PRINCETON UNIVERSITY

ORF 474: High Frequency Trading
Spring 2020
Robert Almgren

Lecture 11a

April 20, 2020

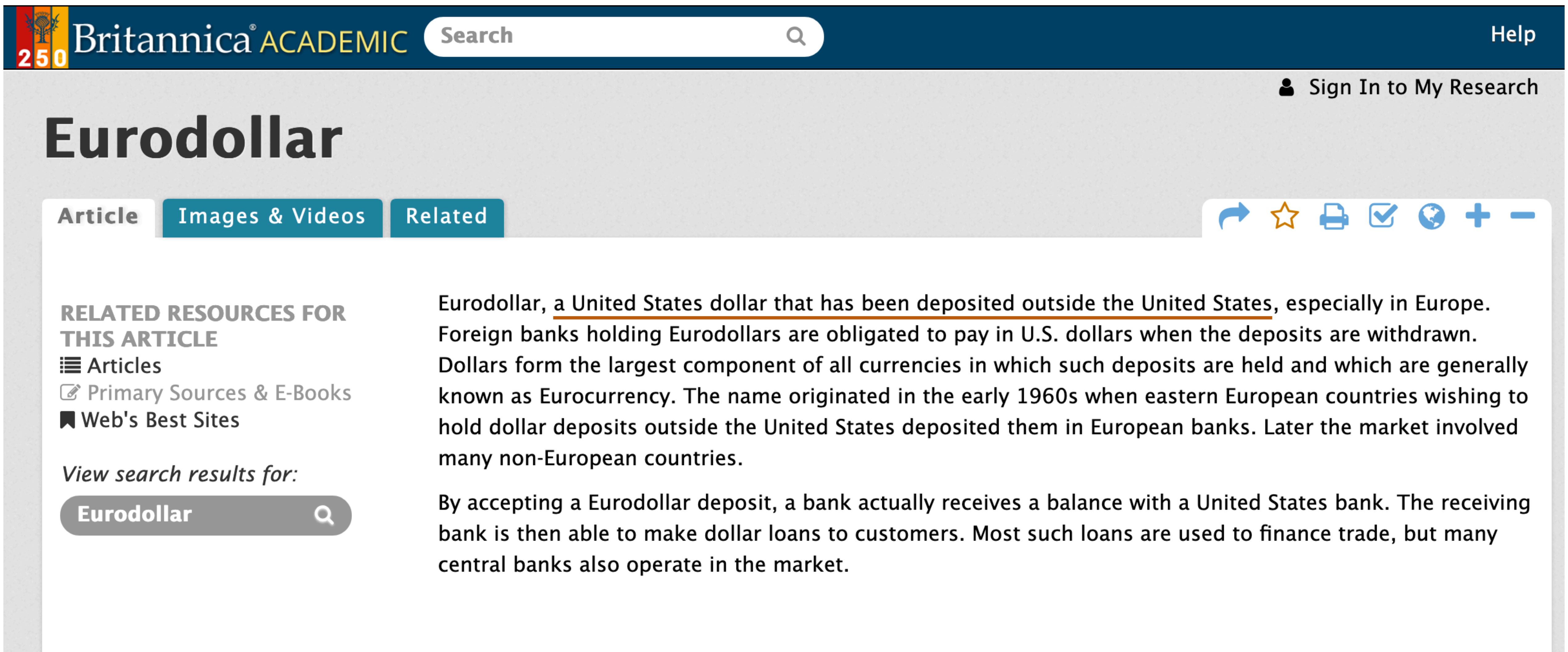
Overall course structure

1. Data and basic empirical facts (2 weeks)
2. Theoretical models for spread (4 weeks)
3. High frequency trading and price signals (4 weeks)
4. Market impact and trade optimization (2 weeks)

   Final "projects" (somewhat more free-form HW)

Eurodollars

1. What is a Eurodollar?
2. What is a Eurodollar future?
3. What is LIBOR?
4. What is replacing LIBOR?



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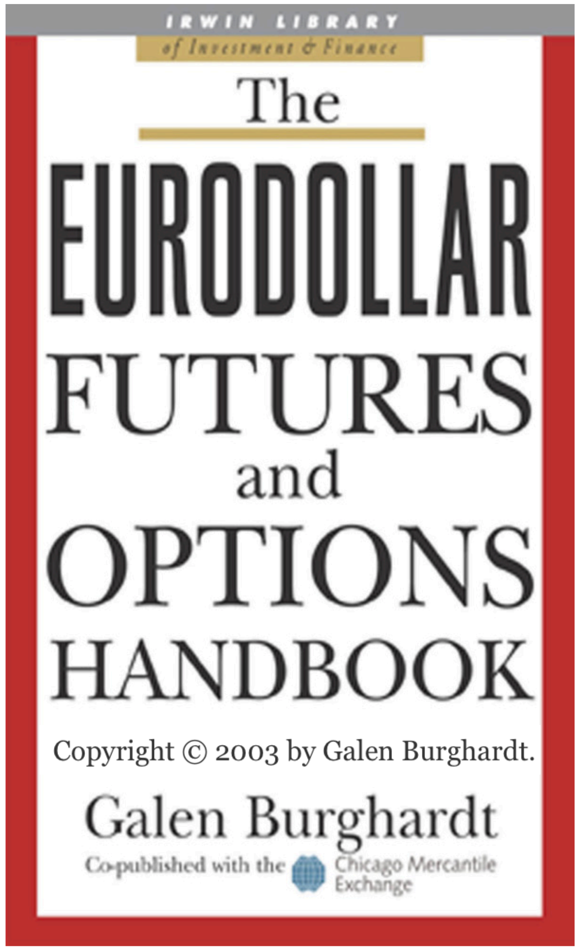
Eurodollar

Eurodollar, a United States dollar that has been deposited outside the United States, especially in Europe. Foreign banks holding Eurodollars are obligated to pay in U.S. dollars when the deposits are withdrawn. Dollars form the largest component of all currencies in which such deposits are held and which are generally known as Eurocurrency. The name originated in the early 1960s when eastern European countries wishing to hold dollar deposits outside the United States deposited them in European banks. Later the market involved many non-European countries.

By accepting a Eurodollar deposit, a bank actually receives a balance with a United States bank. The receiving bank is then able to make dollar loans to customers. Most such loans are used to finance trade, but many central banks also operate in the market.



CHAPTER 1
THE EMERGENCE OF THE EURODOLLAR MARKET

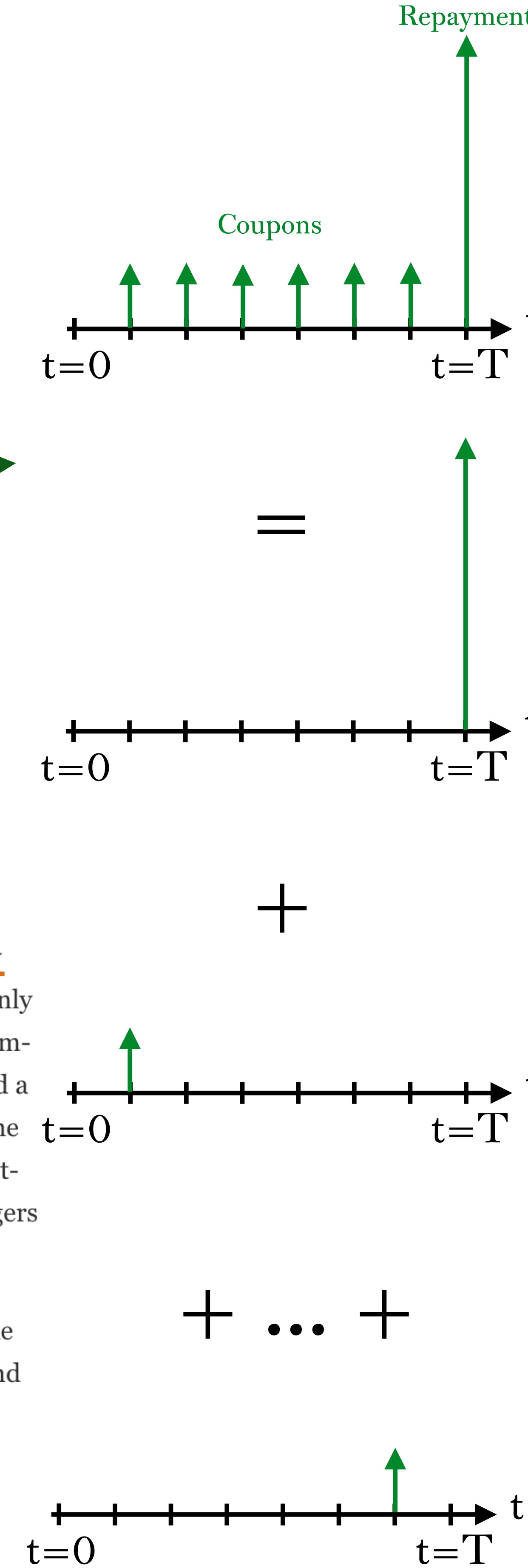
Galen Burghardt, Terry Belton, Morton Lane, Geoffrey Luce, and Rick McVey

The Eurodollar market—the market for dollar-denominated deposits outside of the United States—is perhaps the largest and most liquid of the world's short-term dollar markets. At the same time, swaps based on the London Interbank Offered Rate (LIBOR) and Eurodollar futures, together with their option counterparts, are without question the most liquid and actively traded money market derivatives. For that matter, the LIBOR-based swap market has become so large and liquid that swap rates have largely displaced government bonds as the standard of value against which fixed income instruments are compared.

Two innovations in applied finance matter most for the readers of this book. First was the idea that each cash flow associated with a coupon-bearing bond not only could, but should, be treated as a zero-coupon bond. This insight allowed financial engineers to look at any given bond as simply a collection of zeros, each of which could be valued separately and consistently. Second was the idea that one could trade the price of a commodity without trading the commodity itself. This insight allowed the market to focus in on price or interest rate risk and to develop interest rate derivatives—swaps, forward rate agreements, and futures on bank deposit rates and government bonds. It was interest rate derivatives that finally allowed bankers to understand their interest rate risk and to manage it cheaply and effectively.

The futures market pioneered the idea that one could trade the price separately from the commodity itself. Economists have argued for years that a commodity possesses several useful characteristics, only one of which is its price. And once one could trade the price without actually buying or selling the commodity, the costs of hedging and speculating dropped like a stone. The futures market also pioneered a set of risk management practices that have slowly taken hold in other parts of the financial world. The ideas of requiring collateral to guaranty performance, marking positions to market every day, and settling up gains and losses in cash on a regular basis have had a wonderfully tonic effect on risk managers outside of the futures world.

The idea of financial futures took hold in the early 1970s. Although futures had been traded on a wide range of metals and agricultural commodities since the middle of the 19th century in this country, and had a history reaching back several centuries in other parts of the world, the idea of trading futures contracts on things like foreign currencies and interest rates at first struck people as outrageous.



LIBOR

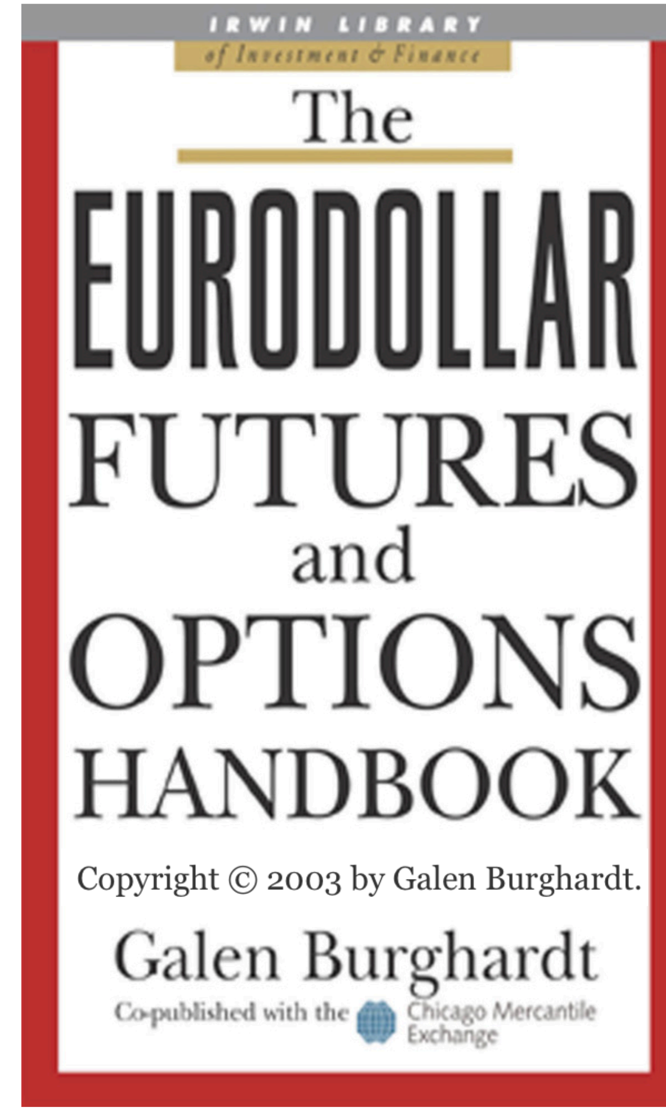
The device that protected the Eurodollar contract from the forces that brought the CD contract down was cash settlement. Initially, the final settlement price on the last day of trading for a Eurodollar contract was determined by the CME, which conducted a poll of banks in London. The CME's survey had two important features:

• The CME asked each bank its perception of the rate at which banks in London were willing to lend to other prime quality banks. The CME avoided in this question asking a bank the rate at which it was lending to any other particular bank.

• The CME threw out high and low responses and calculated the final settlement price using the middle quotes.

The effect of the first of these features was that the survey skirted the problem of individual bank credits. The intended effect of the second was to insulate the final settlement price from capricious manipulation. A bank that misrepresented the market outrageously would have no effect on the final outcome. As a result, the banks polled in the CME's survey likely responded truthfully, although it would be naïve to suppose that their responses never reflected their positions in the cash and futures markets. [1]

1 . Today, the Eurodollar futures final settlement price is based on a survey conducted by the British Bankers' Association of 16 reference banks in London. The banks are asked at what rate they could borrow U.S. dollars for a 3-month term. The responses are placed in rank order, and then the middle two quartiles are arithmetically averaged. The final settlement price is 100 minus this average rate, rounded to 4 decimal places.



Commodity Futures Trading Commission
Office of Public Affairs
Three Lafayette Centre
1155 21st Street, NW
Washington, DC 20581
www.cftc.gov

Examples of Misconduct from Written Communications

Examples of False Reports and Attempts to Manipulate U.S. Dollar LIBOR

March 22, 2005: (emphasis added)

U.S. Dollar LIBOR Submitter:	**if you need something in particular in the libors i.e. you have an interest in a high or a low fix let me know and there's a high chance i'll be able to go in a different level. just give me a shout the day before or send an email from your blackberry first thing.**
New York U.S. Dollar Trader 1:	Thanks - our CP guys have been looking for it a bit higher - not a big deal
U.S. Dollar LIBOR Submitter:	if anything the cash has actually cheapened up since yesterday too albeit by 1/2 tick - true could get some sub 75 days thru the next week

April 1, 2005: (emphasis added)

London U.S. Dollar Trader 1:	**COULD WE PLS HAVE A LOW 6MTH FIX TODAY OLD BEAN?**

September 21, 2005: (emphasis added)

London MMD Manager:	Subject: "$ LIBORS: 83, 89, 96 and 11 **LOWER MATE LOWER !!**
U.S. Dollar LIBOR Submitter:	will see what i can do but it'll be tough as the cash is pretty well bid
London MMD Manager:	**[Another U.S. Dollar Panel Bank] IS DOIN IT ON PURPOSE BECAUSE THEY HAVE THE EXACT OPPOSITE POSITION** - ON WHICH THEY LOST 25MIO SO FAR - LETS TAKE THEM ON!!
U.S. Dollar LIBOR Submitter:	**ok, let's see if we can hurt them a little bit more then**

Examples of False Reports and Attempts to Manipulate Euribor

July 10, 2005: (emphasis added)

London MMD Manager:	HI FRDS **ANY CHANCE TO PUSH UP YOUR CONTRIBUTION TO THE 3MTH EURIBOR FIX?**
Euribor Submitter:	HI [Euribor Submitter] **HERE USUALLY IT WOULD BE 11 ON OUR SIDE SO DO U REALLY NEED A 12 FOR TODAY AS DB CONTRIBUTION?**
London MMD Manager:	EONIA AT 2.068 AND O/N TRADING 2.08 IT WUD MAKE SENSE TO HAVE A HIGHER 3MTH FIX. **WE SHORT A LOT OF JUNES ABOUT 40000 LOTS**
Euribor Submitter:	**OK WE WILL CONTRIBUTE A 12 FOR TODAY AND MONDAY** HAVE A NICE WEEKEND
London MMD Manager:	THX A LOT [. . .]

July 6, 2006: (emphasis added)

Frankfurt Euro Desk Manager:	HIHI [London MMD Manager], **I JUST WANT TO CHECK WHETHER WE HAVE CONFLICTING INTERESTS IN THE JUNE 06 SETTLEMENT. IT DOESN'T MAKE SENSE IF WE TRY TO PUSH ONE WAY AND U WLD LIKE TO HAVE IT THE OTHER WAY AROUND. WE WLD PREFER A LOW 3ME FIXING TO PUSH JUNE06 HIGH. IS THIS UR PREFERENCE AS WELL?**
London MMD Manager:	**THX VM FOR CHECKING** [Frankfurt Euro Desk Manager] - **YES WE WOULD PREFER A LOW FIXING AS WELL**
Frankfurt Euro Desk Manager:	THX [London MMD Manager], **THAT WILL MAKE US MORE POWERFUL IN PUSHING THE FIX WE WANT IT.**

http://www.cftc.gov/ucm/groups/public/@newsroom/documents/file/writtencommunication_deutsche.pdf

Cybersecurity

Libor, Phone-Smashing and Hacking

Also Martin Shkreli, GSE litigation, intraday wealth and blockchain for barter.

By Matt Levine
July 28, 2017, 1:42 PM UTC

Pascal-Emmanuel Gobry responded with a thoughtful Twitter thread about Libor, society and the nature of finance, "which is that all of finance is essentially a gentleman's agreement (that's what a vanilla loan is) and that the agreement matters more than what the agreement is about."

Of course, finance is a set of social conventions because society is a set of social conventions:

> In a sense, the thing about finance isn't that it's this completely made-up thing, it's that it's a completely made-up thing that has a self-correcting mechanism of turning discoverers of fictions that have become too systemically dangerous (which is not the same as "fictions that have become too inaccurate") into billionaires. In that sense, the story of Libor is the perfect finance story. The first layer is that Libor was fake because finance guys are evil, which is often true but not the most important thing. Then the second layer was that Libor was fake for a much more interesting reason. Then the third layer was that it was good that Libor was fake. In this layer lies Enlightenment.

Still, there is a sense in which a "fake" Libor based on surveying banks about their unobservable cost of unsecured short-term borrowing might be better than a "real" one based on transaction data.

https://www.bloomberg.com/opinion/articles/2017-07-28/libor-phone-smashing-and-hacking

Search Bloomberg Opinion

Finance

Libor's Replacement Is a Little Too Real

That means you have to live with the ups and downs of the market.

By Matt Levine
February 13, 2019, 12:00 PM UTC

In actual fact, Libor dealt with this problem in sort of a clever way. Instead of measuring the cost at which banks could borrow, Libor just hypothetically asked them the cost at which they could borrow, and the banks made it up. In normal times, they made up numbers that were pretty close to the rate at which they could borrow. But in 2008, when there was a banking crisis, they made up much lower numbers, because if they said the real high numbers then people would further lose confidence in them.

If your goal is to come up with a regular interest rate, and that rate is 2.46 percent one day, and economic conditions don't change much, it should be 2.45 or 2.46 or 2.47 percent the next day. Big jumps would seem bad. On the other hand, if your goal is to reflect actual reality in a market with many real transactions, then you sort of have to live with the supply and demand dynamics in that market, and sometimes those dynamics reflect the specific technical details of the market participants' businesses rather than some broad abstract notion of the regular interest rate.

https://www.bloomberg.com/opinion/articles/2019-02-13/libor-s-replacement-sofr-is-a-little-too-real

Market impact and trade optimization

- Purpose for trading
  - capture alpha (profit)
  - execute for portfolio manager or external client
- Execution has a cost
  - bid-ask spread (passive fill probability)
  - impact of your trades on market
    - buying pushes price up, getting worse price on remainder
- Determine strategy to achieve best result
  - how large can your assets under management be?
  - what will it cost if that client liquidates? (margin reqts)
  - what size alpha can you capture?
  - how do you structure trades across time?

Ingredients

- How do trades affect market?
- What do you want to achieve?
- What mathematical tools to determine optimum?

Market impact

- "Micro" impact
  - response to individual actions in market
    - market orders, limit orders, cancellation
  - in theory: fundamental facts about market dynamics
  - very affected by endogeneity
    - decision to trade depends on anticipated price motion
- "Macro" impact
  - cost of executing "meta-order" or "parent order"
  - ad hoc estimates for trading purposes
  - impossible without private data

Why is there price impact?

- Consume liquidity (mechanical effect)
- Trades transmit information

Modeling Information Flow in Sequential Double Auctions

Wesley Yuan

Advised by Dr. Robert Almgren

April 15, 2020

Modelling Flash Crashes: A Contagion Approach

Shrishti Asthana

Advisor: Professor Robert Almgren

Applications of impact models

- Pure science: dynamics of markets

  What is the micro response to an action?

  What are optimal trading trajectories?

- Trade cost estimation for portfolio construction

  How much will this trade cost us?

  Is the expected alpha enough to compensate?

- Capacity determination

  How large can our fund grow?

- Margin requirements

  How much will it cost if that client liquidates?

Issues in market impact: Buy vs sell

- "Cost of trading:" people trade to make money
- Every trade has a buyer and a seller
- Which side is market impact?
- Who pays market impact to whom?
- Two possible answers:
  - aggressive trade pays impact
    - only possible approach for micro impact with public data
  - "people like me" pay impact to others
    - typical for macro impact with private trade data

Newedge

Research Snapshot

NEWEDGE PRIME BROKERAGE | INVESTOR RESEARCH

March 30, 2011

Robert Almgren
robertalmgren@quantitativebrokers.com

Galen Burghardt
galen.burghardt@newedge.com

A window into the world of futures market liquidity

The purpose of this snapshot is to call attention to an interesting data set maintained by the Chicago Mercantile Exchange (CME) that affords a unique insight into futures trading costs. As brokers, we use this data to help understand transactions costs and to keep them as low as possible for our clients.

The CME microstructure data allows us to conclude two things. First, those traders whom we traditionally think of as liquidity takers do in fact pay for access to the pool of liquidity afforded by the exchange. Second, the net price paid for liquidity is remarkably small given the size of the bid/ask spread. In this example, which highlights trading in 10-year Treasury note futures, we find that the average price paid by "liquidity takers" is about $3 per contract per round turn, while the value of the bid/ask spread is just over $15.

CTI 1 – An individual member trading for his or her own account, a "local,"

CTI 2 – A member firm trading for its own proprietary account,

CTI 3 – A member firm trading for another member (very little volume), and

CTI 4 – All other participants.

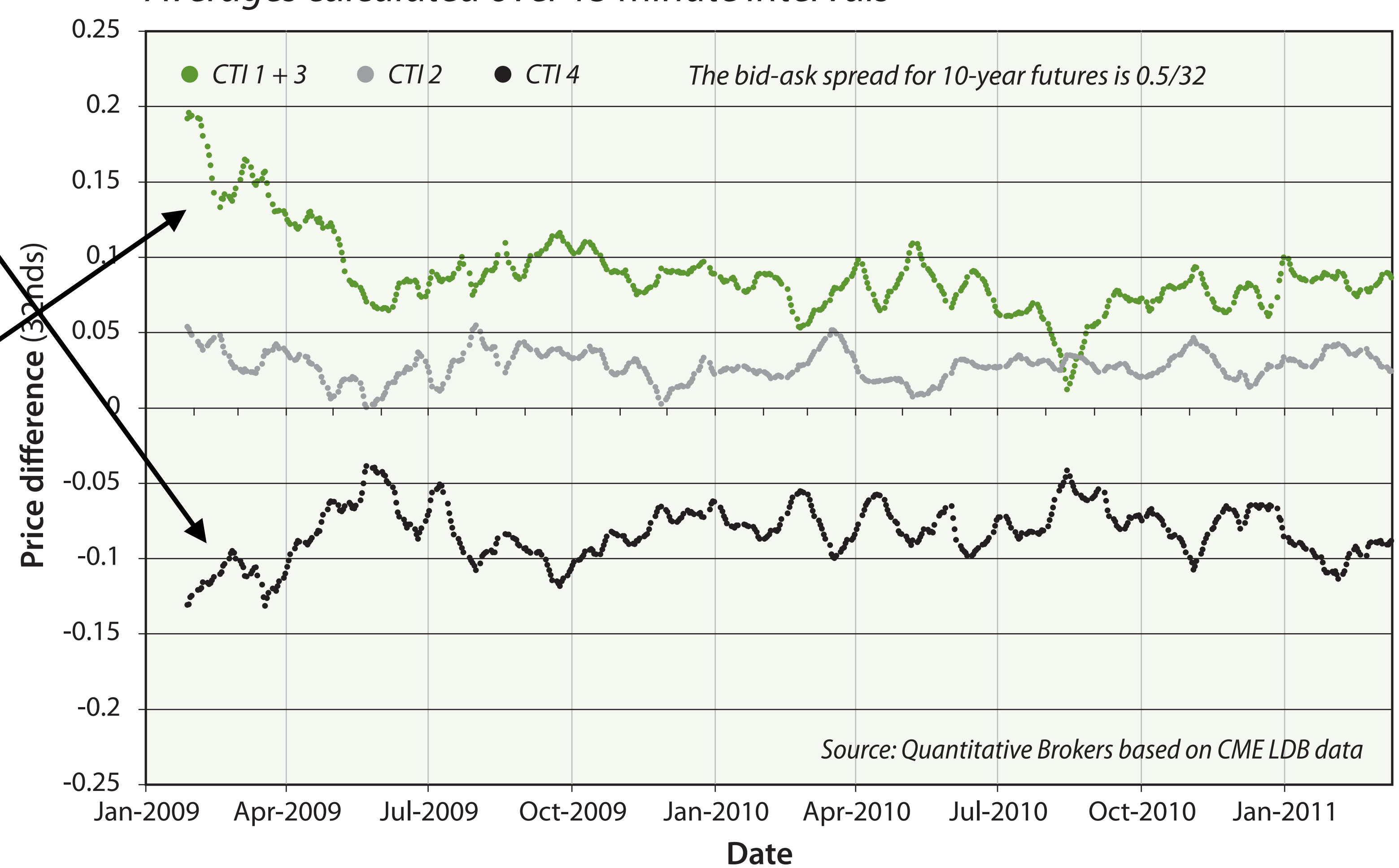
Cost of trading for external participants

... is paid to market makers (member firms)

Exhibit 2

Difference between average sell price and average buy price

Averages calculated over 15 minute intervals



Issues in market impact: Impact vs alpha

You anticipate price increase

↓

You enter a buy order to profit from increase

↓

Price goes up

Impact or alpha?

No action in market is independent
of what came before, nor
of what is expected to come after.

Components of micro impact

- Observed impact = Reaction + prediction
- Reaction impact (pure impact):
  impact of a market order with no information

  very expensive to measure empirically!
- Prediction impact (alpha):
  predicted price motion independently of order
  at times when would submit an order

Issues in market impact: ad hoc models

- What happened last time we did this?
- Avoid disentangling impact and alpha
  - predicted cost includes both
- Impact model may be different
  - for different clients
  - for different strategies (short vs long-term)

Issues in market impact: Multiple assets

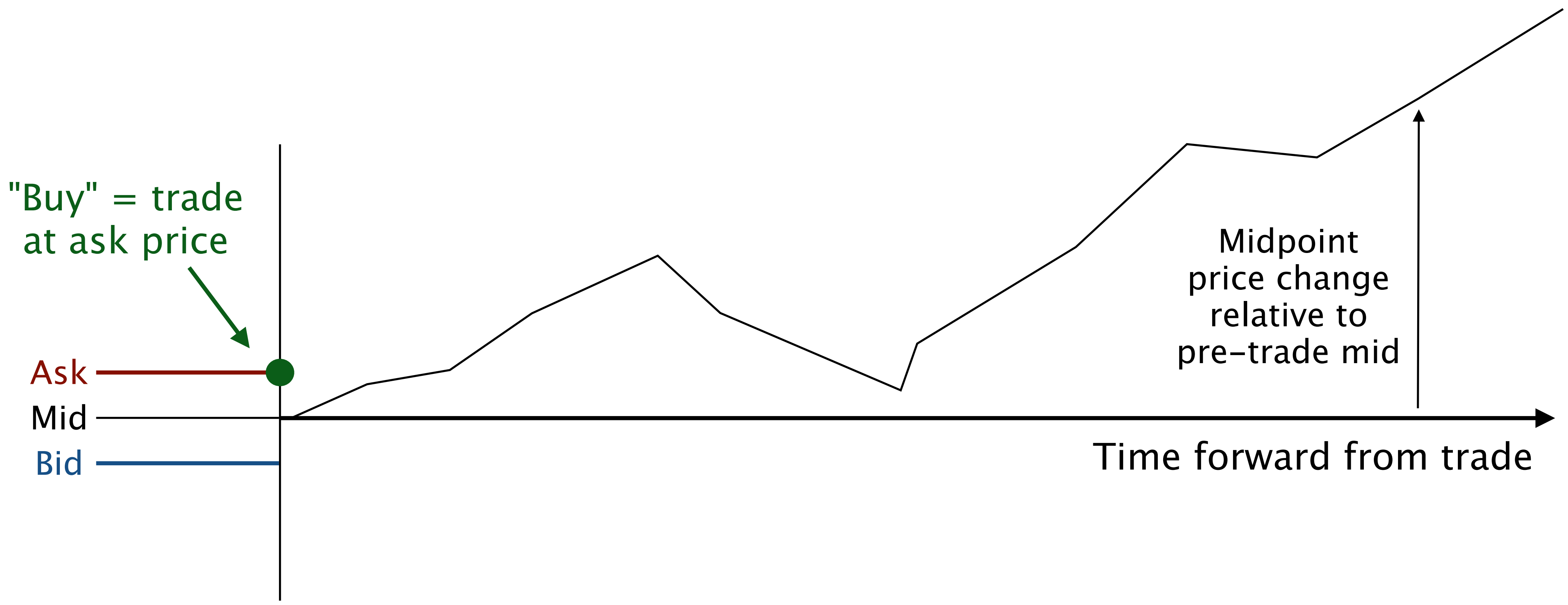
- Many assets are closely coupled
  correlated stocks
  notes/bonds at different durations
  futures on same underlying at different maturities
  futures / cash / swaps
- Trading one affects prices of all
  cross impact modeling
- Impact on one requires impact on all
  what fraction of daily volume are you trading?

Micro impact

trade sign (of market order)
trade size

→

mean price change
as function of time
or number of trades



Expect price to move **up** following a buy, **down** following a sell, on average

Broad recent literature

RESEARCH PAPER
quant.iop.org

QUANTITATIVE FINANCE VOLUME 4 (2004) 176–190
INSTITUTE OF PHYSICS PUBLISHING

Fluctuations and response in financial markets: the subtle nature of 'random' price changes

Jean-Philippe Bouchaud[1,2], Yuval Gefen[3], Marc Potters[2] and Matthieu Wyart[1]

There are two data files for each stock: one gives the list of all successive *quotes*, i.e. the best buy (bid, b) and sell (ask, a) prices, together with the available volume, and the time stamp accurate to the second. A quote can change either as a result of a trade, or because new limit orders appear, or else because some limit orders are cancelled. The other data file is the list of all successive *trades*, with the traded price, traded volume and time stamp, again accurate to the second. Sometimes, several trades are recorded at the very same instant but at different prices: this corresponds to a market order of a size which exceeds the available volume at the bid (or at the ask), and hits limit orders deeper in the order book. In the following, we have grouped all these trades together as a single trade. This allows one to create chronological sequences of trades and quotes, such that between any two trades there is at least one quote.

The last quote before a given trade allows one to define the sign of each trade: if the traded price is above the last midpoint $m = (a + b)/2$, this means that the trade was triggered by a market order (or marketable limit order) to buy, and we will assign to that trade a variable $\varepsilon = +1$. If, on the other hand, the traded price is below the last midpoint $m = (a + b)/2$, then $\varepsilon = -1$. With each trade is also associated a volume V, corresponding to the total number of shares exchanged.

Trades appear at random times, the statistics of which is itself non-trivial (there are intraday seasonalities and also clustering of the trades in time). We will not be interested in this aspect of the problem and always reason in terms of trade time, i.e. time advances by one unit every time a new trade (or a series of simultaneous trades) is recorded.

Relevant textbook

TRADES, QUOTES AND PRICES

Financial Markets Under the Microscope

JEAN-PHILIPPE BOUCHAUD

Capital Fund Management, Paris

JULIUS BONART

University College London

JONATHAN DONIER

Spotify

MARTIN GOULD

Spotify

CAMBRIDGE
UNIVERSITY PRESS

2018

Trade sign

- If your market data has aggressiveness flag, use that
- If your market data does not have aggressiveness flag:
  - match trade price to prevailing bid/ask
    allowing for roundoff error
    tick test for midpoint trades

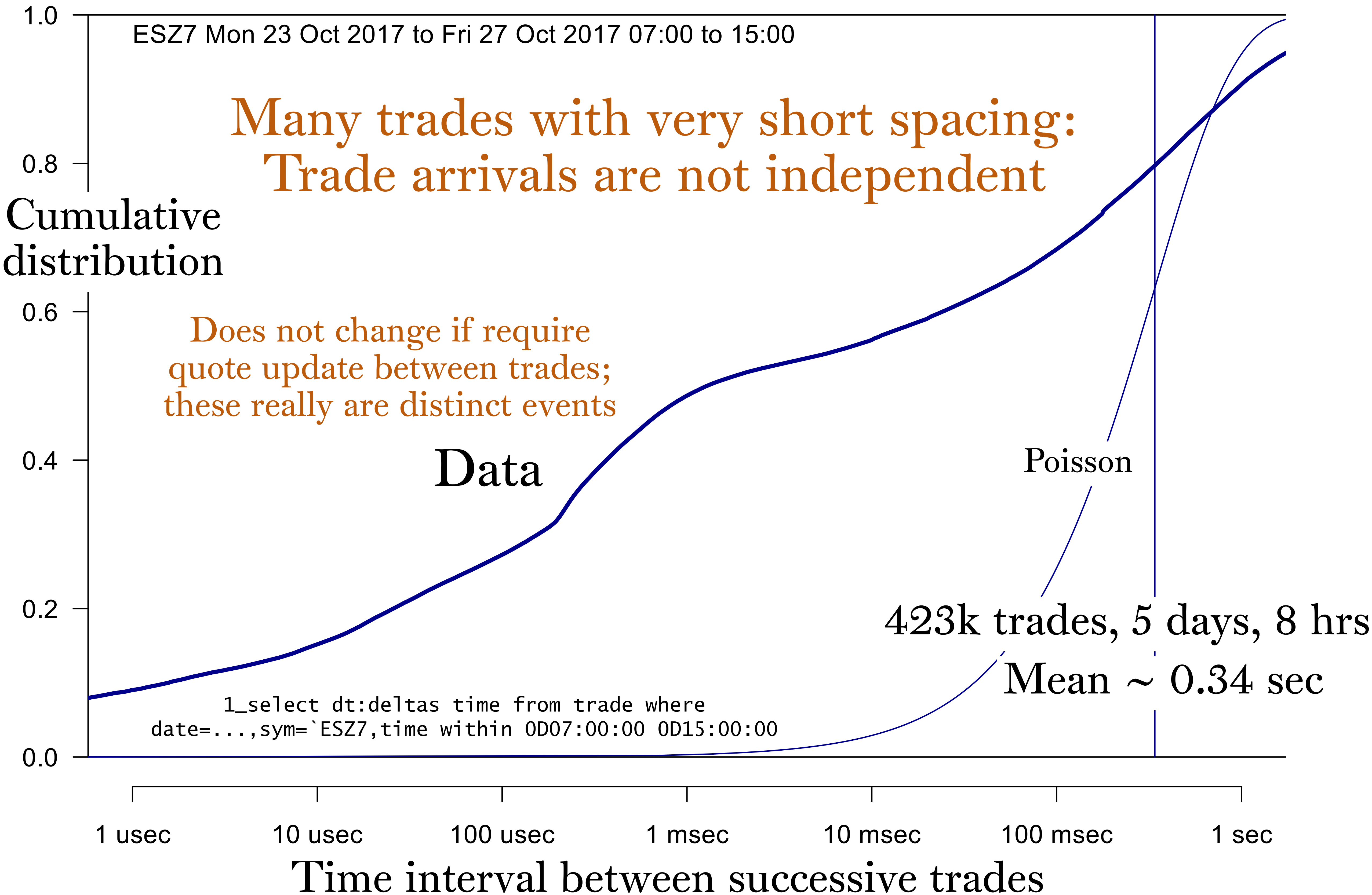
    prc > mid + $\varepsilon \Longrightarrow$ buy

    prc < mid - $\varepsilon \Longrightarrow$ sell
    else $\Longrightarrow$ "tick test" or ignore
  - or use linear functions for partial allocations
- This is sign of aggressive order
  maybe not sign of your order

Trade size

- Multiple prints for each market order
- CME data set aggregates into one trade
  but sometimes still seem to be repeated trades
- Aggregate if no quote update in between

What is a trade?



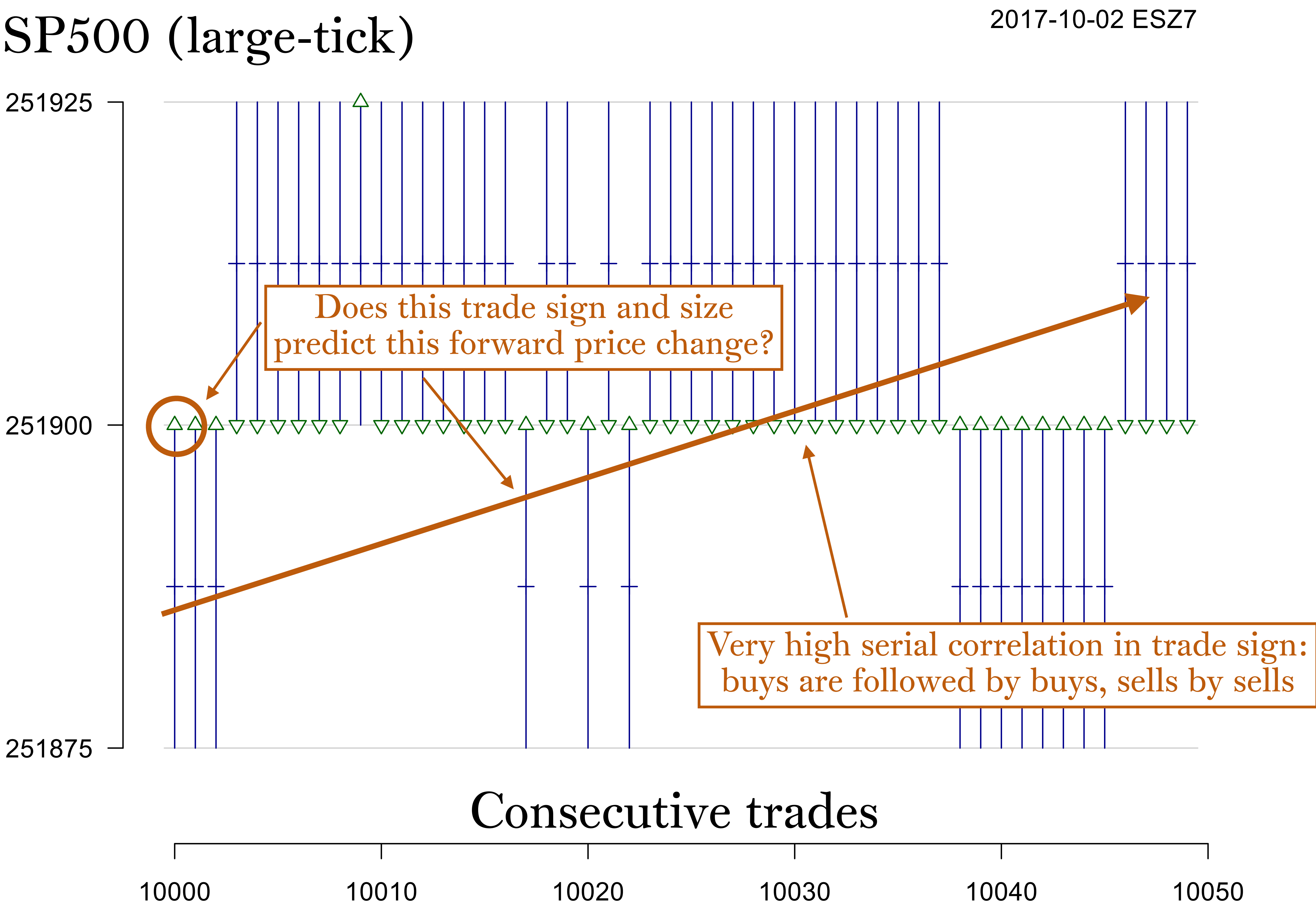
"Time"

- Clock time does not reflect market dynamics:
  intraday profiles
  minute-to-minute fluctuations in activity
- Count time in trade "ticks"
- Depends on trade aggregation

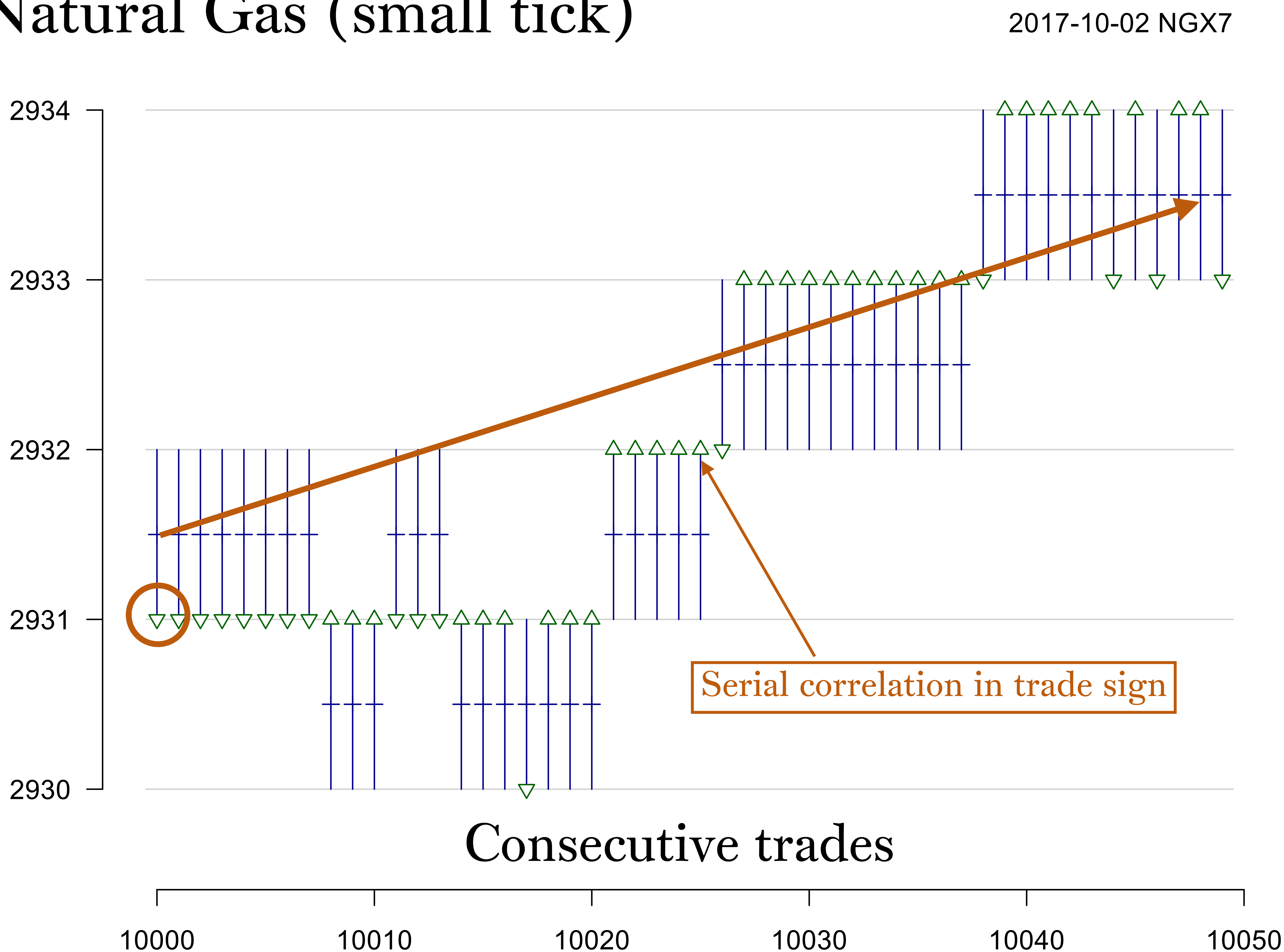
Price change

- Change in bid-ask midpoint price sampled just before discrete trades (join trades and quotes by sequence number, not by timestamp)

SP500 (large-tick)



Natural Gas (small tick)



Trade sizes $q_1, \dots, q_n$

$$q_i > 0 \Longrightarrow \text{Market buy}$$
$$q_i < 0 \Longrightarrow \text{Market sell}$$

$$\epsilon_i \;=\; \operatorname{sign} q_i$$

Midpoint prices just *before* trades $p_1, \dots, p_n$

Return at lag ℓ: $r_{i,\ell} \;=\; p_{i+\ell} - p_i$ (no logs for futures)

Micro cost model: $\boxed{r_{i,\ell} \;=\; f(q_i)\, g(\ell) \;+\; \langle \text{noise} \rangle}$

Depend on sign only

$$f(q) \;=\; \gamma \operatorname{sign}(q)$$

$$R(\ell) = \langle \epsilon_i \cdot (p_{i+\ell} - p_i) \rangle$$

or

Linear in trade size

$$f(q) \;=\; \gamma\, q$$

Regression models (from Week 05)

Differences now:
- Price is bid-ask midpoint (have quote data)
- Look only at constant and linear models
- Look at impact as a function of lags≥1

- Roll

$$\Delta p_t \;=\; \gamma\,\Delta d_t \;+\; \epsilon_t.$$

- Glosten-Milgrom

$$\Delta p_t \;=\; \lambda\,d_t \;+\; \gamma\,\Delta d_t \;+\; \epsilon_t$$

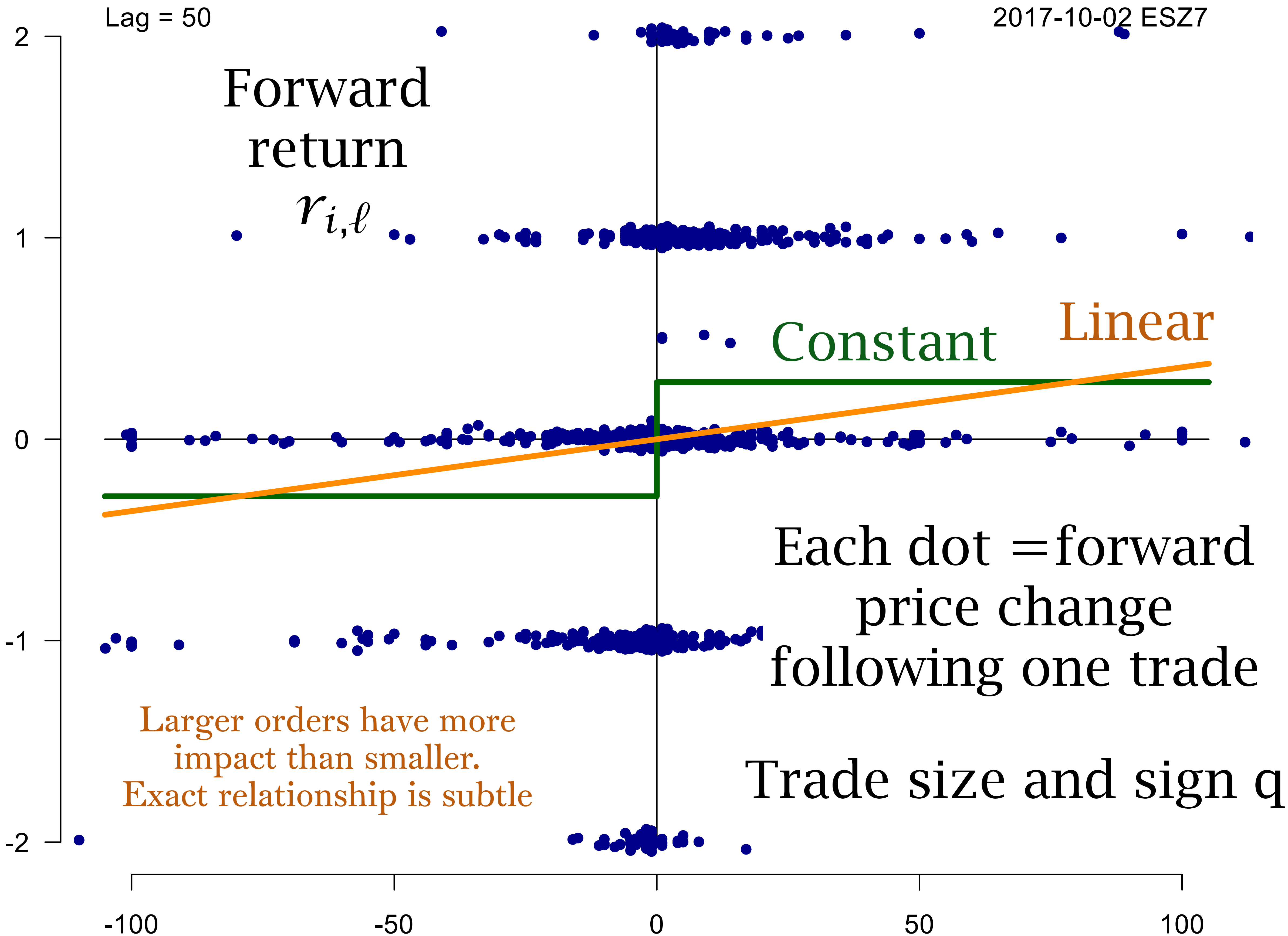
$$\Delta p_t \;=\; \lambda\,q_t \;+\; \gamma\,\Delta d_t \;+\; \epsilon_t$$

- Inventory risk

$$\Delta p_t \;=\; (\lambda+\beta)q_t \;-\; \lambda\phi\,q_{t-1} \;+\; \gamma\,\Delta d_t \;+\; \epsilon_t.$$

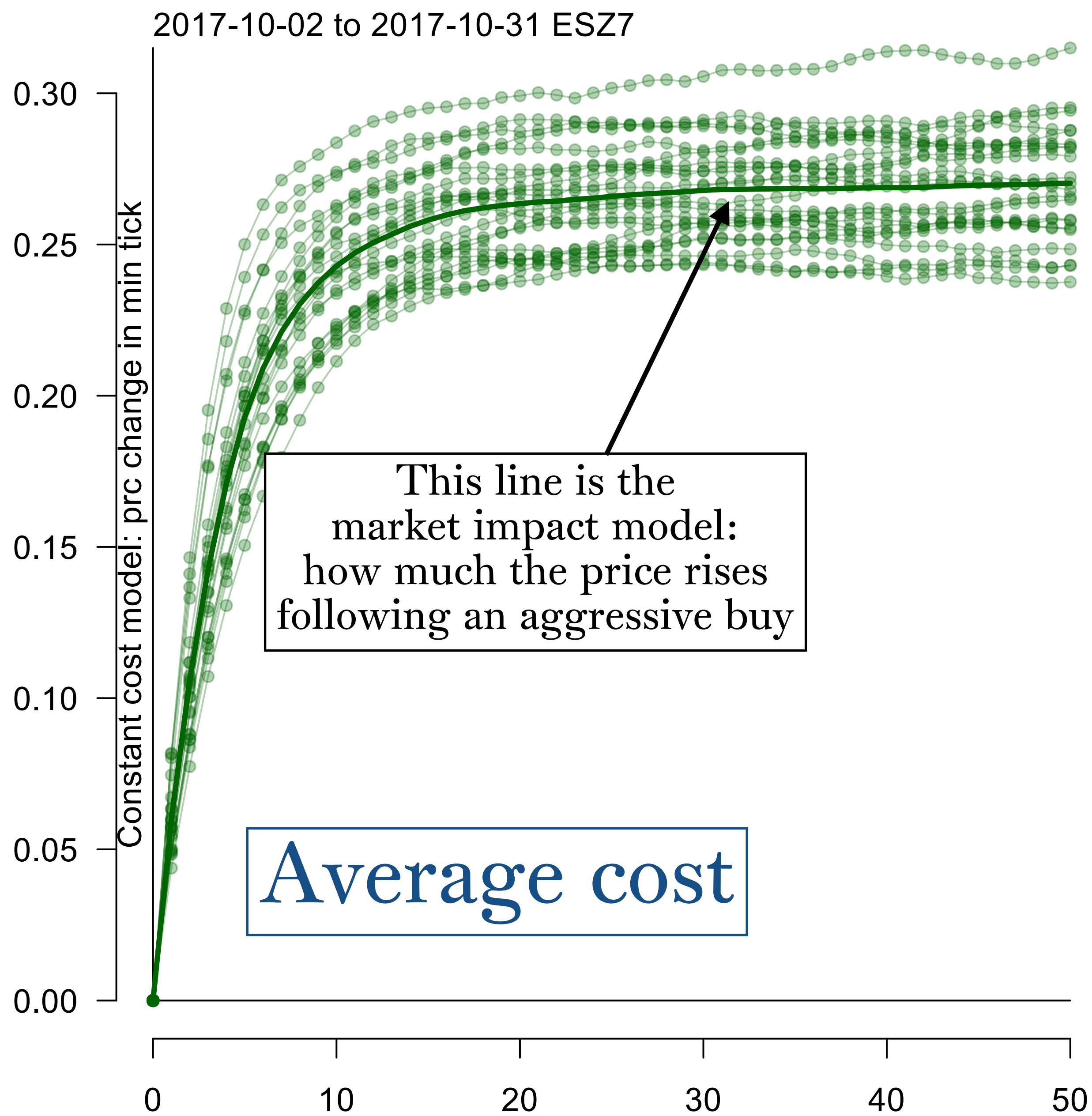
$$q_t \;=\; \phi\,q_{t-1} \;+\; \eta_t$$

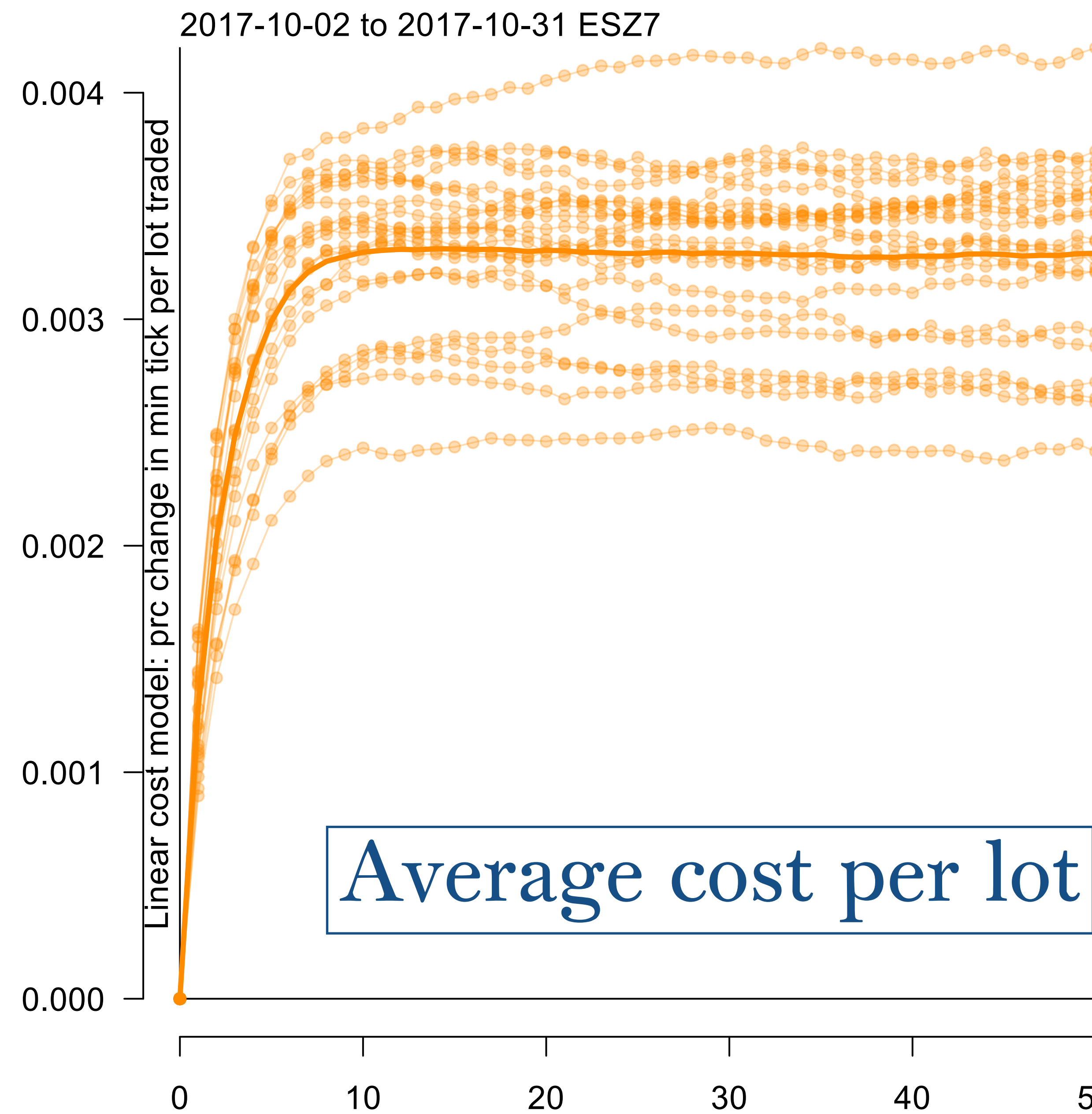
SP500 (large-tick) on one day



SP500 (large-tick) across one month

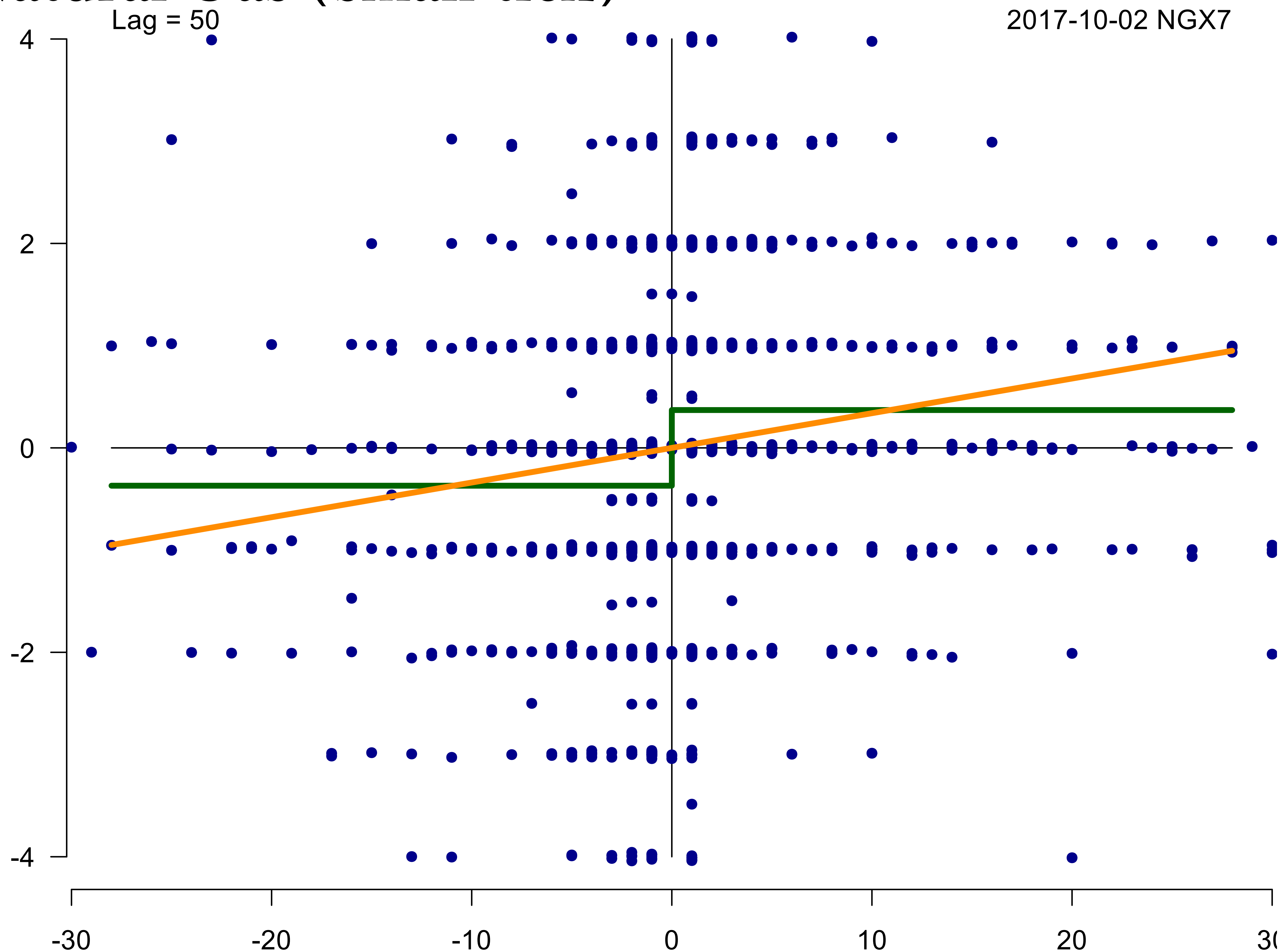
Each line is one date to show variation.
Solid line is mean across dates.



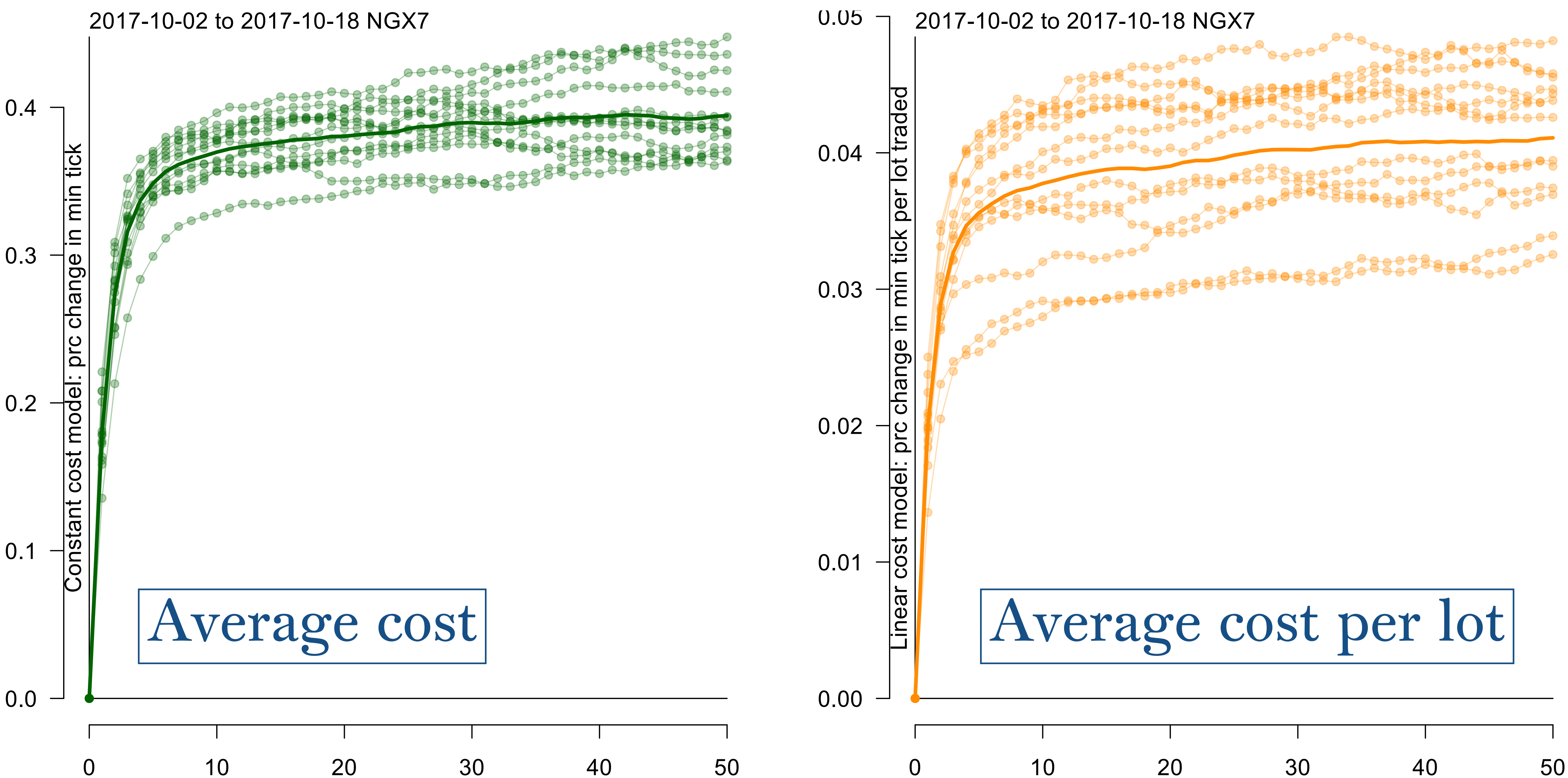


Lag (number of consecutive trades)

Natural Gas (small tick)



Natural Gas (small-tick), across most of a month



Lag (number of consecutive trades)

- Critique of above analysis:

  Serial correlation of trade sign

  Observed impact, not reaction or prediction

$$
\begin{tikzcd}
\text{Buy order} \arrow[rr] \arrow[dr] & & \text{Price increase} \\
& \text{More buy orders} \arrow[ur] &
\end{tikzcd}
$$

- Which buy orders "caused" the price increase?
- What is the impact of a <u>single</u> buy order?

Condition on previous trade sign

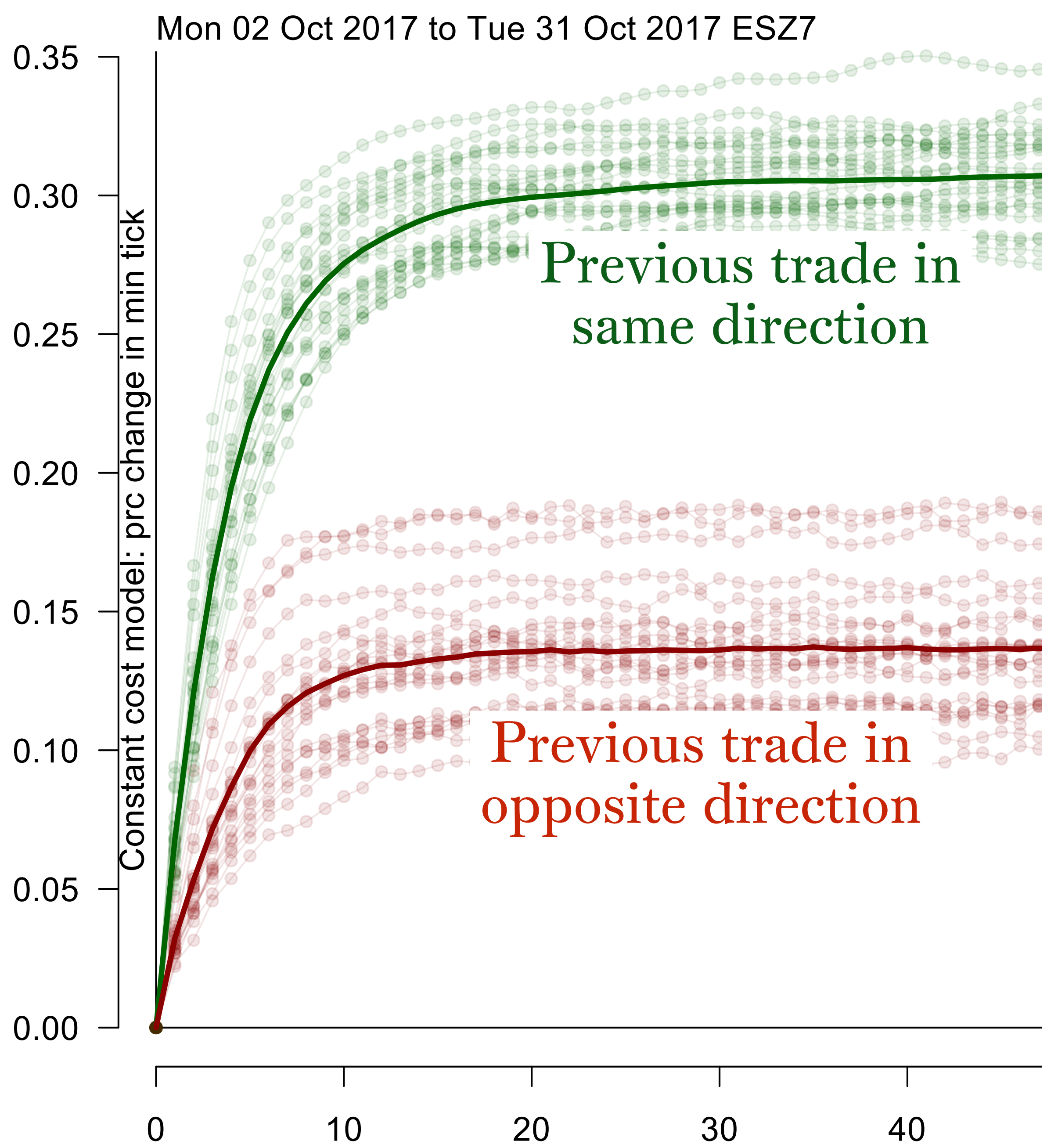
$$R_+(\ell) = \left\langle \epsilon_i \cdot (p_{i+\ell} - p_i) \,\middle|\, \epsilon_{i-1} = \epsilon_i \right\rangle$$

$$R_-(\ell) = \left\langle \epsilon_i \cdot (p_{i+\ell} - p_i) \,\middle|\, \epsilon_{i-1} \neq \epsilon_i \right\rangle$$

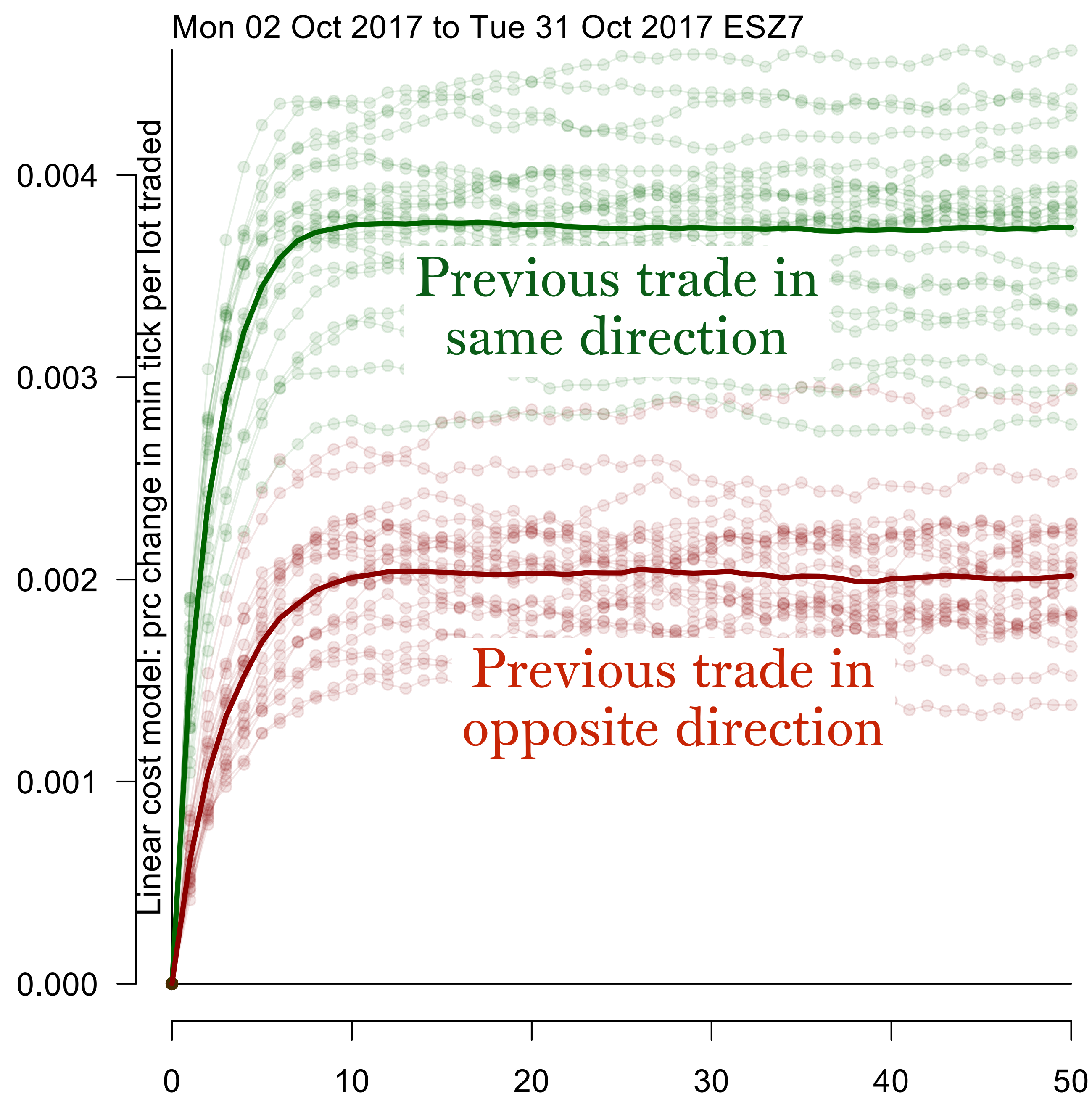
R_+ is larger data sample than R_-

Condition on previous trade sign
ESU7: large tick

Constant model

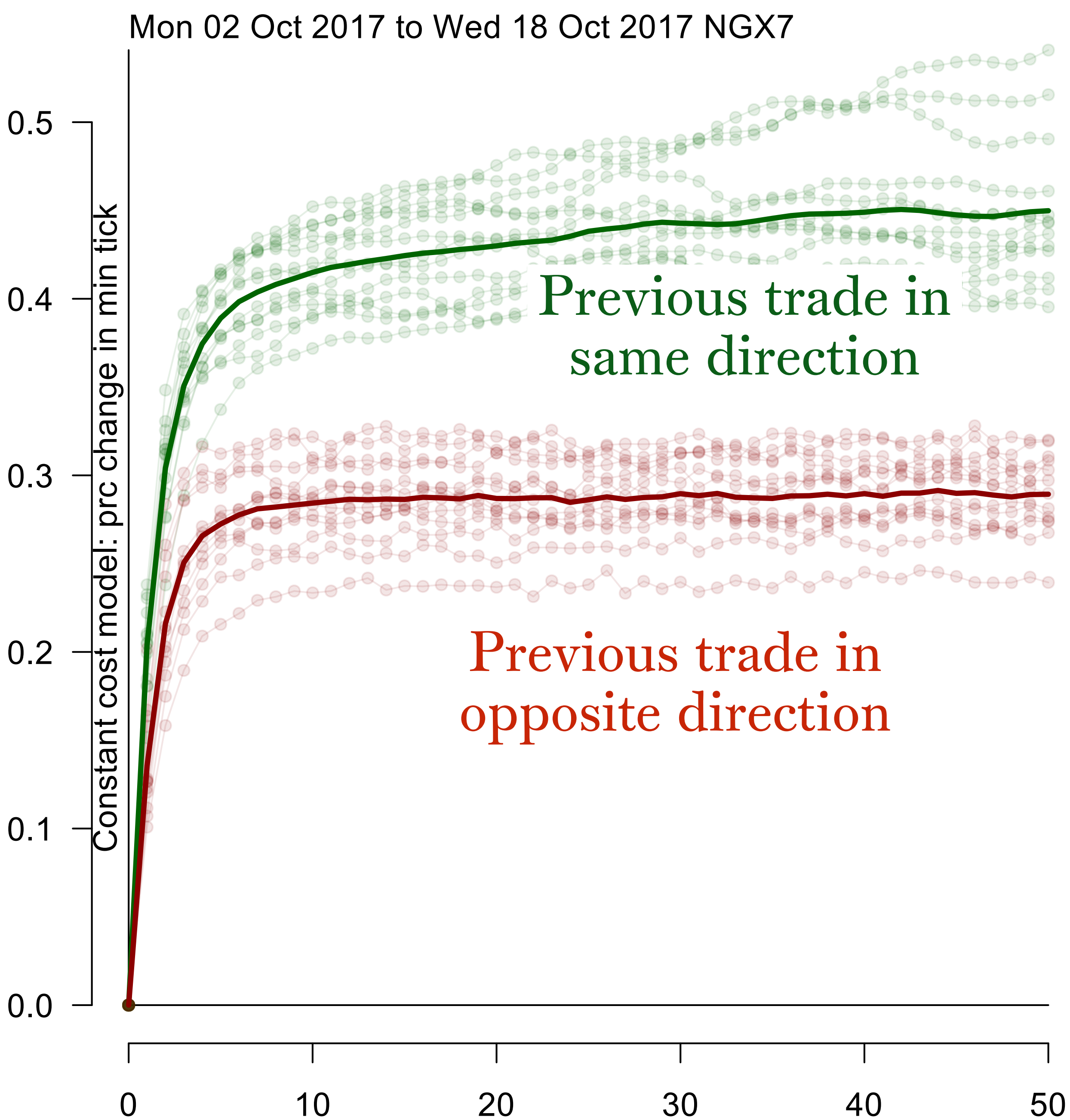


Linear model

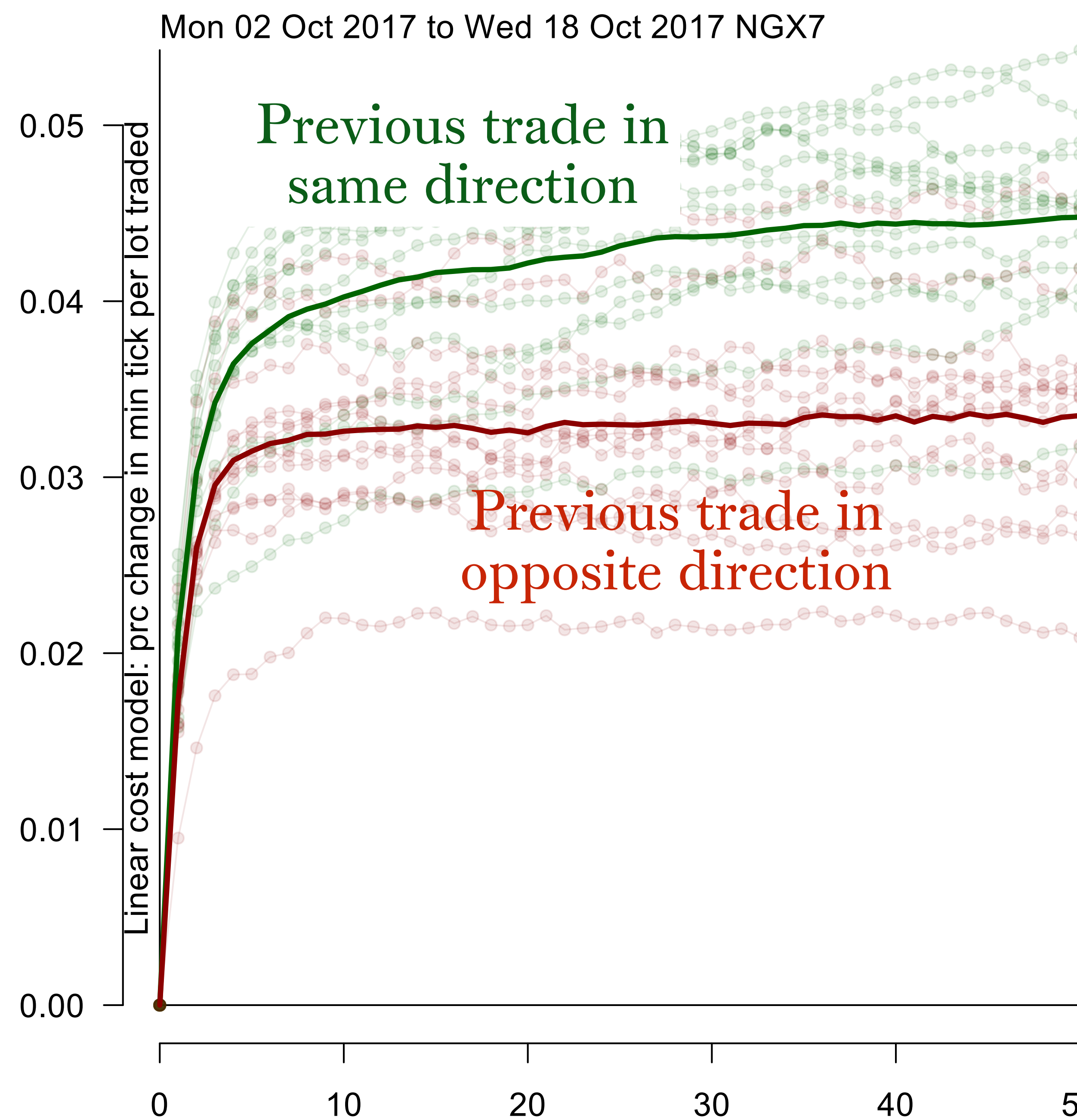


NGX7: small tick

Constant model



Linear model



Formalism

$$\epsilon_i = \operatorname{sign} q_i$$

Return at lag ℓ: $r_{i,\ell} = p_{i+\ell} - p_i$

<> = Mean over i

We have computed $R_\ell = \langle \epsilon_i r_{i,\ell} \rangle_i$

Trade sign autocorrelation $c_\ell = \langle \epsilon_i \epsilon_{i+\ell} \rangle_i$

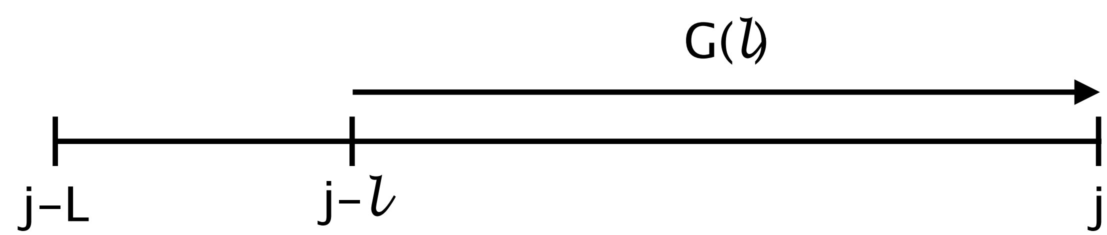
Slow decay

Model for price dynamics

$$p_j \;=\; p_{j-L} + \sum_{\ell=1}^{L} G_\ell\, \epsilon_{j-\ell} + \text{noise}$$

$$G_\ell \;=\; \text{“Bare propagator”}$$

Effect of a single trade



Assumptions:
- additive effects
- independent of size

Calculation of G from R and c

$$r_{j-L,L} \;=\; p_j - p_{j-L} \;=\; \sum_{\ell=1}^{L} G_\ell\, \epsilon_{j-\ell}$$

$$\langle\, \epsilon_{j-L}\, r_{j-L,L}\, \rangle_j \;=\; \sum_{\ell=1}^{L} G_\ell\, \langle \epsilon_{j-\ell}\epsilon_{j-L} \rangle$$

$$R_L \;=\; \sum_{\ell=1}^{L} G_\ell\, c_{L-\ell}$$

$$\begin{aligned} G_1 &= R_1 \\ G_2 &= R_2 - c_1 G_1 \\ G_3 &= R_3 - c_1 G_2 - c_2 G_1 \\ &\vdots \end{aligned}$$

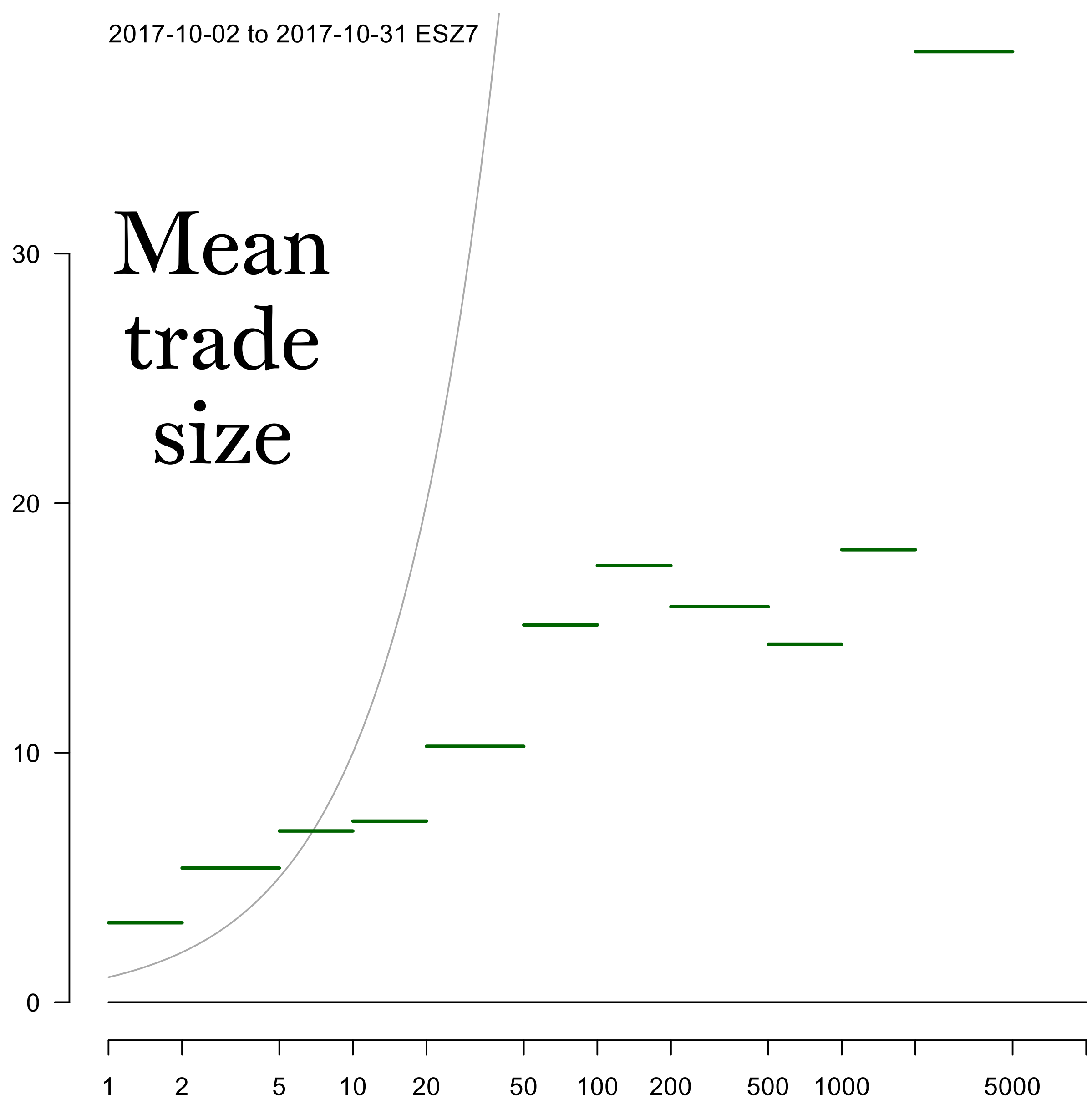
"Yule-Walker" equations
tricky to solve practically

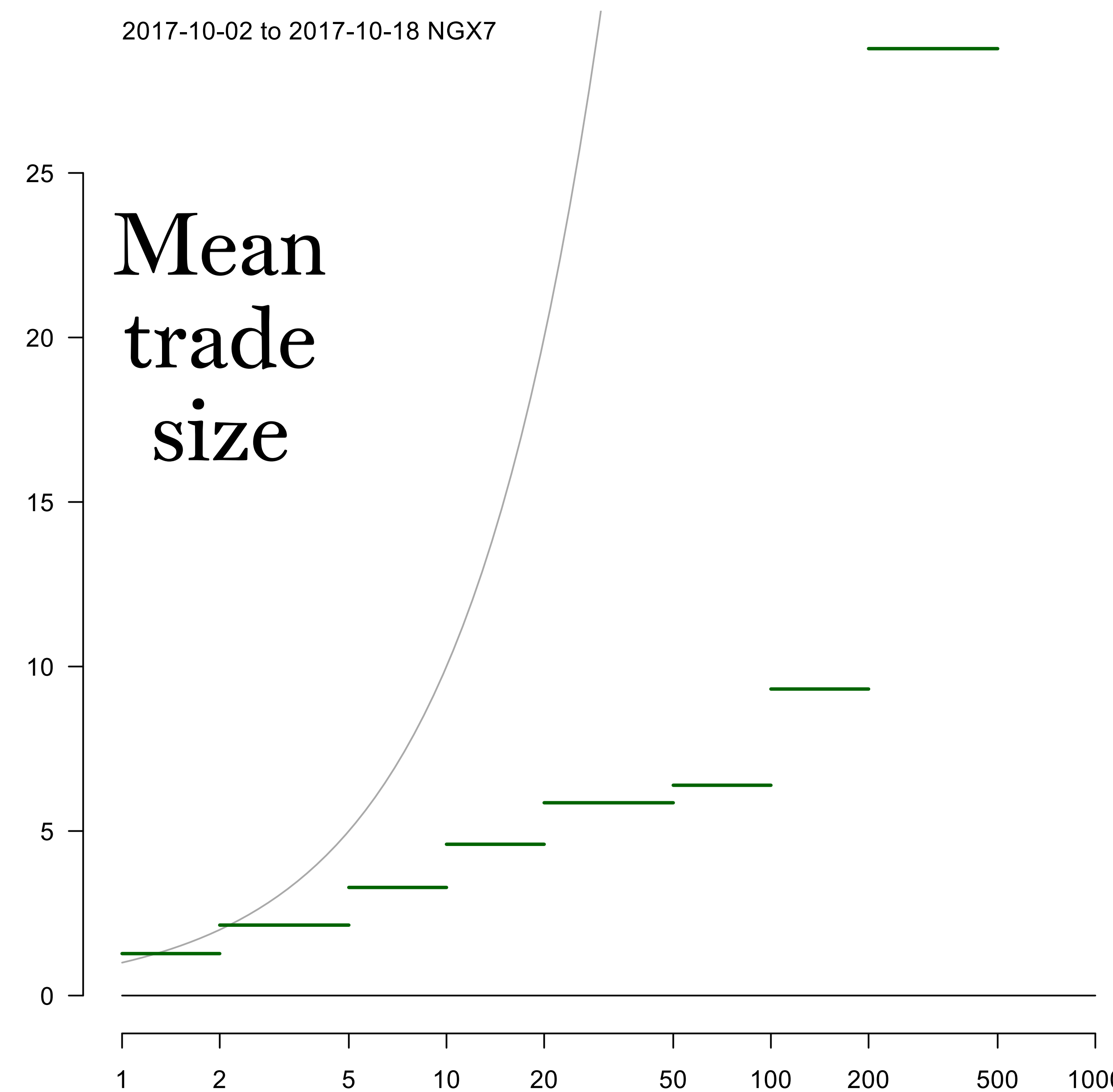
Selective liquidity taking

- Market order size highly dependent on quote size
- Can distinguish between

  market orders that consume all limit size

  market orders that are smaller than limit size
- Or look at average trade size by quote size



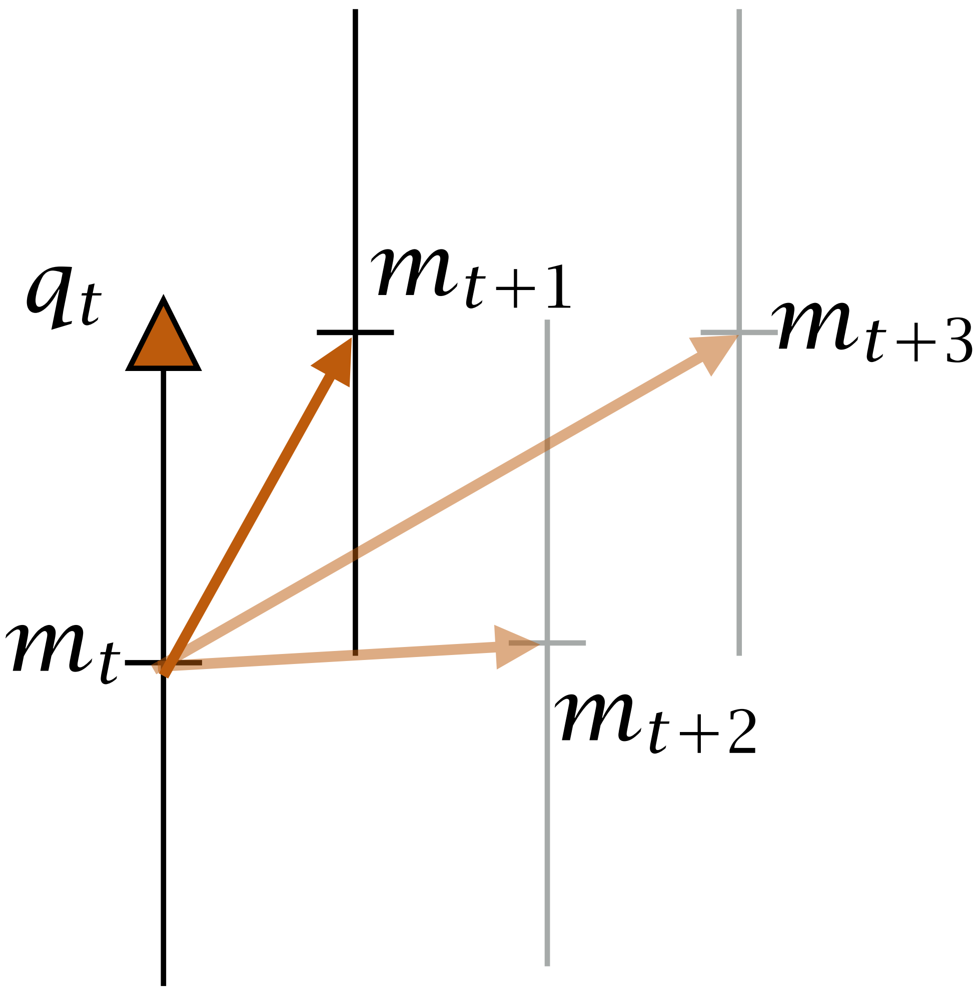


Opposite quote size

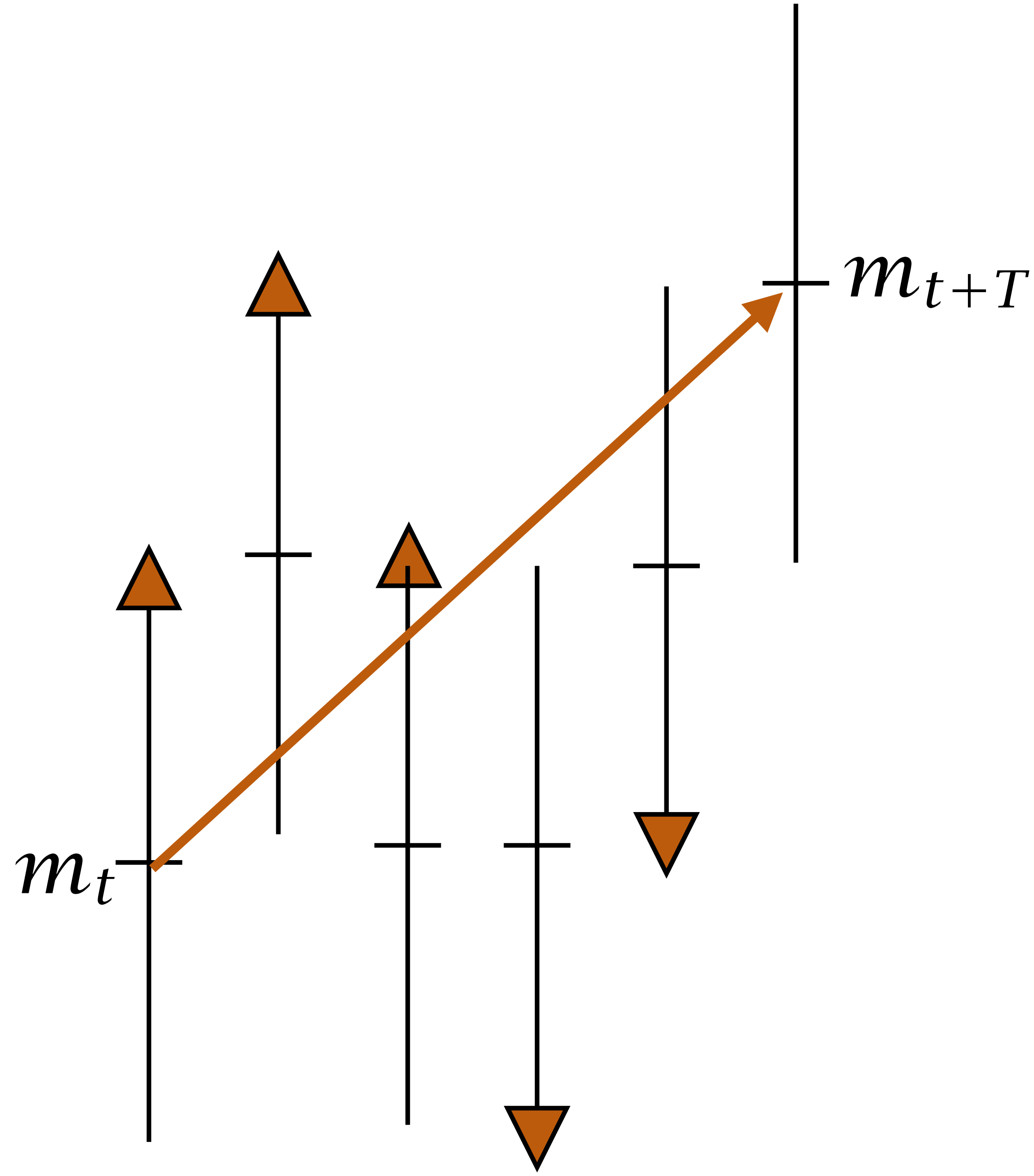
Idea

- Fixed window T (number of trades) (could also use time window)
- Total imbalance on window T $$Q_t = \sum_{s=t}^{t+T-1} q_s$$
- Return across window T $$R_{t,T} = m_{t+T} - m_t$$
- How does $R_{t,T}$ depend on Q_t (averaged across t)? Measure of liquidity

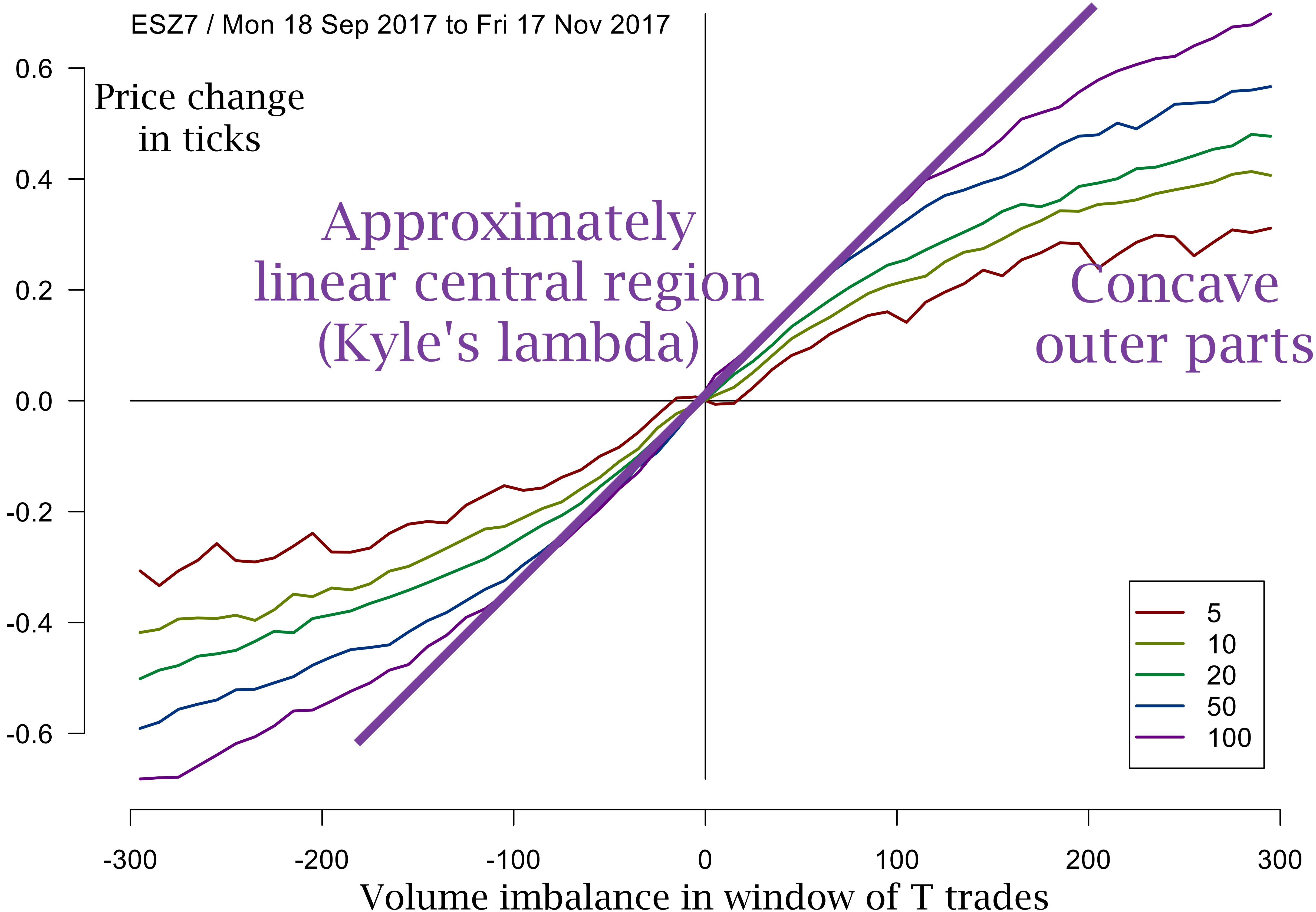
Windowed imbalance



Influence of single trade q_t on future midpoints m_t, m_{t+1}, ...



Influence of net imbalance in some window on overall price change


ESZ7 / Mon 18 Sep 2017 to Fri 17 Nov 2017
Price change
in ticks
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
Approximately
linear central region
(Kyle's lambda)
Concave
outer parts
5
10
20
50
100
-300
-200
-100
0
100
200
300
Volume imbalance in window of T trades

More detailed analysis

PHYSICAL REVIEW E **97**, 012304 (2018)

Universal scaling and nonlinearity of aggregate price impact in financial markets

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How and why stock prices move is a centuries-old question still not answered conclusively. More recently, attention shifted to higher frequencies, where trades are processed piecewise across different time scales. Here we reveal that price impact has a universal nonlinear shape for trades aggregated on any intraday scale. Its shape varies little across instruments, but drastically different master curves are obtained for order-volume and -sign impact. The scaling is largely determined by the relevant Hurst exponents. We further show that extreme order-flow imbalance is not associated with large returns. To the contrary, it is observed when the price is pinned to a particular level. Prices move only when there is sufficient balance in the local order flow. In fact, the probability that a trade changes the midprice falls to zero with increasing (absolute) order-sign bias along an arc-shaped curve for all intraday scales. Our findings challenge the widespread assumption of linear aggregate impact. They imply that market dynamics on all intraday time scales are shaped by correlations and bilateral adaptation in the flows of liquidity provision and taking.

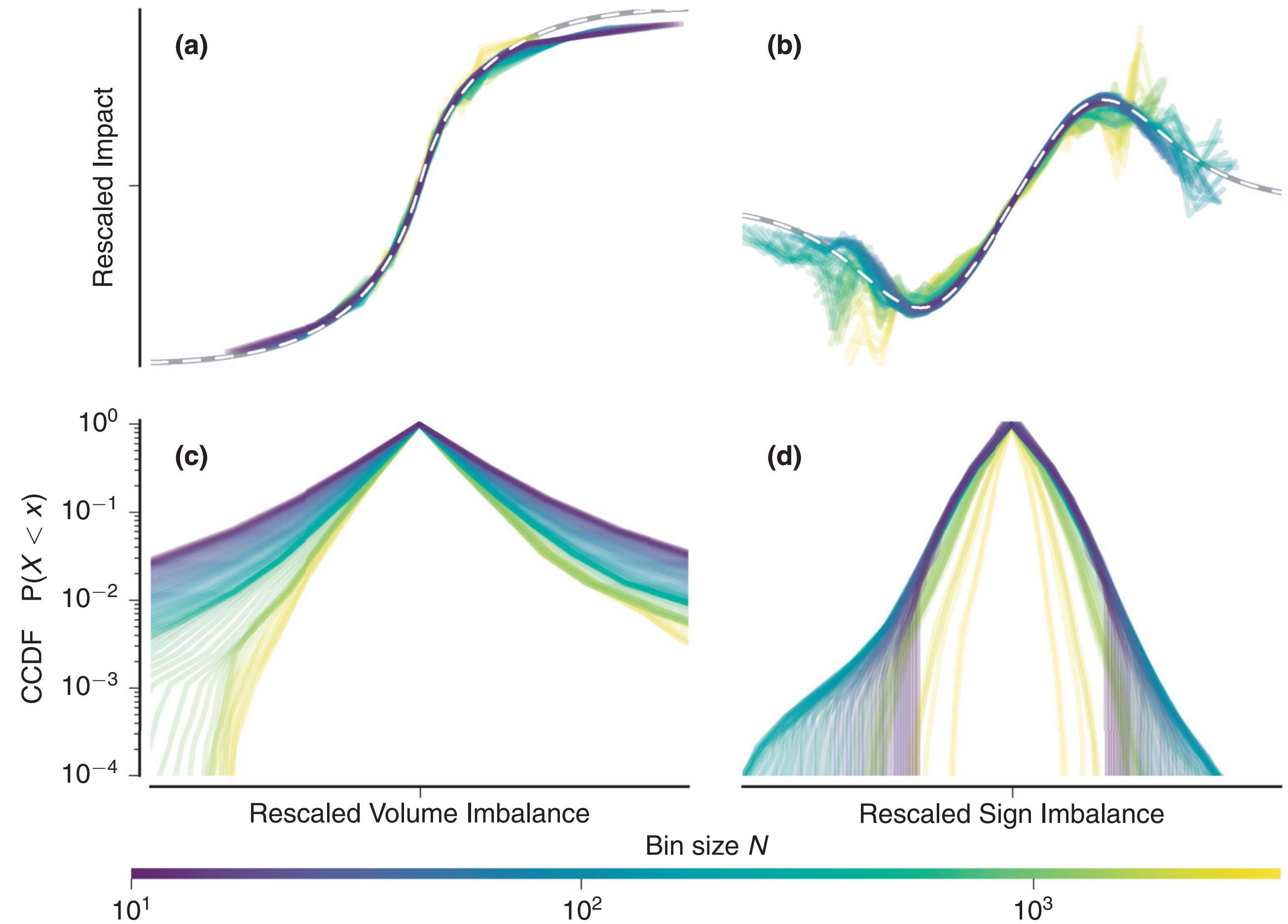


FIG. 1. AAPL on NASDAQ in 2016. (a) Rescaled expected return $\mathcal{R}_N(\mathcal{Q}/N^{\xi})/N^{\psi}$ conditioned on the volume imbalance $\mathcal{Q}$ for different bin sizes N in arbitrary units [see Eq. (2) ff]. The x- and y-axis rescaling exponents are $\xi = 0.84$ and $\psi = 0.53$. (b) Rescaled mean return $\mathcal{R}_N(\mathcal{E}/N^{\psi})/N^{\psi}$ conditioned on the sign imbalance $\mathcal{E}$ [see Eq. (9)]. Here $\xi_\epsilon = 0.69$ and $\psi_\epsilon = 0.48$. (c) and (d) Corresponding complementary cumulative distributions. The positive and negative halves were calculated independently and then binned to smooth out noise and discretization steps for small N. The largest shown N corresponds to the shortest day in the sample.

A. Aggregate impact

As mentioned in the Introduction, we measure the aggregate-volume impact as

$$\mathcal{R}_N(\mathcal{Q}) := \left\langle \ln m_{t+N} - \ln m_t \middle| \mathcal{Q} = \sum_{i=0}^{N-1} q_{t+i} \right\rangle, \tag{2}$$

where m_t is the midprice immediately before the tth transaction, q_t is the signed volume of the tth transaction, and $\langle \cdots \rangle$ denotes an empirical average over all time windows containing N successive trades, executed the same day.

$$\mathcal{R}_N(\mathcal{Q}) \approx R_N \mathscr{F}\left(\frac{\mathcal{Q}}{Q_N}\right), \tag{3}$$

where Q_N and R_N both obey power-law scaling with N,

$$Q_N \approx Q_1 N^{\xi}, \tag{4}$$

$$R_N \approx R_1 N^{\psi}, \tag{5}$$

and the scaling function $\mathscr{F}(x)$ is a sigmoidal function parametrized as

$$\mathscr{F}(x) = \frac{x}{(1+|x|^{\alpha})^{\beta/\alpha}}, \tag{6}$$

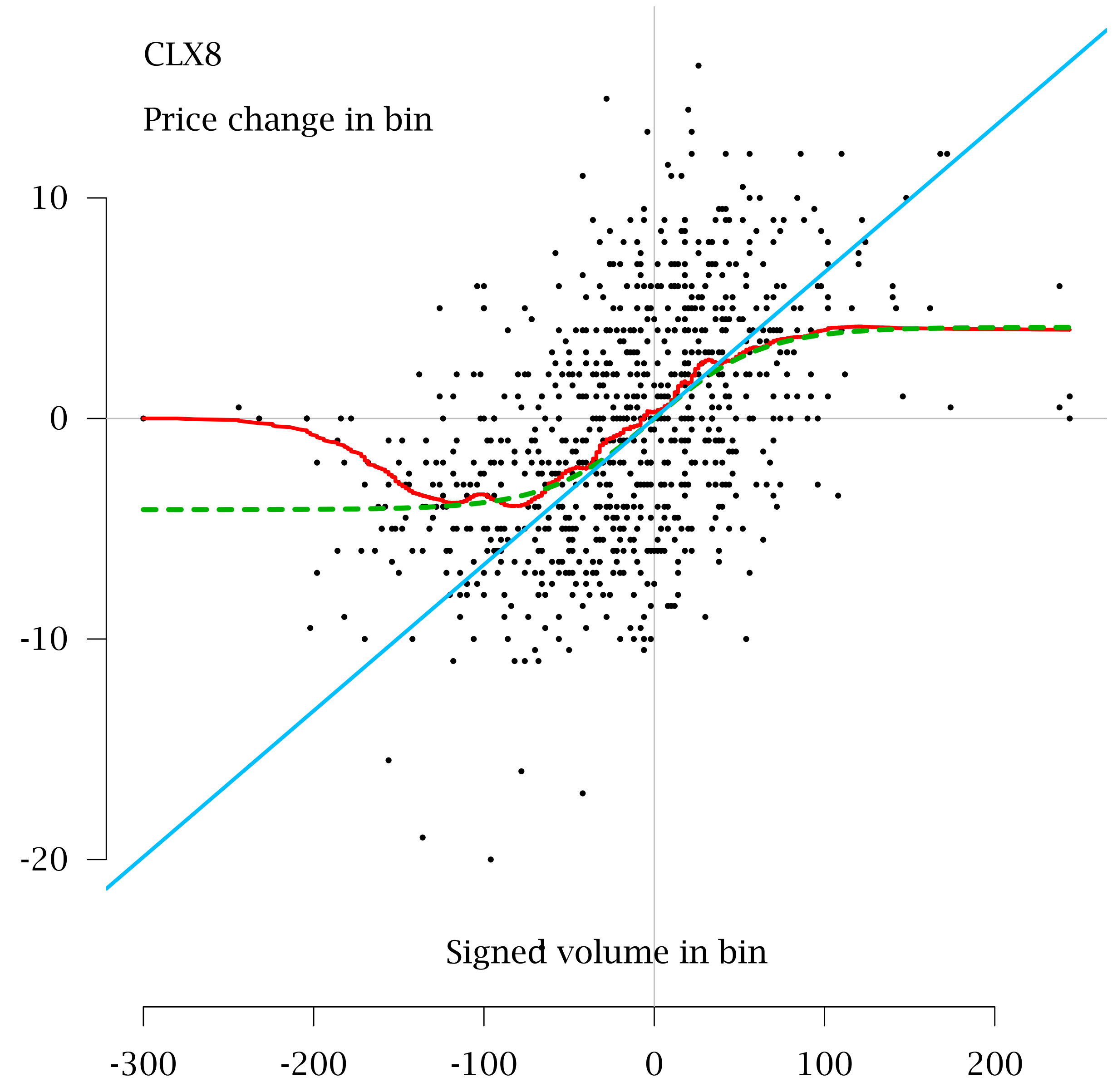
Use in practice

quantitativebrokers

AUGMENTED VOLUME MEASUREMENT FOR PARTICIPATION RATE LIMITS

ISAAC CARRUTHERS
REZA GHOLIZADEH
APRIL 5, 2019

FIGURE 1
Signed volume plotted against price change for the CME Crude Oil Futures with November 2018 expiration, on the 18th of October 2018. Each dot is an overlapping bin during which 300 lots traded on the market. The red line shows the two-sided exponentially smoothed data. The green line shows the best-fit sigmoid function constrained to be centered on the origin. The blue line shows the slope of this sigmoid at the origin.



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Journal of Financial Markets 5 (2002) 31–56

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Illiquidity and stock returns: cross-section and time-series effects☆

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Abstract

This paper shows that over time, expected market illiquidity positively affects ex ante stock excess return, suggesting that expected stock excess return partly represents an *illiquidity premium*. This complements the cross-sectional positive return–illiquidity relationship. Also, stock returns are negatively related over time to contemporaneous unexpected illiquidity. The illiquidity measure here is the average across stocks of the daily ratio of absolute stock return to dollar volume, which is easily obtained from daily stock data for long time series in most stock markets. Illiquidity affects more strongly small firm stocks, thus explaining time series variations in their premiums over time.

JEL classificaion: G12

Keywords: Liquidity and asset pricing; Liquidity premium

"Amihud illiquidity measure"

Stock illiquidity is defined here as the average ratio of the daily absolute return to the (dollar) trading volume on that day, $|R_{iyd}|/VOLD_{iyd}$. R_{iyd} is the return on stock i on day d of year y and $VOLD_{iyd}$ is the respective daily volume in dollars. This ratio gives the absolute (percentage) price change per dollar of daily trading volume, or the daily price impact of the order flow. This follows Kyle's concept of illiquidity—the response of price to order flow—and Silber's (1975) measure of thinness, defined as the ratio of absolute price change to absolute excess demand for trading.

$$ILLIQ_{iy} = 1/D_{iy} \sum_{t=1}^{Diy} |R_{iyd}|/VOLD_{ivyd},$$

Amihud compares total volume, ignoring imbalance (buys+sells), to average absolute return.
We look at signed volume
(buys - sells).

Extensions:

- Nonlinear dependence on trade size

  logarithmic often matches data
- Dependence on time of day
- Model variation with market properties

  quote depth

  recent price trends
- 2 additional research results:

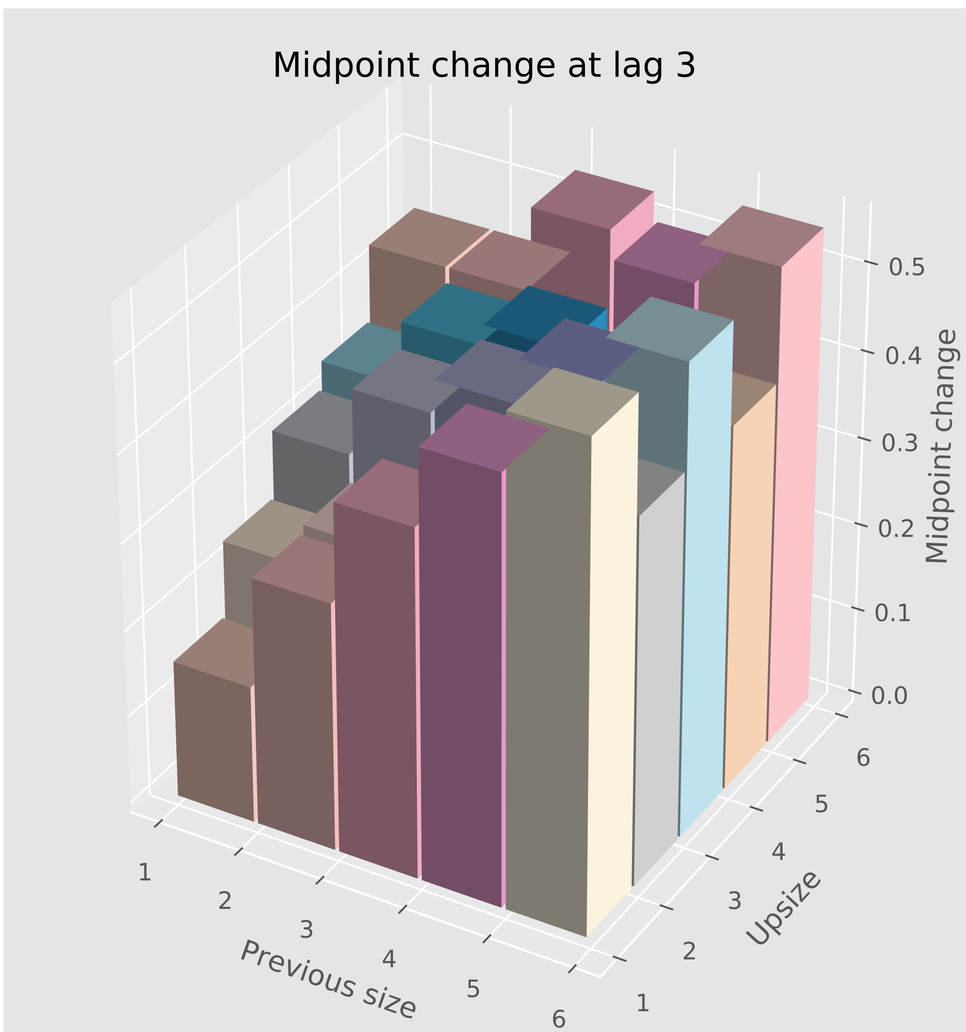
  impact of limit orders

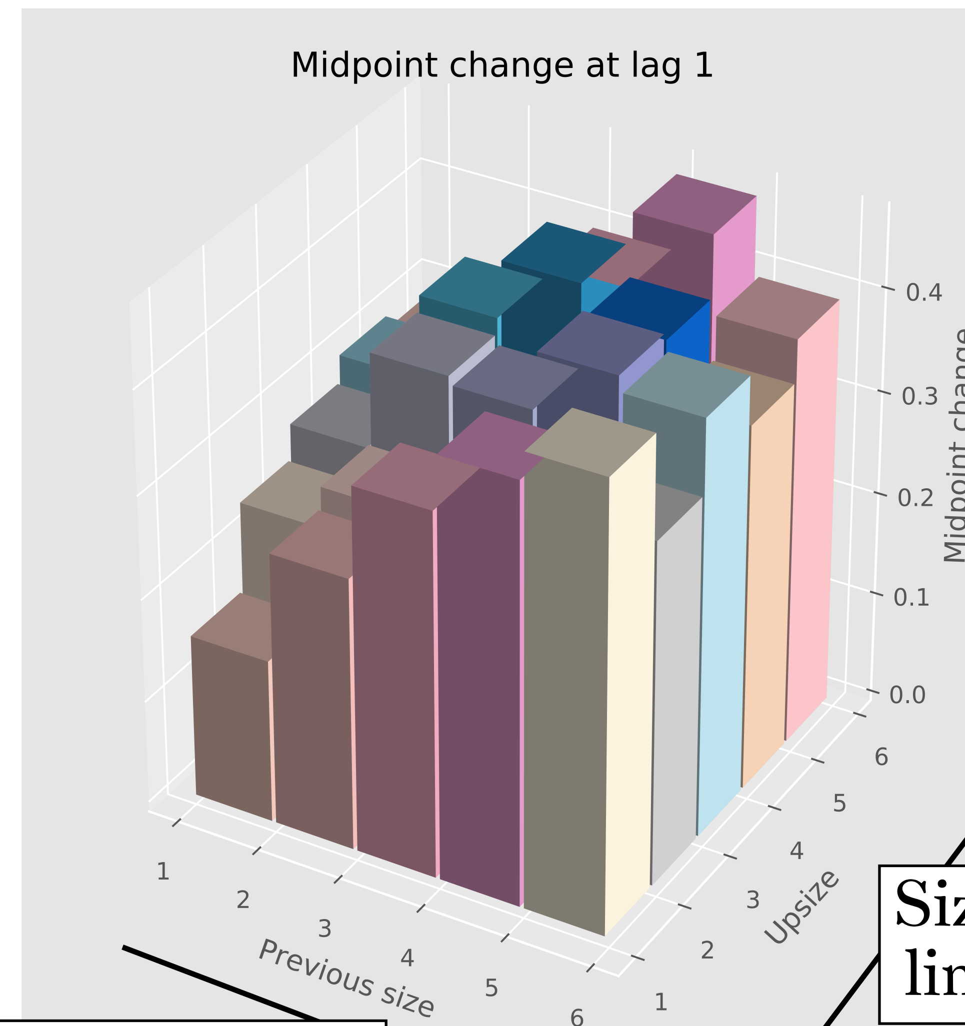
  importance of reason for trading

  significance of round number trade sizes

Impact of limit orders

Anais Ta, Bendheim Center



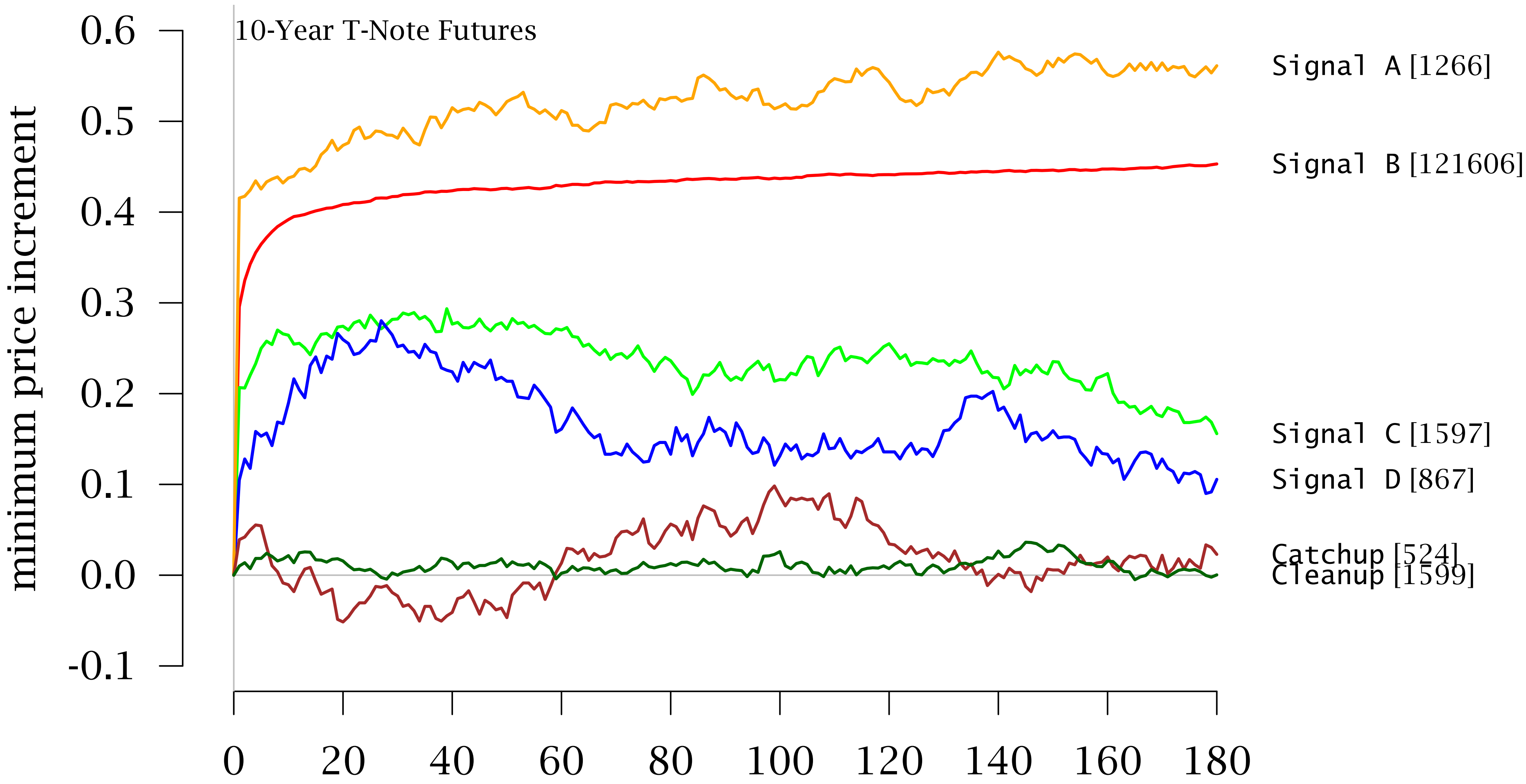


Quote size before new order

Size of new limit order

Market Impact and Alpha

Importance of reason for trading



Sean Zhang, QB

Significance of trade size

Cannot do this with BBO data

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BEST EXECUTION FOR FIXED INCOME AND FUTURES

QB Research
November 12, 2016

Page 1

Humans: Are we that predictable?

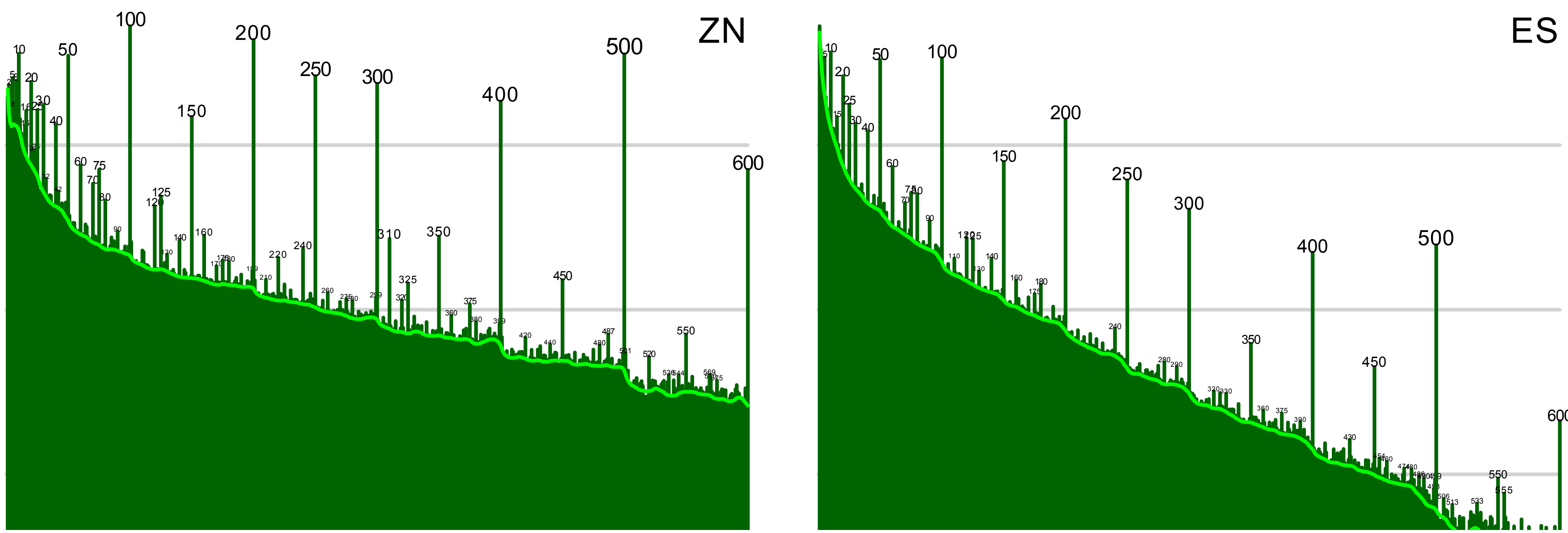

Figure 1: Excess volume is traded in round-numbered sizes. Green bars show total volume at each trade size for 10-year Treasury note futures (ZN) and S&P E-mini futures (ES). Data shown compiles all electronic volume for the year of 2015. The light green lines mark our estimate of the background trade volume at each size. The vertical axis is log-scaled, with horizontal grey lines marking multiples of 10. Many other futures markets show a similar clustering, and the clustering appears to be stable across time.

Isaac Carruthers, QB

Lower impact of round number sizes

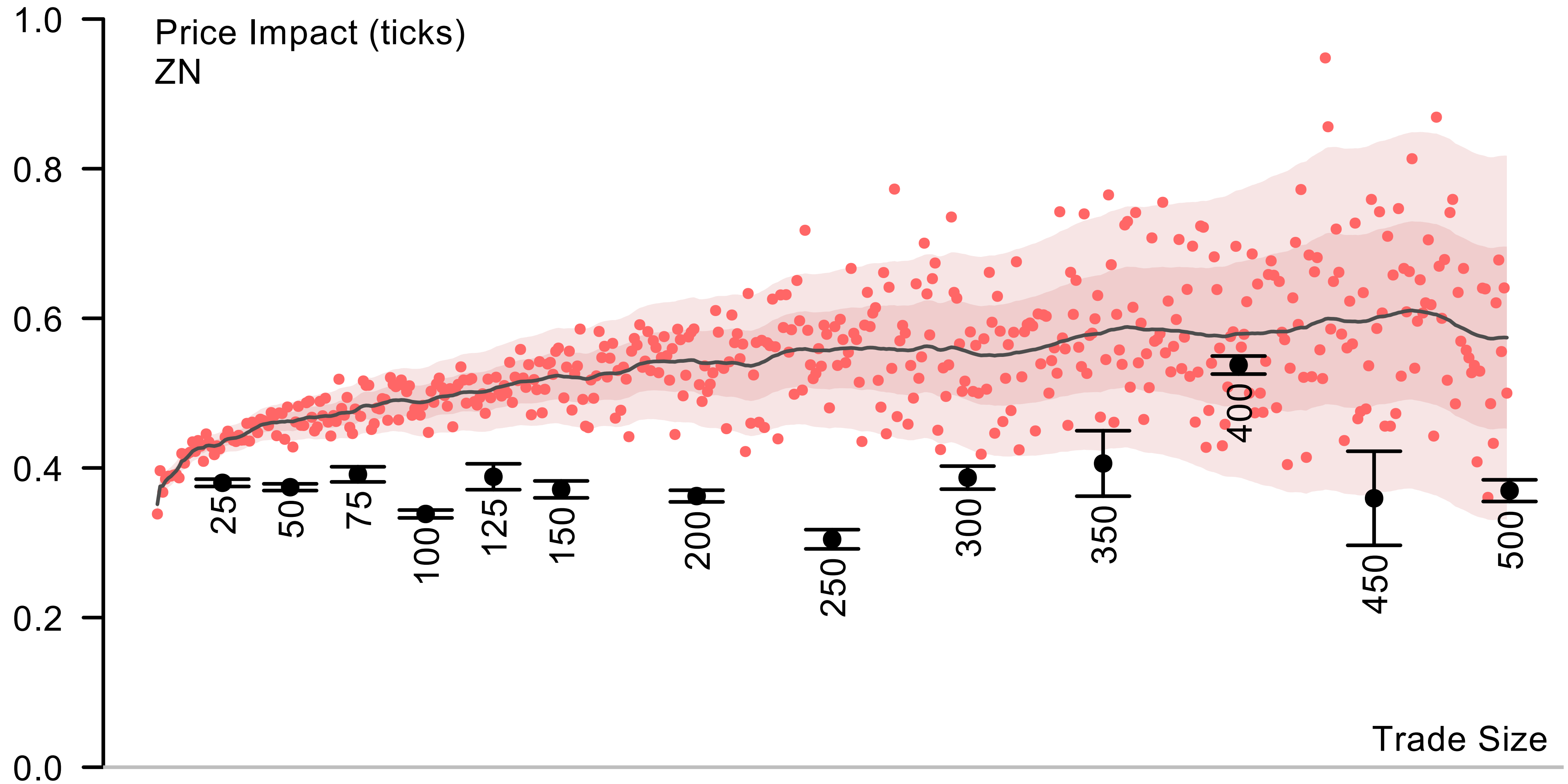

Figure 2: Round lot trades show dramatically lower market impact. Red dots show average 30-second price change following trades of each size. Round numbered sizes are highlighted in black and shown with the standard error of their mean. Shaded bands show regions within 1 and 2 standard deviations of the moving average of the non-round data. The motivation for this report is to explain the effect shown in this figure.

Why do round numbers have lower impact?

- Less correlation with subsequent orders?
- Perceived by market as less informed?
- Submitted into different market conditions?

Not lower correlation with future orders

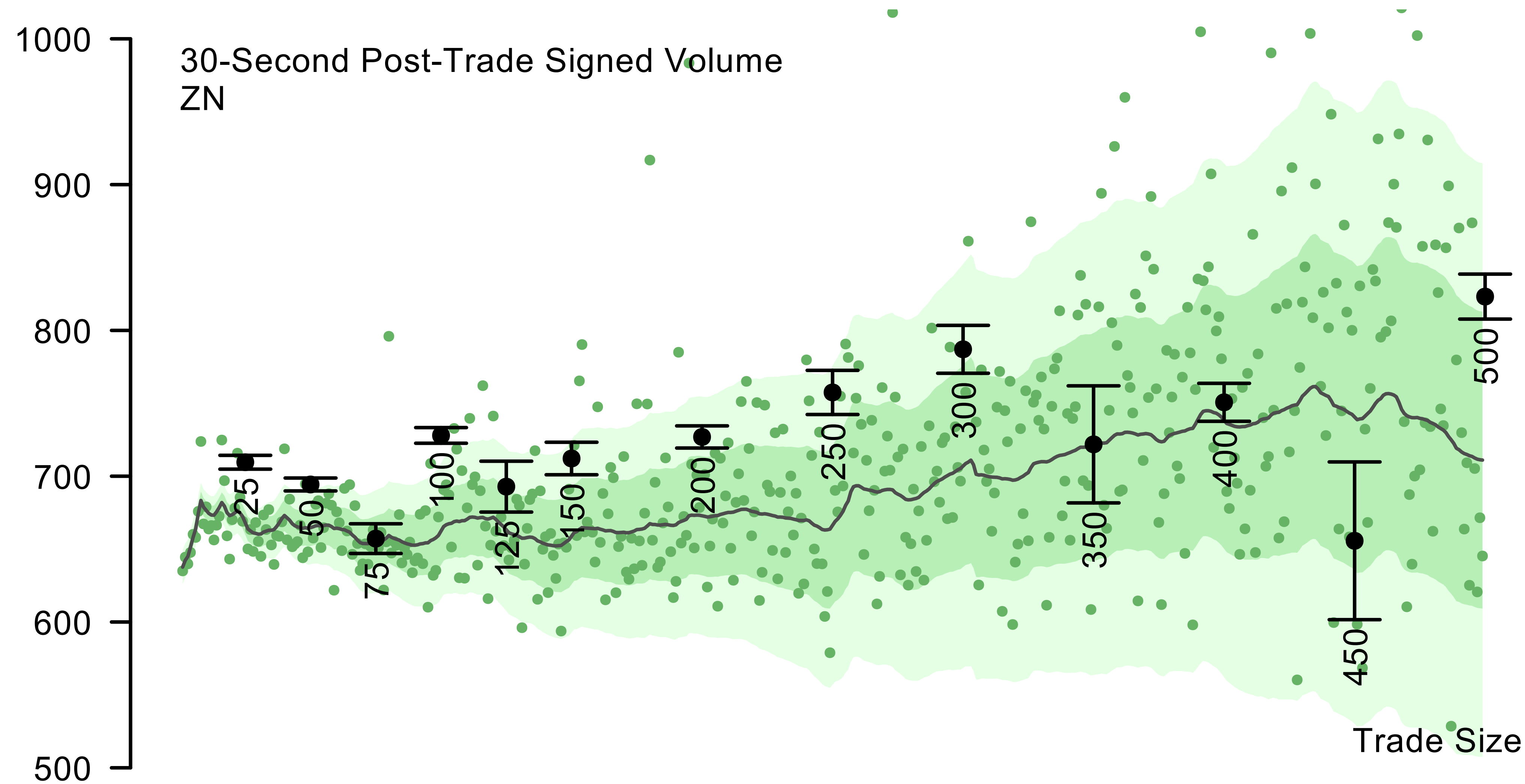

Figure 3: Round lot trades *do not* have lower correlation with future order-flow. Dots show average signed volume in the 30 seconds following trades of different sizes. Round lot trades are followed by *more* volume in the same direction than non-round lot trades, but the effect is not dramatic.

Round number trades go into deeper markets

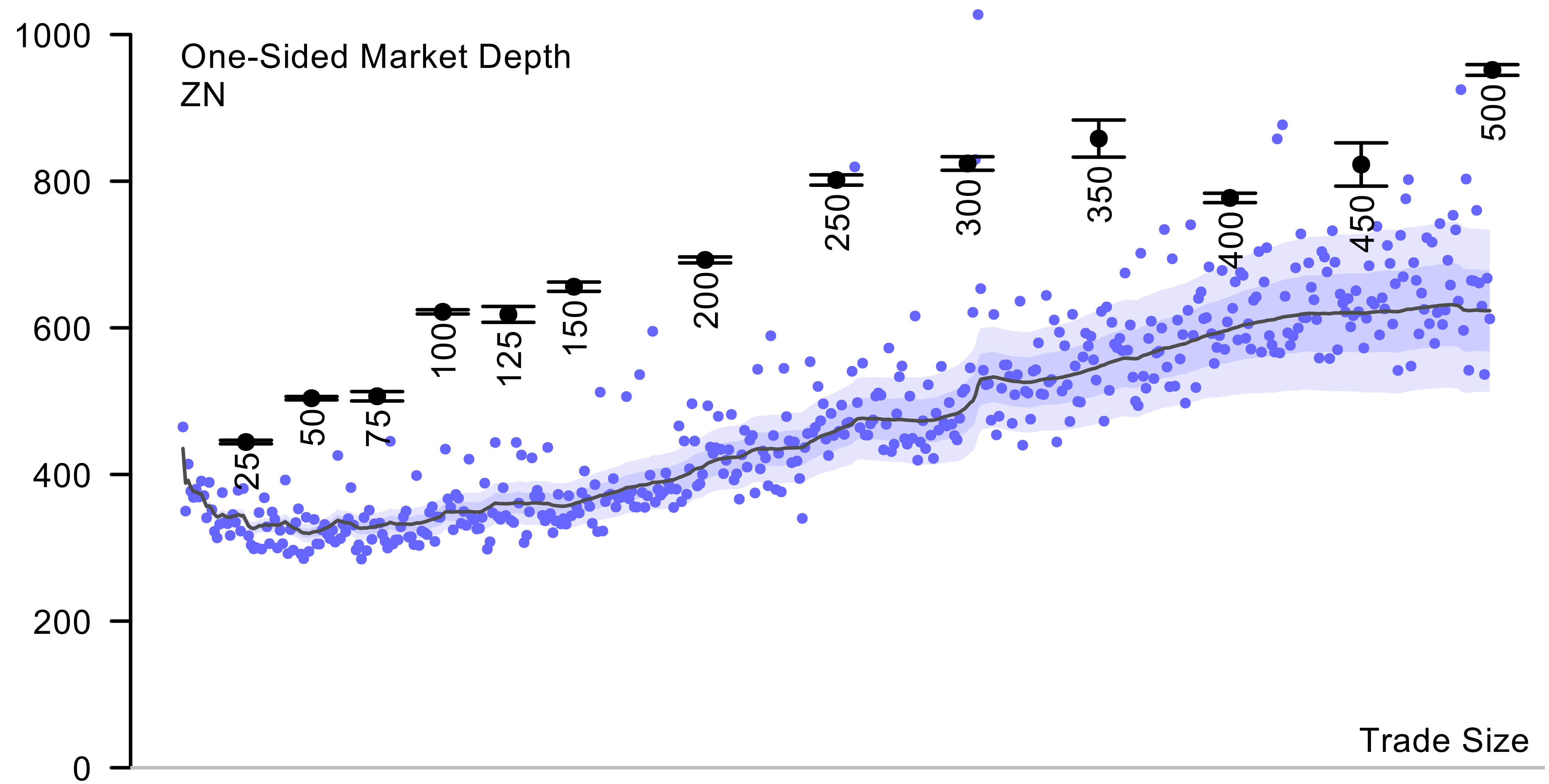

Figure 4: Round lot trades are entered into dramatically deeper markets. Dots show average opposing market depth at trade times within the 30 seconds following trades of different sizes. The higher liquidity that meets round lot trades is a viable explanation of the effect in Figure 2.