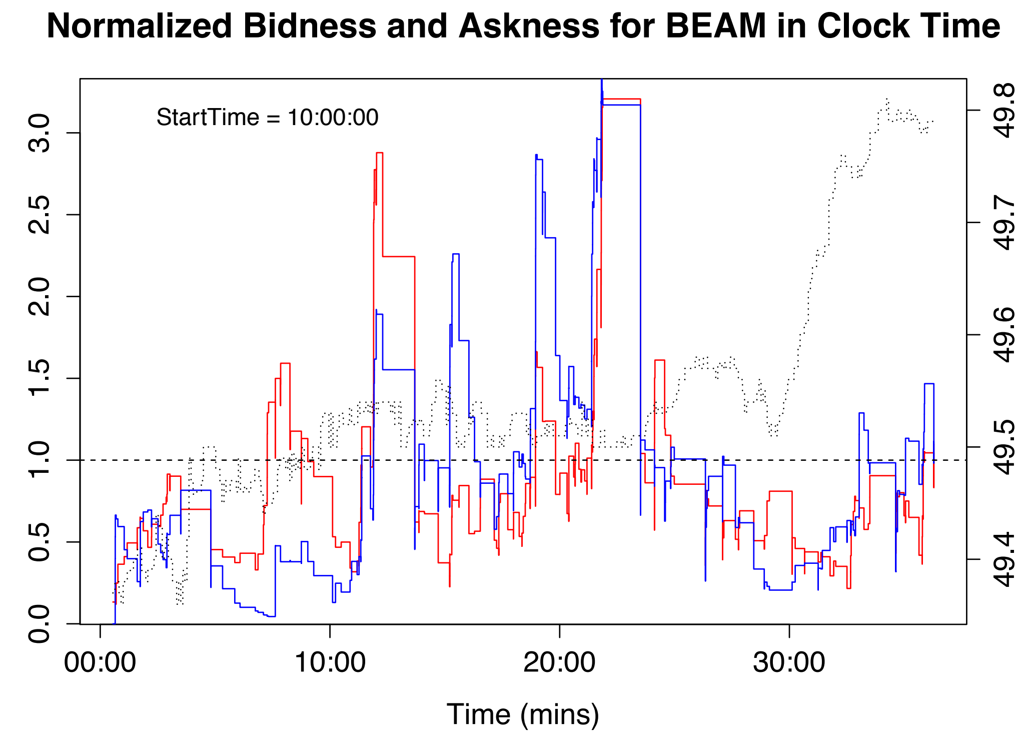
Algorithmic Trading Quantitative Research
Bank of America Merrill Lynch

Specific Examples: Trade Sign Autocorrelation (IV)

What does the signal mean?

- If the order flow was balanced within the window $\tilde{\tau}_w$, then $\bar{A}_n \approx \bar{B}_n \approx 1$. Half of the trades should be BUY and half SELL.
- If we are posted on the bid side and $\bar{B}$ is high, there is a lot of SELL market orders, so we should increase our participation rate (response function in the trading system).





This is one of the signals used by the BAML Instinct® algorithm.

Michael G Sotiropoulos

Algorithmic Trading Quantitative Research
Bank of America Merrill Lynch

Bank of America Unveils Adaptive Implementation(TM) Algorithm for Less Liquid Stocks

Instinct(TM) Aims to Optimize Execution for Small and Mid-Cap Equities

NEW YORK, March 7 2007 /PRNewswire/ -- Bank of America today announced the launch of its latest algorithm, Instinct(TM), designed to optimize execution on stocks whose liquidity may be difficult to predict, including small and mid-cap equities. The algorithm is the ninth available through the company's Electronic Algorithmic Strategy Execution (EASE(TM)) suite of electronic trading tools.

Instinct represents a new breed of Adaptive Implementation algorithms, which analyze and react to market data in real-time during the course of the trade. Unlike earlier generations of algorithms that trade along fixed, predetermined trajectories, Instinct uses a proprietary real-time estimation of available liquidity to continuously modify its execution trajectory throughout the lifetime of the order. This technology is especially suited to less liquid small and mid-cap stocks, as it allows traders to identify and seize liquidity as soon as it becomes available, alleviating slippage.

"These smaller names have long been considered impossible to trade algorithmically," said Dr. Robert Almgren, managing director and head of Quantitative Strategies for Equities. "We believe Instinct will be an exceptional productivity tool for our clients, as it aims to reduce workload and mitigate execution costs by allowing traders to find liquidity."

"Adaptivity is the next frontier in algorithmic trading," Almgren added. "The ability to instantaneously change execution strategies during the course of a trade will give traders greater control over the outcome of that trade. Our aim is to provide algorithms that combine a solid quantitative foundation with the trader's intuition, so that we can continue to help facilitate best execution for our customers and satisfy their unique trading objectives."

Trade imbalance

$$S_t^{\mathrm{T}} = \frac{V_t^+ - V_t^-}{V_t^+ + V_t^-}$$

Based on trades during time T before t

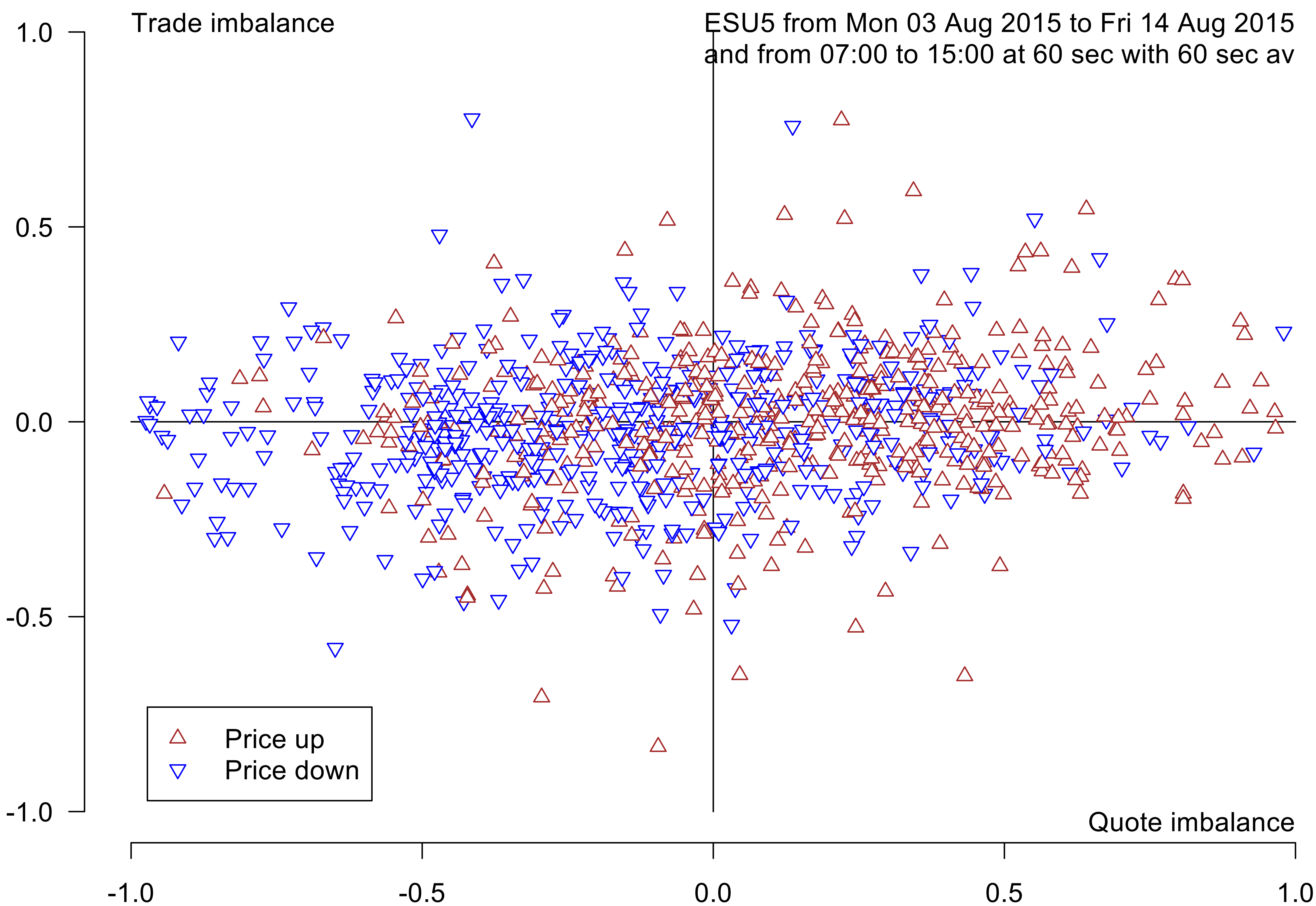
$$S_t^{\mathrm{Q}} = \frac{q_{\mathrm{B}} - q_{\mathrm{A}}}{q_{\mathrm{B}} + q_{\mathrm{A}}}$$

Based on instantaneous bid/ask quote sizes

Both combined may be better indicator of future quote changes than either alone

Trade sign in addition

1. What is likelihood of buy or sell conditional on quote imbalance?
2. Can trade imbalance help to predict direction and timing of next one-tick move?

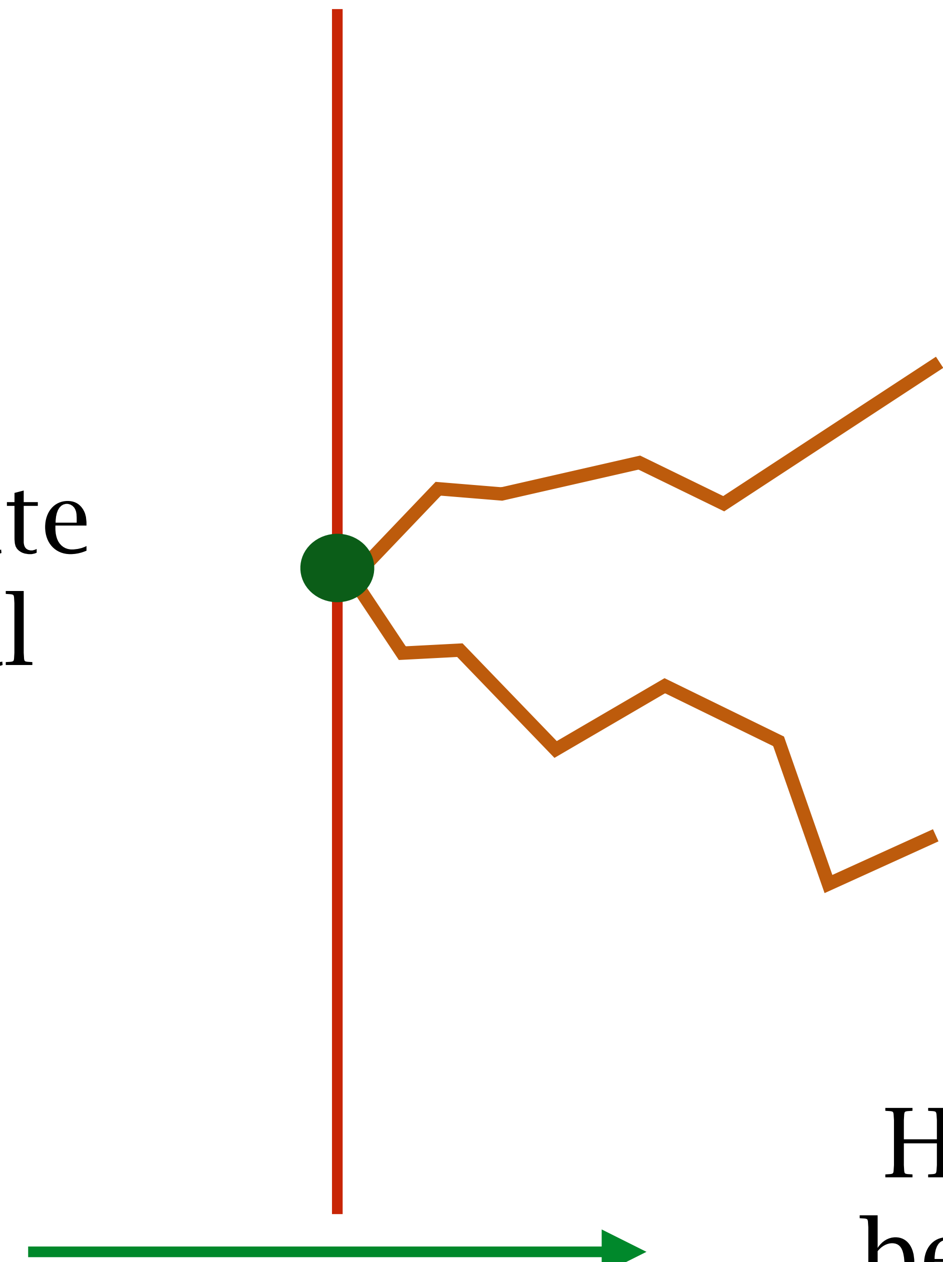

Trade imbalance
ESU5 from Mon 03 Aug 2015 to Fri 14 Aug 2015
and from 07:00 to 15:00 at 60 sec with 60 sec av
1.0
0.5
0.0
-0.5
-1.0
Price up
Price down
Quote imbalance
-1.0
-0.5
0.0
0.5
1.0

Machine learning problem

- Given two signals (quote imbalance, trade imbalance)
- What is the optimal function of both to predict price?
- "Separate red dots from blue dots"

Signal evaluation by forward correlation

Evaluate
signal
$s(t)$



Now

Predict
forward return
$r(t,\tau) = p(t+\tau) - p(t)$

$r(t,\tau)$ should be
correlated with $s(t)$

How does correlation
behave, in presence of
price and signal volatility?

Correlation and regression

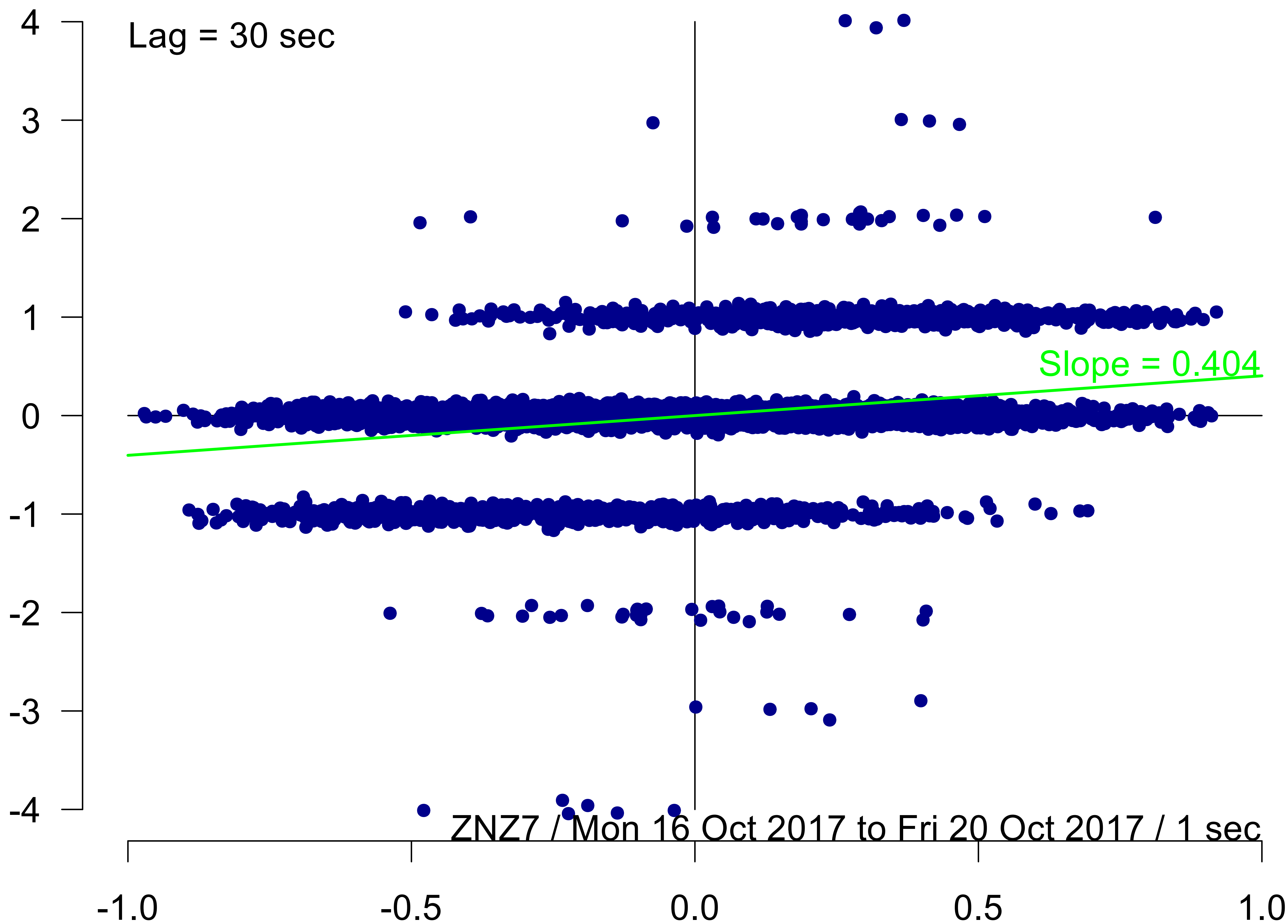
Nick Patterson

[30:06] "...I joined a hedge fund, Renaissance Technologies. I'll make a comment about that. It's funny that I think the most important thing to do on data analysis is to do the simple things right. So, here's a kind of non-secret about what we did at Renaissance: In my opinion, our most important statistical tool was simple regression with one target and one independent variable. It's the simplest statistical model you can imagine. Any reasonably smart high school student could do it. Now we have some of the smartest people around, working in our hedge fund, string theorists we recruited from Harvard, and they're doing simple regression. Is this stupid and pointless? Should we be hiring stupider people and paying them less? And the answer is no. And the reason is nobody tells you what the variables you should be regressing [are]. What's the target? Should you do a nonlinear transform before you regress? What's the source? Should you clean your data? Do you notice when your results are obviously rubbish? And so on. And the smarter you are the less likely you are to make a stupid mistake. And that's why I think you often need smart people who appear to be doing something technically very easy, but actually usually its's not so easy."

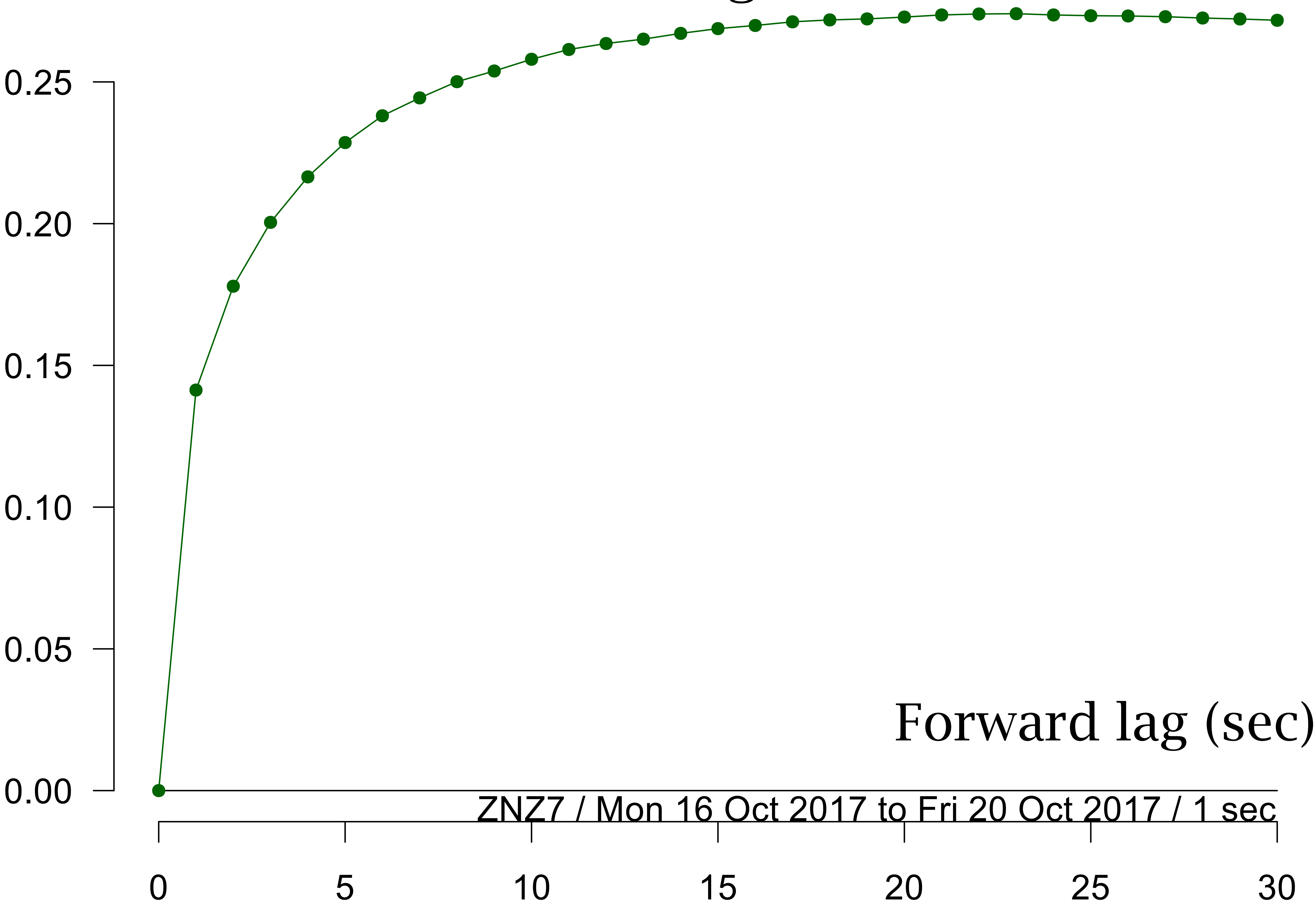
[38:03] "[at] my hedge fund, which was not a very big company, we had 7 Phd's just cleaning data and organizing the databases. That was major. If any of your listeners are thinking they can play the market by modeling financial data, the first thing I'll tell them is the data you can buy will be full of trash."

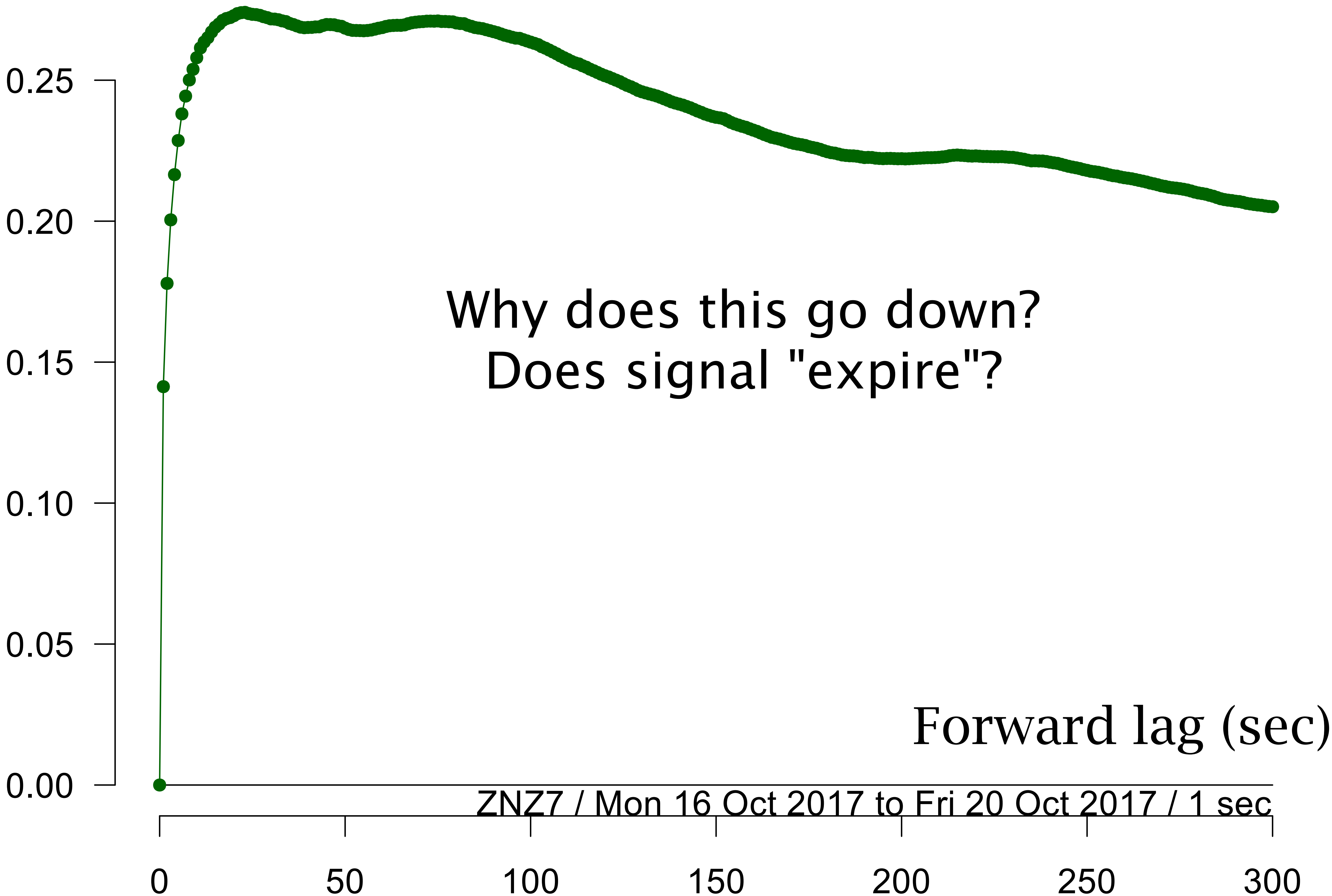
`http://www.thetalkingmachines.com/episodes/ai-safety-and-legacy-bletchley-park`

Forward correlation with imbalance signal



Correlation as function of lag





0.25
0.20
0.15
0.10
0.05
0.00
Why does this go down?
Does signal "expire"?
Forward lag (sec)
ZNZ7 / Mon 16 Oct 2017 to Fri 20 Oct 2017 / 1 sec
0
50
100
150
200
250
300

Relationship between data sets

(x_j, y_j) for $j = 1, \ldots, n$

Assume both are continuous
In practice, price changes are discrete

$$\langle \cdot \rangle = \frac{1}{n} \sum_{j=1}^{n} \cdot \qquad \bar{x} = \langle x \rangle \qquad \sigma_x^2 = \langle (x - \bar{x})^2 \rangle$$

$$\bar{y} = \langle y \rangle \qquad \sigma_y^2 = \langle (y - \bar{y})^2 \rangle$$

- Correction (1:1)

  descriptive statistic, easy to measure between x and y
- Regression (multi:1)

  use when predicting y in terms of one or more variables x
- Principal components (multi)

  determine linear relationship among multiple variables

Correlation vs regression

- Measure correlation of x vs y historically
- Use regression model to predict y in terms of x

Correlation: $$\rho = \frac{\mathrm{Cov}(x,y)}{\sqrt{\mathrm{Var}(x)\ \mathrm{Var}(y)}} = \frac{\big\langle (x-\bar{x})(y-\bar{y}) \big\rangle}{\sqrt{\sigma_x^2\,\sigma_y^2}}$$

ρ is nondimensional

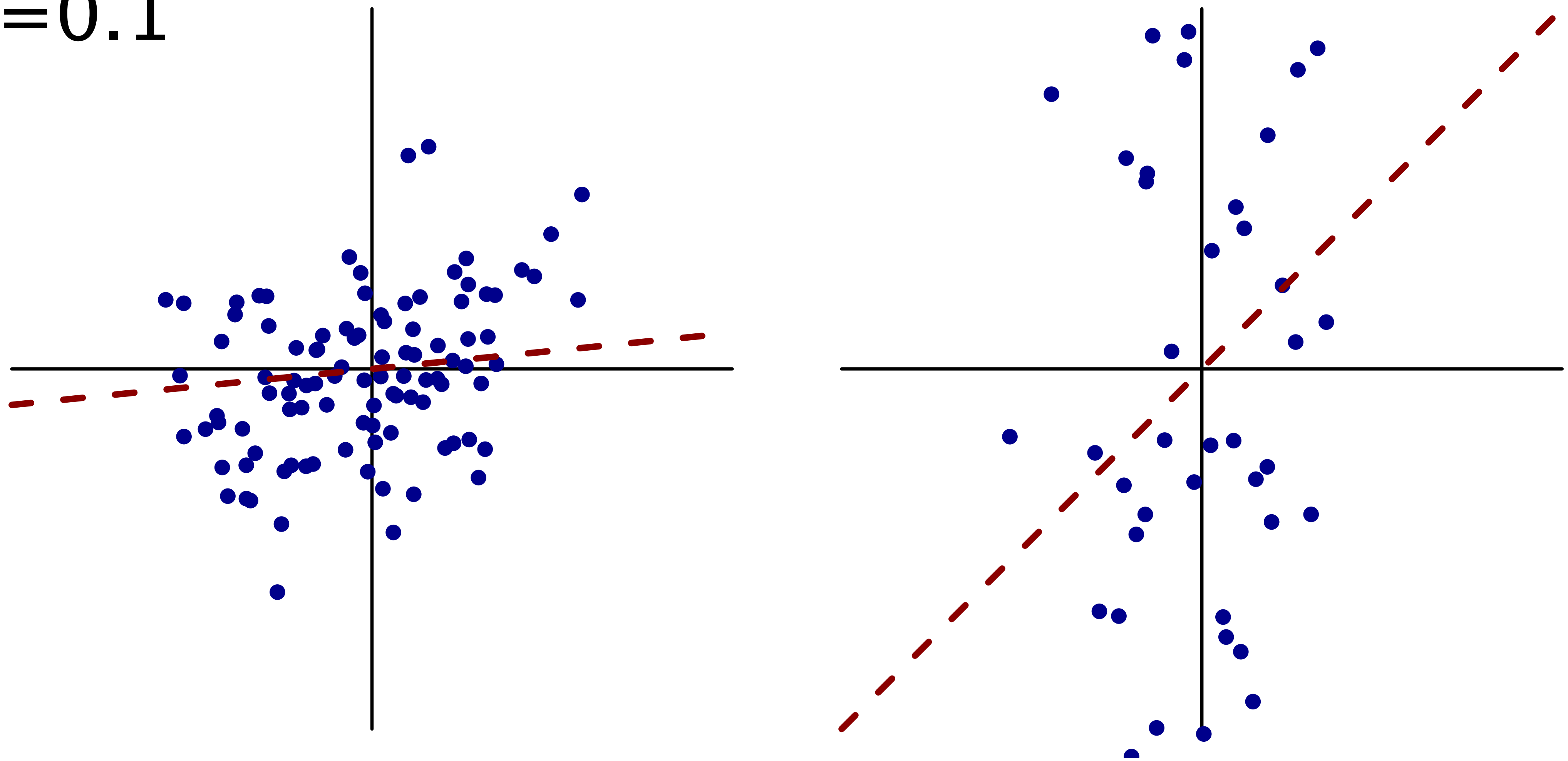
Regression: $$y_j = \beta x_j + \alpha + \epsilon_j.$$

β has units of y/x

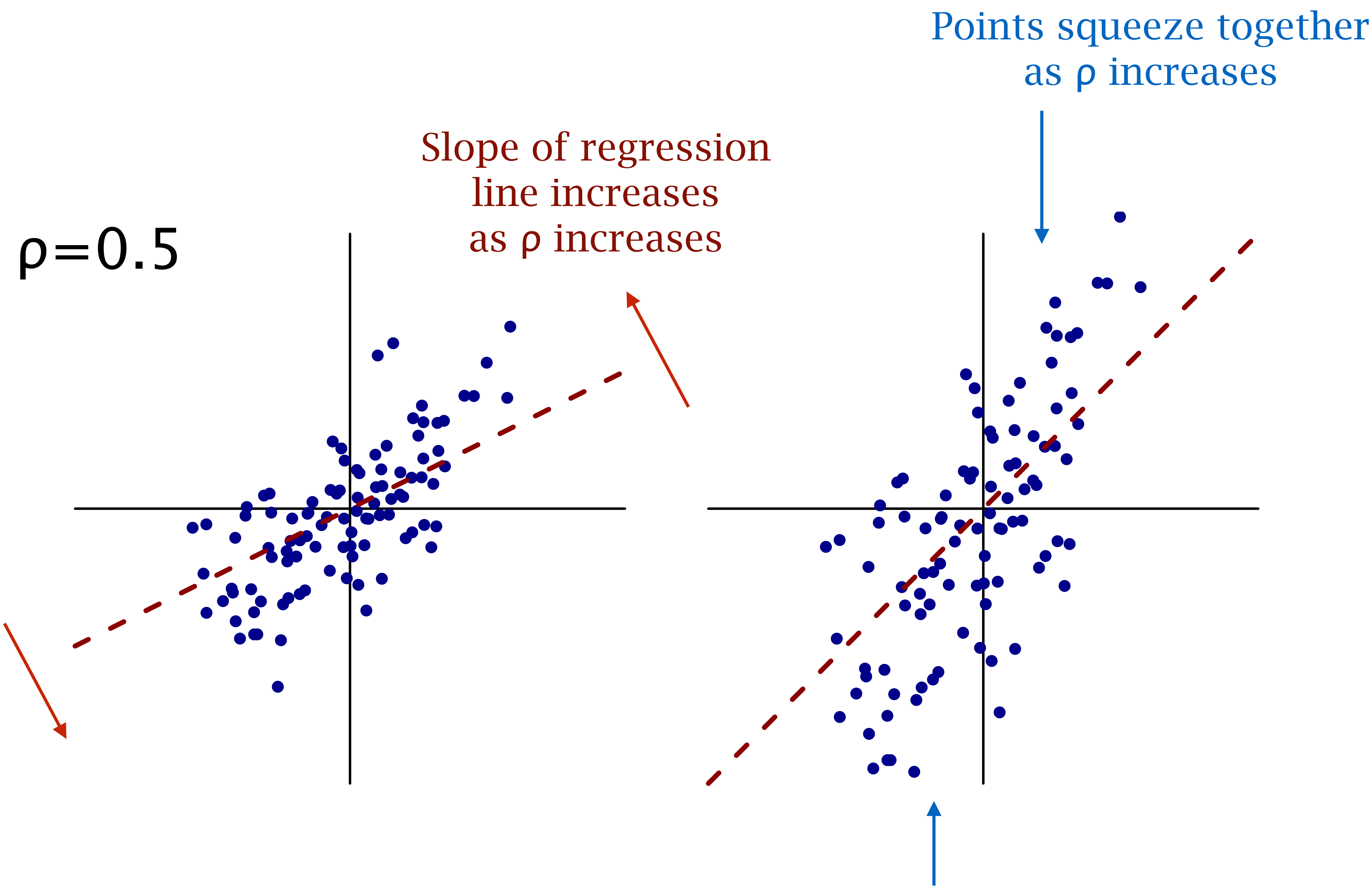
$$\rho = \frac{\sigma_x}{\sigma_y}\beta$$

$$R^2 = 1 - \frac{\mathrm{Var}(\epsilon)}{\mathrm{Var}(y)} = 1 - \left(1 - \frac{\langle xy\rangle^2}{\langle x^2\rangle\langle y^2\rangle}\right) = \rho^2$$

ρ=0.1

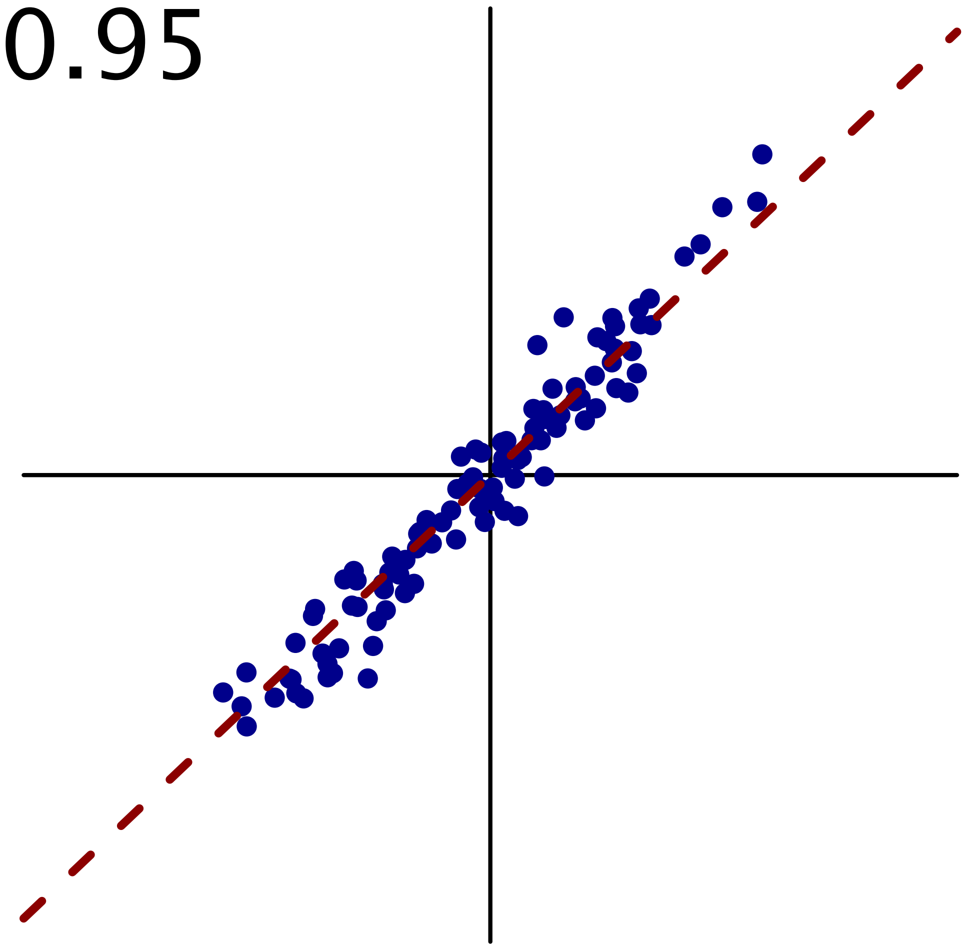


$$\sigma_y = \sigma_x \qquad\qquad \beta = 1$$

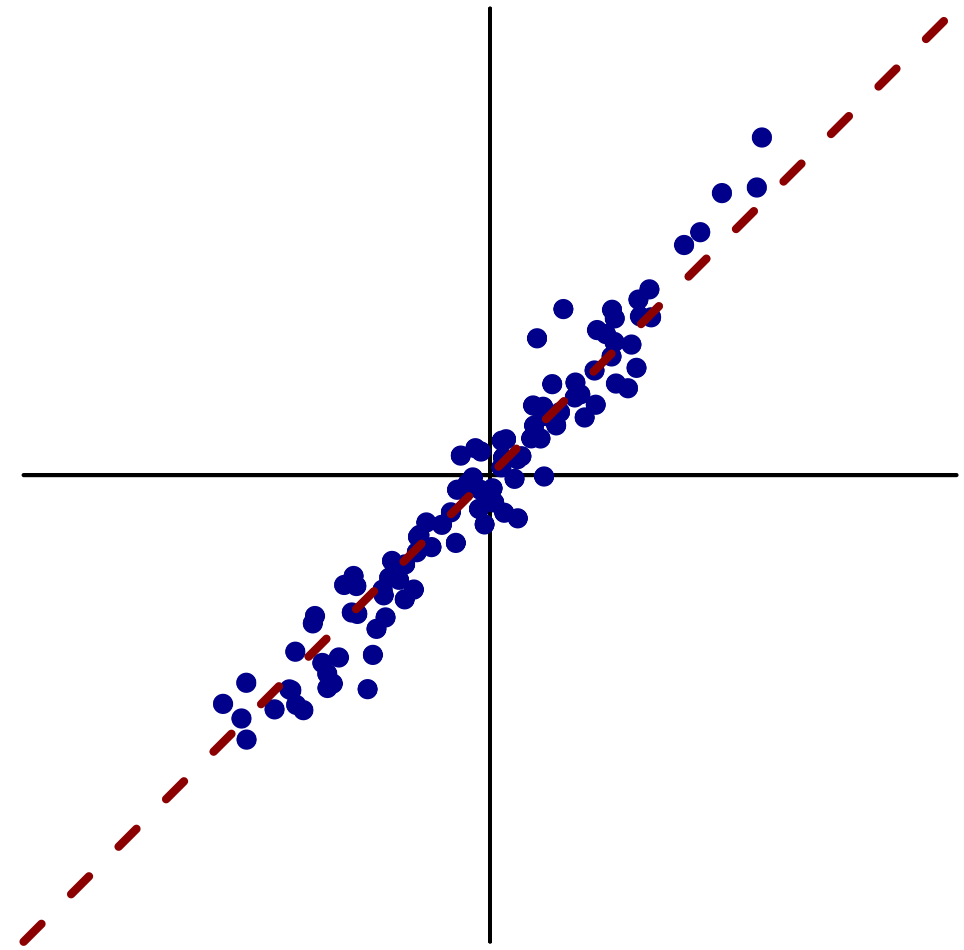


$$\sigma_y = \sigma_x \qquad \beta = 1$$

ρ=0.95



$\sigma_y = \sigma_x$



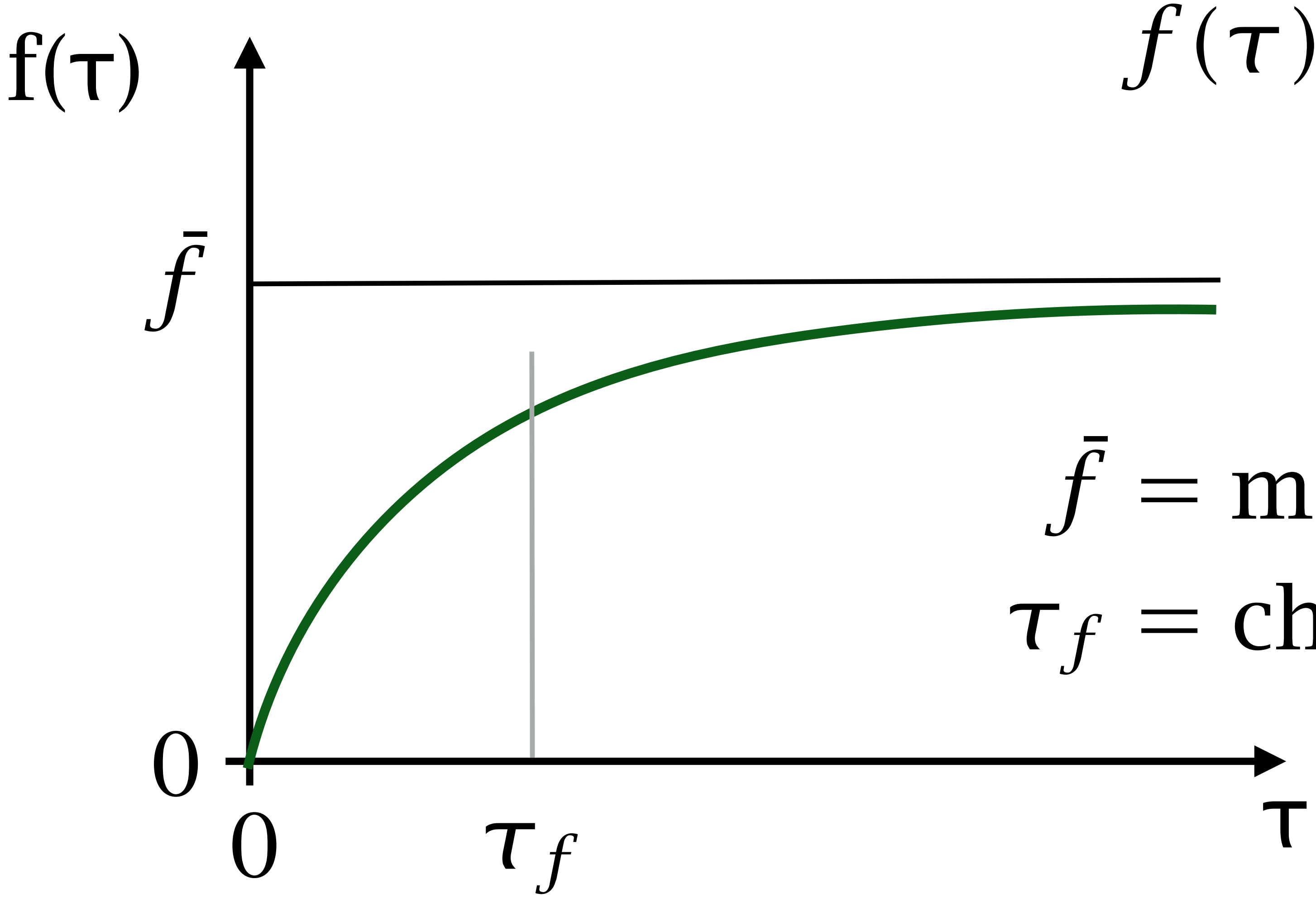
$\beta = 1$

A specific model for signal

$$r(t,\tau) \;=\; f(\tau)\, s(t) \;+\; \sigma\, B(t)$$

Example:

$$f(\tau) \;=\; \bar{f}\,\left(1 - e^{-\tau/\tau_f}\right)$$



$\bar{f}$ = max return for unit s

τ_f = characteristic time

Assume (unconditional moments)

$$\mathbb{E}[\, s(t)\,] = 0 \qquad \mathrm{Var}[\, s(t)\,] = \sigma_s^2$$

Conditional on s(t)

$$\mathbb{E}[\, r(t,\tau) \mid s(t)\,] \;=\; f(\tau)\, s(t)$$

$$\mathrm{Var}[\, r(t,\tau) \mid s(t)\,] \;=\; \sigma^2 \tau$$

Unconditional

$$\mathbb{E}[\, r(t,\tau)\,] \;=\; 0$$

$$\mathrm{Var}[\, r(t,\tau)\,] \;=\; f(\tau)^2 \sigma_s^2 \;+\; \sigma^2 \tau$$

$$\mathrm{Cov}[\, r(t,\tau),\ s(t)\,] \;=\; \mathbb{E}[\, r(t,\tau)\, s(t)\,] \;=\; f(\tau)\,\sigma_s^2$$

$$\rho(\tau) \;=\; \frac{f(\tau)\,\sigma_s}{\sqrt{f(\tau)^2\,\sigma_s^2 \;+\; \sigma^2\,\tau}} \;=\; \frac{1}{\sqrt{1 \;+\; \dfrac{\sigma^2\,\tau}{\sigma_s^2\, f(\tau)^2}}}.$$

Decreases with t

$$\beta \;=\; \frac{\text{standard deviation of } r}{\text{standard deviation of } s}\cdot \rho \;=\; f(\tau)$$

Approaches constant

Nondimensionalization

$$\bar{f} = \text{long-term saturation value of } f(\tau)$$
$$\tau_f = \text{time scale on which } f(\tau) \text{ comes in}$$

Example: $f(\tau) \;=\; \bar{f}\,(1 - e^{-\tau/\tau_f})$

Typical value of r: $\bar{r} = \bar{f}\sigma_s$

Volatility time scale: $\tau_\sigma \;=\; \left(\frac{\bar{r}}{\sigma}\right)^2 \;=\; \left(\frac{\bar{f}\,\sigma_s}{\sigma}\right)^2$

Time for volatility to be as important as signal

Nondimensional signal strength:

$$\kappa = \sqrt{\frac{\tau_\sigma}{\tau_f}} = \frac{\bar{f}\,\sigma_s}{\sigma\sqrt{\tau_f}}$$

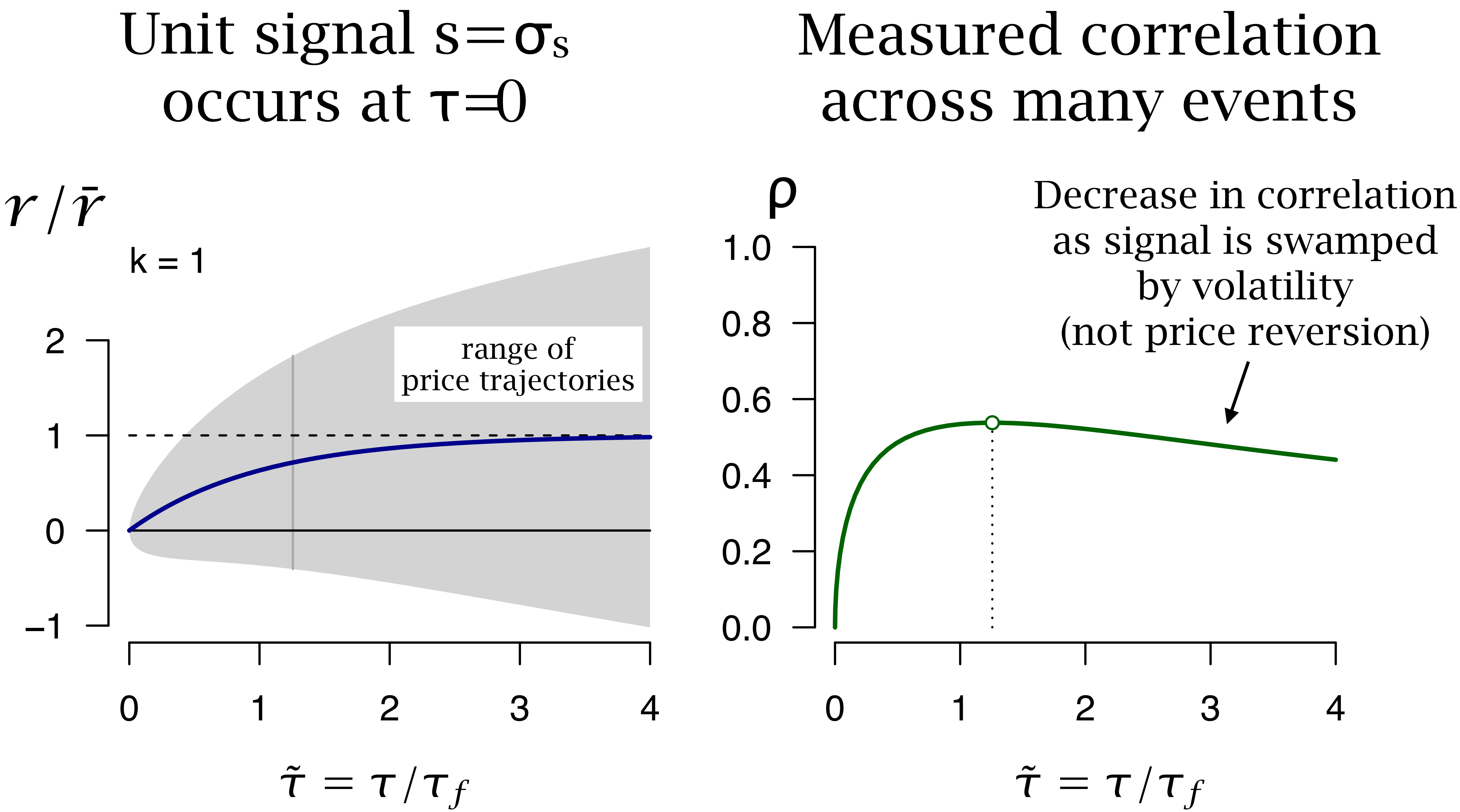
How large is return for average signal strength, compared to volatility on the time interval you have to wait to see that signal?

$$\tau = \tilde{\tau}\,\tau_f \qquad f(\tau) = \bar{f}\,\phi\left(\frac{\tau}{\tau_f}\right)$$

Observed correlation:

$$\rho(\tilde{\tau}) = \frac{1}{\sqrt{1 + \dfrac{1}{\kappa^2}\,\dfrac{\tilde{\tau}}{\phi(\tilde{\tau})^2}}}$$

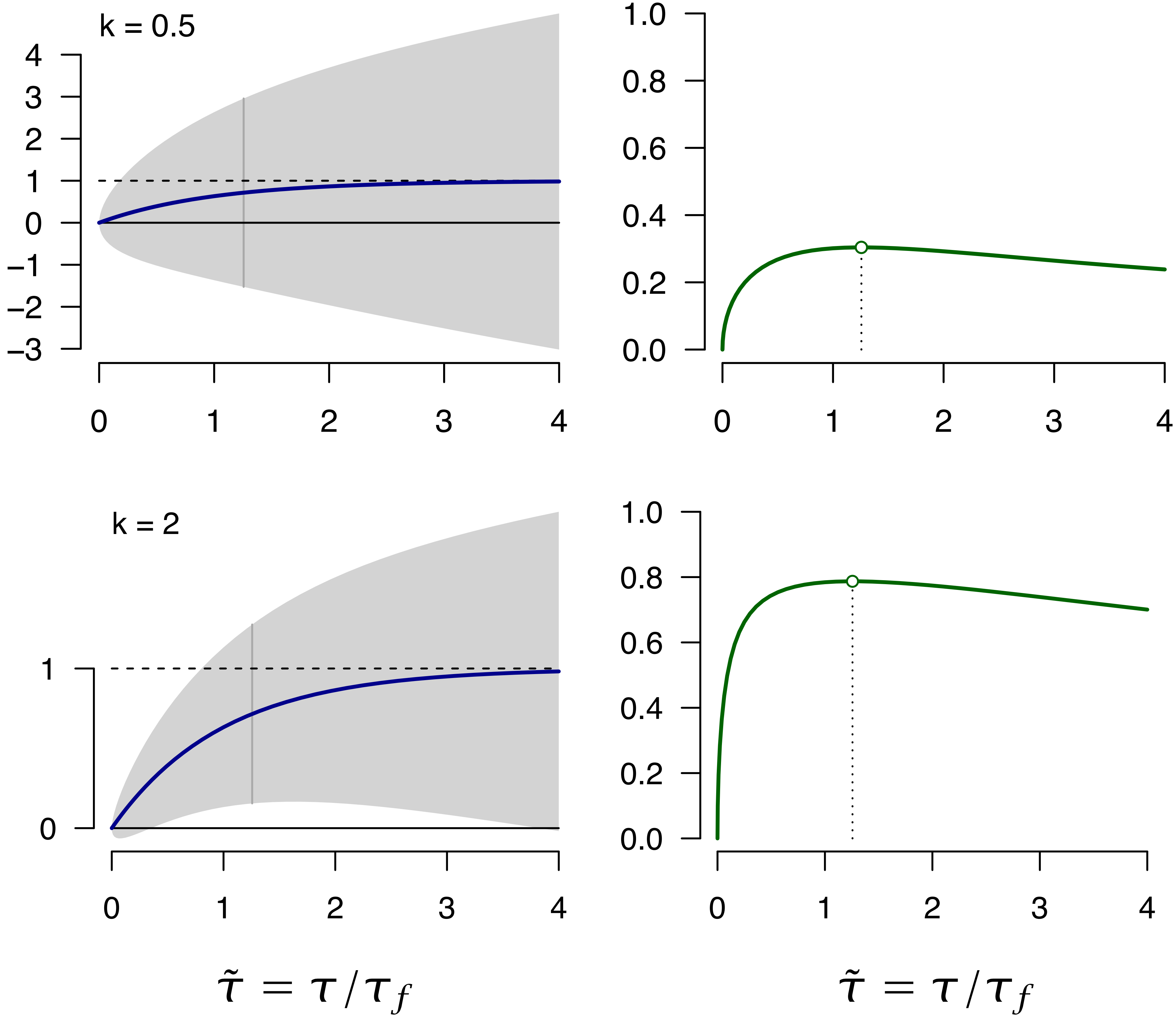
- Decreases to 0 as τ increases
- Peak depends on κ



Unit signal s=σs
occurs at τ=0
Measured correlation
across many events
$r\,/\,\bar{r}$
k = 1
2
1
0
–1
range of
price trajectories
0
1
2
3
4
$\tilde{\tau} = \tau/\tau_f$
ρ
1.0
0.8
0.6
0.4
0.2
0.0
Decrease in correlation
as signal is swamped
by volatility
(not price reversion)
0
1
2
3
4
$\tilde{\tau} = \tau/\tau_f$

Weaker signal

Stronger signal



Possible correlation paths