Vp140 Recitation V

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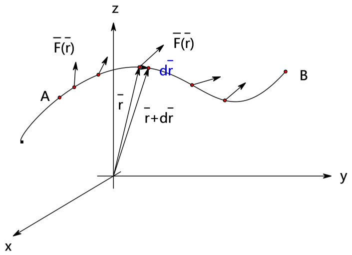
June 27, 2019

Overview

Work

Graph



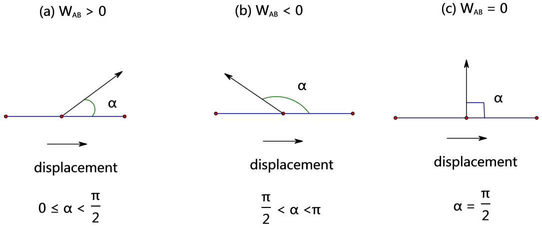
Formula

$$\delta W = \overline{F} \circ d\overline{r}$$

$$W_{AB} = \int_{\Gamma_{AB}} \overline{F} \circ d\overline{r}$$

Positive/Negative work

Graph



Multiple Forces

Formula

$$\delta W = \left(\overline{F}_1 + \overline{F}_2 + \ldots + \overline{F}_N\right) \circ d\overline{r} = \overline{F}_1 \circ d\overline{r} + \overline{F}_2 \circ d\overline{r} + \ldots + \overline{F}_N \circ d\overline{r}$$
$$= \delta W_1 + \delta W_2 + \ldots + \delta W_N$$

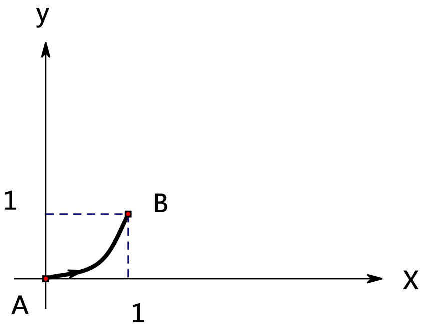
$$\delta W = \overline{F}_{net} \circ d\overline{r}$$

General cases – varying force, curved path

Example from the lecture

Calculate work done by the force $\overline{F}(\overline{r}) = \left(x^2 + y^2\right)\hat{n}_x + x\hat{n}_y\ N$ acting on a particle moving from (0,0) to (1,1) along a segment of parabola $y = x^2$

Graph



Kinetic Energy Theorem

Kinetic Energy

$$K = \frac{1}{2}mv^2$$

Theorem

Work done by the net force on a particle is equal to the change in the particle's kinetic energy.

$$W = \Delta K$$

Power

Instantaneous Power

$$\frac{\delta W}{dt} = \overline{F} \circ \overline{v} = P$$

Average Power

$$\frac{W}{\Delta t} = P_{av}$$

Units

$$[W](Watt)$$

$$1 \text{ hp} = 746\text{W} = 0.746\text{kW}$$

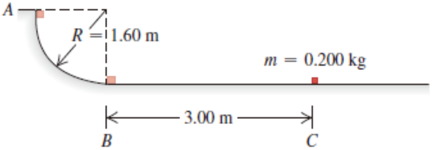
Exercise I

Kinetic Energy Theorem

In a truck-loading station at a post office, a small 0.200-kg package is released from rest at point A on a track that is one quarter of a circle with radius 1.60 m (Figure). The size of the package is much less than 1.60 m, so the package can be treated as a particle. It slides down the track and reaches point B with a speed of 4.80 m/s. From point B, it slides on a level surface a distance of 3.00 m to point C, where it comes to rest.

(a) What is the coefficient of kinetic friction on the horizontal surface?

(b) How much work is done on the package by friction as it slides down the circular arc from A to B?



Conservative Force

Definition

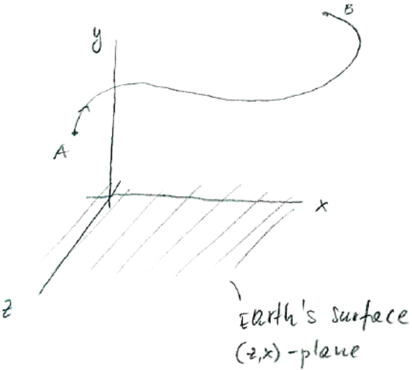
A force with the property that work done by it does not depend on path along which the particle is moved, but only on initial and final positions of the particle.

Examples

1. Gravitational force
2. Elastic force

Displacement along a Curved Path

Figure



Both elastic and gravitational force present

Relationship

$$W = W_{grav} + W_{el} \overset{\text{work-k.e. thm}}{=} K_B - K_A$$

$$\Updownarrow$$

$$U_{grav,A} + U_{el,A} + K_A = U_{grav,B} + U_{el,B} + K_B$$

$$\boxed{E = \underbrace{U}_{= U_{el} + U_{grav}} + K = \text{const}} \quad \text{i.e.} \quad \Delta E = 0$$

Conservative force in 1D

Formula

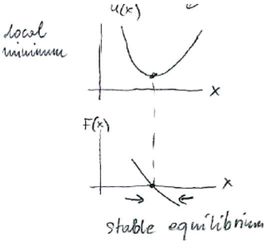
$$F_x = -\frac{du}{dx}$$

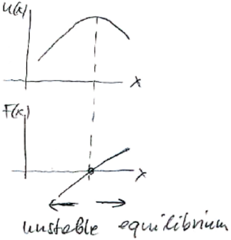
$$\delta W = F_x dx = -\frac{du}{dx} dx = -du$$

$$W_{A \to B} = u(A) - u(B)$$

Conservative force in 1D

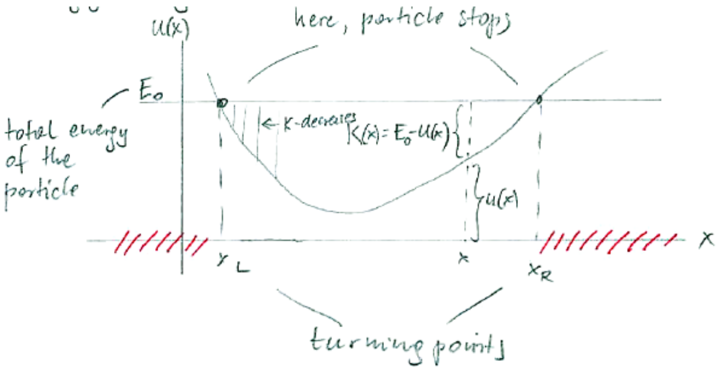
Figures





Potential Well

Figure



Conservative force in 3D

Formula

$$\overline{F} = \left(-\frac{\partial u}{\partial x}, -\frac{\partial u}{\partial y}, -\frac{\partial u}{\partial z}\right) = -\nabla u$$

$$\delta W = \overline{F} \circ d\overline{r} = -\frac{\partial u}{\partial x}dx - \frac{\partial u}{\partial y}dy - \frac{\partial u}{\partial z}dz = -du$$

Exercise II

Potential Energy and Force

Check that the potential energy

$$U(x, y, z) = 3xy^2z - 2yz^2$$

corresponds to the force

$$\mathbf{F}(\mathbf{r}) = \left(-3y^2z, -6xyz + 2z^2, -3xy^2 + 4yz\right)$$

The End

- Office hour: Wed 8:00-10:00 (Discussion Room 326I)
- Email: *zhanghaomeng@sjtu.edu.cn*