1CM30 - Service Supply Chains for Capital Goods Spring 2014 Exam Exercises - Set 4 Condition based maintenance

Hao Peng

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1. The degradation of a machine can be described by a DTM. The engineers collect the time to defect data in the lab by doing reliability testings. The time to defect data statistically suggest that the time to defect follows an exponential distribution with $\lambda = 1$. The time unit is one month. The engineers also collect the data about the duration from the defect time point till failure in the lab. The delay time seems to be a constant value 0.5, since the variance of the delay times are so small.

   For this machine, we will apply a periodic inspection policy with a fixed inspection interval τ. When we detect defects on the machine, we will replace the machine with a new one to prevent unexpected failures. Upon failures we will also replace the machine with a new one. The costs of a replacement are equal to EURO 3000. For a corrective maintenance action additional costs equal to EURO 1000 are incurred because of the disturbance of the production process that depends on the availability of the machine. The inspection cost is 10 Euro.

   Determine the average costs of the periodic inspection policy as a function of τ.

2. The wear-out of a rotating part can be described by a a linear function, $X(t) = \theta t$. The time unit is one revolution. The wear-out can be determined by inspections on wear debris. Based on the wear-out mechanism, θ is dependent on the hardness of the material, the radius of the rotating part, and the force between rubbing surfaces. It is a random parameter among units. The engineers collect the degradation data of 80 units. For each unit, there's an estimated linear function through regression, which has an estimated $\hat{\theta}$. Then there are 80 $\hat{\theta}$ s. The engineers find out the distribution of θ is an exponential distribution with parameter $\hat{\lambda} = 1$, based on the 80 $\hat{\theta}$ s. The failure limit of the wear-out is given as $H = 5000$.

   - Suppose the engineers set a warning limit of $C = 4000$. What is the cdf of the first passage time over C for this degrading component?
   - Conditioned on the fact that the wear-out of the rotating component at 3500 revolutions is 4000, what is the conditional first passage time over H?

   For this component, we will apply a periodic inspection policy with a fixed inspection interval τ and a control limit C. When the degradation level is above C the component will get replaced. The degradation level can only be observed through inspections. Upon failures we will also replace the machine with a new one. The costs of a replacement are equal to EURO 3000. For a corrective maintenance action additional costs equal to EURO 1000 are incurred because of the disturbance of the production process that depends on the availability of the machine. The inspection cost is 10 Euro.

   Determine the average long run cost rate of the periodic inspection policy as a function of τ and C.

Exam Exercises

1. There are three renewal events at the end of a renewal cycle.

① failure renewal before the next inspection comes.

$$\Pr(X+H<\tau) = \Pr(X<\tau-0.5)$$
$$= F_X(\tau-0.5)$$
$$= 1-\exp(-\lambda(\tau-0.5))$$
$$= 1-\exp(-\tau+0.5)$$

② inspection renewal without replacement.

$$\Pr(X>\tau) = 1-F_X(\tau)$$
$$= 1-(1-\exp(-\lambda\tau))$$
$$= \exp(-\tau)$$

③ inspection renewal with replacement

$$\Pr(X<\tau \cap X+H>\tau) = \Pr(X<\tau \cap X>\tau-0.5)$$
$$= \int_{\tau-0.5}^{\tau} f_X(u)\,du$$
$$= \int_{\tau-0.5}^{\tau} \lambda\exp(-\lambda u)\,du$$
$$= \int_{\tau-0.5}^{\tau} \exp(-u)\,du$$

The expected cycle cost $E(CC)$

$$= C_{cm} \underset{①}{Pr(X+H<\tau)} + C_i \underset{②}{Pr(X>\tau)} + (C_i + C_{pm}) \underset{③}{Pr(X<\tau \cap X+H>\tau)}$$

The expected cycle length $E(CL)$

① failure renewal

$X+H$ random $\quad \sum_{t \in (0,\tau)} t \, Pr(X+H=t)$ (expected value discrete version)

$$= \sum_{t \in (0,\tau)} t \, Pr(X=t-0.5)$$

$$= \int_0^\tau t f_X(t-0.5) \, dt$$

$$= \int_0^\tau t \lambda \exp(-\lambda(t-0.5)) \, dt$$

$$= \int_0^\tau t \exp(-(t-0.5)) \, dt$$

② inspection renewal without replacement

$\tau \cdot Pr(X>\tau)$ ------ implies $X+H>\tau$

③ inspection renewal with replacement

$\tau \cdot Pr(X<\tau \cap X+H>\tau)$

$$E(CL) = \int_0^\tau t \exp(-(t-0.5)) dt + \tau \Big(\underbrace{Pr(X>\tau) + Pr(X<\tau \cap X+H>\tau)}_{\text{covers the entire sample space}} \Big)$$

$$= \int_0^\tau t \exp(-(t-0.5)) dt + \tau \, Pr(X+H>\tau)$$

$$= \int_0^\tau t \exp(-(t-0.5)) dt + \tau \, Pr(X > \tau - 0.5)$$

$$= \int_0^\tau t \exp(-(t-0.5)) dt + \tau (1 - F_X(\tau - 0.5))$$

$$= \int_0^\tau t \exp(-(t-0.5)) dt + \tau (1 - (1 - \exp(-\lambda(\tau - 0.5))))$$

$$= \int_0^\tau t \exp(-(t-0.5)) dt + \tau \exp(-(\tau - 0.5))$$

$CR(\tau) = \frac{E(CC)}{E(CL)}$ --- Expected cost rate

Exam Exercise

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The cdf of first passage time T_c

$$\Pr\{T_c \le t\} = \Pr\{\theta t \ge c\}$$
$$= \Pr\{\theta \ge \frac{c}{t}\}$$
$$= 1 - F_\theta(\frac{c}{t})$$

Since, $\theta \sim$ Exponential $(\lambda = 1)$

$$\Pr\{T_c \le t\} = 1 - (1 - \exp(-\lambda \frac{c}{t}))$$
$$= \cancel{1}\exp(-\frac{4000}{t})$$

$$f_{T_c}(t) = \frac{d F_{T_c}(t)}{dt}$$
$$= \exp(-\frac{4000}{t})(+4000) t^{-2}$$

The conditional first passage time T_H, given that. ~~$T_{3500} = 4000$~~ $T_{4000} = 3500$

~~$\theta 3500$~~

$\theta 3500 = 4000$

$\therefore \theta = \frac{4000}{3500}$ this condition specifies θ.

$H = 5000 = \theta T_H = \frac{4000}{3500} T_H$

$\therefore T_H = 5000 \cdot \frac{3500}{4000}$ this condition thus specifies the T_H

$\therefore$ given the condition $T_{4000} = 3500$ revolutions

T_H is a constent number

$$\therefore \Pr\{T_H = 5000 \cdot \frac{3500}{4000} \mid T_{4000} = 3500\} = 1$$

$$\underbrace{\Pr\{T_H < t\}}_{\mid T_c = 3500 \text{ condition}} = \begin{cases} 1 & \text{if } 5000 \cdot \frac{3500}{4000} < t. \\ 0 & \text{otherwise} \end{cases}$$

There are two possible renewal evenes.

① failure renewal in a certain inspection interval $((i-1)\tau, i\tau)$

$$Pr(T_H < i\tau \cap (i-1)\tau < T_c < i\tau)$$

$$= \int_{(i-1)\tau}^{i\tau} f_{T_c}(u)\, Pr(T_H < i\tau \mid T_c = u)\, du$$

$$= \int_{(i-1)\tau}^{i\tau} \exp\left(-\frac{4000}{u}\right) \cdot 4000 \cdot u^{-2}\, Pr(T_H < i\tau \mid T_c = u)\, du$$

$$Pr(T_H < i\tau \mid T_c = u) = \begin{cases} 1 & \text{if } 5000 \cdot \frac{u}{4000} < i\tau \\ 0 & \text{otherwise} \end{cases}$$

② inspection renewal at time point $i\tau$.

$$Pr(T_H > i\tau \cap (i-1)\tau < T_c < i\tau)$$

$$= \int_{(i-1)\tau}^{i\tau} f_{T_c}(u)\, Pr(T_H > i\tau \mid T_c = u)\, du$$

$$= \int_{(i-1)\tau}^{i\tau} \exp\left(-\frac{4000}{u}\right) \cdot 4000 \cdot u^{-2} \cdot Pr(T_H > i\tau \mid T_c = u)\, du$$

$$Pr(T_H > i\tau \mid T_c = u) = \begin{cases} 1 & \text{if } 5000 \cdot \frac{u}{4000} > i\tau \\ 0 & \text{otherwise} \end{cases}$$

The expected cycle length $E(CL)$

① failure renewal in a certain inspection interval. $((i-1)\tau, i\tau)$

$\underline{T_H}$ random

$$\sum_{t\in((i-1)\tau, i\tau)} t \cdot Pr(T_H=t \cap (i-1)\tau < T_c < i\tau)$$

$$= \sum_{t\in((i-1)\tau, i\tau)} t \sum_{u\in((i-1)\tau, i\tau)} Pr(T_c=u) \cdot Pr(T_H=t \mid T_c=u)$$

$$= \sum_{t\in((i-1)\tau, i\tau)} t \int_{(i-1)\tau}^{i\tau} f_{T_c}(u) . Pr(T_H=t \mid T_c=u)\, du$$

where $Pr(T_H=t \mid T_c=u) = \begin{cases} 1 & \text{if } 5000\frac{u}{4000}=t \\ 0 & \text{otherwise.} \end{cases}$

$\boxed{\text{Or}} = \int_{(i-1)\tau}^{i\tau} f_{T_c}(u) \frac{5000u}{4000} Pr(T_H < i\tau \mid T_c=u)\, du$

another expression

where $Pr(T_H < i\tau \mid T_c=u) = \begin{cases} 1 & \text{if } 5000\frac{u}{4000} < i\tau. \\ 0 & \text{otherwise.} \end{cases}$

② inspection renewal at time point $i\tau$

$$i\tau \cdot Pr(T_H > i\tau \cap (i-1)\tau < T_c < i\tau)$$

$$= i\tau \sum_{u\in((i-1)\tau, i\tau)} Pr(T_c=u)\, Pr(T_H > i\tau \mid T_c=u)$$

$$= i\tau \int_{(i-1)\tau}^{i\tau} f_{T_c}(u)\, Pr(T_H > i\tau \mid T_c=u)\, du$$

where $Pr(T_H > i\tau \mid T_c=u) = \begin{cases} 1 & \text{if } 5000\frac{u}{4000} > i\tau. \\ 0 & \text{otherwise} \end{cases}$

The expected cycle cost $E(C)$

① failure renewal.

$$((i-1)C_i + C_{cm}) \cdot Pr(T_H < i\tau \cap (i-1)\tau < T_c < i\tau)$$

② inspection renewal.

$$(iC_i + C_{pm}) \cdot Pr(T_H > i\tau \cap (i-1)\tau < T_c < i\tau)$$

Considering all inspection intervals.

$$E(L) = \sum_{i=1}^{\infty} \left(\int_{(i-1)\tau}^{i\tau} f_{T_c}(u) \frac{J0004}{4000} Pr(T_H < i\tau \mid T_c = u)\, du \right.$$
$$\left. + i\tau \int_{(i-1)\tau}^{i\tau} f_{T_c}(u) Pr(T_H > i\tau \mid T_c = u)\, du \right)$$

$$E(C) = \sum_{i=1}^{\infty} \Big(((i-1)C_i + C_{cm}) \cdot Pr(T_H < i\tau \cap (i-1)\tau < T_c < i\tau)$$
$$+ (iC_i + C_{pm}) \cdot Pr(T_H > i\tau \cap (i-1)\tau < T_c < i\tau) \Big)$$

$$CR(\tau, C) = \frac{E(C)}{E(L)}$$