Bare Demo of IEEEtran.cls for Journals

Michael Shell, *Member, IEEE,* John Doe, *Fellow, OSA,* and Jane Doe, *Life Fellow, IEEE*

***Abstract*—The abstract goes here.**

***Keywords*—IEEEtran, journal, LaTeX, paper, template.**

I. Modeling

A. Bearing Constraints

Suppose we have two sensors at positions $S_1 \in \mathbb{R}^2$ and $s_2 \in \mathbb{R}^2$. When the communication signal is sent from s_1 to s_2, the bearing θ of the vector from s_1 to s_2, can be measured. In addition, we assume there exists an error bound δ for the angle measurement, or in inequalities, $\theta - \delta \le \angle(s_2 - s_1) \le \theta + \delta$. This angular condition can be characterized by two separate linear constraints. The first one is a semi-plane defined by the straight-line corresponding to the angle $\theta + \delta$, or equivalently,

$$[-\sin(\theta+\delta), \cos(\theta+\delta)]x \le 0 \tag{1}$$

where $x \in \mathbf{R}^2$. The condition that the vector $s_2 - s_1$ is within this semi-plane can thus expressed as:

$$[-\sin(\theta+\delta), \cos(\theta+\delta)](s_2 - s_1) \le 0 \tag{2}$$

Similarly, the other semi-planed defined by the line with angle $\theta - \delta$ can be obtained as:

$$[-\sin(\theta-\delta), \cos(\theta-\delta)]x \ge 0 \tag{3}$$

and the constraint for s_1 and s_2 becomes

$$[-\sin(\theta-\delta), \cos(\theta-\delta)](s_2 - s_1) \ge 0 \tag{4}$$

Note that the two semi-plane has different sign in their inequality expression due to the fact that (1) models the lower half of the plane while (3) models the upper half of it.

B. Ranging Constraints

Due to the ranging error, the distance from s_1 to s_2 usually is not identical to the measurement d, but in a range between $d - \eta^- > 0$ and $d + \eta^+$, where $\eta^- \in \mathbb{R}$, $\eta^- > 0$, $\eta^+ \in \mathbb{R}$, $\eta^+ > 0$. The ranging constraint thus becomes,

$$d - \eta^- \le \|s_2 - s_1\| \le d + \eta^+ \tag{5}$$

Note that the the range constraint $d - \eta^- \le \|s_2 - s_1\|$ is not convex. Direct solution of it may end up with local optima, or even violation of this constraint. For this consideration, we relax it to a semi-plane constraint formed by the straight-line passing through the two intersection points of the arc and the two lines associated with angles $\theta \pm \delta$. This semi-plane is described as follows (draw a figure to show it),

$$-[\cos\theta, \sin\theta]x + (d - \eta^-)\cos\delta \le 0 \tag{6}$$

This equation can be obtained directly by noticing that the separation line is normal to the vector $[\cos\theta, \sin\theta]^T$, and has a distance of $(d - \eta^-)\cos\delta$ from the origin. Therefore, we have the following equivalently for s_1 and s_2,

$$-[\cos\theta, \sin\theta](s_2 - s_1) + (d - \eta^-)\cos\delta \le 0 \tag{7}$$

This convex constraint approaches the non-convex constraint $d - \eta^- \le \|s_2 - s_1\|$ when δ goes to zero. Generally, the set formed by (7) includes $\{(s_1, s_2) \in \mathbb{R}^2 \times \mathbb{R}^2, d - \eta^- \le \|s_2 - s_1\|\}$ as a subset. In practice, δ is very small and (7) provides a tight relaxation of $d - \eta^- \le \|s_2 - s_1\|$.

As to the range condition $\|s_2 - s_1\| \le d + \eta^+$, we equivalently write it as follows,

$$(s_2 - s_1)^T(s_2 - s_1) \le (d + \eta^+)^2 \tag{8}$$

C. Restriction on the Variable Space

For beacon-free localization, the translation or the rotation of all coordinates has no impact to the relative ranging and bearing of all pair of sensors. In this paper, we aims to find one feasible solution satisfying the constraints formulated in previous sections. Although a set of coordinates with a large absolute value can also satisfy all the constraints, the accuracy may reduce in this situation due to fixed point operations of large values when an algorithm is realized on a computationally restrictive sensor network. For this consideration, we impose additional constraints as follows,

$$-\epsilon \le s_i \le \epsilon \quad \text{for } i = 1, 2 \tag{9}$$

where $\epsilon \in \mathbb{R}^2$ defines the allowed variable space of s_i. In implementation, the value of ϵ can be assigned according to prior knowledge on the size of the space occupied by the wireless sensor network.

D. Problem Formulation

We are now ready to present the beacon-free localization problem in a mathematical formulation. Taking into account (2), (4), (7), (8), and (10), this problem is defined to find a feasible solution of the following inequality set,

$$
\begin{aligned}
&[-\sin(\theta_{ij}+\delta_{ij}), \cos(\theta_{ij}+\delta_{ij})](x_j - x_i) \le 0 && \text{(10a)}\\
&[-\sin(\theta_{ij}-\delta_{ij}), \cos(\theta_{ij}-\delta_{ij})](x_j - x_i) \ge 0 && \text{(10b)}\\
&-[\cos\theta_{ij}, \sin\theta_{ij}](x_j - x_i) + (d_{ij} - \eta_{ij}^-)\cos\delta_{ij} \le 0 && \text{(10c)}\\
&(x_j - x_i)^T(x_j - x_i) \le (d_{ij} + \eta_{ij}^+)^2 && \text{(10d)}\\
&-\epsilon \le x_i \le \epsilon && \text{(10e)}\\
&\forall j \in \mathcal{N}(i),\ \forall i = 1, 2, .., |V|
\end{aligned}
$$

M. Shell is with the Department of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA, 30332 USA e-mail: (see http://www.michaelshell.org/contact.html).

J. Doe and J. Doe are with Anonymous University.

Manuscript received April 19, 2005; revised January 11, 2007.

where $|V|$ is the total number of sensors in the network $G=(V,E)$ with V and E denoting the vertex set and the edge set respectively. The constraint (10) is composed of 4 scalar inequalities for each communication link $i-j$, and 4 for each sensor i (note that (10e) involves 4 inequalities as x_i is a two-dimensional vector). Overall, there are $4|E|+4|V|$ inequalities for the whole network. With the increase of $|V|$, $|E|$ increases at a speed of $O(|V|^2)$ for some graph topologies, which makes it prohibitive to solve (10) in a centralized manner. In this paper, we therefore explore a distributed computational algorithm for an efficient solution of (10).

II. Iterative Algorithm Design

A. Problem Reformulation

Constraint (10a), (10b) and (10c) can be written into the following compact form,

$$A_{ij}(x_j-x_i)+b_{ij}\leq 0 \tag{11}$$

where

$$A_{ij} = \begin{bmatrix} -\sin(\theta_{ij}+\delta_{ij}) & \cos(\theta_{ij}+\delta_{ij}) \\ \sin(\theta_{ij}-\delta_{ij}) & -\cos(\theta_{ij}-\delta_{ij}) \\ -\cos\theta_{ij} & -\sin\theta_{ij} \end{bmatrix} \tag{12}$$

$$b_{ij} = \begin{bmatrix} 0 \\ 0 \\ (d_{ij}-\eta_{ij}^-)\cos\delta_{ij} \end{bmatrix} \tag{13}$$

To derive theoretical conditions on the solution, we regard this problem as the following optimization with a virtual objective function:

$$\begin{aligned} &\min f(x_1,x_2,...,x_{|V|})=0 && (14a)\\ &\text{s.t.}\quad A_{ij}(x_j-x_i)+b_{ij}\leq 0 && (14b)\\ &(x_j-x_i)^T(x_j-x_i)\leq (d_{ij}+\eta_{ij}^+)^2 && (14c)\\ &-\epsilon\leq x_i\leq\epsilon && (14d)\\ &\forall j\in\mathcal{N}(i),\ \forall i=1,2,..,|V| \end{aligned}$$

B. Dual Space Analysis

Define $\lambda_{ij}\in\mathbb{R}^3$, $\lambda_{ij}\geq 0$ as the dual variable associated with the constraint (14b), and $\mu_{ij}\in\mathbb{R}$, $\mu_{ij}\geq 0$ as the dual variable associated with the constraint (14c). Accordingly, define a Lagrangian function as

$$\begin{aligned} L \;=\; &\frac{1}{2}\sum_{i=1}^{|V|}\sum_{j\in\mathcal{N}(i)}\Bigg(k_0\lambda_{ij}^T\big(A_{ij}(x_j-x_i)+b_{ij}\big)\\ &+\frac{1}{2}\mu_{ij}\big((x_j-x_i)^T(x_j-x_i)-(d_{ij}+\eta^+)^2\big)\Bigg) \end{aligned} \tag{15}$$

where $k_0>0$. Using KKT condition, we get the following equivalent form of the problem,

$$\sum_{i=1}^{|V|}(y_i-x_i)^T\frac{\partial L}{\partial x_i}\geq 0\,\forall y_i\in\Omega \tag{16a}$$

$$\begin{cases}(\lambda_{ij})_k>0 & \text{if } \big(A_{ij}(x_j-x_i)+b_{ij}\big)_k=0\\ (\lambda_{ij})_k=0 & \text{else if } \big(A_{ij}(x_j-x_i)+b_{ij}\big)_k<0\end{cases} \tag{16b}$$

$$\forall\, k=1,2,3,4 \tag{16c}$$

$$\begin{cases}\mu_{ij}>0 & \text{if } (x_j-x_i)^T(x_j-x_i)-(d_{ij}+\eta^+)^2=0\\ \mu_{ij}=0 & \text{else if } (x_j-x_i)^T(x_j-x_i)-(d_{ij}+\eta^+)^2<0\end{cases} \tag{16d}$$

where the set Ω is defined as $\Omega=\{x\in\mathbb{R}^2,-\epsilon\leq x\leq\epsilon\}$, $(\lambda_{ij})_k$ denotes the k-th element of the vector λ_{ij}, and $\big(A_{ij}(x_j-x_i)+b_{ij}\big)_k$ denotes the k-th element of the vector $A_{ij}(x_j-x_i)+b_{ij}$. $\frac{\partial L}{\partial x_i}$ is computed as

$$\frac{\partial L}{\partial x_i}=-\sum_{j\in\mathcal{N}(i)}\big(k_0A_{ij}^T\lambda_{ij}+\mu_{ij}(x_j-x_i)\big) \tag{17}$$

According to the properties of projection to convex set, (16) is equivalently converted to,

$$x_i=\mathrm{P}_\Omega(x_i-k_1\frac{\partial L}{\partial x_i}) \tag{18a}$$

$$\lambda_{ij}=\big(\lambda_{ij}+k_2A_{ij}(x_j-x_i)+k_2b_{ij}\big)^+ \tag{18b}$$

$$\mu_{ij}=\big(\mu_{ij}+k_3(x_j-x_i)^T(x_j-x_i)-k_3(d_{ij}+\eta^+)^2\big)^+ \tag{18c}$$

where $k_1>0$, $k_2>0$, $k_3>0$, $P_\Omega(x)$ is defined as the projection of $x\in\mathbb{R}^2$ to the set Ω. In concrete,

$$y=P_\Omega(x)\Leftrightarrow y_i=\begin{cases}\epsilon_i & \text{if } x_i\geq\epsilon_i\\ -\epsilon_i & \text{if } x_i\leq-\epsilon_i\\ x_i & \text{otherwise}\end{cases}$$
$$\text{for } i=1,2 \tag{19}$$

where $y=[y_1,y_2]^T\in\mathbb{R}^2$, $\epsilon=[\epsilon_1,\epsilon_2]^T$. For $x=[x_i]\in\mathbb{R}^n$, $y=[y_i]\in\mathbb{R}^n$ with n being $n=1,2,3,...$, $y=x^+$ is defined in entry-wise as $y_i=\max\{x_i,0\}$. The equation set (18) is derived using KKT condition and projection theorems. It is an equivalent solution of (10). The solution of (10) cannot be directly solved due to the nonlinearity in it. Instead, we use the following procedure, the equilibrium point of which exactly follows (18), to solve (10) iteratively,

$$\begin{aligned} x_i(t) &= (1-c_0)x_i(t-1)+c_0\mathrm{P}_\Omega\bigg(x_i(t-1) \\ &\quad +k_1\sum_{j\in\mathcal{N}(i)}[k_0A_{ij}^T\lambda_{ij}(t-1)+\mu_{ij}x_j(t-1) \\ &\quad -\mu_{ij}x_i(t-1)]\bigg) \qquad (20a) \\ \lambda_{ij}(t) &= (1-c_0)\lambda_{ij}(t-1)+c_0[\lambda_{ij}(t-1) \\ &\quad +k_2A_{ij}x_j(t-1)-k_2A_{ij}x_i(t-1)+k_2b_{ij}]^+ \qquad (20b) \\ \mu_{ij}(t) &= (1-c_0)\mu_{ij}(t-1)+c_0[\mu_{ij}(t-1) \\ &\quad +k_3\|x_j(t-1)-x_i(t-1)\|^2-k_3(d_{ij}+\eta^+)^2]^+ \qquad (20c) \end{aligned}$$

where the expression of $\frac{\partial L}{\partial x_i(t-1)}$ in (17) is substituted into (20a).

III. Convergence Analysis

Theorem 1: For any small value $\epsilon_0 > 0$, there always exist $\delta_0 > 0$, $n_0 > 0$ and x_i^* (for $i = 1, 2, ..., |V|$), which is a solution of (10), such that for all $0 < c_0 < \delta_0$ and $t > n_0$, $\|x_i(t) - x_i^*\| \leq \epsilon_0$ for $i = 1, 2, ..., |V|$, where $x_i(t)$ is the solution generated by the dynamic iteration (20) at time step t with $k_0, k_1, k_2, k_3 > 0$ and any feasible initial values $x_i(0) \in \Omega$, $\lambda_{ij}(0) \geq 0$, $\mu_{ij}(0) \geq 0$, $\forall i = 1, 2, ..., |V|$, $j \in \mathcal{N}(i)$.

A. Subsection Heading Here

Subsection text here.

1) Subsubsection Heading Here: Subsubsection text here.

IV. Conclusion

The conclusion goes here.

Appendix A
Proof of the First Zonklar Equation

Appendix one text goes here.

Appendix B

Appendix two text goes here.

Acknowledgment

The authors would like to thank...

References

[1] H. Kopka and P. W. Daly, *A Guide to LATEX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.

PLACE PHOTO HERE

Michael Shell Biography text here.

John Doe Biography text here.

Jane Doe Biography text here.