#### ERASMUS SCHOOL OF ECONOMICS --

Exam Expercises 1. There are three renewal evenes at the end of a renewal cycle. O failure renewal before the next inspection comes. Pr(X+H < Z) = Pr(X < Z-05) = Fx(Z-015) = |-exp(-)(7-0.7)= 1 - exp(-7 + 0.5)@ inspection renewal without replacement.  $Pr(X>z) = 1 - F_X(z)$  $= |-(1-\exp(-\lambda z))|$ = exp(-7) 3) inspection renewal with replacement Pr (X<Z () X+H>Z) = Pr(X<Z () X>Z-05)  $= \int_{\tau-\rho_{1}}^{\tau} f_{x}(u) du$  $= \int_{7-0.5}^{7} \lambda \exp(-\lambda u) du$  $=\int_{7-0.5}^{2} \exp(-u) du$ 





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The expected cycle cost ECCC) = Ccm Pr(X+H < C) + Ci Pr(X>Z)+ CCi+(pm)Pr(X<Z/1 X+H>Z) The expected cycle length ECCL) O failure renewal Σ t Pr(X+H=t) (expected value t6(0,τ) discrete version) XTH Vandom = \\ \text{te(0,\z)} \text{tfr(X=t-0.5)}  $= \int_0^{\pi} t f_{\mathsf{X}}(t-0.5) dt.$ = ( t 1 exp(-1(t-0,5)) dt. =  $\int_{0}^{2} t \exp(-(t-0.5)) dt$ inspection renewal without replacement T.Pr(X7T) --- implies X+H>T inspection renewal with replacement Z. Pr (XCZ () XtH>Z) E(CL)= 5 = fot exp(-(t-05))dt + 7 (Pr(X>Z)+Pr(X<T), = fot exp(-(t-0,5))dt+7 Pr(X+H>7) Sample space = fo texp(-(t-0,5))dt + zpr(x> z-0,5)  $= \int_0^{\infty} e^{-2\pi i x} \left( -(t-o_{1}x) \right) dt + T \left( 1 - F_{x}(\tau-o_{1}x) \right)$ =  $\int_0^{2} + \exp(-(t-0.5))dt + 7(1-(1-\exp(-)(\tau-0.5)))$ 100 years INTI foct exp(-(t-0x)) dt + T € exp (-\*(T-0x)), (R(T)= ECCC) --- Expected cost vate

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Exam Exercise

The cdf of first passage time Tc Pr{Tc < t} = Pr { Ot > C} = Pr \ 6 > \frac{2}{7}  $= \left| -F_{6}\left( \frac{c}{+} \right) \right|$ Since, on Exponential ( $\lambda=1$ )  $Pr\{Tc \leq t\} = 1 - (1 - exp(-\lambda = 1)) | frc(t) = \frac{dF_{tc}(t)}{dt}$  $=\exp(-\frac{4000}{4})(+4000)$ =  $\frac{1}{4} \exp\left(-\frac{4000}{4}\right)$ The conditional first passage time TH, given that . 73500 4000 14000=3500 : 0= 4000 this condition specifies 0. H= 5000 = BTH = 4000 TH TH= Joon, 3 Tas this covelipion thus specifies given the condition Tyooo = 3500 varolutions THis a constant number : Pr {TH = Joon. 3500 | T4000 = 3700 } = |  $Pr \{TH < t\} = \{I \text{ if } Iooo \} \{Iooo < t\}$ 

Condition





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There are two possible renewal evenes.
O failure renewal. in a certain inspection interval
Pr(TH < ir. ∩ (i+) < Tc < ir)
= fiz fre(u) Pr(TH< ic Te=u) du.
= 52 exp(-4000, 4000, tu Pr (TH<2T / Tc=4) du
$=\int_{\overline{(z-1)}}^{z} \exp(-\frac{4000}{t}u) \cdot 4000 \cdot tu \int_{\overline{(z-1)}}^{2}  r(T_{4} < it  T_{c=u}) du$ $Pr(T_{4} < it  T_{c=u}) = \int_{\overline{(z-1)}}^{2}  r(T_{4} < it  T_{c=u}) du$ $O  \text{Other Wise}.$
E) inspection renewal at time point 22.  Pr (TH7ic. (16-1)Z <tc<iz)< td=""></tc<iz)<>
= Sit fac(u) Pr(TH>iz  Tc=u) du
$= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot \Pr(T_{H} > iz T_{c} = u) du$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u^{2} \cdot u$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u$ $= \int_{\widehat{z}-1/z}^{2z} \exp(-\frac{4000}{u}) \cdot 4000 \cdot u$
Pr (TH>27 (Tc=4) = \$ 1 if. 5000, 4000 >27
( V VIII S C .





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The expected cycle length. E(CL)
O failure renewal in a certain inspection interval.
TH > t. Pr(TH=t ( & Dz < Tc < iz). rendom to ((inte, iz)
$= \sum_{t \in (D/R, i\tau)} \Pr(T_{t} = t \mid T_{c} = u)$ $t \in (D/R, i\tau)$
= \( \tau \) \( \frac{\frac{1}{\tau}}{\tau} \) \( \frac{1}{\tau} \) \( \
where $Pr(TH=t te=u)=51$ if $\frac{1}{4000}=t$ 0 otherwise.
$\frac{ Or }{ Or } = \int_{e-Dr}^{ic} f_{c}(u) \frac{1000  \text{U}}{4000}  \text{lr}(T_{H} < iz T_{c} = u)  du$ auther expression
where Pr(Ty<22 T=1)= } It = 5000 < 20.
E) inspection renewal at time point. ic
ic. Pr(TH7ix(16H)c< tc <ic) =="" \(="" \subseteq="" \text{pr(tc="u)" ic="" pr(th="">ic Tc=u)} \) = ic \( \subseteq \text{Pr(Tc=u) Pr(TH&gt;ic Tc=u)} \)</ic)>
Zie je fre(u) Pr(TH7it Tc=u) du
where $P_{r}(T_{H} > i\tau   T_{c} = u) = \begin{cases} 1 & \text{if } \frac{1}{\sqrt{000}} = \frac{1}{2}i\tau. \end{cases}$ $100 \text{ years IMPACT} = \begin{cases} 2000 & \text{otherwise} \end{cases}$
1913 - 2013  The state of the s

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The expected cycle cost ECCC)
os fature revenual.
(2-1) (2+ Ccm) Pr(TH<22 (61)2 <tc<27)< td=""></tc<27)<>
E) inspection Veneral.
(ici+Cpm).Pr(TH>ic (in)c <tc<ic)< td=""></tc<ic)<>
Considering all inspection intervals.
ECC6)= = ( Six fr(u) Torry Pr(TH<22/Tc=4)dy.
+. it fr(W) Pr(TH7i2   Tc=u) du)
$E(CC) = \sum_{i \ge 1} \left( (2-1)(i+Ccm) \cdot \Pr(TH < iT \cap (i-1) < T < iT \right)$
+ (iCi+Cpm).Pr(TH>22(1.(i-1)Z <tc<ic))< th=""></tc<ic))<>
$(R(\tau, c) = \underbrace{E(cc)}_{E(cl)}$
ECCL)



