Office hour 1/11/2021

36#

20,000

4 gps

20 units

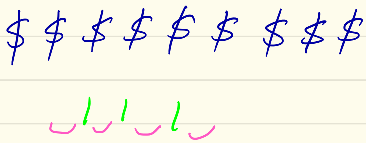
gp1 2 3 4

2 unit 2 unit 3 unit 4 units

a) all four gps not empty.

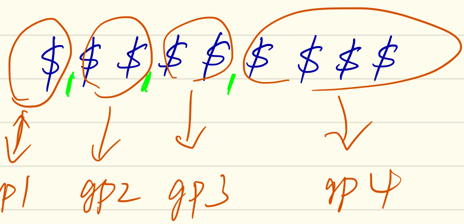
$20 - 2 - 2 - 3 - 4 = 9$

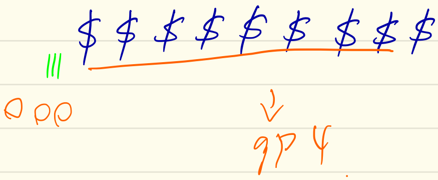
9 units to 4 gps



← 9 units

← 3 blockes.





$$\underbrace{|\,|\,|}\ \underbrace{\$\,\$\,\$\,\$\,\$\,\$\,\$\,\$\,\$}$$

12 objects

$$\frac{12!}{3!\,9!}$$

$$\binom{n+r-1}{r-1}$$

when n is the # of units to assign

r is the gps to assign to

- n units are the same.
- gps can be empty.

→ one way is shown above

→ 2nd way bijections.

Prerequisite #2

$$1-\frac{1}{x} \le \log x \le x-1$$

$$\left(1-\frac{\lambda}{n}\right)^n \longrightarrow \quad \text{as } n \to \infty$$

$$1-\frac{1}{x} \longleftrightarrow 1-\frac{\lambda}{n}$$

#1 lhs x $\quad x=\frac{n}{\lambda}$

plug in the inequality.

$$1-\frac{1}{x} \le \log x$$

$$\downarrow \qquad \downarrow$$

$$1-\frac{\lambda}{n} \le \log\frac{n}{\lambda}$$

$$\downarrow$$

$$\left(1-\frac{\lambda}{n}\right)^n \le (\log n-\log\lambda)^n$$

stuck!

#2

- $1 - \frac{1}{x} \le \log x \le x - 1$

- $\left(1 - \frac{\lambda}{n}\right)^n$

$\left(1 - \frac{\lambda}{n}\right)^n$ e?

$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$

$= \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^{-n}$

#2 $x = 1 - \frac{\lambda}{n}$

$$n\left(1 - \frac{1}{\left(1 - \frac{\lambda}{n}\right)}\right) \le n\left(\log\left(1 - \frac{\lambda}{n}\right)\right) \le n\left(1 - \frac{\lambda}{n} - 1\right)$$

$$n\left(\frac{\cancel{1} - \frac{\lambda}{n} \cancel{-1}}{1 - \frac{\lambda}{n}}\right) = \frac{-\lambda}{1 - \frac{\lambda}{n}} \to -\lambda$$

$$\log\left(1 - \frac{\lambda}{n}\right)^n \to -\lambda$$

$$n\left(\frac{-\lambda}{n}\right) = -\lambda$$

log

$\log(\text{target}) \longrightarrow -\lambda$ as $n \to \infty$

$\Downarrow$ $\text{target} \longrightarrow e^{-\lambda}$ cts

log

target $\longrightarrow (\log^{-1})(-\lambda)$

$= e^{-\lambda}$

$\log f(x) \longrightarrow a$

$\Downarrow$ $f(x) \longrightarrow e^{a}$

if $\underset{\log}{g}(\underset{(1-\frac{\lambda}{n})^n}{f(n)}) \longrightarrow \underset{-\lambda}{a}$ as $n \to \infty$

is $f(n) \longrightarrow g^{-1}(a)$ as $n \to \infty$?

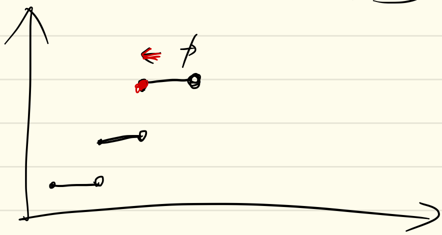
$f(n) \longrightarrow e^{-\lambda}$

calculus.

g is cts

cdf

#4

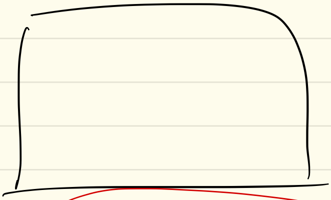


continuity $\not\Rightarrow$ differentiable

$\Leftarrow$

$$\boxed{\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)}$$

#23



3U 4R

7 decisions to make

$$\binom{7}{3} = \frac{7!}{3!\,4!}$$

7 3
choose

3U 4R

$$\underbrace{UUU}\underbrace{RRRR}$$

$$\frac{7!}{3!\,4!}$$

X is normal. $X \sim N(\mu, \sigma)$

μ the mean

σ is sd.

#1 (a)

$$x = \frac{u-\mu}{\sigma} \qquad \frac{X-\mu}{\sigma} \sim N(0,1)$$

$$dx = \frac{du}{\sigma}$$

plug in $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f($$

(b) $\int_0^{\infty} \frac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}\,dx$

$$= \int_0^{\infty} \frac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\underbrace{\int_0^{\infty} x^{\alpha-1} e^{-x}\,dx}_{\lambda=1}}\,dx$$

$u = \lambda x$

Second part

\#12 3 xyz

[0,9]. 2 of them equal.

$\boxed{\text{Leading \# is not 0 except 0 itself.}}$

Forget about this for now

(x, y, z) 2 of them equal.

- pick 2 positions to be equal.

$$\binom{3}{2}$$

- assign values

$$10 \times 9$$

$$\binom{3}{2} \times 10 \times 9.$$

2 positions equal

For second

0 _ 0 ← 9 1~9

0 0 _ ← 9 1~9

0 _ _ ← 9 1~9

same

$$\binom{3}{2} \cdot 10 \cdot 9 - 3 \times 9$$

First part

① at least 2 digits are equal

\# of valid xyz numbers $-$ \# of xyz s.t. x, y, z are all different.

x $(1\sim 9)$, y $(0\sim 9)$, z $(0\sim 9)$

x $(1\sim 9)$, y 0 and rest, z rest

$$9 \times 10 \times 10 - 9 \times 9 \times 8$$

#33   10   RFEU

FE   R U

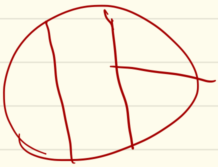
FE/EF

# of cases ~~R~~FE seated together $\rightarrow$ $9! \cdot 2!$ $-$ # of cases RU seated together and FE seated together

RU

$8! \cdot 2! \cdot 2!$

$=$



#3 $S = \sum_{k=1}^{\infty} k(1-p)^{k-1}$

$$\underbrace{S_n = \sum_{k=1}^{n} k(1-p)^{k-1}}$$

Solution 1

$$S = \lim_{n\to\infty} S_n$$

- $S_n = 1\cdot(1-p)^0 + 2\cdot(1-p)^1 + \cdots + n(1-p)^{n-1}$

$$(1-p)\cdot S_n = 1\cdot(1-p)^1 + 2\cdot(1-p)^2 \cdots + n(1-p)^n$$

$$S_n - (1-p)\cdot S_n = \underbrace{(1-p)^0 + 1\cdot(1-p)^1 + \cdots (1-p)^{n-1}} - \underbrace{n(1-p)^n}$$

$$pS_n = \frac{1-(1-p)^n}{1-(1-p)} - n(1-p)^n$$

$$pS_n = \frac{1-(1-p)^n}{p} - n(1-p)^n$$

$$S_n = \frac{1-(1-p)^n}{p^2} - \frac{n(1-p)^n}{p}$$

$(1-p)^n \to 0$, $\frac{n(1-p)^n}{p} \to 0$

$$S = \lim_{n\to\infty} S_n = \frac{1}{p^2}$$

Solution 2 Use differentiable on infinite sums.