Exercise 2

time to defect   delay time

$X \sim$ Uniform $H \sim$ Exponential (λ)

The probability of failure renewal in a certain inspection interval $((i-1)\tau, i\tau)$:

$$\int_{(i-1)\tau}^{i\tau} f_X(u) F_H(i\tau-u)\,du$$

$$= \int_{(i-1)\tau}^{i\tau} \frac{1}{10}\left(1-\exp(-\lambda(i\tau-u))\right) du$$

(when $0 \le (i-1)\tau \le i\tau \le 10$) otherwise it equals 0.

The probability of inspection renewal with replacement at time point $(i\tau)$:

$$\int_{(i-1)\tau}^{i\tau} f_X(u)\left(1-F_H(i\tau-u)\right) du$$

$$= \int_{(i-1)\tau}^{i\tau} \frac{1}{10} \exp(-\lambda(i\tau-u))\, du$$

(when $0 \le (i-1)\tau \le i\tau \le 10$) otherwise it equals 0

$$E(CL) = \sum_{i=1}^{\infty} \left\{ \int_{(i-1)\tau}^{i\tau} t \int_{(i-1)\tau}^{i\tau} f_X(u) f_H(t-u)\, du\, dt + i\tau \int_{(i-1)\tau}^{i\tau} f_X(u)\left(1-F_H(i\tau-u)\right) du \right\}$$ (lecture note equation (7))

$$= \sum_{i=1}^{\lfloor \frac{10}{\tau} \rfloor} \left\{ \int_{(i-1)\tau}^{i\tau} t \int_{(i-1)\tau}^{i\tau} \frac{1}{10} \lambda \exp(-\lambda(t-u))\, du\, dt + i\tau \int_{(i-1)\tau}^{i\tau} \frac{1}{10} \exp(-\lambda(i\tau-u))\, du \right\}$$ (periods before 10)

$$+ \int_{\lfloor \frac{10}{\tau} \rfloor \tau}^{(\lfloor \frac{10}{\tau} \rfloor+1)\tau} t \int_{\lfloor \frac{10}{\tau} \rfloor \tau}^{10} \frac{1}{10} \lambda \exp(-\lambda(t-u))\, du\, dt + \tau\left(\left\lfloor \frac{10}{\tau} \right\rfloor+1\right) \int_{\lfloor \frac{10}{\tau} \rfloor \tau}^{10} \frac{1}{10} \exp\left(-\lambda\left(\left(\left\lfloor \tfrac{10}{\tau} \right\rfloor+1\right)\tau-u\right)\right) du$$

(the last period till 10)

For the renewal cycle cost, when failure renewal happens, the renewal cycle cost is the sum of the inspection cost $(i-1)C_i$ and the corrective maintenance cost C_{cm}.

Then the expected cycle cost due to failure renewals is

$$\sum_{i=1}^{\infty}\left\{((i-1)C_i+C_{cm})\int_{(i-1)\tau}^{i\tau} f_X(u)F_H(i\tau-u)\,du\right\}$$ (lecture note equation (8))

$$=\sum_{i=1}^{\lfloor\frac{10}{\tau}\rfloor}\left\{((i-1)C_i+C_{cm})\int_{(i-1)\tau}^{i\tau}\frac{1}{10}(1-\exp(-\lambda(i\tau-u)))\,du\right\}$$ (periods before 10)

$$+\left(\lfloor\tfrac{10}{\tau}\rfloor C_i+C_{cm}\right)\int_{\lfloor\frac{10}{\tau}\rfloor\tau}^{10}\frac{1}{10}\left(1-\exp\left(-\lambda\left(\left(\lfloor\tfrac{10}{\tau}\rfloor+1\right)\tau-u\right)\right)\right)du$$ (the last periods till 10)

When the inspection renewal happens at time point $i\tau$, the renewal cycle cost equals the sum of the inspection cost iC_i and the preventive maintenance cost C_{pm}.

Then the expected cycle cost due to inspection renewals is

$$\sum_{i=1}^{\infty}\left\{(iC_i+C_{pm})\int_{(i-1)\tau}^{i\tau} f_X(u)(1-F_H(i\tau-u))\,du\right\}$$ (lecture note equation (9))

$$=\sum_{i=1}^{\lfloor\frac{10}{\tau}\rfloor}\left\{(iC_i+C_{pm})\int_{(i-1)\tau}^{i\tau}\frac{1}{10}\exp(-\lambda(i\tau-u))\,du\right\}$$ (periods before 10)

$$+\left(\left(\lfloor\tfrac{10}{\tau}\rfloor+1\right)C_i+C_{pm}\right)\int_{\lfloor\frac{10}{\tau}\rfloor\tau}^{10}\frac{1}{10}\exp\left(-\lambda\left(\left(\lfloor\tfrac{10}{\tau}\rfloor+1\right)\tau-u\right)\right)du$$ (the last period till 10)

Then,

$E(CC)=$ the expected cycle cost due to failure renewals + the expected cycle cost due to inspection renewals.

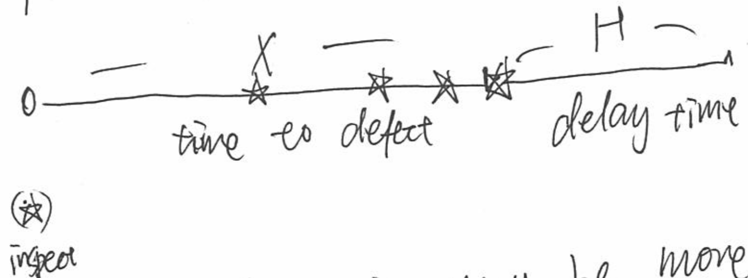
$$CR(\tau)=\frac{E(CC)}{E(CL)}$$

the numerical result will be given in another file.

For exams, only the derivation part is required.

exercise 3 (open question.)

Since the defect appears at a later stage for this single-component system, it may be more efficient to have an inspection schedule that has decreasing intervals.



In other words, it may be more efficient to have more frequent inspections at a later stage of the life cycle.

You ~~can~~ could specify a certain form of inspection schedules. Then derive the E(CL) and E(CC) of a renewal cycle.

The derivation part is required for exams.