STA137HW5

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R Markdown

1.

a). $x_n = x_{90} = 12$. $w_t = x_t - \mu$ $\quad w_{90} = x_{90} - \mu = 12 - 14 = -2$

$\hat{w}_{n+1} = \phi w_n + \theta \hat{\varepsilon}_n = 0.8\cdot(-2) + 0.9(-2) = -1.6 - 1.8 = -3.4$

$\hat{w}_{n+2} = \phi \hat{w}_{n+1} = 0.8(-3.4) = -2.72$ $\quad \hat{w}_{n+3} = \phi \hat{w}_{n+2} + \theta \hat{\varepsilon}_{n+2} = 0.8(-2.72) = -2.176$

$\hat{x}_{n+1} = \mu + \hat{w}_{n+1} = 14 + (-3.4) = 10.6$ $\quad \hat{x}_{n+2} = \mu + \hat{w}_{n+2} = 14 - 2.72 = 11.28$

$\hat{x}_{n+3} = \mu + \hat{w}_{n+3} = 14 - 2.176 = 11.824$

b) $x_n = Y_n - Y_{n-1} = 1312 - 1300 = 12$ $\quad w_n = x_n - \mu = 12 - 10.1 = 1.9$

$\hat{w}_{n+1} = \phi w_n + \theta \hat{\varepsilon}_n = 0.8 \times 1.9 + 0.9 \times (-2) = 1.52 - 1.8 = -0.28$

$\hat{w}_{n+2} = \phi \hat{w}_{n+1} + \theta \hat{\varepsilon}_{n+1} = 0.8(-0.28) + 0 = -0.224$

$\hat{w}_{n+3} = \phi \hat{w}_{n+2} + \theta \hat{\varepsilon}_{n+2} = 0.8(-0.224) + 0 = -0.1792$

$\hat{x}_{n+1} = \mu + \hat{w}_{n+1} = 10.1 - 0.28 = 9.82$ $\quad \hat{x}_{n+2} = \mu + \hat{w}_{n+2} = 10.1 - 0.224 = 9.876$

$\hat{x}_{n+3} = \mu + \hat{w}_{n+3} = 10.1 - 0.1792 = 9.9208$ $\quad \hat{Y}_{n+1} = \hat{x}_{n+1} + Y_n = 9.82 + 1312 = 1321.82$

$\hat{Y}_{n+2} = \hat{x}_{n+2} + \hat{Y}_{n+1} = 9.876 + 1321.82 = 1331.696$

$\hat{Y}_{n+3} = \hat{x}_{n+3} + \hat{Y}_{n+2} = 9.9208 + 1331.696 = 1341.6168$

2.

a) $w_{90} = x_{90} - \mu = 12 - 14 = -2$ $\quad \hat{w}_{n+1} = \phi w_n + \theta_1 \hat{\varepsilon}_n + \theta_2 \hat{\varepsilon}_{n-1} = 0.8(-2) + (-1.2) \times 0.6 + 0.9 \times (0.3)$

$= -1.6 + (0.72) + 0.27 = -2.05$

$\hat{w}_{n+2} = \phi \hat{w}_{n+1} + \theta_1 \hat{\varepsilon}_{n+1} + \theta_2 \hat{\varepsilon}_n = 0.8(-2.05) + 0 + 0.9(0.6) = -1.64 + 0.54 = -1.10$

$\hat{w}_{n+3} = \phi \hat{w}_{n+2} + \theta_1 \hat{\varepsilon}_{n+2} + \theta_2 \hat{\varepsilon}_{n+1} = 0.8(-1.1) + 0 + 0 = -0.88$

$\hat{x}_{n+1} = \mu + \hat{w}_{n+1} = 14 - 2.05 = 11.95$ $\quad \hat{x}_{n+2} = 14 - 1.1 = 12.9$ $\quad \hat{x}_{n+3} = 14 - 1.6 = 13.12$

b) $\hat{w}_{n+1} = \phi w_n + \theta \hat{\varepsilon}_n + \theta_2 \hat{\varepsilon}_{n-1} = 0.8 \times (1.9) + (-1.2) \cdot 0.6 + 0.9 \times 0.3 = 1.52 - 0.72 + 0.27 = 1.07$

$\hat{w}_{n+2} = \phi \hat{w}_{n+1} + \theta_1 \hat{\varepsilon}_{n+1} + \theta_2 \hat{\varepsilon}_n = 0.8(1.07) + 0 + 0.9(0.6) = 0.856 - 0.54 = 0.316$

$\hat{w}_{n+3} = \phi \hat{w}_{n+2} + \theta_1 \hat{\varepsilon}_{n+2} + \theta_2 \hat{\varepsilon}_{n+1} = 0.8(0.316) + 0 + 0 = 0.2528$

$\hat{x}_{n+1} = \mu + \hat{w}_{n+1} = 10.1 + 1.07 = 11.17$ $\quad \hat{x}_{n+2} = \mu + \hat{w}_{n+2} = 10.1 + 0.316 = 10.416$

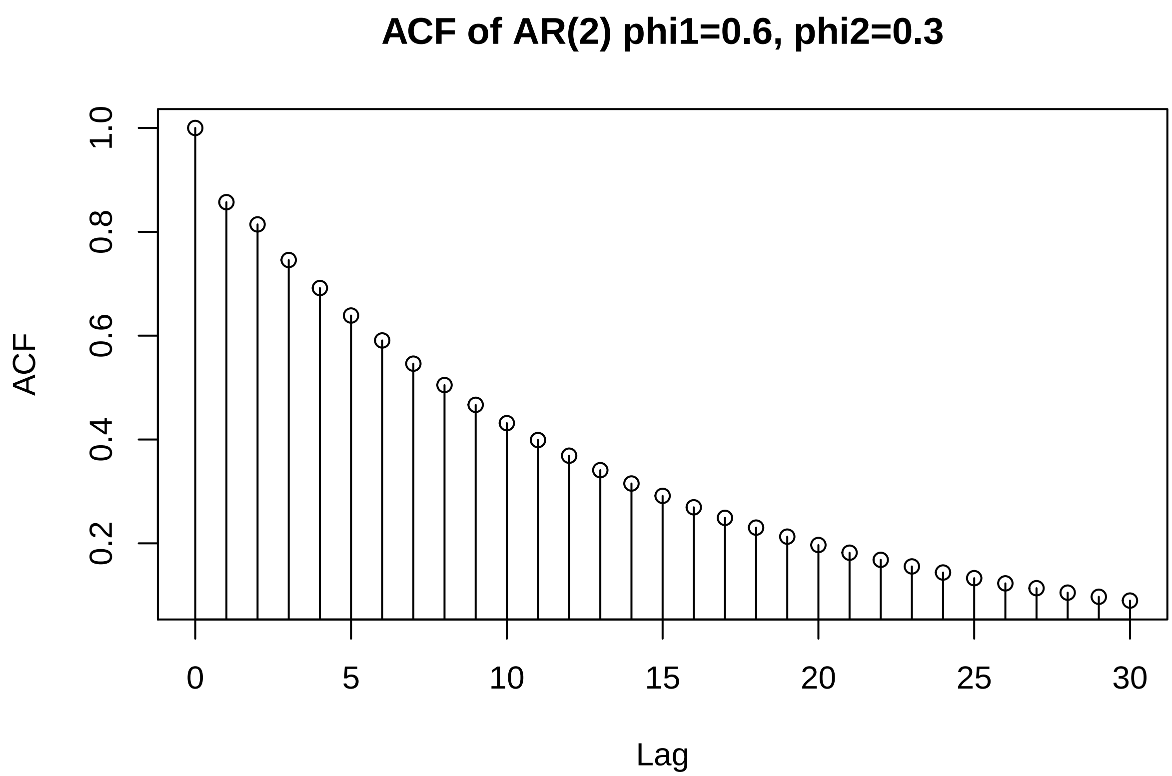
$\hat{x}_{n+3} = \mu + \hat{w}_{n+3} = 10.1 + 0.2528 = 10.3528$ $\quad \hat{Y}_{n+1} = \hat{x}_{n+1} + Y_n = 11.17 + 1312 = 1323.17$

$\hat{Y}_{n+2} = \hat{x}_{n+2} + \hat{Y}_{n+1} = 10.416 + 1323.17 = 1333.586$ $\quad \hat{Y}_{n+3} = \hat{x}_{n+3} + \hat{Y}_{n+2} = 10.3528 + 1333.586$

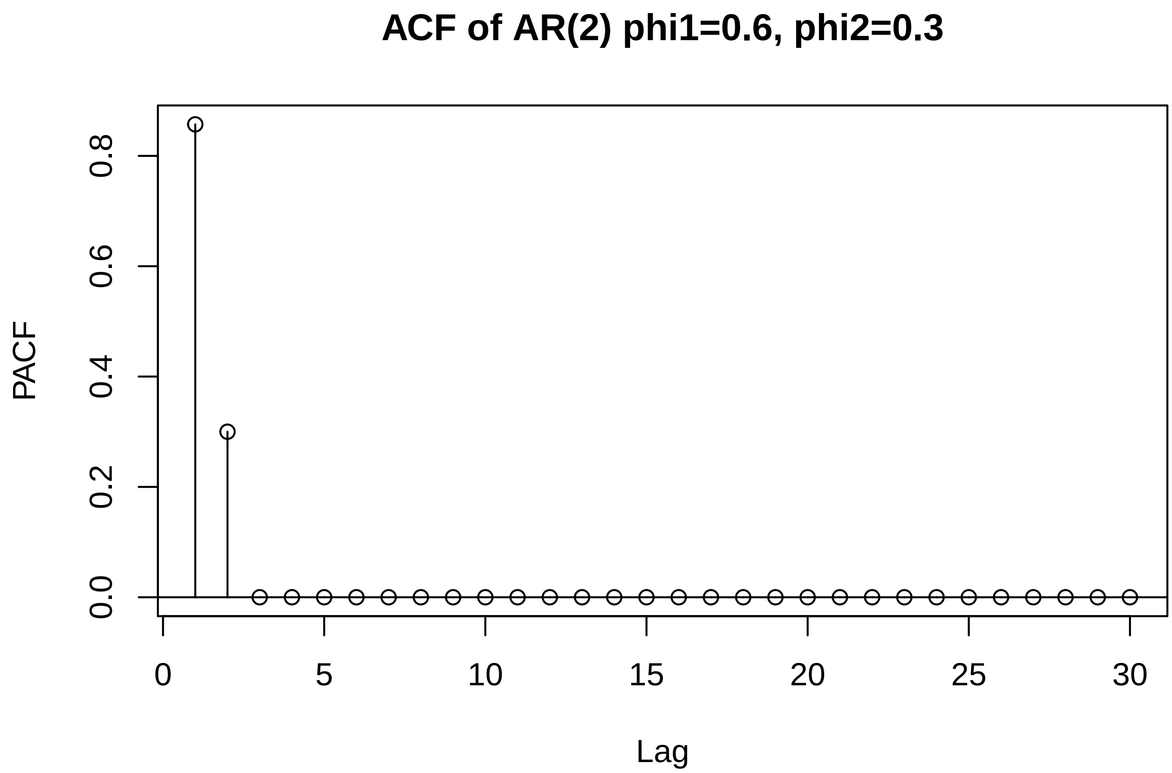
$= 1343.9388$

3a Shape of plots: Skewed Right

```r
r<- ARMAacf(ar = c(0.6, 0.3), lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=0.6, phi2=0.3", ylab = "ACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

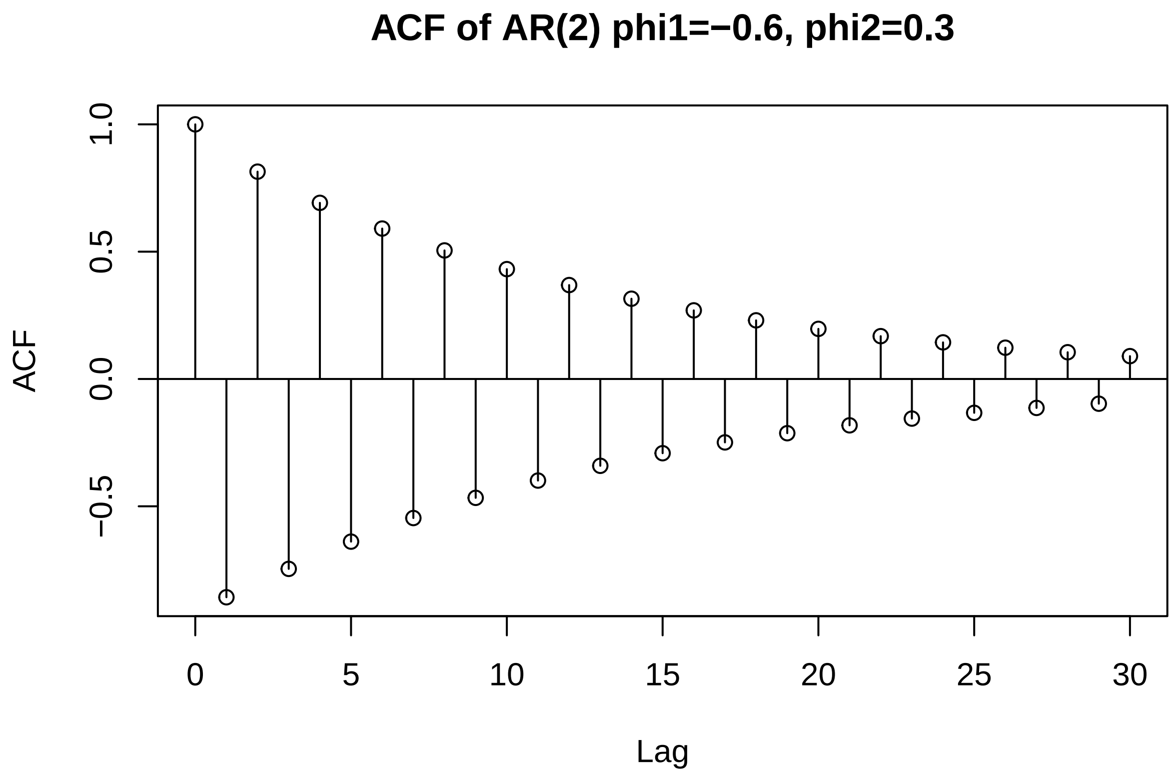



```r
r<- ARMAacf(ar = c(0.6, 0.3), lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=0.6, phi2=0.3", ylab = "PACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

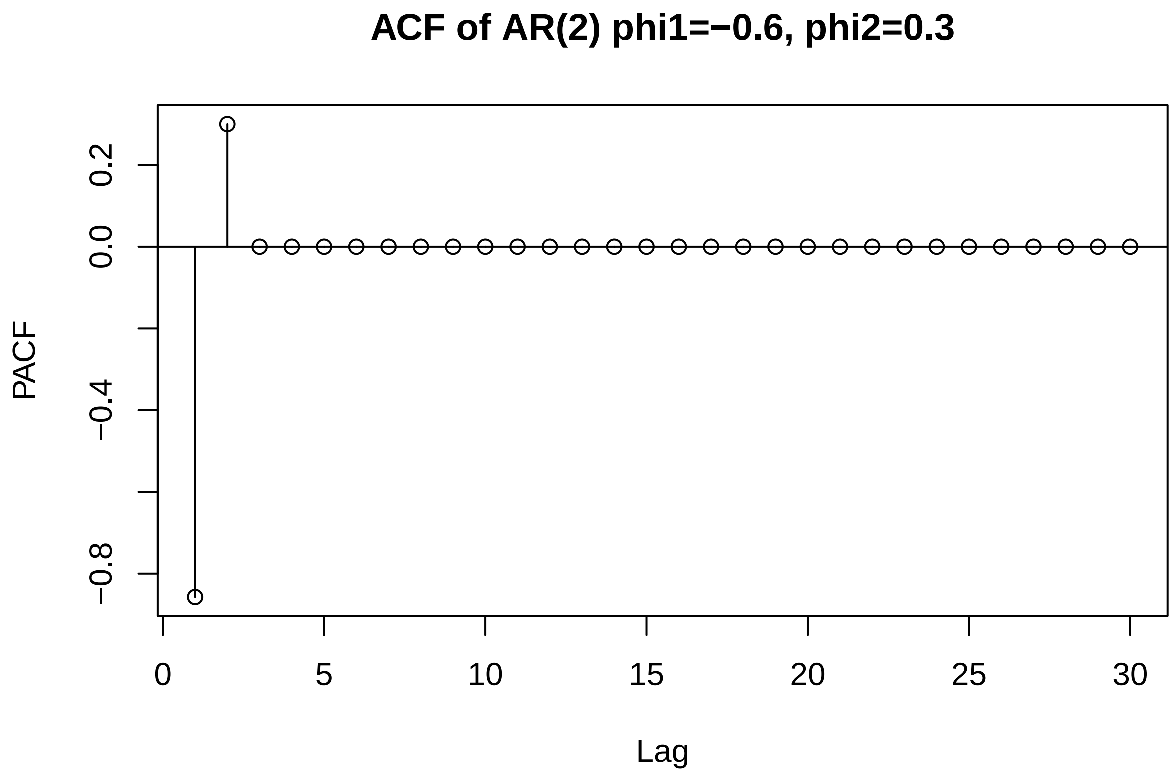


3b Shape of plots: Skewed Right

```
r<- ARMAacf(ar = c(-0.6, 0.3), lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=-0.6, phi2=0.3", ylab = "ACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

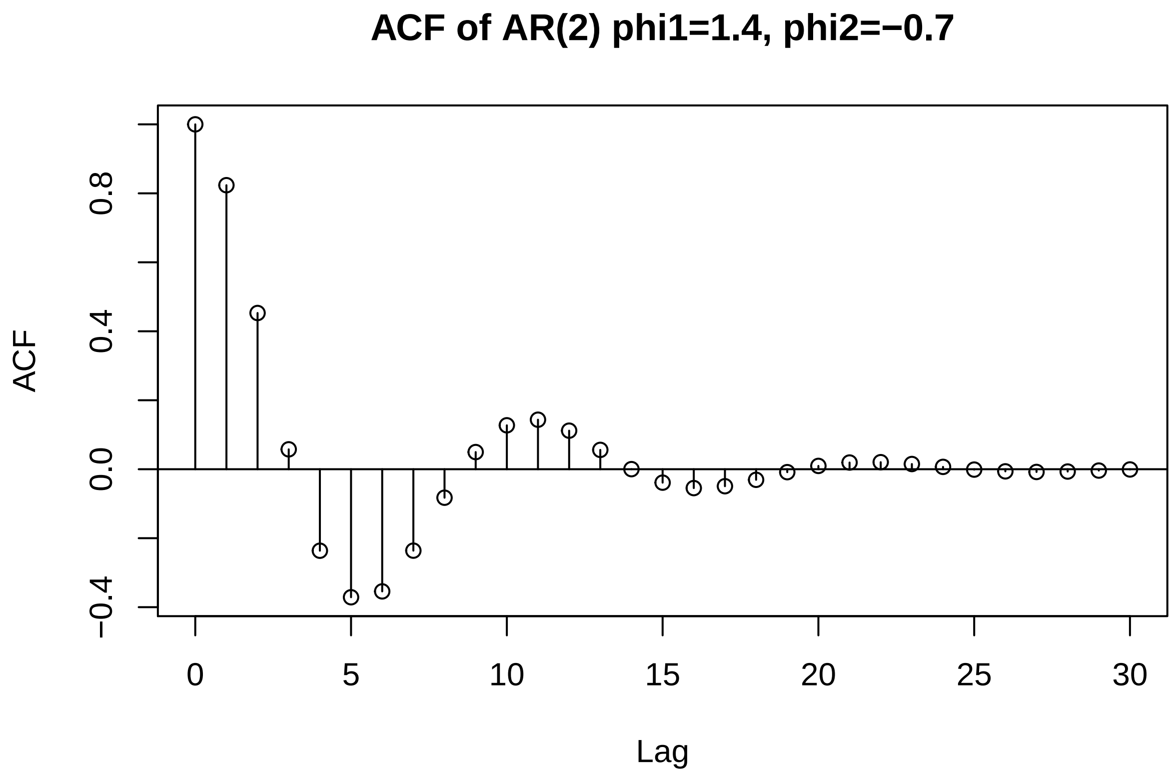



```r
r<- ARMAacf(ar = c(-0.6, 0.3), lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=-0.6, phi2=0.3", ylab = "PACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

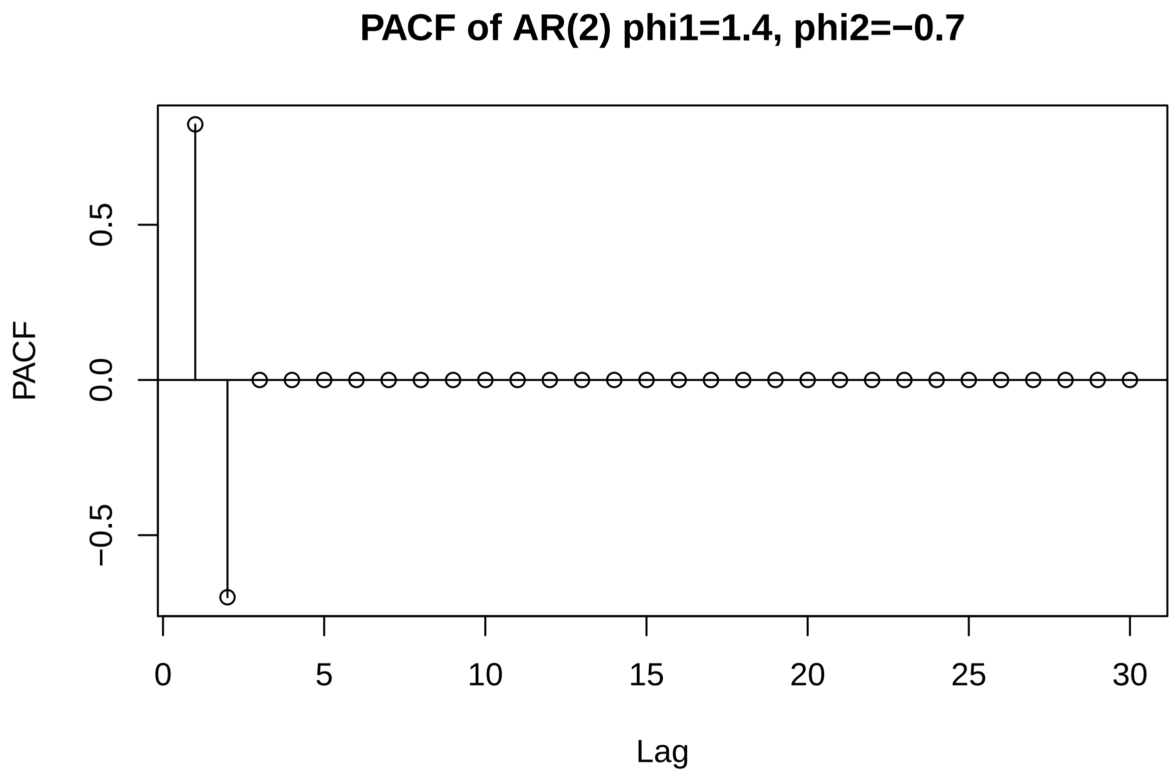


3c Shape of plots: Skewed Right

```
r<- ARMAacf(ar = c(1.4, -0.7), lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=1.4, phi2=-0.7", ylab = "ACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

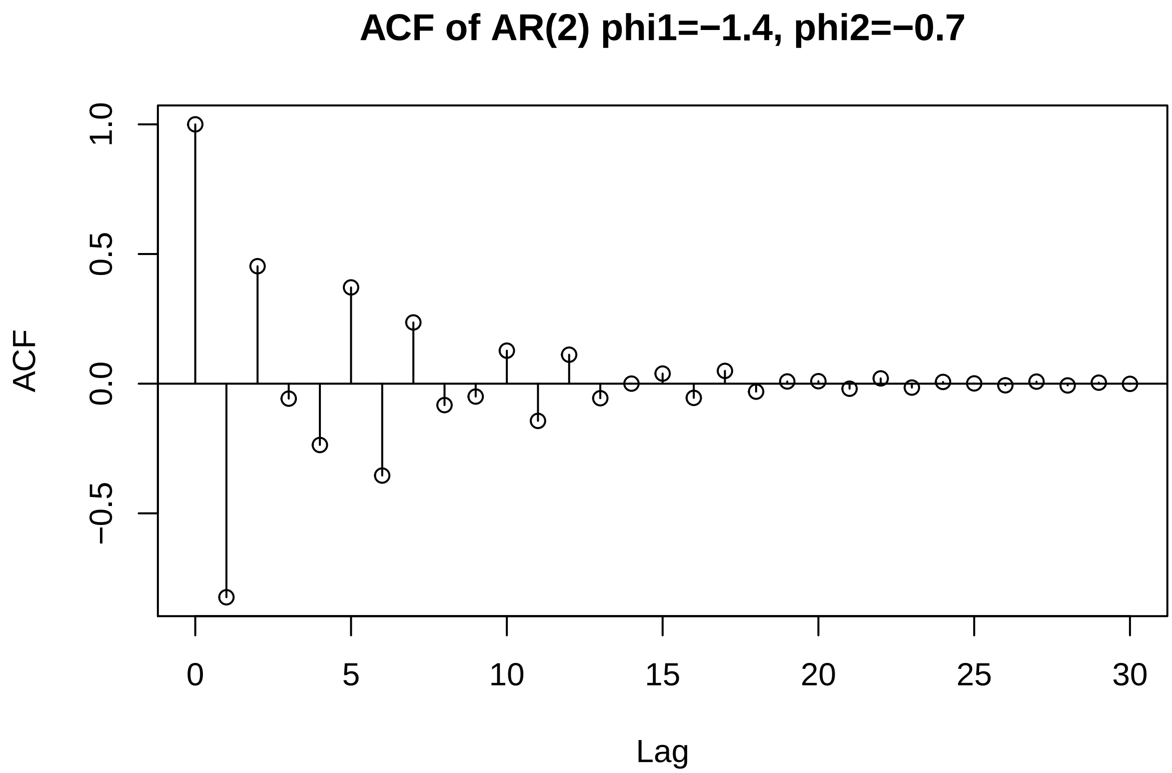



```r
r <- ARMAacf(ar = c(1.4, -0.7), lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "PACF of AR(2) phi1=1.4, phi2=-0.7", ylab = "PACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

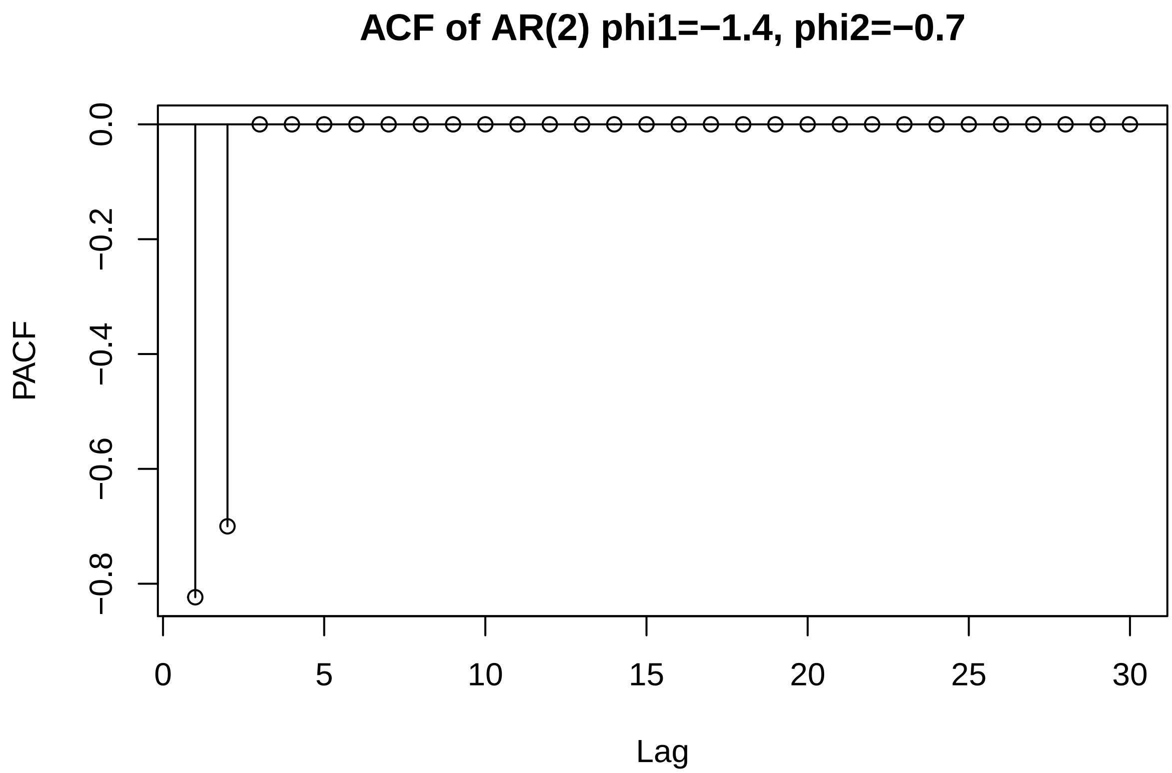


3d Shape of plots: Skewed Right

```
r<- ARMAacf(ar = c(-1.4, -0.7), lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=-1.4, phi2=-0.7", ylab = "ACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

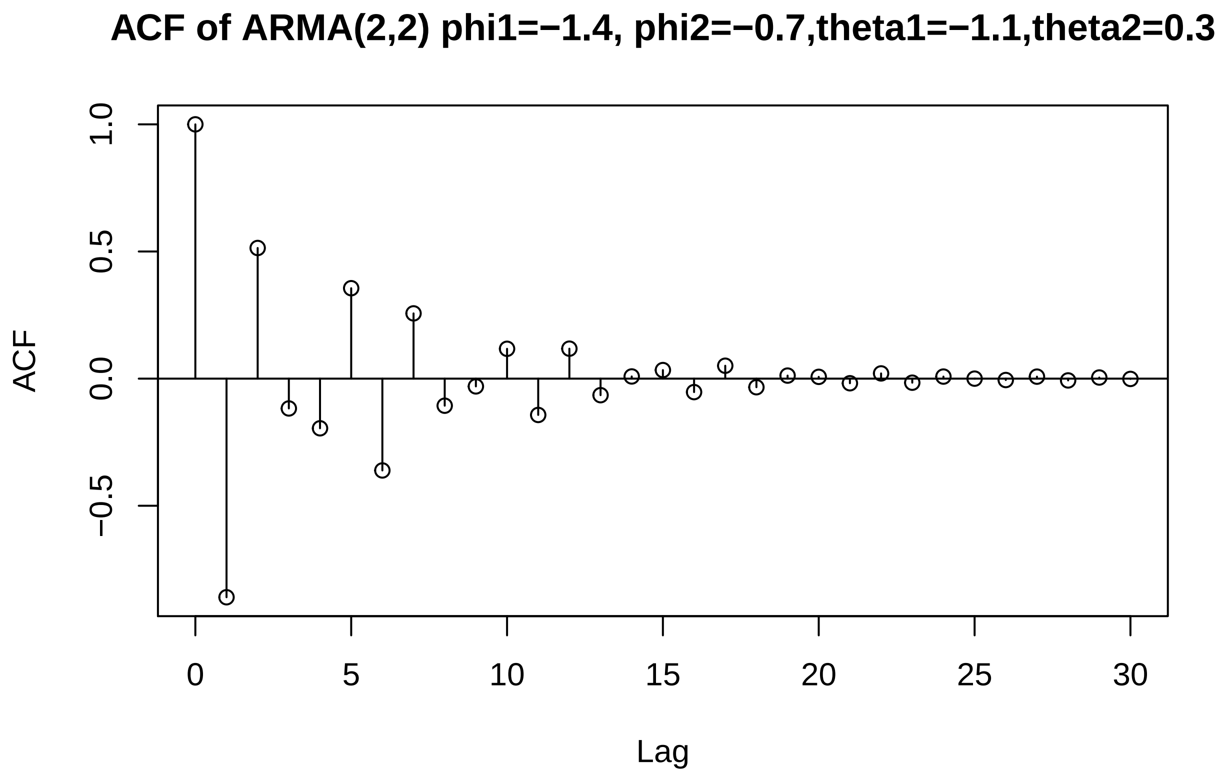



```r
r<- ARMAacf(ar = c(-1.4, -0.7), lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=-1.4, phi2=-0.7", ylab = "PACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

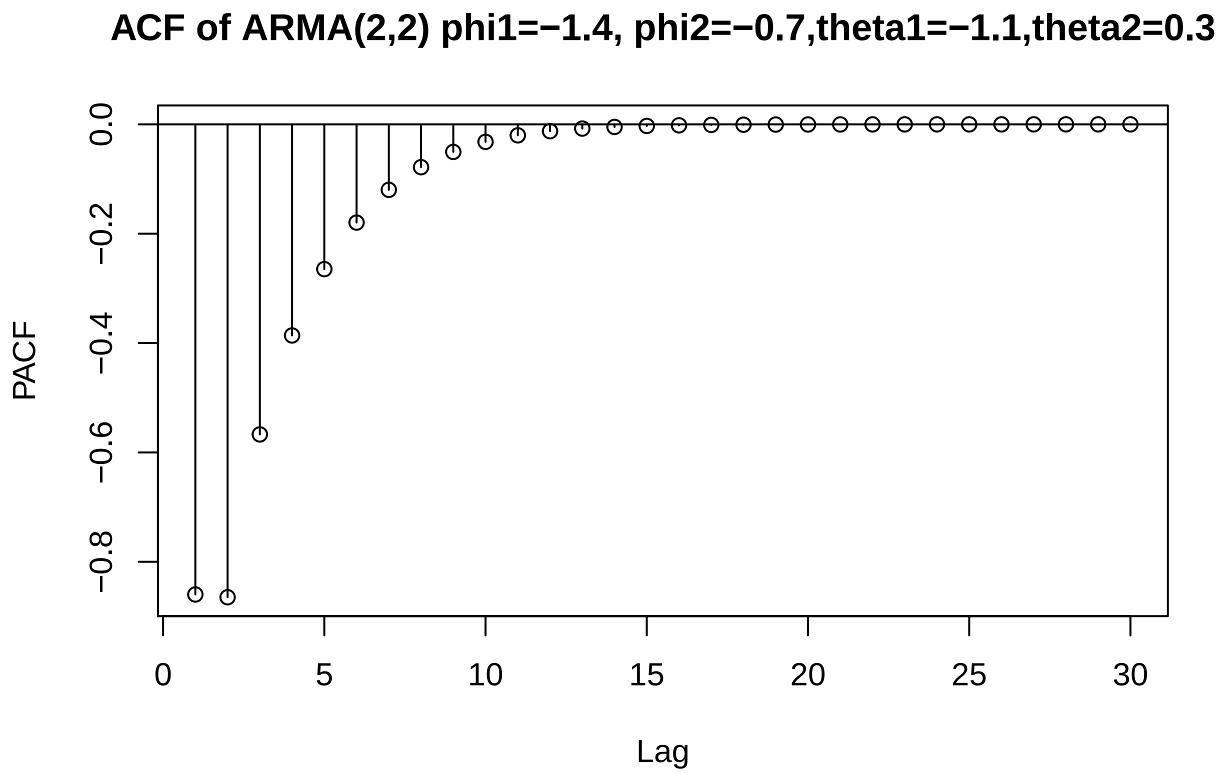


3e Shape of plots: Skewed Right

```
r<- ARMAacf(ar = c(-1.4, -0.7), ma=c(-1.1,0.3),lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of ARMA(2,2) phi1=-1.4, phi2=-0.7,theta1=-1.1,theta2=0.3", ylab = "ACF
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

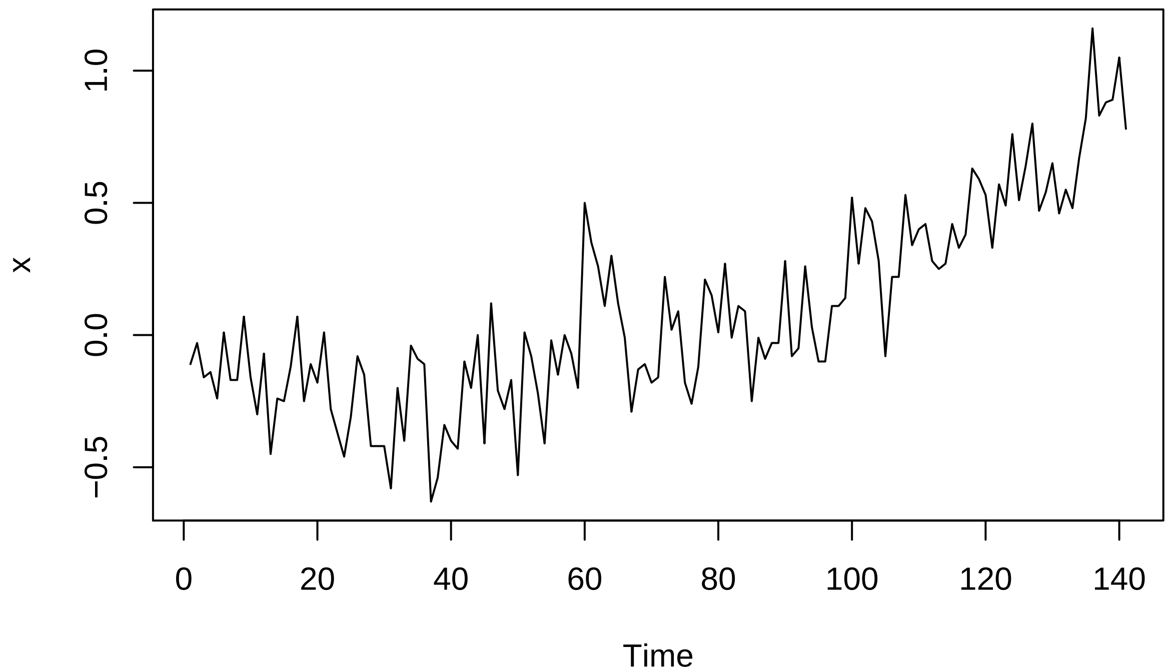



```
r<- ARMAacf(ar = c(-1.4, -0.7), ma=c(-1.1,0.3),lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "ACF of ARMA(2,2) phi1=-1.4, phi2=-0.7,theta1=-1.1,theta2=0.3", ylab = "PA
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
```

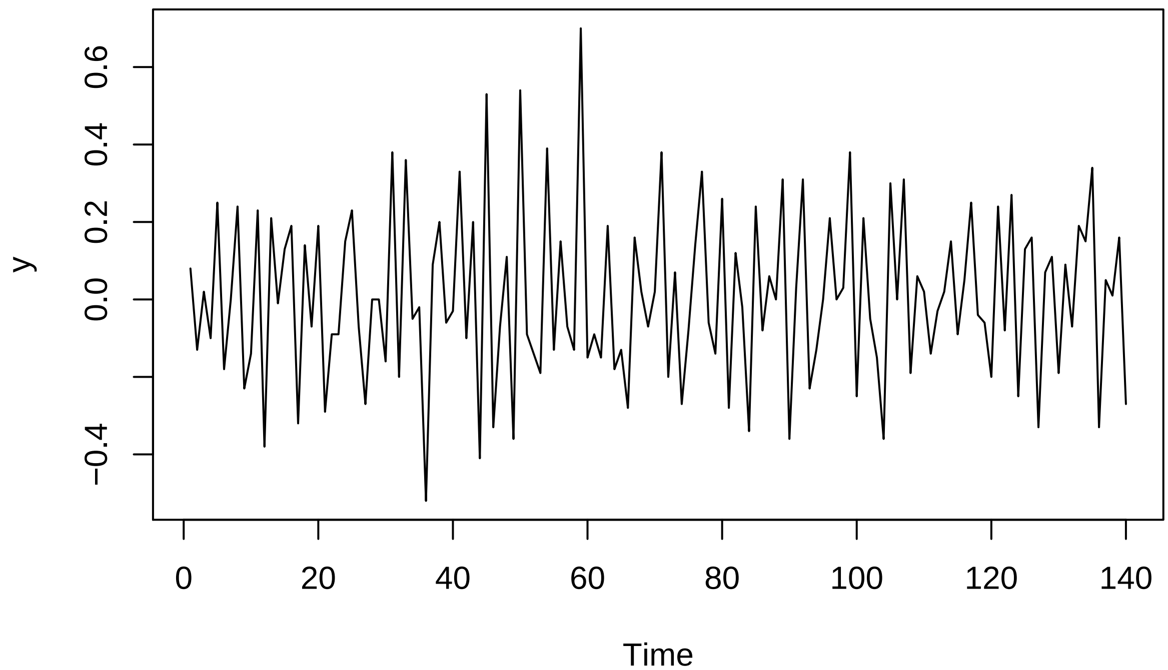


4a finding: The overall series is increasing. All values of lag are outside of 0. Only lag1 are outside of 0 and others are inside of PACF.

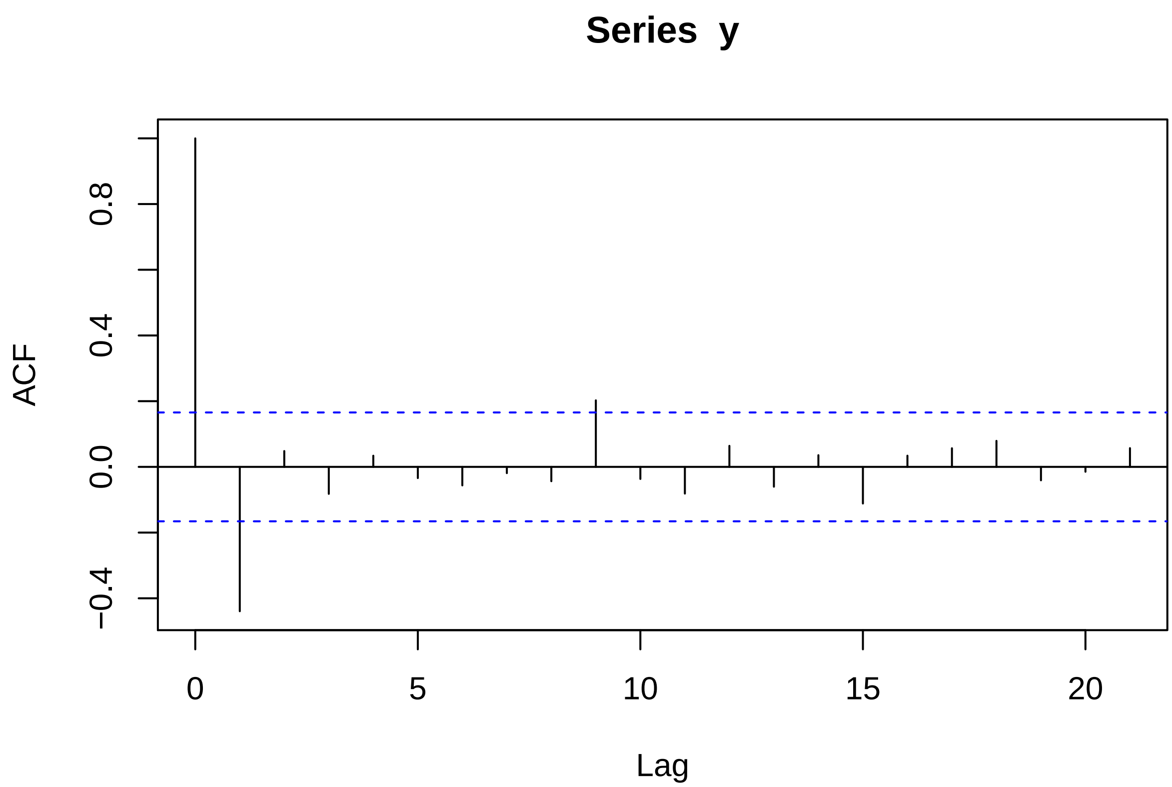
```
x <- read.csv("C:/Users/Administrator/Desktop/sta137/GlobTempNASA_2020.csv" header=TRUE)
x <- x[,2]
y<-diff(x,1)
#Plot the series against time.
plot.ts(x)
```



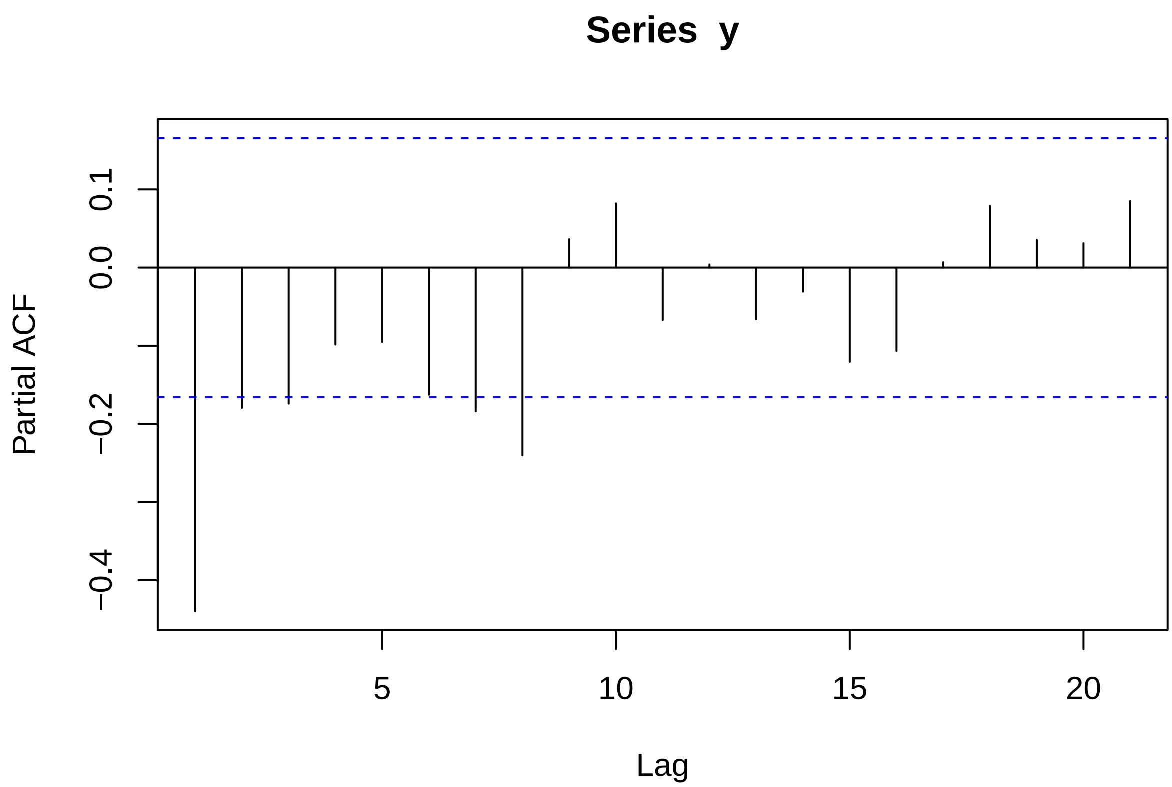

```
plot.ts(y)
```




```
#plot ACF
acf(y)
```




```
#plot PACF
pacf(y)
```



4b ARIMA(3,1,2) has the smallest aicc so it is the most appropriate model.The residuals from this model can be described as white noise because base on the ACF plot most ACF of lag are inside 0 except lag0 and lag9.

```
library(astsa)
```

```
## Warning: package 'astsa' was built under R version 4.0.3
```

```
AICc<-matrix(0,4,4)
for (i in 1:4){
    for (j in 1:4){
       AICc[i,j]<-sarima(x,p=i-1,d=1,q=j-1,details=FALSE)$AICc
  }
}
```

```
## Warning in sqrt(diag(fitit$var.coef)): NaNs produced

## Warning in sqrt(diag(fitit$var.coef)): NaNs produced
```

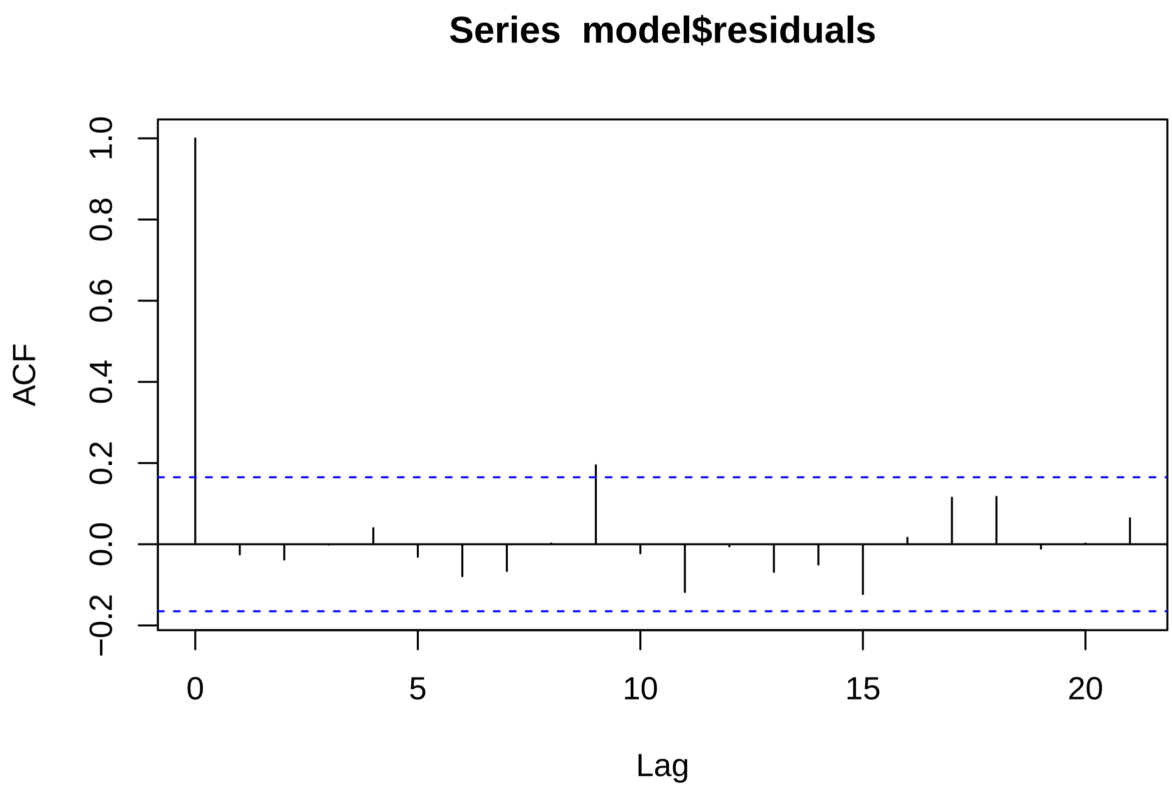
```
AICc
```

```
##            [,1]       [,2]       [,3]       [,4]
## [1,] -0.1884868 -0.4925925 -0.4930382 -0.4965466
## [2,] -0.3897800 -0.4992842 -0.4906390 -0.4814520
## [3,] -0.4073453 -0.4955839 -0.4820290 -0.4664774
## [4,] -0.4226023 -0.4823401 -0.5123697 -0.4506333
```

```r
sarima(x,p=3,d=1,q=2,details=FALSE)
```

```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##     Q), period = S), xreg = constant, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ar2      ar3      ma1     ma2  constant
##       1.2481  -0.1252  -0.2409  -1.9581  1.0000    0.0074
## s.e.  0.0823   0.1329   0.0845   0.0542  0.0551    0.0051
##
## sigma^2 estimated as 0.03033:  log likelihood = 43.18,  aic = -72.36
##
## $degrees_of_freedom
## [1] 134
##
## $ttable
##          Estimate     SE  t.value p.value
## ar1        1.2481 0.0823  15.1647  0.0000
## ar2       -0.1252 0.1329  -0.9420  0.3479
## ar3       -0.2409 0.0845  -2.8528  0.0050
## ma1       -1.9581 0.0542 -36.1346  0.0000
## ma2        1.0000 0.0551  18.1327  0.0000
## constant   0.0074 0.0051   1.4423  0.1515
##
## $AIC
## [1] -0.516881
##
## $AICc
## [1] -0.5123697
##
## $BIC
## [1] -0.3697989
```

```r
model<-arima(x,order=c(3,1,2))
acf(model$residuals)
```



4c We can see that the fitted model line is close fitted to the true model line, so i think this is a good fit

```
library(forecast)
```

```
## Warning: package 'forecast' was built under R version 4.0.4
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
##
## Attaching package: 'forecast'
```

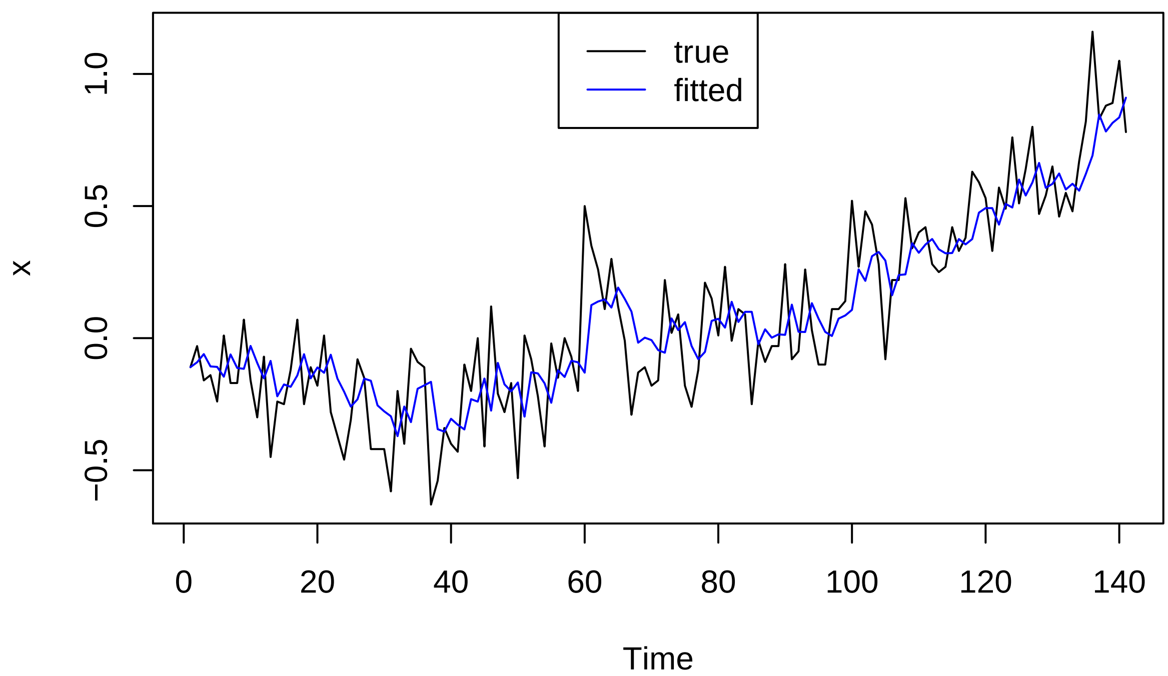
```
## The following object is masked from 'package:astsa':
##
##     gas
```

```
model <- auto.arima(x, stepwise=F, approximation=F, ic="aicc")
model
```

```
## Series: x
## ARIMA(1,1,1) with drift
##
## Coefficients:
```

```
##          ar1      ma1   drift
##       0.2033  -0.8294  0.0072
## s.e.  0.1099   0.0636  0.0034
##
## sigma^2 estimated as 0.03405:  log likelihood=39.04
## AIC=-70.08   AICc=-69.78   BIC=-58.31
```

```
n <- length(x)
plot.ts(x)
lines(1:n, model$fitted, col = "blue")
legend("top", legend = c("true","fitted"), lty=c(1, 1), col = c("black","blue"))
```



4d We use 95% confidence Interval to predict the value,but the trend should go upward,therefore it should be a quality prediction.

```
#split data
xnew <- x[1:(n-5)]
xlast <- x[(n-4):n]
#fit
model1 <- arima(xnew,order = c(3,1,2))
#prediction
h <- 5
m <- n - h
fcast <- predict(model1, n.ahead=h)
fcast
```

```
## $pred
```

```
## Time Series:
## Start = 137
## End = 141
## Frequency = 1
## [1] 0.8811239 0.8548309 0.7832710 0.7610984 0.7494578
##
## $se
## Time Series:
## Start = 137
## End = 141
## Frequency = 1
## [1] 0.1781046 0.1868427 0.1942899 0.1959672 0.1968989
```

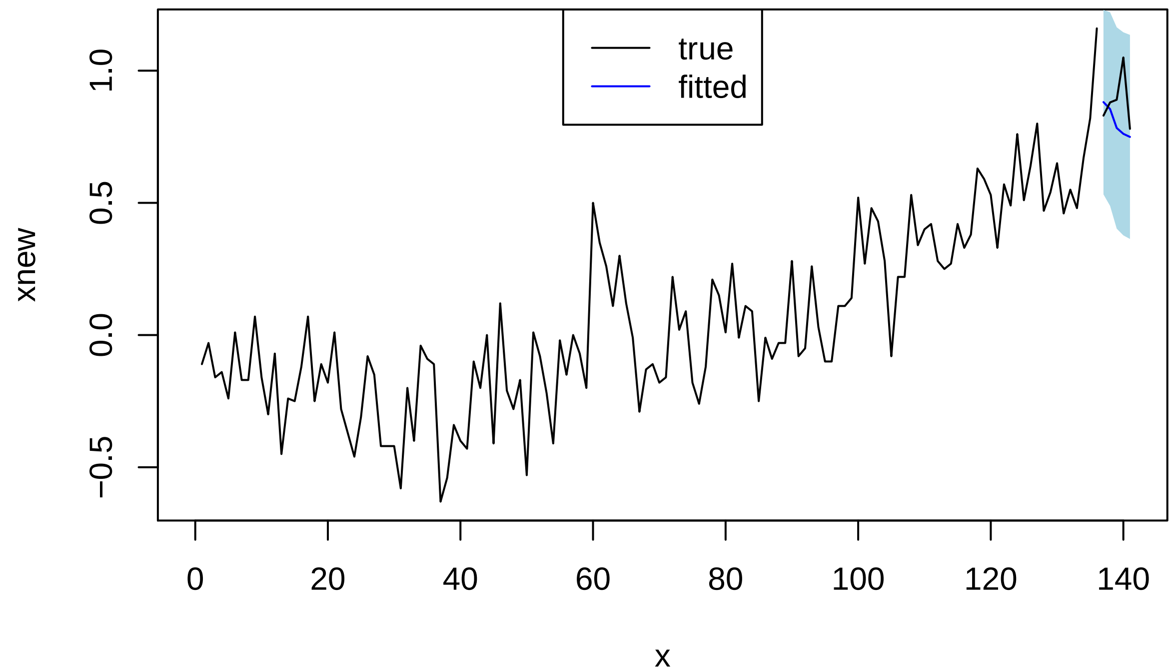
```
upper <- fcast$pred+1.96*fcast$se
upper
```

```
## Time Series:
## Start = 137
## End = 141
## Frequency = 1
## [1] 1.230209 1.221043 1.164079 1.145194 1.135380
```

```
lower <- fcast$pred-1.96*fcast$se
lower
```

```
## Time Series:
## Start = 137
## End = 141
## Frequency = 1
## [1] 0.5320388 0.4886192 0.4024629 0.3770026 0.3635360
```

```
#plot
plot.ts(xnew, xlim = c(0,n), xlab = "x")
polygon(x=c(m+1:h,m+h:1), y=c(upper,rev(lower)), col='lightblue', border=NA)
lines(x=m+(1:h), y=fcast$pred,col='blue')
lines(x=m+(1:h), y=xlast,col='black')
legend("top", legend = c("true","fitted"), lty=c(1, 1), col = c("black","blue"))
```




```r
knitr::opts_chunk$set(echo = TRUE)
r<- ARMAacf(ar = c(0.6, 0.3), lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=0.6, phi2=0.3", ylab = "ACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
r<- ARMAacf(ar = c(0.6, 0.3), lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=0.6, phi2=0.3", ylab = "PACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}

r<- ARMAacf(ar = c(-0.6, 0.3), lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=-0.6, phi2=0.3", ylab = "ACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
r<- ARMAacf(ar = c(-0.6, 0.3), lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=-0.6, phi2=0.3", ylab = "PACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
r<- ARMAacf(ar = c(1.4, -0.7), lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=1.4, phi2=-0.7", ylab = "ACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
```

```r
abline(h=0)}
r <- ARMAacf(ar = c(1.4, -0.7), lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "PACF of AR(2) phi1=1.4, phi2=-0.7", ylab = "PACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
r<- ARMAacf(ar = c(-1.4, -0.7), lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=-1.4, phi2=-0.7", ylab = "ACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
r<- ARMAacf(ar = c(-1.4, -0.7), lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "ACF of AR(2) phi1=-1.4, phi2=-0.7", ylab = "PACF",xlab = "Lag")
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
r<- ARMAacf(ar = c(-1.4, -0.7), ma=c(-1.1,0.3),lag.max = 30, pacf = F)
l <- 0:30
{plot(l,r,type = "p", main = "ACF of ARMA(2,2) phi1=-1.4, phi2=-0.7,theta1=-1.1,theta2=0.3", ylab = "AC
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
r<- ARMAacf(ar = c(-1.4, -0.7), ma=c(-1.1,0.3),lag.max = 30, pacf = T)
l <- 1:30
{plot(l,r,type = "p", main = "ACF of ARMA(2,2) phi1=-1.4, phi2=-0.7,theta1=-1.1,theta2=0.3", ylab = "PA
segments(x0=l, y0=0, x1=l, y1=r)
abline(h=0)}
x <- read.csv("C:/Users/Administrator/Desktop/sta137/GlobTempNASA_2020.csv" header=TRUE)
x <- x[,2]
y<-diff(x,1)
#Plot the series against time.
plot.ts(x)
plot.ts(y)
#plot ACF
acf(y)
#plot PACF
pacf(y)
library(astsa)
AICc<-matrix(0,4,4)
for (i in 1:4){
    for (j in 1:4){
       AICc[i,j]<-sarima(x,p=i-1,d=1,q=j-1,details=FALSE)$AICc
  }
}
AICc
sarima(x,p=3,d=1,q=2,details=FALSE)
model<-arima(x,order=c(3,1,2))
acf(model$residuals)
library(forecast)
model <- auto.arima(x, stepwise=F, approximation=F, ic="aicc")
model
n <- length(x)
plot.ts(x)
lines(1:n, model$fitted, col = "blue")
```

```r
legend("top", legend = c("true","fitted"), lty=c(1, 1), col = c("black","blue"))
#split data
xnew <- x[1:(n-5)]
xlast <- x[(n-4):n]
#fit
model1 <- arima(xnew,order = c(3,1,2))
#prediction
h <- 5
m <- n - h
fcast <- predict(model1, n.ahead=h)
fcast
upper <- fcast$pred+1.96*fcast$se
upper
lower <- fcast$pred-1.96*fcast$se
lower
#plot
plot.ts(xnew, xlim = c(0,n), xlab = "x")
polygon(x=c(m+1:h,m+h:1), y=c(upper,rev(lower)), col='lightblue', border=NA)
lines(x=m+(1:h), y=fcast$pred,col='blue')
lines(x=m+(1:h), y=xlast,col='black')
legend("top", legend = c("true","fitted"), lty=c(1, 1), col = c("black","blue"))
```