exercise 5.

$$\Pr\{T_C < t\} = \Pr\{\phi - \theta t < C\}$$
$$= \Pr\{\theta t > \phi - C\}$$
$$= \Pr\{\theta > \frac{\phi - C}{t}\}$$
$$= 1 - F_\theta\left(\frac{\phi - C}{t}\right)$$

Since $\theta \sim \text{Lognormal}(\mu = 1, \sigma = 0.2)$

$$\Pr\{T_C < t\} = 1 - \Phi\left(\frac{\ln\left(\frac{\phi - C}{t}\right) - \mu}{\sigma}\right)$$

c.d.f.

$$= 1 - \Phi\left(\frac{\ln\left(\frac{5000 - 2000}{t}\right) - 1}{0.2}\right)$$

given that $T_{2000} = 3500$.

$$\phi - \theta 3500 = 2000$$

$\because \phi = 5000$ $\therefore \theta = \frac{5000 - 2000}{3500}$

then $\because T_H = \frac{-H + \phi}{\theta}$ $(\phi - \theta T_H = H)$.

$$T_H = (5000 - 1000)\frac{3500}{(5000 - 2000)}$$

$\therefore T_H$ is a consecent number given the condition.

$$\Pr\{T_H = \frac{(5000 - 1000)}{(5000 - 2000)} 3500\} = 1$$

$$\Pr\{T_H < t\} = \begin{cases} 1 & \text{if } \frac{(5000 - 1000)}{(5000 - 2000)} 3500 < t \\ 0 & \text{otherwise.} \end{cases}$$

~~c.d.f~~

exercise 6.

If the failures are not self-announcing, the equations for E(CL) will be different from Equation 11 (lecture note)

$$E(CL) = \sum_{i=1}^{\infty} \{ iz \int_{(i-1)z}^{iz} f_{TC}(u)\,du \}$$

This is because no matter what happens at the end of the ~~renewal~~ renewal cycles, whether it fails or not, we can only detect it at the inspection time points. (iz)

The equation for E(C) should also be modified.

If it is a failure renewal at the end of the renewal cycle, and this failure renewal happens at inspection interval $((i-1)z, iz)$, we will detect this failure at the inspection time point iz, Then we should pay iC_i for inspection cost, and the corrective maintenance cost C_{cm}.

The renewal cycle cost when inspection renewal occurs is the same as in Equation (14) (lecture note).

$$\therefore E(C) = \sum_{i=1}^{\infty} \{ (iC_i + C_{cm}) \int_{(i-1)z}^{iz} f_{TC}(u) F_{TH|TC=u}(iz)\,du + (iC_i + C_{pm}) \int_{(i-1)z}^{iz} f_{TC}(u)(1 - F_{TH|TC=u}(iz))\,du \}$$

The numerical study is not required for the exam.

Using the equations for E(C) and E(CL),

$$CR(z, C) = \frac{E(C)}{E(CL)}.$$

Notice that $C_i = 10$, $C_{pm} = 3000$, $C_{cm} = 4000$.