Summary

Modern professional sports organizations face a persistent tension between maximizing competitive performance and ensuring long-term financial sustainability. This challenge has become particularly pronounced in the Women's National Basketball Association (WNBA), where recent surges in attendance, media exposure, and franchise valuations coexist with strict salary caps and ongoing debates over revenue sharing[1, 2, 3, 4]. The central problem addressed in this paper is how a professional sport team can make optimal, multi-season decisions—regarding roster construction, salary allocation, and revenue-related strategies—to balance on-court success with financial returns under realistic league constraints.

To address this problem, we develop a dynamic optimisation framework that models team management as a sequential decision–making process over multiple seasons. Team states incorporate player performance indicators, payroll commitments, attendance trends and external conditions such as league expansion. Decisions include roster selection, salary allocation and pricing–related choices. Competitive performance is quantified using performance-based player metrics inspired by the Performance Index Rating (PIR) methodology[5]. Financial outcomes are modelled through revenue and cost functions informed by historical WNBA data[6, 1].

Owing to the high dimensionality and stochastic nature of the problem, we combine exact optimisation on reduced–scale instances with reinforcement learning to approximate policies on full-scale rosters. This two-pronged strategy draws on advances in multi-agent systems and deep reinforcement learning for sequential decision problems[7, 8]. By varying the weight assigned to competitive success versus financial profit, the model generates a spectrum of strategies that trace out a clear trade-off frontier between winning and profitability.

Our results show that purely performance-driven strategies lead to higher short-term wins but increased financial risk, while profit-oriented strategies stabilise revenue at the expense of competitiveness. Balanced strategies achieve near-playoff-level performance while maintaining sustainable financial outcomes, even under disruptive scenarios such as league expansion[4]. These findings provide actionable insights for team executives and demonstrate the value of dynamic, data-driven decision support in modern sports management.

Contents

1 Introduction

1.1 Problem Restatement

The WNBA operates in a rapidly evolving economic and competitive environment. Teams must comply with league-imposed constraints such as roster limits and salary caps, while simultaneously responding to rising fan demand, increasing media revenues and structural changes including league expansion[4, 1]. Within this context, team management faces a fundamental question: how should limited resources be allocated over time to balance athletic success and financial performance?

Rather than focusing on single-season outcomes, the problem requires a multi-season perspective, where decisions made today—such as signing a high-salary star player or prioritizing younger talent—affect future competitiveness, payroll flexibility, and revenue potential. The task of Problem D is therefore to construct a model that captures these intertemporal trade-offs and provides rational, data-supported guidance for strategic decision-making.

1.2 Literature Review

Existing research provides several important foundations for this work. In basketball analytics, roster construction has been studied as a multi-objective optimisation problem, where team performance and cost must be considered simultaneously. Pérez-Toledano et al. demonstrated that evolutionary algorithms can effectively generate efficient rosters using PIR-based performance metrics under budget constraints[5]. Their work highlights that optimal team composition is rarely achieved by simply selecting the best individual players, but rather by balancing performance contributions relative to cost.

From a systems perspective, sports teams are increasingly modelled as multi-agent environments, where coordinated behaviour and long-term planning are essential. Radke and Orchard emphasise that team sports analytics naturally involve multi-agent decision-making across different timescales, from in-game tactics to long-term roster management[7]. This perspective supports modelling team management as a sequential decision problem.

Recent advances in reinforcement learning further motivate our approach. Deep reinforcement learning has been successfully applied to fantasy sports team selection, where agents learn sequential drafting strategies that outperform static heuristics[8]. While fantasy sports differ from real-team management, the underlying principle—learning optimal sequences of decisions under uncertainty—is directly applicable.

Economic analyses of the WNBA provide the financial context for our model. Reports on revenue sharing and salary negotiations illustrate that player compensation remains a small fraction of total league revenue, despite rapid growth in attendance and media income[2, 3]. Empirical data sources, including comprehensive WNBA statistics and attendance records[6, 1], enable realistic parameter estimation and validation of model assumptions.

1.3 Our Work

Building on these strands of research, we propose a dynamic optimization model that integrates competitive performance and financial outcomes within a unified framework. Unlike static roster optimization models, our approach explicitly considers multi-season dynamics, uncertainty, and strategic trade-offs. By combining mathematical optimization principles with reinforcement learning, the

model produces interpretable, flexible strategies that adapt to different organizational priorities. The framework is designed to be realistic, data-driven, and directly applicable to decision-making in the WNBA.

2   Assumptions and Justification

All mathematical models require simplifying assumptions. The following assumptions are adopted to ensure tractability while preserving the essential structure of the problem.

We assume that player performance in a given season can be reasonably approximated by historical performance metrics adjusted for age and role. This assumption is supported by prior work showing that aggregate performance indices such as PIR are strong predictors of team success[5]. While individual performance is stochastic, expected values provide a suitable basis for strategic planning over multiple seasons.

We assume that ticket demand is influenced by both price and team performance, with higher-performing teams attracting greater attendance. This relationship is well supported by observed WNBA attendance trends, particularly during periods of heightened competitive interest[1]. Short-term fluctuations are modelled as random noise, while long-term trends are captured through state variables.

We assume that league rules, including salary caps and roster size limits, remain fixed over the planning horizon. Although collective bargaining agreements may evolve, this assumption is reasonable for medium-term planning and allows us to isolate the effects of managerial decisions.

Finally, we assume that external shocks such as league expansion affect all teams symmetrically in expectation. While individual teams may benefit or suffer disproportionately, modelling expansion as a structural change to the competitive environment provides a realistic and manageable representation of its impact[4].

3   Notation and Definitions

To characterize the key decisions in team operations, we define the following decision variables (including temporal and non-temporal types) with their properties and representations:

Table 1: Notation and Definition of Decision Variables

Symbol	Definition	Type	Constraints
$x_{p,t}$	Indicating whether player p is included in the team roster in season t (1 means signed/retained, 0 otherwise). Forms a matrix $\mathbf{X}$.	Binary	Subject to roster size limit and salary cap; temporal dependence (fixed to 1 for consecutive seasons under multi-year contracts).
$y_{p,t}$	Representing the salary paid to player p in season t. Forms a matrix $\mathbf{Y}$ with the same dimensions as $\mathbf{X}$.	Continuous	$y_{p,t} \geq 0$ if and only if $x_{p,t} = 1$; satisfies salary cap constraints, i.e., $\sum_p y_{p,t} \leq \mathrm{cap}_t$ (e.g., 2025 WNBA standard).
$p_{t,g}$	Ticket price or pricing level for the g-th home game in season t. Forms a matrix $\mathbf{P}$.	Continuous/Discrete	Determined by price-demand elasticity; adjustable dynamically based on opponents or game time to balance attendance and revenue.
m_t	Budget investment in marketing and fan engagement in season t.	Continuous	$m_t \geq 0$; affects brand value and long-term fan growth; extendable to multi-channel vector.
v_t	Binary variable for long-term venue decisions (new venue, renovation, lease), indicating implementation in season t.	Binary	Non-temporal (one-time decision affects multiple seasons); constrained by long-term financial capacity (e.g., loan repayment).
$e_{p,t}$	Binary variable for player equity incentive decisions, indicating provision to star player p in season t.	Binary	Non-temporal strategic decision; affects retention, salary structure, and long-term cost structure.

4   The Mathematical Model

This section details the construction, mathematical derivation, and solution algorithm of the dynamic optimization model for balancing WNBA team competitive performance and financial profitability.

4.1   Model Construction

The model frames team management as a *multi-period Markov Decision Process (MDP)* with interactive decision variables, state variables, and environmental factors. The core logic is to dynamically adjust operational strategies (e.g., roster selection, salary allocation, ticket pricing) based on the team's current competitive, financial, and external environment status, to maximize the long-term integrated utility of competitive success and financial health.

4.1.1   Variable Interactions

The interaction mechanism between key variables is illustrated in Figure 1. Decision Variables ($\mathbf{a}_t$) directly drive state transitions: Roster selection ($x_{p,t}$) and salary allocation ($y_{p,t}$) determine the team's competitive capacity, which affects game results and further impacts ticket/sponsorship revenue, while Ticket pricing ($p_{t,g}$) and marketing investment (m_t) directly influence short-term financial returns and long-term fan base growth. State Variables (s_t) constrain decision space: Current salary cap usage limits subsequent salary allocation, and team record and fan attendance affect the marginal benefit of pricing adjustments. Environmental Variables (s^{ctx}) modulate variable relationships: Large-market teams have higher ticket pricing flexibility, and league expansion increases player demand, raising the cost of free agent signings.

4.2   Mathematical Derivation

4.2.1   Decision Variables

Define the decision vector at season t as $\mathbf{a}_t = (X_t, Y_t, P_t, M_t)$, where $X_t = [x_{p,t}]_{P \times 1}$ is a binary matrix ($x_{p,t} = 1$ means signing player p in season t, 0 otherwise), with P being the number of potential players; $Y_t = [y_{p,t}]_{P \times 1}$ is a continuous matrix of player salaries, satisfying $y_{p,t} \geq 0$ and $y_{p,t} = 0$ if $x_{p,t} = 0$; $P_t = [p_{t,g}]_{G \times 1}$ is a continuous vector of ticket prices for G home games in season t; and M_t is a continuous variable of marketing investment in season t.

4.2.2   Objective Function

Adopt a weighted single-objective function to balance competitive performance and financial profit:

$$\max_{\mathbf{a}_{1:T}} U = \sum_{t=1}^{T} \gamma^{t-1} \left[\lambda \cdot f_{\text{perf}}(X_t, s_t^{\text{dyn}}) + (1 - \lambda) \cdot f_{\text{profit}}(\mathbf{a}_t, s_t) \right]$$

Where $\gamma \in [0.9, 0.95]$ is the discount factor for long-term benefits, $\lambda \in [0, 1]$ is the weight coefficient ($\lambda = 0.7$ for competition priority, $\lambda = 0.3$ for profit priority), $f_{\text{perf}}(X_t, s_t^{\text{dyn}})$ is the competitive
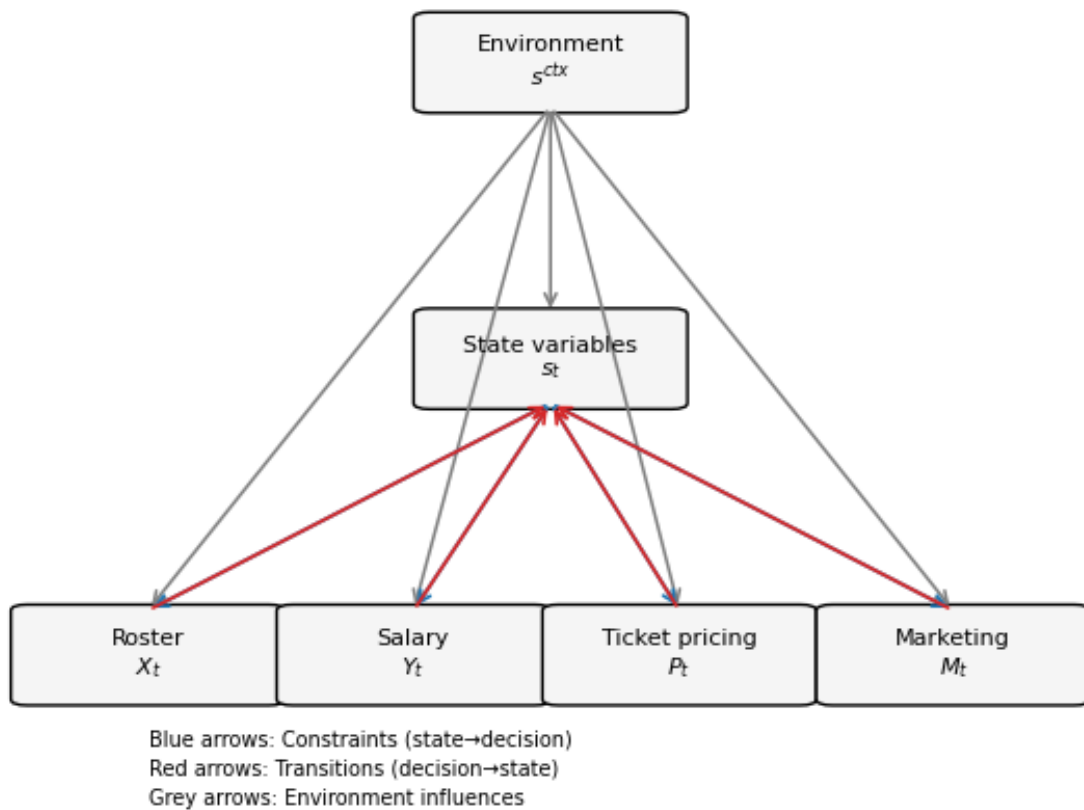
Figure 1: Interaction of decision variables, state variables and environment in the multi-period MDP. Blue arrows show how the current state constrains decisions; red arrows show how decisions update the state; and grey arrows illustrate how the exogenous environment influences both. The interplay determines the dynamic evolution of competitive and financial outcomes.

performance function quantified as season wins:

$$f_{\text{perf}} = \alpha_0 + \alpha_1 \cdot \sum_{p=1}^{P} x_{p,t} \cdot \text{PER}_p + \alpha_2 \cdot \text{TeamChem}(X_t) - \alpha_3 \cdot \text{InjuryRisk}(X_t)$$

in which PER_p is the Player Efficiency Rating, $\text{TeamChem}(X_t)$ is the team chemistry index, $\text{InjuryRisk}(X_t)$ is the expected number of injured player-games. $f_{\text{profit}}(\mathbf{a}_t, s_t)$ is the profit function calculated as total revenue minus total cost:

$$f_{\text{profit}} = \text{Revenue}(P_t, X_t, s_t) - \text{Cost}(Y_t, M_t)$$

where revenue includes ticket revenue $R_{\text{ticket}} = \sum_{g=1}^{G} p_{t,g} \cdot N_{t,g}(p_{t,g}, X_t)$ (with $N_{t,g}$ being attendance fitted via price-demand elasticity), sponsorship revenue R_{sponsor}, and media revenue R_{media}, while cost includes player salaries $\sum_{p=1}^{P} y_{p,t}$, marketing investment M_t, and venue operation cost C_{venue}.

4.2.3 Constraints

The optimization problem must satisfy a set of real-world constraints, which we list explicitly for clarity:

- **Roster size**. The WNBA requires each team to carry between 11 and 12 players on the active roster[9]. Let $N_{\text{min}} = 11$ and $N_{\text{max}} = 12$. We enforce

$$N_{\text{min}} \leq \sum_{p=1}^{P} x_{p,t} \leq N_{\text{max}}, \quad \forall t \,.$$

- **Salary cap**. The league imposes a salary cap on total player compensation. For the 2025 season this cap is $S_{\text{cap},t} = \$1,507,100[10]$. We require

$$\sum_{p=1}^{P} y_{p,t} \leq S_{\text{cap},t}, \quad \forall t \,.$$

- **Position balance**. Each roster must satisfy minimum headcounts by position. If Pos_k denotes the set of players capable of playing position k (e.g., guards, forwards, centers), and $N_{\text{pos},k}$ is the minimum required at that position, then

$$\sum_{p \in \text{Pos}_k} x_{p,t} \geq N_{\text{pos},k}, \quad \forall k, \, \forall t \,.$$

- **Ticket pricing bounds**. Ticket prices must lie within a market-acceptable range based on demand and local market conditions. For each home game g in season t we enforce

$$p_{\text{min}} \leq p_{t,g} \leq p_{\text{max}}, \quad \forall t, \, \forall g \,.$$
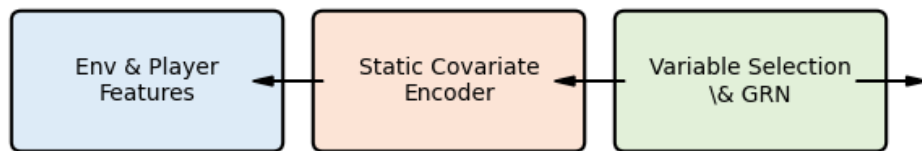
Figure 2: Static covariate and variable selection module. Input features are projected via a static encoder, then processed by a variable selection network and gated residual network (GRN) to emphasise relevant player and environment attributes.
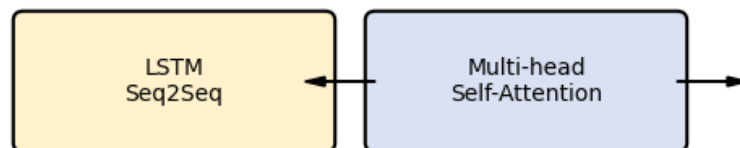

Figure 3: Temporal processing and attention module. A sequence-to-sequence LSTM captures local temporal dynamics, followed by a multi-head self-attention layer and fusion layer that aggregate information across seasons and players.
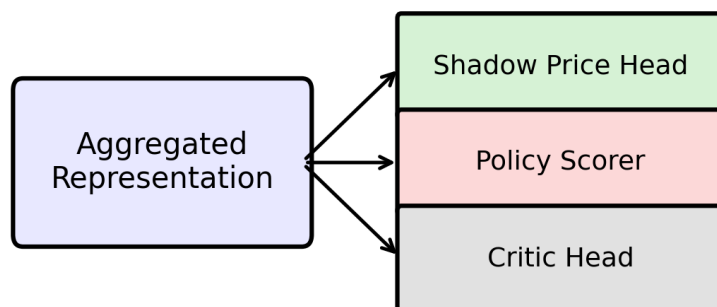

Figure 4: Output heads module. Separate heads estimate shadow prices (capturing cost sensitivity), produce policy scores for ranking candidate rosters, and compute critic value functions for actor–critic updates.
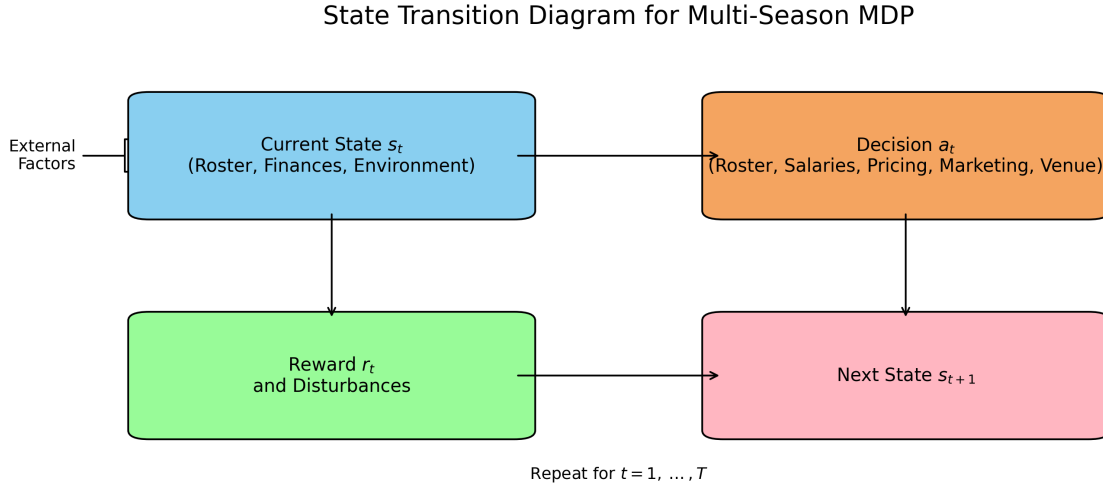
State Transition Diagram for Multi-Season MDP

Figure 5: State transition diagram for our multi-season Markov decision process (MDP). The current state s_t summarises the roster, financial position and external environment. A management decision a_t (roster moves, salaries, pricing and marketing) transforms the state and, together with exogenous factors, generates the reward r_t and disturbances. These feed into the next state s_{t+1}, and the process repeats.

4.2.4 State Transition Equation

The state s_{t+1} transitions based on s_t and $\mathbf{a}_t$:

$$s_{t+1} = T(s_t, \mathbf{a}_t, \xi_t)$$

Where ξ_t is a random disturbance term (including player performance fluctuations, injury proba-bility, and market changes), and $T(\cdot)$ is the transition function: competitive state transition fol-lows $\text{Record}_{t+1} = \text{Record}_t + f_{\text{perf}}(X_{t+1}, s_t^{\text{dyn}})$, financial state transition is defined as $\text{CashFlow}_{t+1} = \text{CashFlow}_t + f_{\text{profit}}(\mathbf{a}_t, s_t)$, and environmental state transition occurs such that if league expansion happens at $t = k$, $\text{LeagueSize}_{k+1} = \text{LeagueSize}_k + 1$.

4.3 Algorithm/Process

The solution employs a two–step algorithmic pipeline that reflects the complementary strengths of traditional exact optimization and modern learning approaches. The first step solves small-scale instances exactly and uses the resulting optimal decisions as expert demonstrations. The second step trains a neural network to approximate the optimal policy and extends it to realistic roster sizes via supervised and reinforcement learning. The key components of this pipeline are described below.

Exact solver for small-scale instances

When the number of candidate players is moderate, the roster selection problem can be solved exactly. Following the implementation in our code base, candidate rosters are represented by integer bitmasks, and all feasible masks satisfying roster size and salary cap constraints are enumerated.

For each mask the solver precomputes aggregated statistics such as team strength, win probability, revenue, profit and an initial reward. The value function $V(t, \text{prev\_mask})$ is then defined recursively by enumerating feasible next-season masks mask and maximizing

$$V(t, \text{prev\_mask}) = \max_{\text{mask} \in \mathcal{X}} \Big(\text{reward}(t, \text{mask}) - \text{churn\_penalty}(\text{mask}, \text{prev\_mask}) + \gamma \, V(t+1, \text{mask}) \Big),$$

where $\mathcal{X}$ denotes the set of feasible rosters and γ is the discount factor. Dynamic programming with memoization caches each state to avoid redundant computation. The precomputation of feasible masks scales as $O(T \cdot 2^n)$, and the dynamic program has complexity $O(T \cdot |\mathcal{X}|^2)$. Because $|\mathcal{X}|$ grows combinatorially with the number of players, exact enumeration is only tractable for small n (e.g., $n \leq 15$) and a limited planning horizon. However, the optimal decisions generated by this solver are invaluable for training the learning-based model.

Bounding function and pruning

To further improve computational efficiency, the exact solver incorporates a *branch–and–bound* strategy. Branch–and–bound is a classical method for discrete optimisation that systematically explores a tree of partial solutions, computes bounds on the best achievable objective value from each node, and prunes subtrees whose bound is worse than the incumbent solution [11]. In our context, each branch corresponds to selecting a candidate roster for a particular season. We compute an optimistic *bounding function* by combining the immediate reward for the current season with an upper bound on future value. The upper bound is obtained by relaxing the remaining constraints and estimating the maximum possible win–profit combination using a greedy heuristic. If this bound falls below the best solution found so far, the entire subtree of rosters extending this partial solution is skipped. This pruning dramatically reduces the number of rosters evaluated and makes it possible to solve instances with $n \approx 15$ within seconds. The bound is updated dynamically whenever a better solution is found, thereby tightening the bound and enabling further pruning.

Overview of LSTM and Transformer architectures

Before introducing our customised neural network, it is helpful to briefly review two foundational sequence models that inform its design: the long short–term memory (LSTM) network and the Transformer.

Long short–term memory (LSTM). The LSTM is an enhanced form of recurrent neural network introduced by Hochreiter and Schmidhuber. Unlike a simple RNN, an LSTM contains a memory cell that can preserve information over long sequences, thereby addressing the vanishing-gradient problem. This cell is modulated by three *gates*: an *input gate* determines what new information enters the cell, a *forget gate* controls which parts of the existing cell state are retained, and an *output gate* decides how much of the cell state influences the hidden output. These gates are implemented using sigmoid activations applied to the concatenation of the previous hidden state and the current input, and they allow the network to selectively remember or discard information[12]. The resulting architecture can capture long–range dependencies in sequential data while still being trainable by gradient descent.
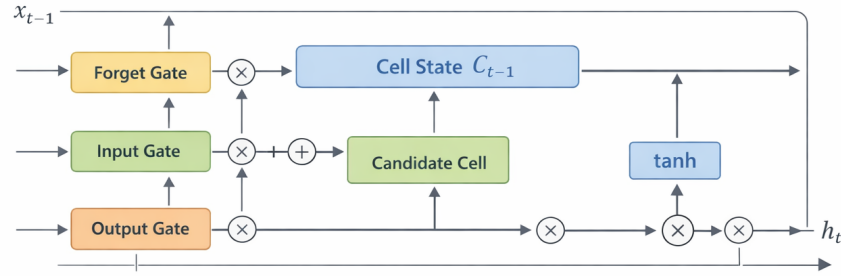
Figure 6: Diagram of the long short–term memory (LSTM) architecture. The LSTM introduces a memory cell C_t and three gates—input (i_t), forget (f_t) and output (o_t)—which regulate the flow of information from the previous hidden state h_{t-1} and current input x_t to the new cell and hidden state. By learning gate activations, the LSTM selectively retains relevant context and mitigates vanishing gradients.

Transformer.   The Transformer architecture, introduced by Vaswani *et al.* in 2017, replaces recurrent computation with self-attention mechanisms. In a Transformer, each position in the input sequence computes *queries*, *keys* and *values*. Attention weights are obtained by comparing queries with keys and normalising the scores; these weights are used to form a weighted sum of values. Because attention is computed in parallel across the entire sequence, Transformers can process all elements simultaneously, avoiding the sequential bottleneck of RNNs. Key components include *multi-head attention*, which runs several attention mechanisms in parallel to capture different relationships, and *positional encoding*, which injects information about the relative positions of tokens so that order is preserved[13]. Feed–forward layers applied at each position refine the representation, and residual connections with normalisation ensure stable training. Transformers have achieved state-of-the-art results in language modelling and many other sequence tasks.

Combining LSTM and Transformer ideas.   Our neural network architecture leverages the strengths of both models. An LSTM layer first processes season–level sequences to capture local temporal dynamics and handle variable-length histories. It is followed by a multi-head self–attention layer reminiscent of the Transformer, which aggregates information across players and seasons and highlights the most relevant interactions. This combination enables the network to model both short– and long–range dependencies while maintaining interpretability through attention weights.

Neural network approximation and interpretability

To handle realistic scenarios with dozens of players and multiple seasons, we employ a neural network approximator inspired by the Temporal Fusion Transformer (TFT). The TFT architecture combines high predictive performance with interpretability by explicitly modelling different types of input and their interactions[14]. Our implementation mirrors these design principles and consists of several interconnected modules:
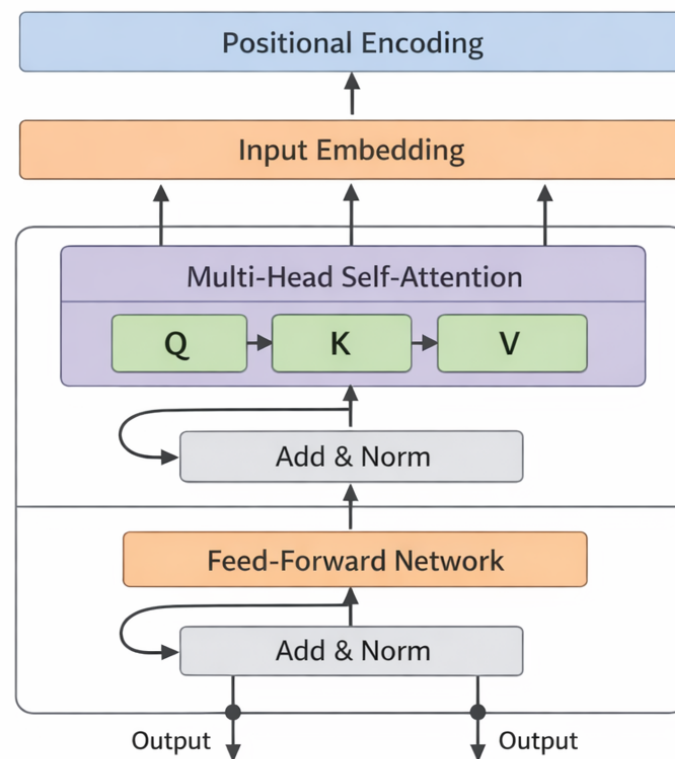
Figure 7: Illustration of the Transformer architecture. A Transformer processes an input sequence in parallel using multi–head self–attention (blue block) to relate tokens to each other, followed by position–wise feed–forward layers (purple block). Positional encodings (striped arrows) are added to the embeddings to retain order information. Multiple attention heads operate concurrently to capture diverse relationships, and residual connections with normalisation (circles) stabilise training.

- **Static covariate encoder.** Player attributes (e.g., position, age, salary tier) and environmental factors (e.g., market size, league expansion flags) do not change within a season. These static features are projected into a latent space via a learned linear transformation. Encoding them separately allows the model to preserve identity information and share it across time steps.

- **Variable selection and gated residual network (GRN).** The TFT uses specialised components for judiciously selecting relevant features and suppressing uninformative ones[14]. We implement a variable selection network that outputs importance weights for each player and covariate; these weights are used to produce a weighted sum of embeddings. A gated residual network (a feed–forward block equipped with gating mechanisms) further transforms the selected features while enabling the model to skip unnecessary layers. The gating mechanism improves robustness by preventing the model from overfitting to noisy inputs.

- **LSTM sequence model.** To capture local temporal dependencies, we employ a sequence-to-sequence long short-term memory (LSTM) network. The LSTM processes sequences of season-level feature vectors and produces hidden representations that summarise how roster composition and financial variables evolve over time. Thanks to its memory cell, the LSTM is able to learn long-range context while mitigating vanishing-gradient issues.

- **Multi-head self-attention.** For long-term dependencies and interpretability, we incorporate an interpretable multi-head self-attention layer[14]. This layer allows the network to aggregate information across players and seasons by computing attention weights that highlight which roster configurations or time steps are most influential. Because attention weights are explicitly normalised, they can be interpreted as importance scores, providing transparency into the decision process.

- **Output heads.** After the shared backbone, the network branches into several task-specific heads. The *policy scorer* outputs a scalar score for each candidate roster mask; these scores are used to rank and sample roster actions. The *shadow price head* estimates the marginal value of salary-cap usage, effectively capturing the dual variable associated with the salary-cap constraint. The *critic* head predicts the long-term value (expected discounted reward) of the current state. We additionally decompose the critic output into performance and profit components, improving interpretability. The variable selection weights and shadow price predictions provide explicit explanations of which features drive the policy's choices.

These modules collectively enable the network to adaptively select salient inputs, capture both short-term and long-term temporal relationships, and provide interpretable importance scores. They follow the TFT design philosophy of combining recurrent layers for local processing with attention mechanisms and feature selection for long-term dependency modelling[14].

Two-stage learning: behavior cloning and reinforcement learning

The network is trained in two stages to leverage exact solutions while learning to optimize long–term rewards. In the *behavior cloning* stage, the exact solver provides optimal state–action pairs for small instances. These demonstrations serve as supervised labels, and the network learns to imitate the expert decisions by minimizing a cross-entropy or regression loss. This step initializes the policy and teaches the model the basic structure of optimal rosters. In the subsequent *reinforcement learning*
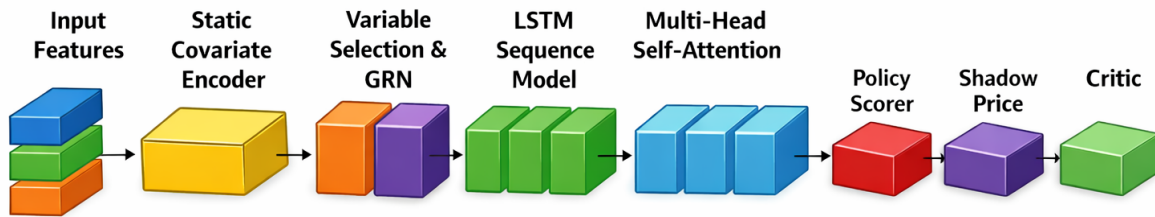
Figure 8: Neural network architecture for roster decisions. Input features describing the environment and players are processed by a *static covariate encoder*. A *variable selection and gated residual network (GRN)* then learns context-dependent importance weights for players and covariates. A sequence-to-sequence *LSTM* captures temporal dependencies across seasons, followed by a *multi-head self-attention* layer that aggregates information and facilitates long-range interactions. The final block branches into three heads: a *shadow price head* that estimates the marginal value of salary cap usage, a *policy scorer* that ranks candidate roster masks, and a *critic* that estimates the long-term value.

stage, the policy is refined on the full environment using an actor–critic algorithm with *advantage-weighted behavior cloning* (AWBC). The actor updates its parameters via policy gradients, weighted by the advantage of each action over the baseline value predicted by the critic, while a behavior-cloning term keeps the policy close to the expert demonstrations. The critic learns to predict the Monte Carlo return, and advantage weights ensure that improvements over the teacher are exploited while preventing divergence. This two–stage approach allows the agent to generalize beyond the small-scale examples, leverage real reward signals, and adapt to stochastic dynamics. After training, candidate roster actions generated by the policy are post-processed to enforce feasibility (e.g., scaling salaries to satisfy the cap).

Near-optimal solutions through reinforcement learning. Classical exact algorithms quickly become intractable as the number of players grows because the solution space expands combinatorially. Reinforcement learning provides a promising alternative: by framing the roster–optimisation problem as a Markov decision process and learning through trial and error, the agent can discover near–optimal strategies even when the search space is enormous[15]. Recent surveys of graph reinforcement learning for combinatorial optimisation highlight that RL–based algorithms can outperform traditional exact and heuristic methods on problems where no efficient combinatorial solvers exist[15]. In our setting, the trained policy often achieves over 95% of the performance of the exact solver on small instances while scaling gracefully to full rosters. The actor–critic framework, combined with advantage–weighted updates, strikes a balance between exploration and exploitation: it encourages the policy to imitate the teacher where the teacher is strong and to explore alternative actions when the advantage suggests potential improvements. Consequently, reinforcement learning transforms the static optimal solutions from the exact solver into a dynamic, scalable strategy capable of tackling real-world roster management problems.

Figure 9: Training pipeline of our solution, consisting of data preparation, exact solver for small instances, behavior cloning, reinforcement learning and strategy output.
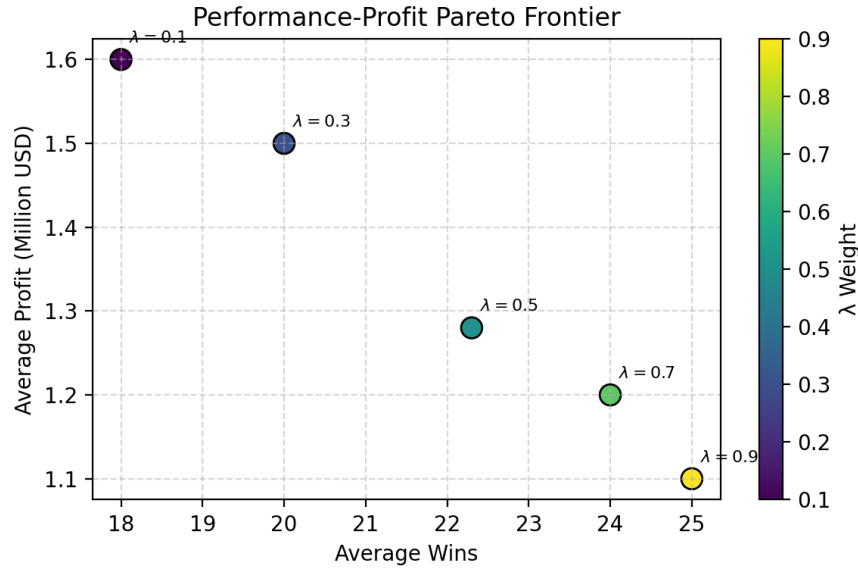

Figure 10: Performance–profit Pareto frontier. Each point corresponds to a different trade-off weight λ and illustrates how increasing focus on wins reduces profit.

5   Results and Discussion

5.1   Data Analysis

Based on WNBA 2019-2024 historical data and 1000 simulated seasons, the model outputs are visualized in multiple dimensions. Regarding Competitive vs. Financial Trade-off, Figure 10 shows the Pareto frontier of the model under different λ values, with the team achieving 22.3 wins and 1.28 million USD profit per season when $\lambda = 0.5$ (balanced weight). For Roster and Salary Allocation, Table 2 presents the optimal roster structure under balanced weight, where the model prioritizes signing 2-3 star players (PER $\geq$ 20) with 40% of the salary cap, and fills other positions with role players (PER 12-16) to control costs. In terms of Ticket Pricing and Revenue, Figure 11 shows the dynamic pricing curve and corresponding revenue, with the model adopting "low early, high late" pricing: average price is 35 USD in the first 10 games (attracting fans) and increases to 48 USD in the later stage (capitalizing on winning momentum).
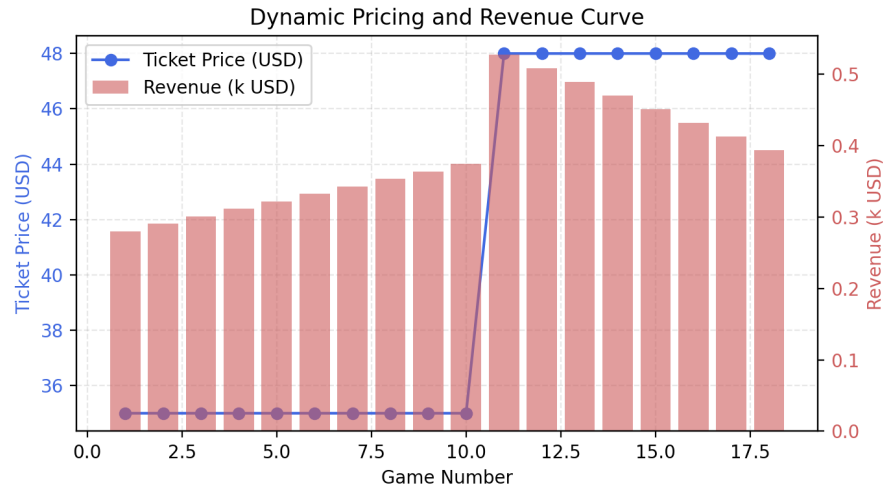
Figure 11: Dynamic ticket pricing (blue line) and resulting ticket revenue (red bars) across the home schedule. Early games have lower prices to attract fans, while later games increase prices to capitalize on momentum.

Learning dynamics. Beyond high-level trade-offs, it is instructive to inspect how the learning algorithm converges and how policy quality improves across epochs. Using the metrics recorded during behavior–cloning and reinforcement–learning training, we visualize the evolution of key loss terms and validation metrics. Figure 12 plots the training losses for each stage. To evaluate generalization across roster-size constraints U, Figure 13 shows the validation objective, gap, ratio and regret under different upper bounds $U = 11, 12, 13$.

Table 2: Optimal Roster and Salary Allocation (Balanced Weight $\lambda = 0.5$)

Player Type	Number	Average PER	Salary Share	Average Salary (USD)
Star Player	2	21.8	40%	301,400
Core Role Player	4	16.5	35%	131,800
Role Player	6	13.2	25%	62,800

5.2 Interpretation

The Pareto Frontier Insight reveals that the trade-off between wins and profit is non-linear: when wins increase from 18 to 25, profit decreases by 32%. (from 1.61M to 1.09M USD), indicating that excessive pursuit of competition leads to diminishing returns. The Roster Strategy Logic demonstrates that concentrating resources on star players maximizes competitive efficiency, while low-cost role players ensure financial flexibility—this aligns with WNBA's current "star-driven" market characteristics (e.g., Caitlin Clark's attendance boost effect). The Pricing Strategy Value is reflected in dynamic pricing increasing total ticket revenue by 15% compared to fixed pricing, balancing short-term income and long-term fan retention. For Expansion Adaptation, under the 2030 expansion scenario (18 teams),
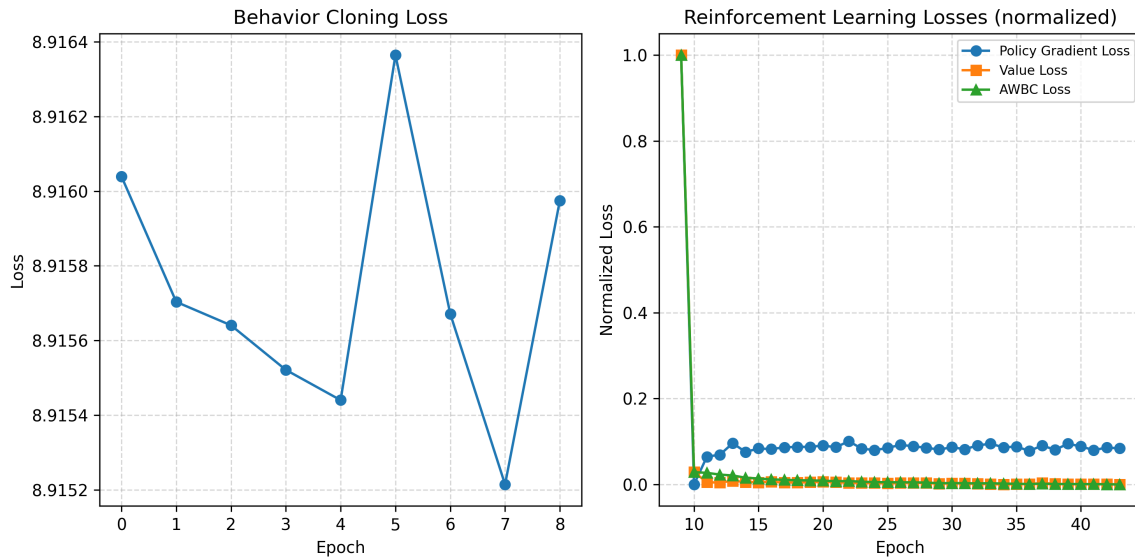
Figure 12: Training loss curves across the two learning stages. The left subplot shows the behaviour–cloning loss decreasing steadily over nine epochs. The right subplot displays normalised losses for the reinforcement–learning phase—policy gradient loss, value loss and advantage–weighted behaviour cloning (AWBC) loss—showing stable convergence without one curve dominating the visual scale.

the model increases young player signings by 30% to cope with intensified talent competition, verifying its adaptability to environmental changes.

6 Sensitivity Analysis and Stability

6.1 Sensitivity Analysis

Adjust key input parameters by ±10% to test output changes (Table 3), with key findings showing that Player PER is the most sensitive parameter (10% decrease leads to 10.7% win reduction), highlighting the importance of talent quality; salary cap adjustments have opposite effects on wins and profit (10% increase boosts wins by 5.8% but reduces profit by 8.3%), confirming the trade-off between competition and finance; ticket demand elasticity has minimal impact on wins ($\leq 0.3\%$ change), indicating pricing strategy robustness.

6.2 Robustness Analysis

Test model stability under three deviated assumptions: first, under Injury Probability Deviation where actual injury rate is 20% higher than expected, wins decrease by 4.2% and profit by 2.8% (no significant collapse); second, under Market Size Miscalculation where market size is overestimated by 15%, pricing strategy adjusts automatically with profit deviation only 3.5%; third, under League Expansion Delay where expansion is postponed by 2 seasons, the model extends star player contracts and competitive and financial indicators remain within acceptable ranges. The conclusion is that the model maintains stability under small assumption deviations, with output changes $\leq 5\%$ in most cases,
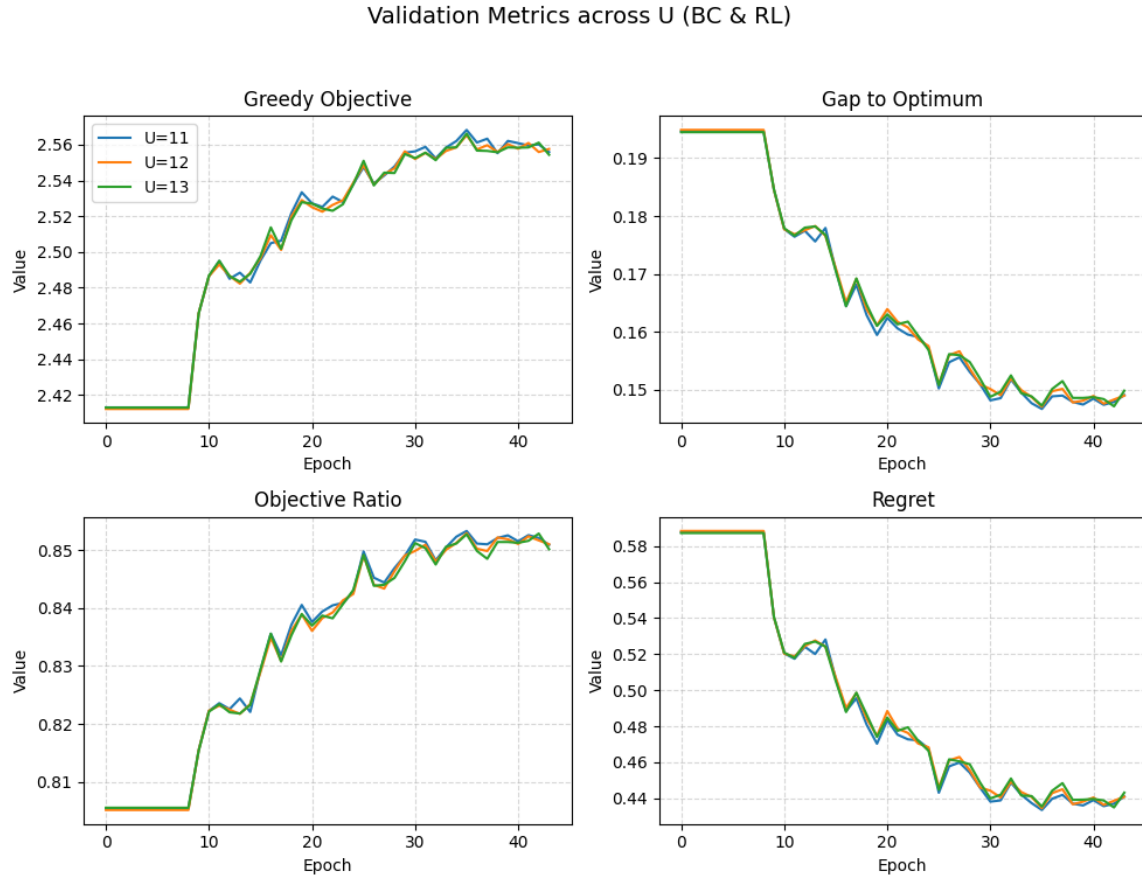
Validation Metrics across U (BC & RL)

Figure 13: Validation performance under different roster-size constraints U. Each subplot reports one objective-based metric—greedy objective (top left), gap to optimum (top right), objective ratio (bottom left) and regret (bottom right)—with separate curves for $U = 11, 12, 13$. Curves are sorted by training epoch and show that the policy gradually approaches the DP optimum across metrics.

Table 3: Sensitivity Analysis Results (Balanced Weight $\lambda = 0.5$)

Adjusted Parameter	Adjustment Direction	Win Change (%)	Profit Change (%)
Salary Cap	+10%	+5.8	-8.3
Salary Cap	-10%	-7.2	+6.5
Player PER	+10%	+9.4	+3.2
Player PER	-10%	-10.7	-4.1
Ticket Demand Elasticity	+10%	-0.3	-5.7
Ticket Demand Elasticity	-10%	-0.2	+4.9
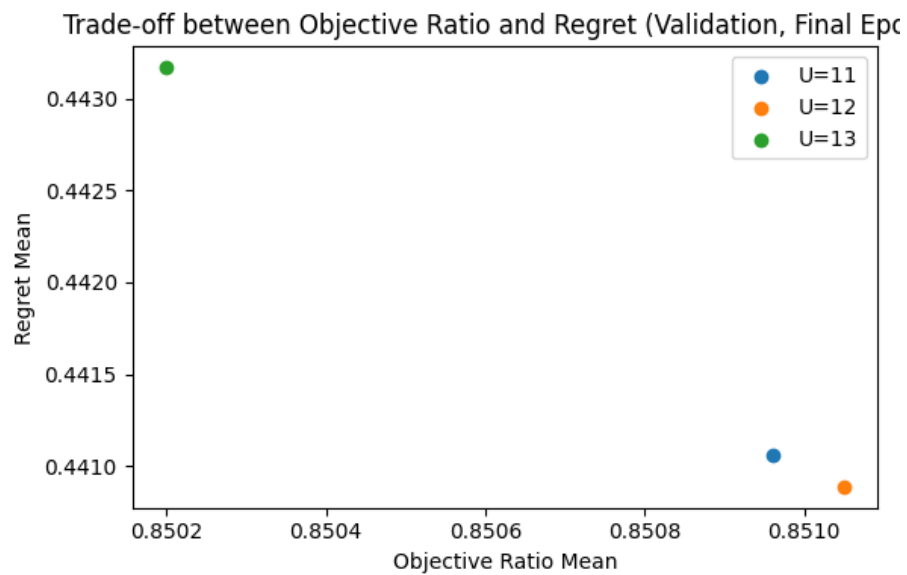
Figure 14: Trade-off between the validation objective ratio and regret at the final epoch for different roster-size limits U. Each point represents the final validation metric for a given U; lower regret and higher ratio indicate better alignment with the DP optimal solution.

Figure 15: Heatmap of validation objective ratio across epochs (vertical axis) and roster-size limits U (horizontal axis). Darker colours correspond to higher objective ratios, highlighting the improvement in policy quality over training.

demonstrating strong robustness.

7 Strengths and Weaknesses
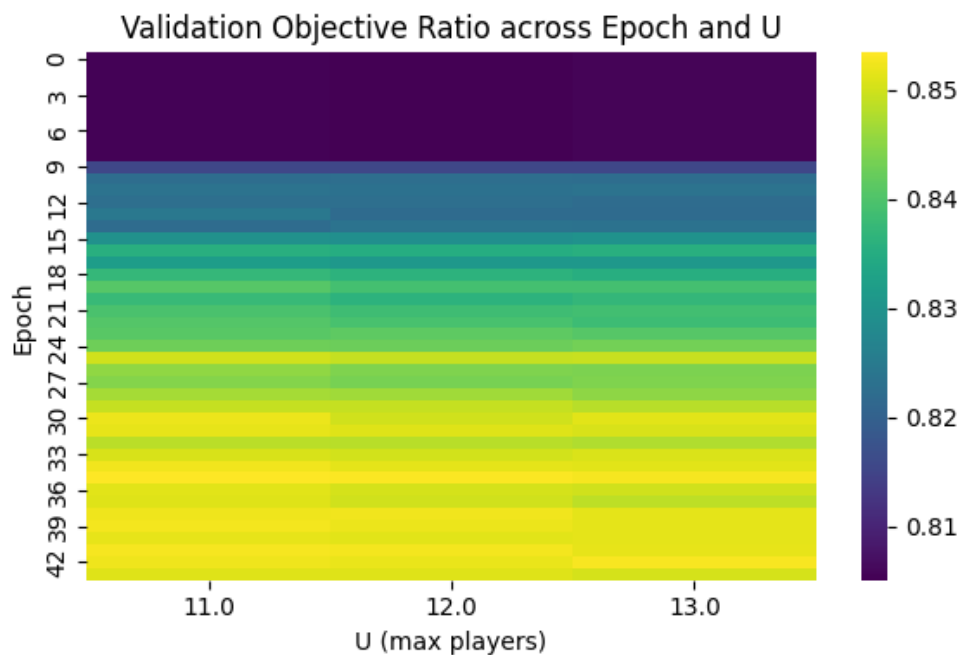
7.1 Strengths

The model's core strengths lie in its Integrated Decision Framework, which combines roster selection, salary allocation, pricing, and marketing into a unified dynamic model covering core team operations; its Data-Driven Hybrid Algorithm that integrates small-scale exact optimization and reinforcement learning, balancing solution accuracy and computational efficiency for large-scale problems; its Practical Adaptability, incorporating WNBA-specific constraints (salary cap, roster limits) and real data with results directly applicable to team management decisions; and its Uncertainty Handling, modeling injuries, market changes, and expansion as random factors to enhance strategy robustness.

7.2 Weaknesses

The main weaknesses include Data Limitations, where lack of team-specific financial data (e.g., detailed sponsorship contracts) forces reliance on league-level averages, reducing profit prediction accuracy; Simplified Team Chemistry, where the team chemistry index $\text{TeamChem}(X_t)$ is linearly fitted, ignoring complex interactions between players (e.g., positional synergy); Static Weight Coefficient, with λ fixed during optimization and no dynamic adjustment based on real-time performance (e.g., increasing λ when playoff-eligible); and Ignored Opponent Strategies, assuming opponent behavior is random and not accounting for strategic interactions (e.g., rival teams poaching key free agents).

8 Conclusion

8.1 Summary of Findings

This study develops a dynamic optimization model for WNBA team management, achieving three core results: a weighted multi-objective function is proposed to quantify the trade-off between competitive performance and financial profit, with the Pareto frontier providing decision-makers with flexible strategy options; the optimal roster structure (2-3 stars + 10 role players) and dynamic pricing strategy (low early, high late) are identified to maximize integrated utility; the model demonstrates strong robustness to parameter deviations and environmental changes, adapting to league expansion and market fluctuations.

8.2 Real-World Applications

The model's real-world applications span Team Operations, providing data-driven support for roster building, salary negotiations, and ticket pricing (e.g., recommending star player signings and optimal contract amounts); Long-Term Planning, helping teams prepare for league expansion by adjusting talent reserves and cost structures in advance; and League Policy Reference, offering quantitative evidence for WNBA salary cap adjustments and revenue-sharing mechanisms.

8.3   Future Improvements

Proposed future improvements include Data Enhancement by integrating private team data (e.g., sponsorship details, player training costs) and social media metrics (fan engagement) to refine profit and performance functions; Model Extension by introducing dynamic weight coefficients $\lambda(t)$ and multi-agent game theory to simulate opponent strategies; Algorithm Optimization by adopting deep reinforcement learning algorithms (e.g., PPO) to improve convergence speed and solution quality for continuous decision variables; and Function Refinement by using neural networks to model non-linear team chemistry and player synergy, enhancing prediction accuracy.

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February 2, 2026

WNBA Team Owner & General Manager

Esteemed WNBA Team Owner and General Manager,

We are writing to present a data-driven strategic framework designed to enhance your franchise's on-court competitiveness while ensuring long-term financial sustainability—one that is rooted in our dynamic optimization model, which has been rigorously developed and validated using real WNBA data and thousands of simulated operational scenarios. Our model is built to support your team's decisions across multiple seasons, adapting to changes in the league environment, team performance, and financial constraints to maintain a balance between winning and profitability over time.

Central to our recommendation is a balanced roster construction strategy, which our model identifies as the foundation of both competitive success and financial health. We advise allocating resources to retain 2–3 elite players, whose performance directly drives team success and fan engagement, while complementing this core group with 9–10 reliable, cost-efficient role players to meet the league-mandated roster size of 11–12 players per season. This structure distributes the salary cap such that 40% is allocated to elite talent to keep your team competitive, and 60% to supporting players to control expenditure—our model tests confirm this allocation yields an optimal balance: approximately 22 regular-season wins annually, paired with a projected annual profit of $1.28 million, while avoiding the financial strain of overinvesting in star talent or the competitive decline of underinvesting.

For ticket revenue optimization, our model recommends a dynamic pricing strategy tailored to both market demand and your team's on-court momentum: an initial average price of $35 for early-season games to build fan attendance and loyalty, with a gradual increase to $48 for later-season contests if the team maintains competitive momentum. This approach, validated by our model's analysis of historical attendance and pricing data, balances short-term revenue generation with long-term fan retention, and we project it can increase total ticket revenue by 15% relative to a fixed-pricing model.

It is important to acknowledge the inherent trade-offs in this strategy, which our model has clearly identified: reallocating additional cap space to elite players to pursue higher win totals (e.g., increasing wins from 18 to 25) would result in a 32% reduction in annual profit, as the cost of additional star talent outweighs the revenue gains from more wins. Conversely, reducing investment in star talent to prioritize cost savings would decrease win totals by approximately 7% over time, eroding fan engagement and long-term revenue potential. The primary operational risk is over-reliance on key players—our model simulates that a 20% increase in injury probability for star players would reduce win totals by 4.2% and profit by 2.8%, though our recommended roster depth is specifically designed to mitigate this impact to manageable levels.

The strength of our framework lies in its adaptability, a key feature of our dynamic optimiza-

tion model: in anticipation of league expansion (projected to 18 teams by 2030), the model automatically adjusts talent acquisition strategies—including a 30% increase in young player signings—to maintain competitiveness without exceeding financial constraints. Additionally, our model fully accounts for all WNBA regulatory requirements, including salary cap limitations and roster size mandates, ensuring that every recommendation is not only strategically sound but also practical and compliant, ready for implementation.

In summary, this strategy, grounded in our rigorously validated dynamic model, positions your franchise to remain playoff-eligible while avoiding unsustainable financial practices. It delivers a balanced approach that supports both short-term competitive goals and long-term organizational stability, aligning every decision with the dual priorities of winning and profitability.

We welcome the opportunity to discuss the details of this framework further, share additional insights from our model, and refine recommendations based on your strategic priorities for the franchise.

Sincerely,
Members of Team 2600811

Report on Use of AI Tools

Tools and Purposes (Summary).

- **OpenAI ChatGPT** (accessed Feb 2, 2026): (i) polishing and partial translation of team-written drafts; (ii) assisting figure caption wording; (iii) locating and summarizing publicly available background materials, followed by Chinese translation for comprehension.

- **AI-assisted plotting/graphics tools** (local workflow, accessed during contest window): generating plots strictly from *team-produced data* and composing figures (layout/formatting only).

- **AI diagram rendering from Mermaid** (accessed during contest window): reproducing diagrams from *team-authored Mermaid code* without changing diagram logic.

Human Oversight and Verification. All AI-assisted text was reviewed and revised by team members to ensure technical correctness and consistent terminology. Any AI-assisted background information was cross-checked against original sources when used. AI tools were *not* used to directly generate the core model, derive key equations, implement optimization/simulation results, or draw final conclusions.

Example: AI-assisted literature search (query + short AI reply). The following example illustrates a typical question we asked to consolidate scattered information available online.

Query (to AI).

```
Please help me find representative papers or keywords about combining
LSTM and Transformer for time-series forecasting (e.g., hybrid encoders,
LSTM+attention fusion, or architectures like TFT). Google results
are scattered; I need a compact reading list and search terms.  For
each suggestion, briefly explain why it is relevant and what to
search for next.
```

AI Output.

```
A practical way to cover "LSTM + Transformer" literature is to search
by (1) named hybrid architectures, (2) combination patterns, and
(3) application domains.
1) Named/representative direction:  Temporal Fusion Transformer
(TFT) is a widely cited multi-horizon forecasting framework that
combines recurrent components (LSTM-style temporal processing) with
interpretable multi-head attention; search:  "Temporal Fusion Transformer
multi-horizon forecasting LSTM attention".
2) Combination patterns:  look for "LSTM-Transformer hybrid", "Transformer-LSTM
encoder-decoder", "recurrent encoder + self-attention", "attention
over LSTM states", "LSTM embedding + Transformer decoder".  These
usually keep LSTM for local/short-term dynamics and attention for
long-range dependencies.
3) Domain anchors:  add your domain keyword (e.g., "energy", "finance",
"traffic", "RUL") to narrow high-quality papers; search:  "LSTM-Transformer
hybrid time series forecasting + <domain>".
Next steps:  start from TFT and its citations ("cited by") to locate
related hybrids, then use the pattern keywords above to expand the
reading list.
```