Vp140 Recitation I

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Overview

1. Uncertainty and Significant Figures
2. Unit
3. Vectors and Basic Vector Operations
4. Cartesian coordinate system
5. Kinematics in 1D

Uncertainty

Some examples

1. 56.47 ± 0.02 mm
2. 1.6454(21)
3. 47 ohms $\pm$ 10%

- You do not need to know the detailed calculation methods in this course.
- The way to calculate the uncertainty will be carefully discussed in your Vp141 lab.

Significant figures

Multiplication or division

Result can have no more significant figures than the factor with the fewest significant figures.

Example

1. $\frac{0.745 \times 2.2}{3.885} = 0.42$
2. $1.32578 \times 10^7 \times 4.11 \times 10^{-3} = 5.45 \times 10^4$

Significant figures

Addition or subtraction

Number of significant figures is determined by the term with the largest uncertainty (i.e., fewest digits to the right of the decimal point).

Example

1. $27.153 + 138.2 - 11.74 = 153.6$

Significant notation

Example

1. $384,000,000\mathrm{m} = 3.84 \times 10^8\mathrm{m}$
2. $384,000,000\mathrm{m} = 3.8400 \times 10^8\mathrm{m}$

- For the textbook, it usually gives numerical value with 3 significant figures. Be careful about your online homework.
- You must round, not truncate.

Unit prefixes

Example

$$
\begin{aligned}
1 \text{ kilometer} &= 1 \text{ km} &= 10^3 \text{ meters} &= 10^3 \text{ m} \\
1 \text{ kilogram} &= 1 \text{ kg} &= 10^3 \text{ grams} &= 10^3 \text{ g} \\
1 \text{ kilowatt} &= 1 \text{ kW} &= 10^3 \text{ watts} &= 10^3 \text{ W}
\end{aligned}
$$

Example

TABLE 1.1 Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and other cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for a car moving at freeway speed to travel 3 cm)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^{3} m (distance in a 10-minute walk)		

Unit conversions

- An equation must always be dimensionally consistent.

Example

SOLUTION

EXAMPLE 1.1 CONVERTING SPEED UNITS

The world land speed record of 763.0 mi/h was set on October 15, 1997, by Andy Green in the jet-engine car *Thrust SSC*. Express this speed in meters per second.

SOLUTION

IDENTIFY, SET UP, and EXECUTE: We need to convert the units of a speed from mi/h to m/s. We must therefore find unit multipliers that relate (i) miles to meters and (ii) hours to seconds. In Appendix E we find the equalities 1 mi = 1.609 km, 1 km = 1000 m, and 1 h = 3600 s. We set up the conversion as follows, which ensures that all the desired cancellations by division take place:

$$\begin{aligned} 763.0\ \text{mi/h} &= \left(763.0\ \frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{1.609\ \cancel{\text{km}}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right) \\ &= 341.0\ \text{m/s} \end{aligned}$$

EVALUATE: This example shows a useful rule of thumb: A speed expressed in m/s is a bit less than half the value expressed in mi/h, and a bit less than one-third the value expressed in km/h. For example, a normal freeway speed is about 30 m/s = 67 mi/h = 108 km/h, and a typical walking speed is about 1.4 m/s = 3.1 mi/h = 5.0 km/h.

Exercise I

Exercise I

A simple pendulum consists of a light inextensible string AB with length L, with the end A fixed, and a point mass M attached to B. The pendulum oscillates with a small amplitude, and the period of oscillation is T. It is suggested that T is proportional to the product of powers of M, L and g, where g is the acceleration due to gravity. Use dimensional analysis to find this relationship.

Vectors

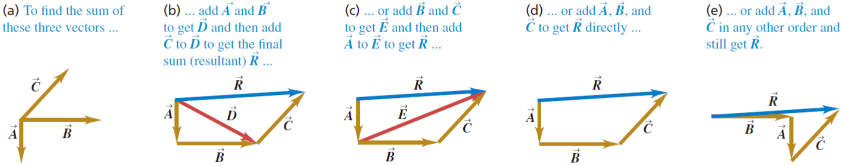
- Magnitude + Direction
- Notation: $\vec{A}$

Example

1. Displacement
2. Force

Addition and subtraction

Addition

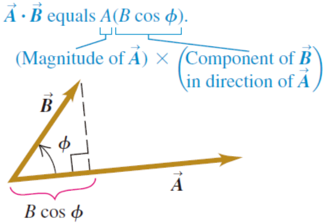


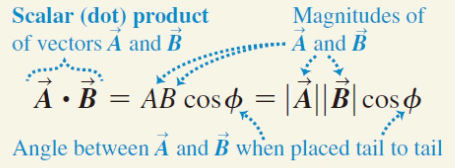
1.13 Several constructions for finding the vector sum $\vec{A} + \vec{B} + \vec{C}$.

Subtraction

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Scalar product (Scalar)

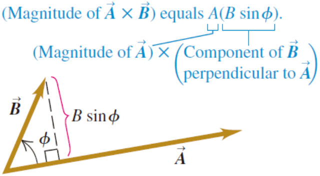




- Scalar product of two perpendicular vectors is always zero.
- Commutative law : $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Vector product (Vector)

Magnitude



Magnitude of **vector (cross) product** of vectors $\vec{B}$ and $\vec{A}$

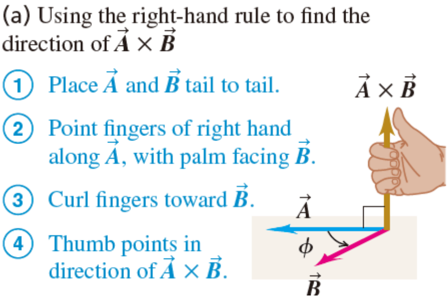
$$C = AB \sin\phi$$

Magnitudes of $\vec{A}$ and $\vec{B}$

Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

- The vector product of two parallel or antiparallel vectors is always zero.
- The vector product of any vector with itself is zero.

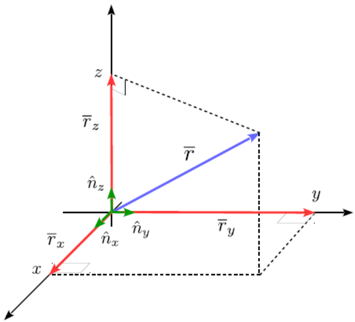
Direction



- The vector product is not commutative but instead is anticommutative : $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Cartesian coordinate system

Cartesian



$$\overline{r} = x\hat{n}_x + y\hat{n}_y + z\hat{n}_z$$

Exercise II

Exercise II

Consider two vectors $\mathbf{u} = 3\hat{n}_x + 4\hat{n}_y$ and $\mathbf{w} = 6\hat{n}_x + 16\hat{n}_y$. Find the components of the vector $\mathbf{w}$ that are, respectively, parallel and perpendicular to the vector $\mathbf{u}$.

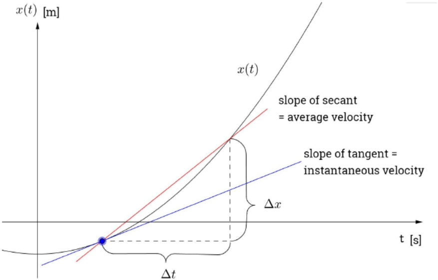
Exercise III

Exercise III

A river flows from south to north at 5 km/h. On this river, a boat is heading east to west, perpendicular to the current at 7 km/h. As viewed by an eagle hovering at rest over the shore, how fast and in what direction is this boat traveling?

Average & instantaneous quantities

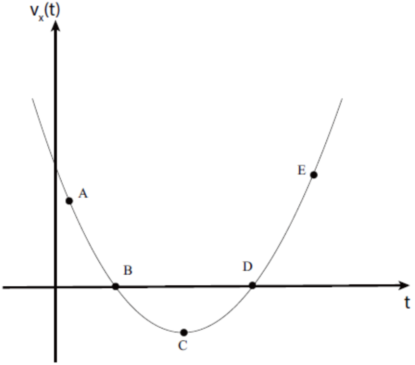
Velocity



- Average velocity: $v_{\mathrm{av},x} \overset{def}{=} \frac{x(t+\Delta t)-x(t)}{\Delta t}$
- Instantaneous velocity: $\lim_{\Delta t\to 0} \frac{x(t+\Delta t)-x(t)}{\Delta t} = \frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x}(t) \overset{def}{=} v_x(t)$

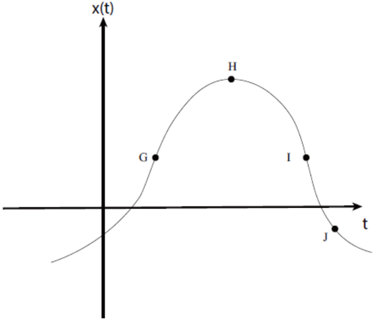
Analysis the graphs

$v_x(t)$



Analysis the graphs

$x(t)$



Exercise IV

Exercise IV

A car is moving in one direction along a straight line. Find the average velocity of the car if:

- it travels half of the **journey time** with velocity v_1 and the other half with velocity v_2.
- it covers half of the **distance** with velocity v_1 and the other one with velocity v_2.

Here both v_1 and v_2 are constants.

Example from slides

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Example. Motion with Varying Acceleration

The acceleration of an iron object moving in a straight line close to a magnet turns out to be related to its position with respect to the magnet, $x > 0$, as $a_x = -k/x$, where k is a positive constant. It has been experimentally determined that the velocity of the object is $\sqrt{2}v_0 > 0$, when the object is at $x = x_0 > 0$ and it is v_0 when $x = 2x_0$.
What is the velocity of this particle when $x = 3x_0$?

Note that $a_x = \frac{dv_x}{dt} \overset{\text{chain rule}}{=} \frac{dv_x}{dx}\underbrace{\frac{dx}{dt}}_{v_x} = \frac{dv_x}{dx} v_x$.

Hence $-\frac{k}{x} = v_x \frac{dv_x}{dx}$. Integrating and using the information provided

$$\int_{\sqrt{2}v_0}^{v_0} v_x \, dv_x = -\int_{x_0}^{2x_0} \frac{k}{x} \, dx \qquad \Longrightarrow \qquad \frac{1}{2}v_0^2 - v_0^2 = -k \ln 2,$$

so that $\boxed{k = \frac{v_0^2}{2 \ln 2}.}$

Example from slides

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Example. Motion with Varying Acceleration (contd)

To find out the answer, integrate with the corresponding limits

$$\int_{\sqrt{2}v_0}^{v_x(3x_0)} v_x\, dv_x = -\int_{x_0}^{3x_0} \frac{k}{x}\, dx$$

to get

$$\frac{1}{2}v_x^2(3x_0) - v_0^2 = -k\ln 3 = -\frac{v_0^2}{2}\frac{\ln 3}{\ln 2}.$$

Solving for $v_x(3x_0)$ yields

$$\boxed{v_x(3x_0) = v_0\sqrt{2 - \frac{\ln 3}{\ln 2}}.}$$

The End

- Office hour: Wed 8:00-10:00 (Discussion Room 326I)
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