Vp140 Recitation VI

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Overview

Momentum

Definition

$$\overline{p} = m\overline{v}$$

Units

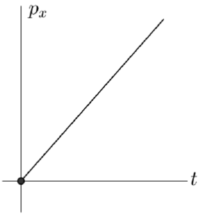
$$\left[kg \cdot m/s^2\right]$$

Momentum and Force

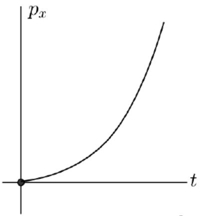
Equation

$$\overline{F} = \frac{\mathrm{d}\overline{p}}{\mathrm{d}t}$$

Graph



constant force $F_x = \text{const} > 0$



increasing force $F_x = \frac{\mathrm{d}p_x}{\mathrm{d}t} > 0$

Momentum-Impulse Theorem

Formula

$$p_2 - p_1 = \int_{t_1}^{t_2} F_x \mathrm{d}t$$

Kinetic Energy vs. Momentum.

Example from the lecture

Suppose we want to bring two objects to a stop.

$$
\begin{aligned}
v_A &= 10\ [m/s] & \qquad\qquad & & v_B &= 5\ [m/s] \\
m_A &= 400\ [kg] & & & m_B &= 800\ [kg] \\
p_A &= 4000\ [kg \cdot m/s] & & = & p_B &= 4000\ [kgm/s \\
K_A &= 20\ [kJ] & & & K_B &= 10kJ
\end{aligned}
$$

What is the average stopping force if both A and B have stopped

* after travelling a distance $s = 100$ m? $\Delta K = -F \cdot s$

$$F_A = 200N \qquad F_B = 100N$$

* after travelling for $t = 10$ s? $\Delta p = -F \cdot t$

$$F_A = 400N \qquad F_B = 400N$$

Conservation of Momentum

Definition

If the sum of all external forces on the system is equal to zero, then the total momentum of the system is constant.

Generalization

If the sum of external forces on a system is equal to zero, then the total momentum of the system is constant.

Energy in Collisions

Different cases

$$K_{\text{final}} = K_{\text{initial}} + Q \quad \begin{cases} = 0 & \text{(elastic)} \\ > 0 & \text{(superelastic)} \\ < 0 & \text{(inelastic)} \end{cases}$$

Elastic vs. Inelastic Collisions

1. Elastic: Mechanical energy is conserved. The total momentum of the system is conserved.
2. Inelastic: Mechanical energy is not conserved (total energy is conserved, but a part of mechanical energy is transformed irreversibly into internal energy). The total momentum of the system is conserved.

   Completely inelastic collisions:colliding particles move as one object after the collision, stick to each other.

Exercise I

Conservation of Momentum

A 0.100-kg stone rests on a frictionless, horizontal surface. A bullet of mass 6.00 g, traveling horizontally at 350 m/s, strikes the stone and rebounds at right angles (downwards) to its original direction with a speed of 250 m/s.

1. Compute the magnitude and direction of the velocity of the stone after it is struck.
2. Is the collision perfectly elastic?

Exercise II

Conservation of Momentum

A neutron at rest decays (breaks up) to a proton and an electron. Energy is released in the decay and appears as kinetic energy of the proton and electron. The mass of a proton is 1836 times the mass of an electron. What fraction of the total energy released goes into the kinetic energy of the proton?

Center of Mass

Definition

$$\overline{r}_{cm} = \frac{\sum_{i=1}^{N} m_i \overline{r}_i}{\sum_{i=1}^{N} m_i}$$

$$M\overline{v}_{cm} = \sum_{i=1}^{N} \overline{p}_i = \overline{p} \quad , \; M = \sum_{i=1}^{N} m_i$$

$$\frac{\mathrm{d}\overline{p}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(M\overline{v}_{cm}\right) = M\frac{\mathrm{d}\overline{v}_{cm}}{\mathrm{d}t}$$

Conclusion

1. The total momentum of the system is equal to the momentum of a hypothetical particle of mass M moving with velocity $\overline{v}_{\mathrm{cm}}$.
2. If the sum of all external forces acting on the system is equal to zero, the center of mass moves with a constant velocity.

Exercise III

Continuous center of mass

For a solid object whose mass distribution does not allow for a simple determination of the center of mass by symmetry, we can generalize to integrals

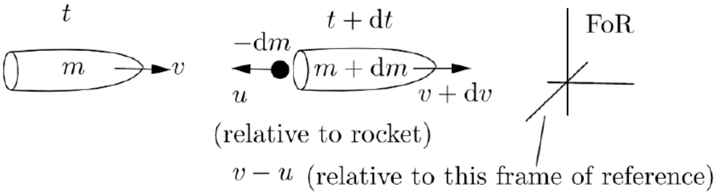
$$x_{\mathrm{cm}} = \frac{1}{M}\int x dm \qquad y_{\mathrm{cm}} = \frac{1}{M}\int y dm$$

Consider a thin rod of length L, mass M, and cross-sectional area A. Let the origin of the coordinates be at the left end of the rod and the positive x-axis lie along the rod.

1. If the density $\rho = M/V$ of the object is uniform, perform the integration described above to show that the x-coordinate of the center of mass of the rod is at its geometrical center.
2. If the density of the object varies linearly with x. That is, $\rho = ax$, where a is a positive constant. Calculate the x-coordinate of the rod's center of mass.

Example-Rocket Propulsion

Figure



The End

- Office hour: Wed 8:00-10:00 (Discussion Room 326I)
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