

1. **Modus Ponens:** Show that $\{P, P \Rightarrow Q\} \models Q$.
2. **Wumpus World (Simple):** Consider the following facts using the Wumpus World predicates from AIMA:

$$\begin{aligned} &\neg P_{1,1} \\ &B_{1,1} \iff (P_{1,2} \vee P_{2,1}) \\ &B_{2,1} \iff (P_{1,1} \vee P_{2,2} \vee P_{3,1}) \\ &\neg B_{1,1} \\ &B_{2,1} \end{aligned}$$

Prove whether $P_{1,2}$ is true or not (that is, whether there is a pit at location [1,2]).

The textbook gives several other examples of knowledge bases and queries using the Wumpus World that you can also try.

3. **Horn Clauses** (Russell & Norvig) If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.
 - (a) Can we prove that the unicorn is mythical?
 - (b) Can we prove that the unicorn is magical?
 - (c) Can we prove that the unicorn is horned?
4. **Liars and Truth-tellers** (adapted from OSSMB): Three people, Amy, Bob, and Cal, are each either a liar or a truth-teller. Assume that liars always lie, and truth-tellers always tell the truth.

(a) OSSMB 82-12:

- Amy says, "Cal and I are truthful."
- Bob says, "Cal is a liar."
- Cal says, "Bob speaks the truth or Amy lies."

What can you conclude about the truthfulness of each?

(b) OSSMB 83-11:

- Amy says, "Cal is not honest."
- Bob says, "Amy and Cal never lie."
- Cal says, "Bob is correct."

What can you conclude about the truthfulness of each?

5. **More Liars and Truth-tellers** (adapted from JRM14 392): At Liars Anonymous, there was a gathering of liars in extreme denial and fully reformed truth-tellers. Each will profess his/her truthfulness. However, when they are all together, their statements about each other reveal the truth:

- Amy says, "Hal and Ida are truth-tellers."
- Bob says, "Amy and Lee are truth-tellers."
- Cal says, "Bob and Gil are truth-tellers."
- Dee says, "Eli and Lee are truth-tellers."
- Eli says, "Cal and Hal are truth-tellers."
- Fay says, "Dee and Ida are truth-tellers."
- Gil says, "Eli and Jay are liars."
- Hal says, "Fay and Kay are liars."
- Ida says, "Gil and Kay are liars."

- Jay says, "Amy and Cal are liars."
- Kay says, "Dee and Fay are liars."
- Lee says, "Bob and Jay are liars."

Which are liars and which are truth-tellers?

6. **The Doors of Enlightenment** (from CRUX 357): There are four doors X , Y , Z , and W leading out of the Middle Sanctum. At least one of them leads to the Inner Sanctum. If you enter a wrong door, you will be devoured by a fierce dragon. Well, there were eight priests A , B , C , D , E , F , G , and H , each of whom is either a knight or a knave. (Knights always tell the truth and knaves always lie.) They made the following statements to the philosopher:

- A : X is a good door.
- B : At least one of the doors Y or Z is good.
- C : A and B are both knights.
- D : X and Y are both good doors.
- E : X and Z are both good doors.
- F : Either D or E is a knight.
- G : If C is a knight, so is F .
- H : If G and I (meaning H) are knights, so is A .

(a) Smullyan's problem: Which door should the philosopher choose?

(b) Liu's problem: The philosopher lacked concentration. All he heard was the first statement (A 's) and the last statement (H 's) plus two fragments:

- C : A and ... are both knights.
- G : If C is a knight, ...

Prove that he had heard enough to make a decision.