Vp140 Recitation VII

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Overview

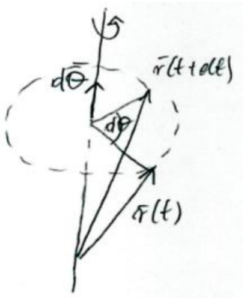
1. Basic kinematic quantities in rotational motion
2. Moment of inertia
3. Dynamics for Rigid Body Rotation
4. Work and Power in rotational motion
5. angular momentum

Angular velocity

Formula

$$\overline{\omega} = \frac{\mathrm{d}\overline{\theta}}{\mathrm{d}t} \text{ [rad/s]}$$

Figure

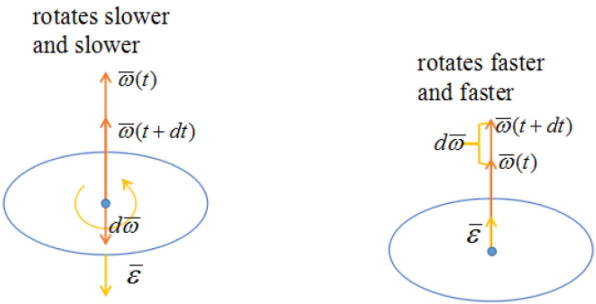


Angular acceleration

Formula

$$\overline{\varepsilon} = \frac{\mathrm{d}\overline{\omega}}{\mathrm{d}t}$$

Figure

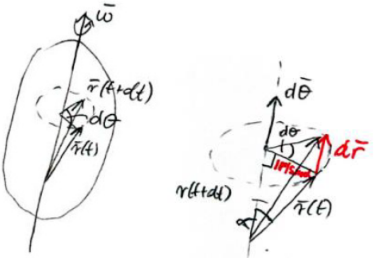


Velocity in a rotational motion

Formula

$$\overline{v} = \overline{\omega} \times \overline{r}$$

Figure



Acceleration in a rotational motion

Formula

$$\overline{a} = \overline{\varepsilon} \times \overline{r} + \overline{\omega} \times (\overline{\omega} \times \overline{r})$$

Magnitude

$$|\overline{a}_{\text{tan}}| = |\overline{\varepsilon}| \cdot |\overline{r}_{\perp}|$$

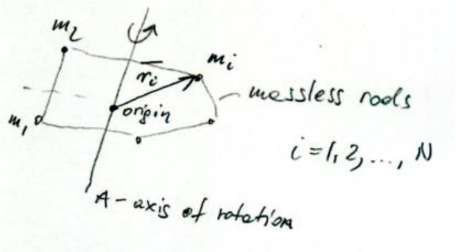
$$|\overline{a}_{\text{ centr }}| = |\overline{\omega} \times (\overline{\omega} \times \overline{r}_{\perp})| = |\overline{\omega}| \, |\overline{\omega} \times \overline{r}_{\perp}| = \omega^2 \, |\overline{r}_{\perp}|$$

Total Kinetic Energy

Formula

$$K = \sum_{i=1}^{N} \frac{1}{2} m_i \omega^2 r_{i\perp}^2 = \frac{1}{2}\left(\sum_{i=1}^{N} m_i r_{i\perp}^2\right)\omega^2 = \frac{1}{2} I \omega^2$$

Figure



Moment of Inertia

Formula

$$I = \sum_{i=1}^{N} m_i r_{i\perp}^2$$

$$I_A = \int_{\text{object}} r_\perp^2 \mathrm{d}m$$

Figure

Calculate the Moment of Inertia

Figure

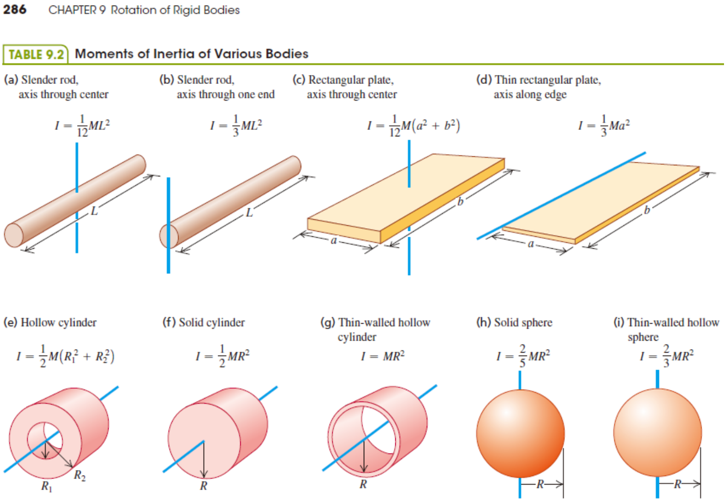

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TABLE 9.2 Moments of Inertia of Various Bodies

(a) Slender rod, axis through center
$I = \frac{1}{12}ML^2$

(b) Slender rod, axis through one end
$I = \frac{1}{3}ML^2$

(c) Rectangular plate, axis through center
$I = \frac{1}{12}M(a^2 + b^2)$

(d) Thin rectangular plate, axis along edge
$I = \frac{1}{3}Ma^2$

(e) Hollow cylinder
$I = \frac{1}{2}M(R_1^2 + R_2^2)$

(f) Solid cylinder
$I = \frac{1}{2}MR^2$

(g) Thin-walled hollow cylinder
$I = MR^2$

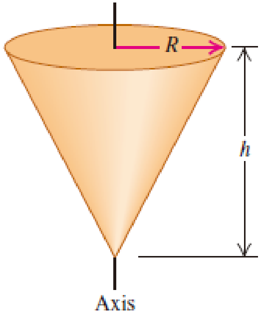
(h) Solid sphere
$I = \frac{2}{5}MR^2$

(i) Thin-walled hollow sphere
$I = \frac{2}{3}MR^2$

Exercise I

Calculate the moment of inertia

Calculate the moment of inertia of a uniform solid cone about an axis through its center. The cone has mass M and altitude h. The radius of its circular base is R.



Exercise II

Moment of Inertial Application

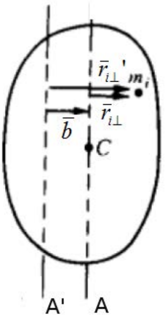
A 12.0-kg mass is released from rest and falls, causing the uniform 10.0-kg cylinder of diameter 30.0 cm to turn about a frictionless axle through its center. How far will the mass have to descend to give the cylinder 480 J of kinetic energy?

Parallel axis theorem (Steiner's theorem)

Formula

$$I_{A'} = I_A + mb^2$$

Figure



Torque

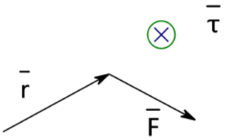
Formula

$$\overline{\tau} = \overline{r} \times \overline{F}$$

$$|\overline{\tau}| = |\overline{r}_{\perp}|\,|\overline{F}|$$

$$|\overline{\tau}| = |\overline{r}||\overline{F}| \sin\angle(\overline{r}, \overline{F}) = |\overline{r}_{\perp}|\,|\overline{F}| = |\overline{r}|\,\left|\overline{F}_{\perp}\right|$$

Figure



right-hand rule

units: N·m (=joule)

The Second Law of Dynamics for a Rigid Body Rotating about a Fixed Axis

Formula

$$\tau_z = I_z \varepsilon_z$$

Compares with Particle Motion

Compare with the 2^{nd} law of dynamics for (1D) motion of a particle

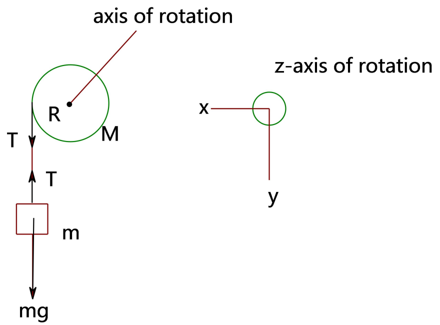
- particle: *net force* $\Rightarrow$ *acceleration* which is $\propto$ *force*, and $\propto \frac{1}{mass}$
- rigid body: net torque along axis of rotation $\Rightarrow$ angular acceleration which is $\propto$ torque, and $\propto \frac{1}{moment\ \ of\ \ inertia}$

Application of the Second Law of Dynamics

Examples from the lecture

The cylinder has mass M, radius R, can rotate about its axis of symmetry. Find the acceleration of the block a_y and the tension T in the string as it unwinds from the cylinder.

Figure



Rigid body rotation about a moving axis

Formula

$$K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

Conclusion

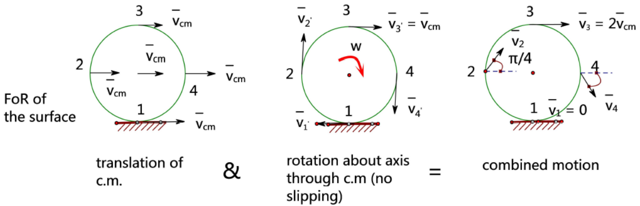
The rigid body's kinetic energy is the sum of $\frac{1}{2}Mv_{\mathrm{cm}}^2$, associated with the translational motion of the center of mass, and $\frac{1}{2}I_{cm}\omega^2$ associated with rotation about an axis through the center of mass.

Rolling Without Slipping

Relationship

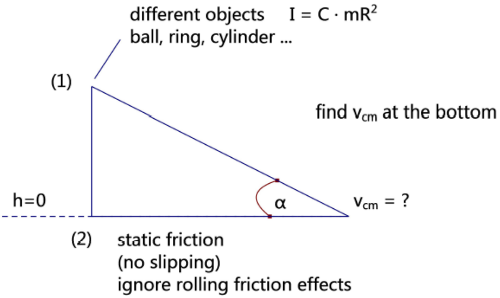
$$v_{cm} = \omega R$$

Figure



Rigid body race -(Example from slides)

Figure



Conclusion

$$v_{cm} = \sqrt{\frac{2gh}{1+C}}$$

Work and power in rotational motion

Work

$$W_{\theta_1 \to \theta_2} = \int_{\theta_1}^{\theta_2} \tau_z \mathrm{d}\theta$$

Power

$$P = \tau_Z \omega_Z$$

Angular Momentum

Definition

$$\overline{L} = \overline{r} \times \overline{p}$$

Relation with torque

$$\overline{\tau} = \frac{\mathrm{d}\overline{L}}{\mathrm{d}t}$$

Angular Momentum Conservation

Formula

$$\frac{dL_z}{dt} = \tau_z^{ext}$$

Conclusion

When the net external torque on a system is zero, then the total angular momentum of the system is conserved.

The End

- Office hour: Wed 8:00-10:00 (Discussion Room 326I)
- Email: *zhanghaomeng@sjtu.edu.cn*