Vp140 Recitation IV

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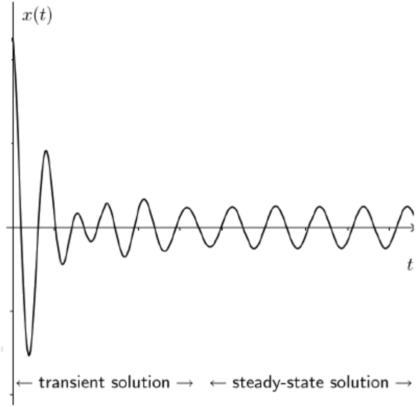
Overview

1. Forced Oscillations
2. Dynamics in Non-Inertial Frames of Reference
3. Earth as a Frame of Reference

Forced Oscillations

Graph



Forced Oscillations

Equations

$$x_s(t) = A\cos\left(\omega_{dr} t + \phi\right)$$

$$A\left(\omega_{dr}\right) = \frac{F_0}{m\sqrt{\left(\omega_0^2 - \omega_{dr}^2\right)^2 + \left(\frac{b\omega_{dr}}{m}\right)^2}}$$

$$\tan\phi = \frac{b\omega_{dr}}{m\left(\omega_{dr}^2 - \omega_0^2\right)}$$

Observation

Peak in the curve $A = A\left(\omega_{dr}\right)$ at the resonance frequency:

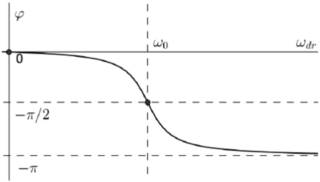
$$\omega_{dr} = \omega_{res} = \sqrt{\omega_0^2 - b^2/2m^2}$$

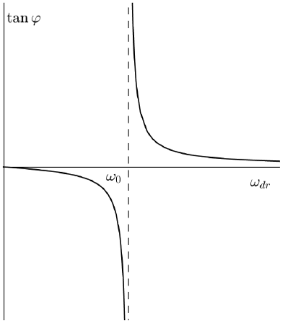
Phase shift

Equation

$$\tan \phi = \frac{b\omega_{dr}}{m\left(\omega_{dr}^2 - \omega_0^2\right)}$$

Graph





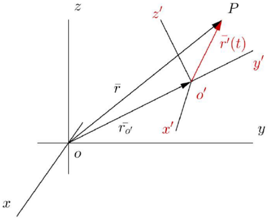
Exercise I

Forced Oscillation

A sinusoidally varying driving force is applied to a damped harmonic oscillator of force constant k and mass m. If the damping constant has a value b_1, the amplitude is A_1 when the driving angular frequency equals $\sqrt{k/m}$. In terms of A_1, what is the amplitude for the same driving frequency and the same driving force amplitude Fmax, if the damping constant is (a) $3b_1$ and (b) $b_1/2$?

Equation of Motion in a Non-Inertial FoR

Figure



Position relation

$$\overline{r} = \overline{r}_{O'} + \overline{r'}$$

Equation of Motion in a Non-Inertial FoR

Velocity relation

$$\overline{v} = \overline{v}_{O'} + \overline{v}' + (\overline{\omega} \times \overline{r}')$$

Comment

The arbitrary motion of $x'y'z'$ can be decomposed into a translational motion and a rotational motion about an instantaneous axis of rotation; the last term is due to the latter.

Equation of Motion in a Non-Inertial FoR

Acceleration relation

$$\overline{a} = \overline{a}_{O'} + \overline{a}' + 2\overline{\omega} \times \overline{v}' + \frac{\mathrm{d}\overline{\omega}}{\mathrm{d}t} \times \overline{r}' + \overline{\omega} \times \left(\overline{\omega} \times \overline{r}'\right)$$

$$m\overline{a}' = \overline{F} - m\overline{a}_{o'} - m\frac{\mathrm{d}\overline{\omega}}{\mathrm{d}t} \times \overline{r'} - 2m\left(\overline{\omega} \times \overline{v'}\right) - m\overline{\omega} \times \left(\overline{\omega} \times \overline{r'}\right)$$

Comment

Pseudo forces (also called fictitious forces or forces of inertia) — kinematic corrections (have units of [N]) that are due to the fact that we describe dynamics in a non-inertial FoR.

Equation of Motion in a Non-Inertial FoR

Fictitious Forces

$$
\begin{aligned}
& -m\bar{a}_{o'} && \text{d}'\text{Alembert ''force''} \\
& -m\frac{\mathrm{d}\omega}{\mathrm{d}t}\times\bar{r}' && \text{Euler ''force''} \\
& -2m\bar{\omega}\times\bar{v}' && \text{Coriolis ''force''} \\
& -m\bar{\omega}\times(\bar{\omega}\times\bar{r}') && \text{centrifugal ''force''}
\end{aligned}
$$

Comment

These "forces" must never appear in inertial FoRs!

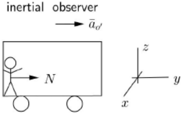
Examples

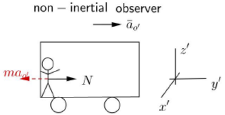
Kinematic Equation

$$m\bar{a}' = \overline{F} - m\bar{a}_{o'} - m\frac{\mathrm{d}\overline{\omega}}{\mathrm{d}t} \times \overline{r'} - 2m\left(\overline{\omega} \times \overline{v'}\right) - m\overline{\omega} \times \left(\overline{\omega} \times \overline{r'}\right)$$

Accelerating Car Moving along a Straight Line

Assume $\bar{a}_{o'} \neq 0$, $\bar{\omega} = 0, \bar{v'} = 0$; e.g. acclerating car moving along a straight line.





$$ma_{o'} = N$$

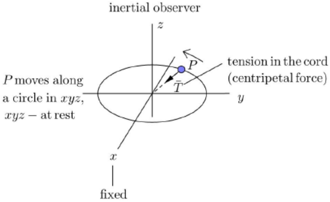
$$0 = N - ma_{o'}$$

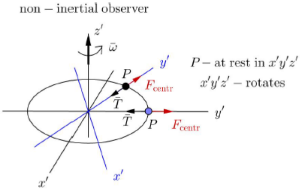
Examples

Kinematic Equation

$$m\overline{a}' = \overline{F} - m\overline{a}_{o'} - m\frac{\mathrm{d}\overline{\omega}}{\mathrm{d}t} \times \overline{r'} - 2m\left(\overline{\omega} \times \overline{v'}\right) - m\overline{\omega} \times \left(\overline{\omega} \times \overline{r'}\right)$$

Uniform Circular Motion

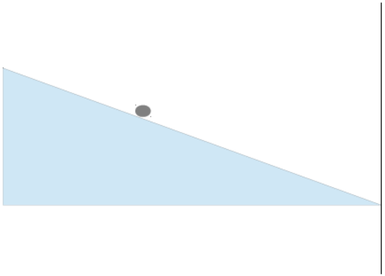




Exercise II

Centripetal Force

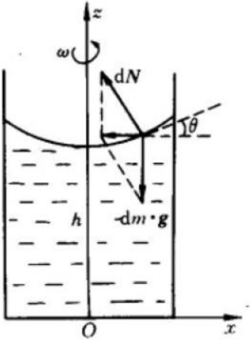
A plane, inclined at an angle α to the horizontal, rotates with constant angular speed ω about a vertical axis (see the figure). Where on the inclined plane should we place a particle, so that it remains at rest? The plane is frictionless.



Exercise III

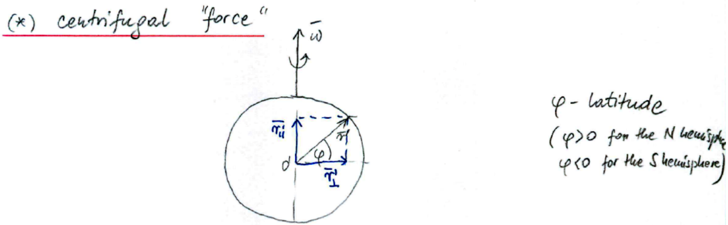
Shape of surface (Credit to Jiadi)

A bucket of water is rotating about its central vertical axis at constant angular velocity ω. Try to prove that when the water is static relative to the bucket, the upper surface of the water is paraboloid.



Centrifugal Force

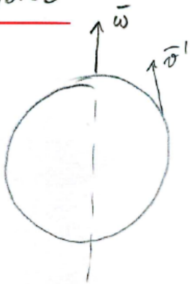
Figure



Corioli Force

Figure

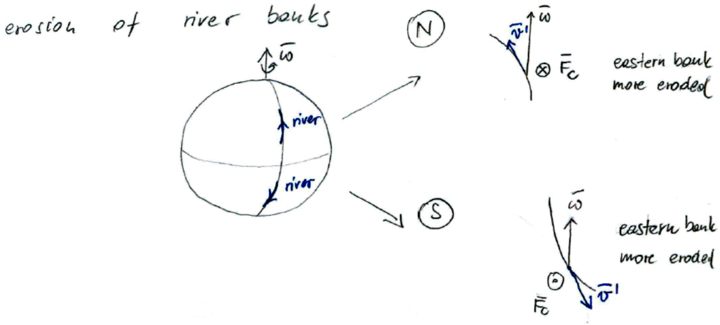
(*) the Coriolis' force



$$\bar{F}_c = -2m\left(\bar{\omega} \times \bar{v}'\right)$$

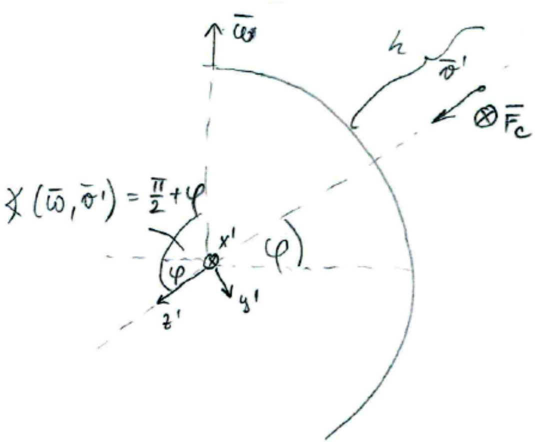
Corioli Force

Erosion of bank



Corioli Force

Free fall



The End

- Office hour: Wed 8:00-10:00 (Discussion Room 326I)
- Email: *zhanghaomeng@sjtu.edu.cn*