



Multi-Model Federated Learning

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Outline

- Introduction

- Federated learning (FL)

- Multi-model federated learning (MMFL)

- Variance-reduced client sampling in a simple MMFL system
- Modeling computational heterogeneity in MMFL
- Experiments

Federated Learning

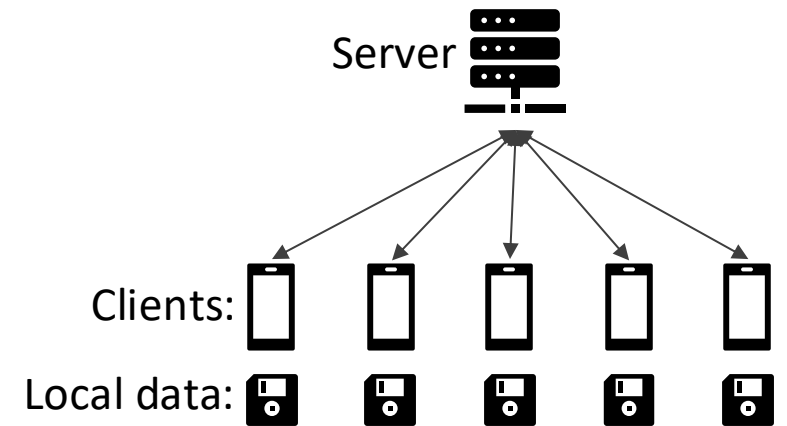
Distributed learning with unshared local data

Server:

- 1 Receive updates from clients
- 2 Aggregate local updates for a better global model
- 3 Broadcast new model parameters to clients

Local client (device):

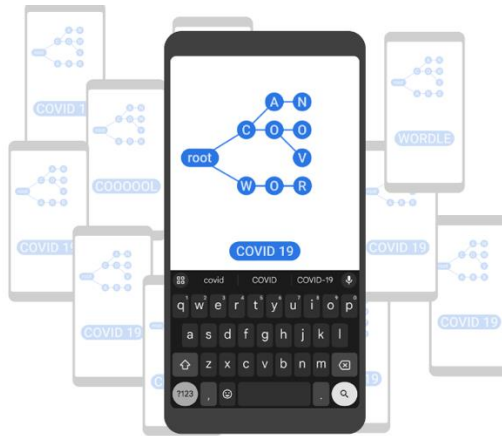
- 1 Get global model parameters
- 2 Train model parameters with local data
- 3 Send updated parameters to the server



Multi-Model Federated Learning

Examples: Multiple FL applications on one device.

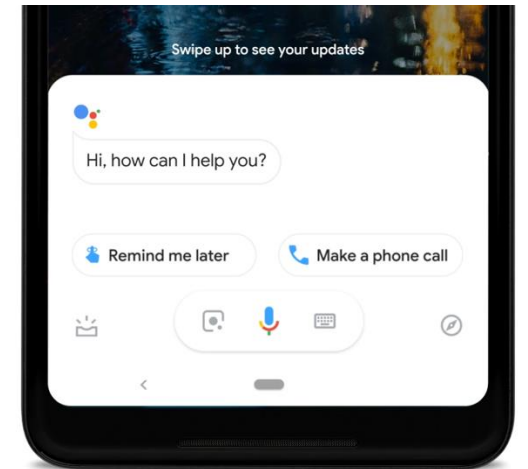
Keyboard prediction



Predicting text selection

Sounds good. Let's meet at 350 Third Street,
Cambridge later then

Speech model



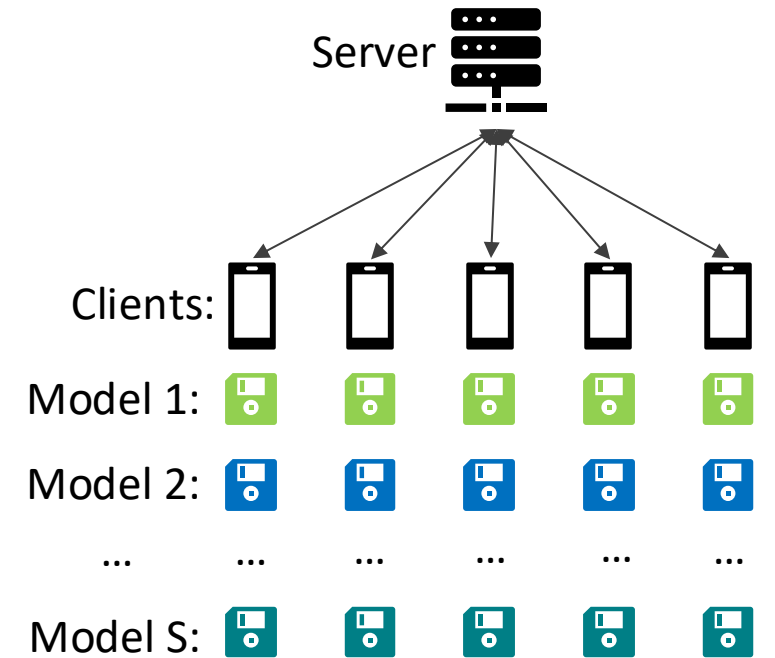
Source: federated.withgoogle.com

Multi-Model Federated Learning

Key assumptions from previous work [1]

In each round, the server only allows partial participation, and each active client can only train one model.

- 1) Partial Participation: reduce communication cost
- 2) Only train one model: computational constraints



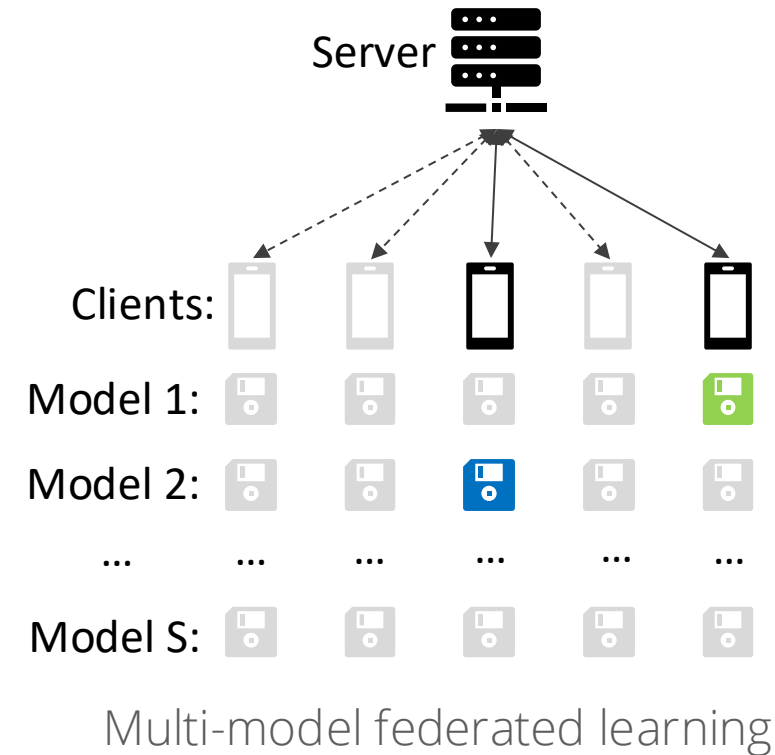
Multi-model federated learning

Multi-Model Federated Learning

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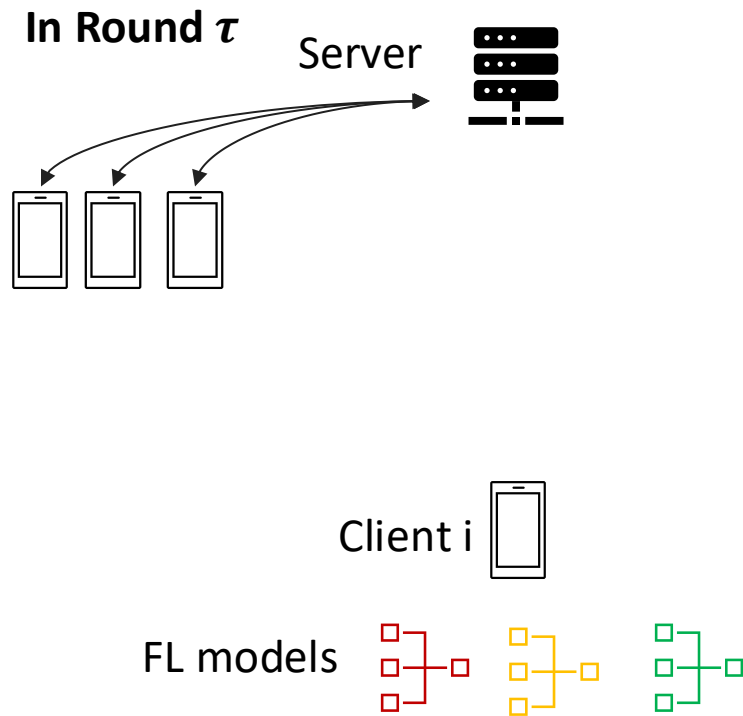
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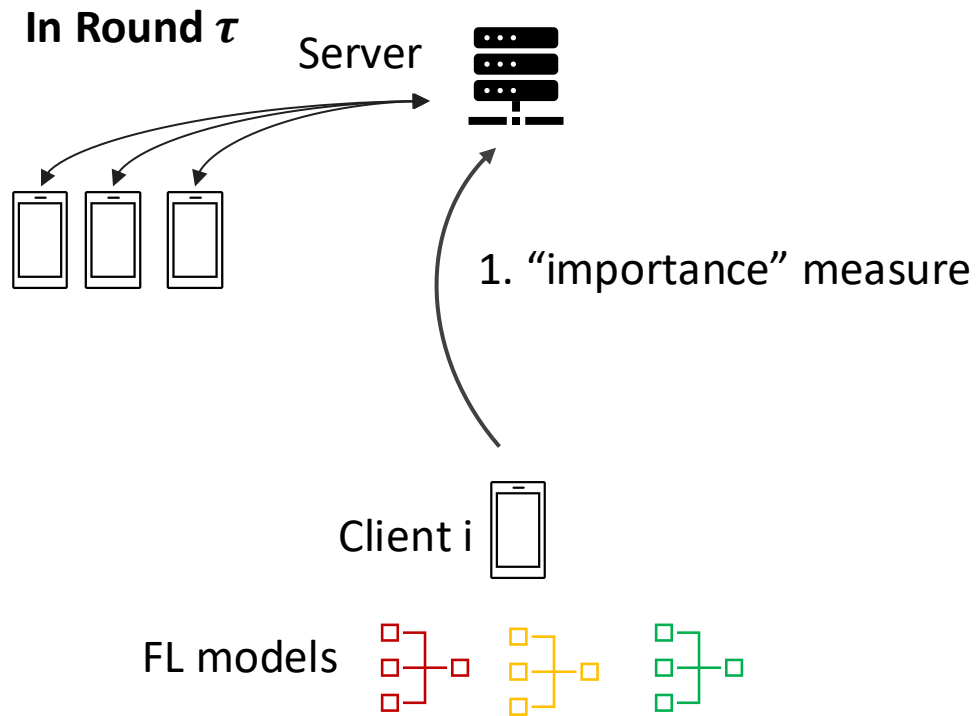
MMFL Optimal Variance-Reduced Sampling

Idea: the server prefers selecting more “important” clients.



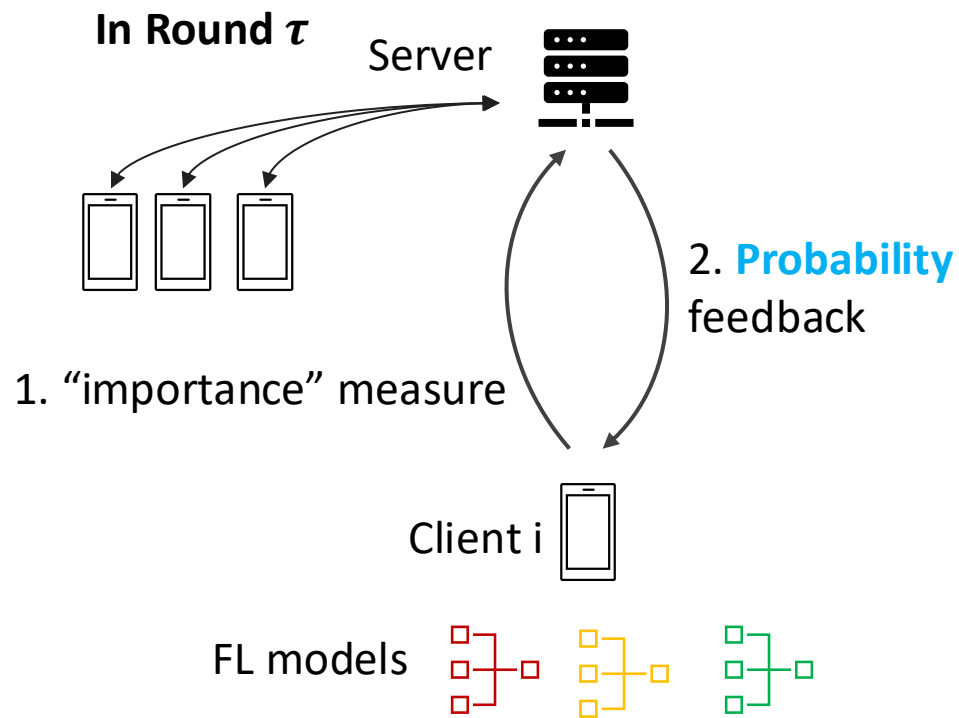
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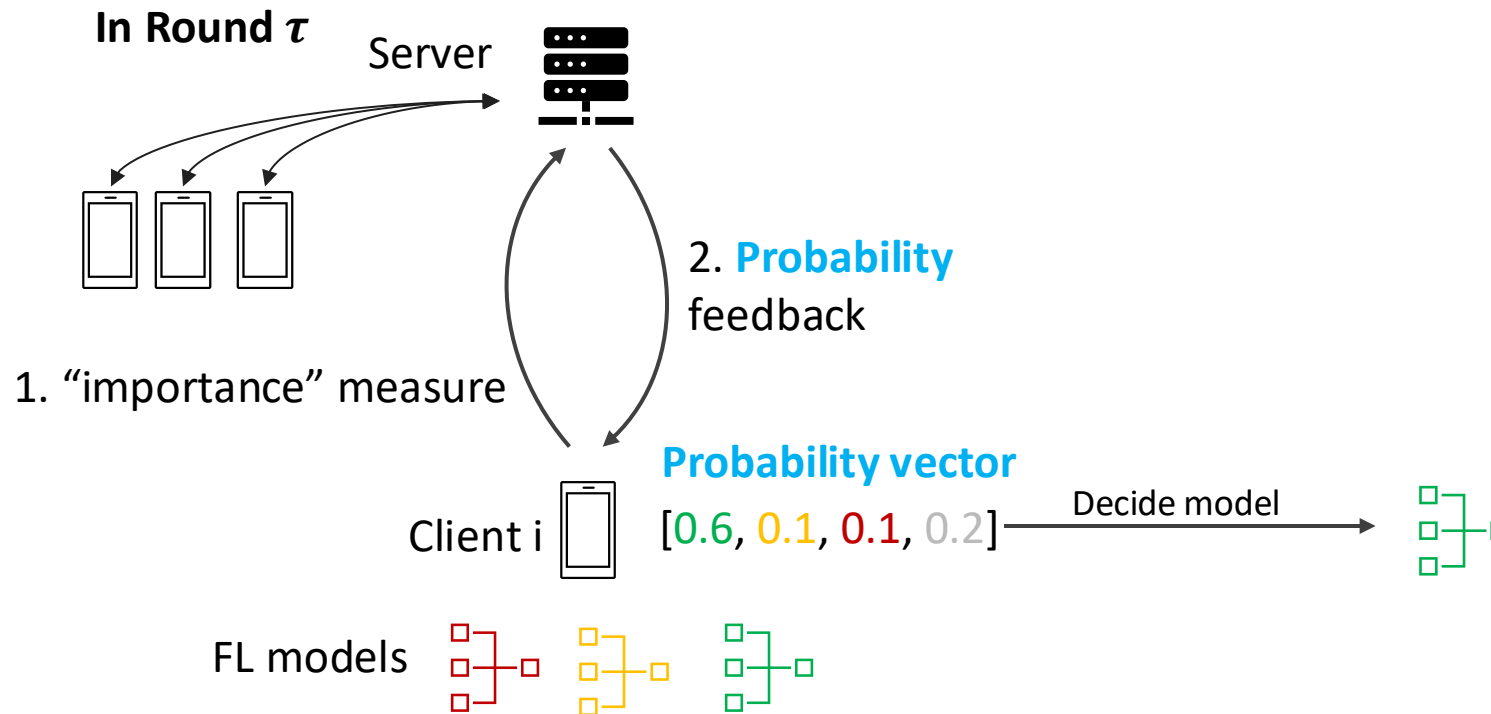
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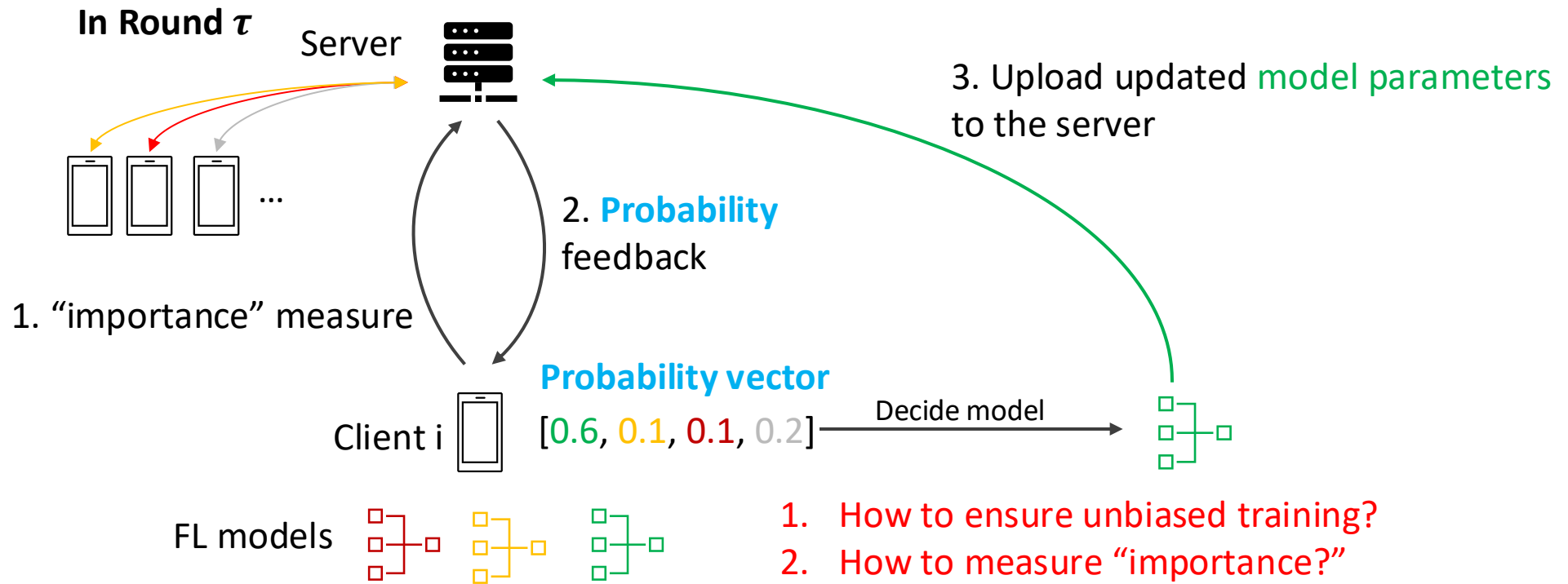
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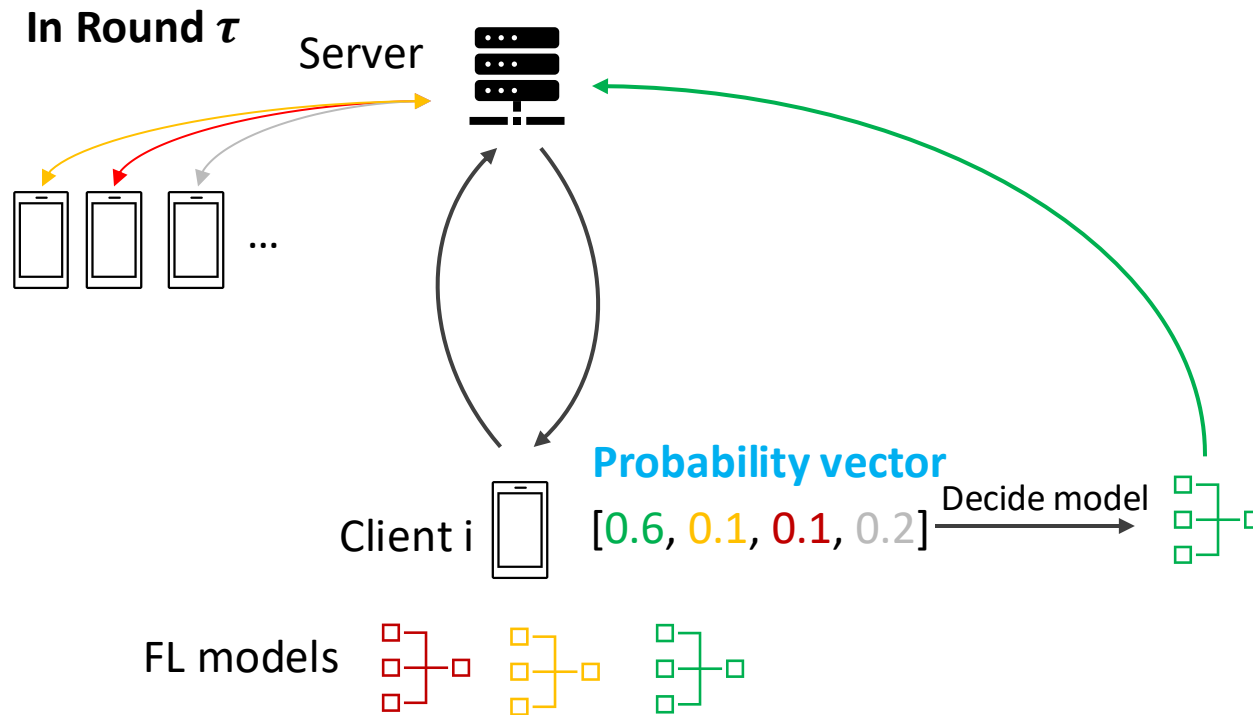


MMFL Optimal Variance-Reduced Sampling

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MMFL Optimal Variance-Reduced Sampling



In each global round (Aggregation):

$$w_s^{\tau+1} = w_s^\tau - \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^\tau} U_{i,s}^\tau$$

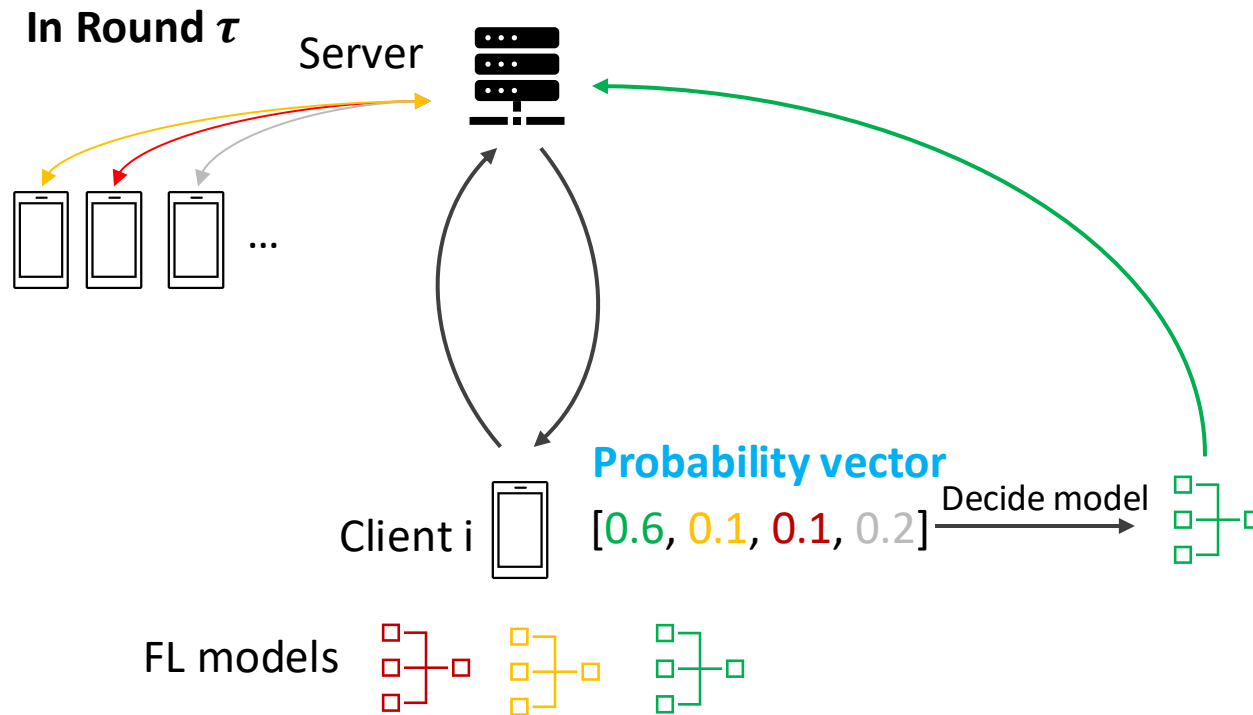
$d_{i,s} = \frac{n_{i,s}}{\sum_{j=1}^N n_{j,s}}$: dataset size ratio.

$U_{i,s}^\tau = \eta_\tau \sum_{t=1}^K \nabla f_{i,s}^{t,\tau}$: local update.

$p_{s|i}^\tau$: probability of assigning client i to model s .

$\mathcal{A}_{\tau,s}$: set of assigned clients for model s .

MMFL Optimal Variance-Reduced Sampling



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$$w_S^{\tau+1} = w_S^\tau - \sum_{i \in \mathcal{A}_{\tau,S}} \frac{d_{i,S}}{p_{S|i}^\tau} U_{i,S}^\tau$$

Unbiased Training:

$$\begin{aligned} & \mathbb{E} \left[\sum_{i \in \mathcal{A}_{\tau,S}} \frac{d_{i,S}}{p_{S|i}^\tau} U_{i,S}^\tau \right] \\ &= \mathbb{E} \left[\sum_{i=1}^N \frac{d_{i,S}}{p_{S|i}^\tau} U_{i,S}^\tau \mathbf{1}_{i \in \mathcal{A}_{\tau,S}} \right] \\ &= \sum_{i=1}^N d_{i,S} U_{i,S}^\tau \end{aligned}$$

MMFL optimal variance-reduced sampling

Aggregation:

$$w_s^{\tau+1} = w_s^{\tau} - \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^{\tau}} U_{i,s}^{\tau}$$

Random Variable X

$\mathbb{E}[X]$ is given.

MMFL optimal variance-reduced sampling

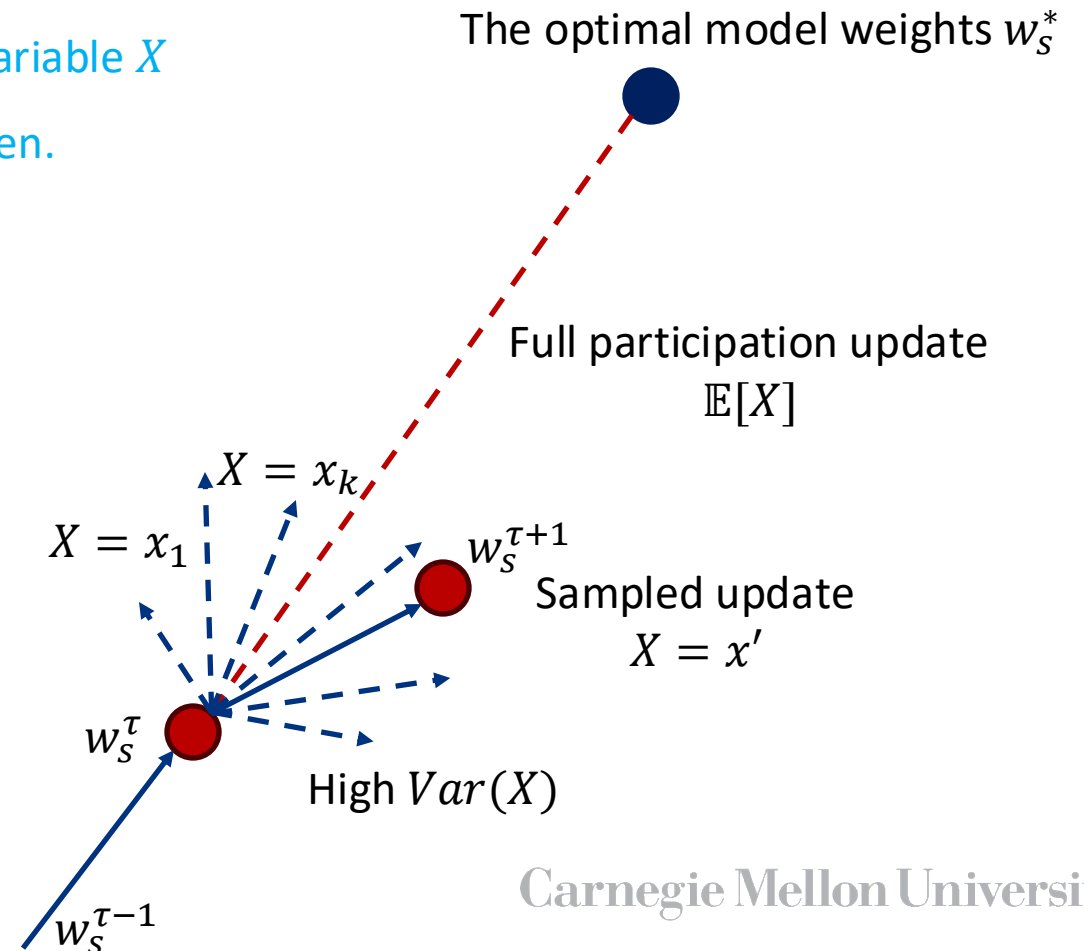
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Random Variable X
 $\mathbb{E}[X]$ is given.

High variance of X can make the training unstable...
Therefore, define our objective:

$$\min_{\{p_{s|i}^\tau\}} \sum_{s=1}^S \mathbb{E}_{\mathcal{A}_{\tau,s}} \left[\left\| \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^\tau} U_{i,s}^\tau - \sum_{i=1}^N d_{i,s} U_{i,s}^\tau \right\|^2 \right]$$



MMFL optimal variance-reduced sampling

Aggregation:

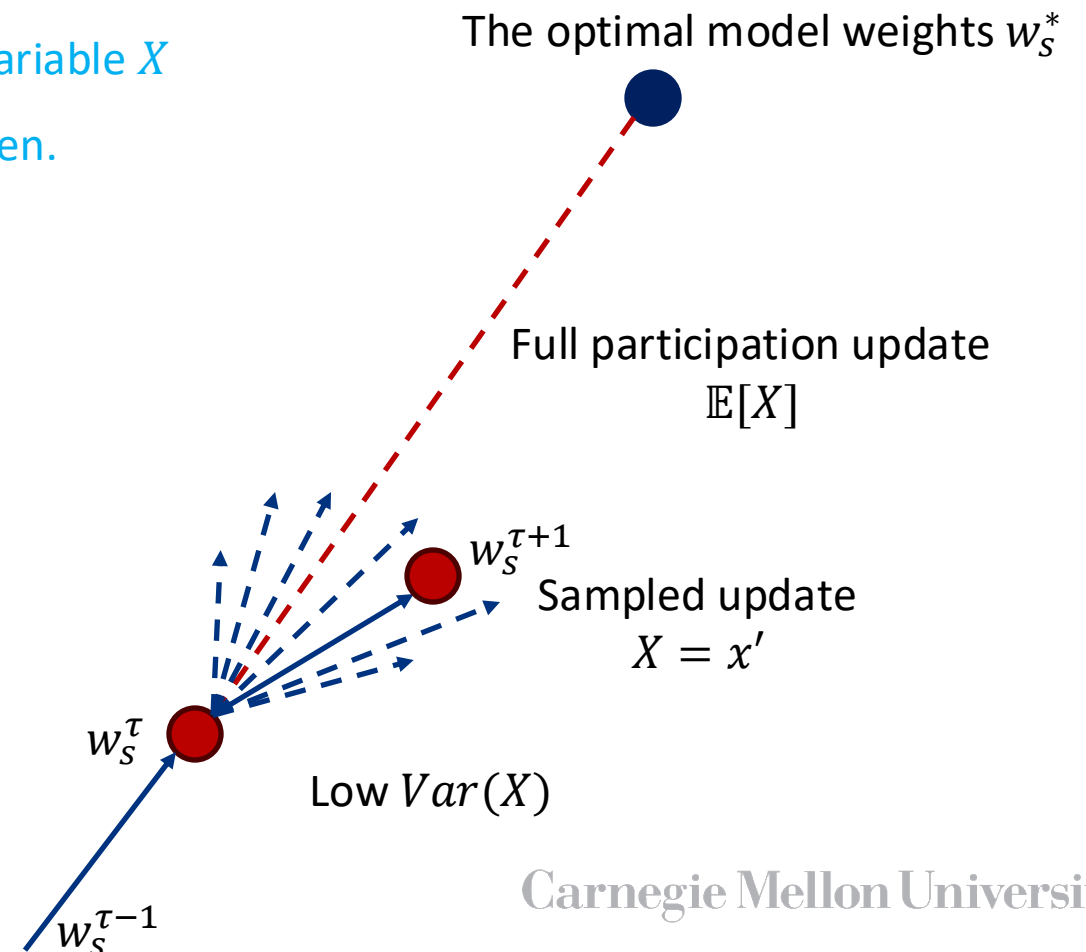
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Notice: variance is an ideal objective to stabilize the training, but there could be other factors...
(will further discuss later)



MMFL Optimal Variance-Reduced Sampling

Minimizing the variance of update

$$\begin{aligned} \min_{\{p_{s|i}^\tau\}} \quad & \sum_{s=1}^S \mathbb{E}_{\mathcal{A}_{\tau,s}} \left[\left\| \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^\tau} U_{i,s}^\tau - \sum_{i=1}^N d_{i,s} U_{i,s}^\tau \right\|^2 \right] \\ \text{s.t.} \quad & p_{s|i}^\tau \geq 0, \sum_{s=1}^S p_{s|i}^\tau \leq 1, \sum_{s=1}^S \sum_{i=1}^N p_{s|i}^\tau = m \quad \forall i, s \end{aligned}$$

τ : global round number
 i : client index
 s : model index
 m : expected number of active clients
 $d_{i,s}$: dataset size ratio
 t : local epoch number
 $\mathcal{A}_{\tau,s}$: set of active clients

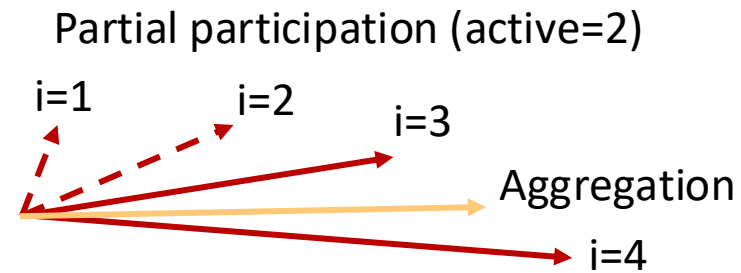
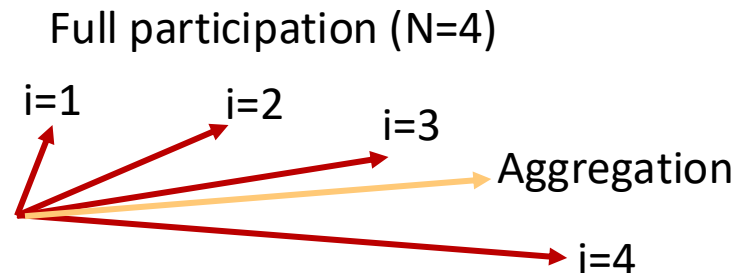
MMFL Optimal Variance-Reduced Sampling

Closed-form solution of the problem

$$p_{s|i}^{\tau} = \begin{cases} (m - N + k) \frac{\|\tilde{U}_{i,s}^{\tau}\|}{\sum_{j=1}^k M_j^{\tau}} & \text{if } i = 1, 2, \dots, k, \\ \frac{\|\tilde{U}_{i,s}^{\tau}\|}{M_i^{\tau}} & \text{if } i = k + 1, \dots, N. \end{cases} \quad (5)$$

where $\|\tilde{U}_{i,s}^{\tau}\| = \|d_{i,s} U_{i,s}^{\tau}\|$ and $M_i^{\tau} = \sum_{s=1}^S \|\tilde{U}_{i,s}^{\tau}\|$. We reorder clients such that $M_i^{\tau} \leq M_{i+1}^{\tau}$ for all i , and k is the largest integer for which $0 < (m - N + k) \leq \frac{\sum_{j=1}^k M_j^{\tau}}{M_k^{\tau}}$.

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Gradient-based Variance-Reduce Sampling (GVR)

Computing the gradient norm is too expensive on the client side!

Reduce computational cost

Computing the gradient norm is too expensive on the client side.

Client i loss value

$$\min_{\{p_{s|i}^\tau\}} \sum_{s=1}^S \mathbb{E}_{\mathcal{A}_{\tau,s}} \left[\left\| \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^\tau} U_{i,s}^\tau - \sum_{i=1}^N d_{i,s} U_{i,s}^\tau \right\|^2 \right]$$
$$\text{s.t. } p_{s|i}^\tau \geq 0, \sum_{s=1}^S p_{s|i}^\tau \leq 1, \sum_{s=1}^S \sum_{i=1}^N p_{s|i}^\tau = m \quad \forall i, s$$

τ : global round number
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Loss-based **V**ariance-**R**educed Sampling (**LVR**)

Reduce computational cost

Computing the gradient norm is too expensive on the client side.

Client i loss value

$$\min_{\{p_{s|i}^\tau\}} \sum_{s=1}^S \mathbb{E}_{\mathcal{A}_{\tau,s}} \left[\left\| \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^\tau} U_{i,s}^\tau - \sum_{i=1}^N d_{i,s} U_{i,s}^\tau \right\|^2 \right]$$
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Now we have two methods to optimize the sampling distribution.
Can we analyze their influence on convergence speed?

Convergence proof

Based on some common assumptions (L-smoothness, μ -strongly convex, etc.)

We modified and adapted the proof from [2].

Theorem 4 (Convergence). *Let w_s^* denote the optimal weights of model s . If the learning rate $\eta_\tau = \frac{16}{\mu} \frac{1}{(\tau+1)K+\gamma}$, then*

$$\mathbb{E}(\|w_s^\tau - w_s^*\|^2) \leq \frac{V_\tau}{(\tau K + \gamma_\tau)^2} \quad (413)$$

Here we define $\gamma_\tau = \max\{\frac{32L}{\mu}, 4K \sum_{i \in \mathcal{N}_s} \mathbb{1}_i^{s,\tau} P_{i,s}^\tau\}$

$V_\tau = \max\{\gamma_\tau^2 \mathbb{E}(\|w_s^0 - w_s^*\|^2), (\frac{16}{\mu})^2 \sum_{\tau'=0}^{\tau-1} z_{\tau'}\},$

$z_{\tau'} = \mathbb{E}[Z_g^{\tau'} + Z_l^{\tau'} + Z_p^{\tau'}],$

$\mathbb{E}[Z_g^\tau] = K \sum_{i \in \mathcal{N}_s} \frac{(d_{i,s} \sigma_{i,s})^2}{p_{s|i}^\tau} + 4LK \sum_{i \in \mathcal{N}_s} d_{i,s} \Gamma_{i,s} + \max(\frac{1}{d_{i,s}}) \mathbb{E}[\sum_{i \in \mathcal{N}_s} \frac{(d_{i,s})^2 \sum_{t=1}^K \|\nabla f_{i,s}(w_{i,s}^{t,\tau})\|^2}{p_{s|i}^\tau}],$

$\mathbb{E}[Z_l^\tau] = R \mathbb{E}[|\mathcal{N}_s| \sum_{i \in \mathcal{N}_s} (\mathbb{1}_i^{s,\tau} P_{i,s}^\tau f_{i,s}(w_s^\tau) - d_{i,s} f_{i,s}(w_s^\tau))^2],$ where $R = \frac{2K^3 \bar{\sigma}^2}{e_w^2 e_f^2 \theta},$

$\mathbb{E}[Z_p^\tau] = (\frac{2}{\theta} + K(2 + \frac{\mu}{2L}))K^2 \bar{\sigma}^2 + \frac{2K^3 \bar{\sigma}^2}{\theta} \mathbb{E}[(\sum_{i \in \mathcal{N}_s} \mathbb{1}_i^{s,\tau} P_{i,s}^\tau - 1)^2].$

Convergence proof

Based on some common assumptions (L-smoothness, μ -strongly convex, etc.)

We modified and adapted the proof from [2].

$$\mathbb{E}[Z_g^\tau] = K \sum_{i \in \mathcal{N}_s} \frac{(d_{i,s} \sigma_{i,s})^2}{p_{s|i}^\tau} + 4LK \sum_{i \in \mathcal{N}_s} d_{i,s} \Gamma_{i,s} + \max\left(\frac{1}{d_{i,s}}\right) \mathbb{E}\left[\sum_{i \in \mathcal{N}_s} \frac{(d_{i,s})^2 \sum_{t=1}^K \|\nabla f_{i,s}(w_{i,s}^{t,\tau})\|^2}{p_{s|i}^\tau}\right],$$

$$\mathbb{E}[Z_l^\tau] = R \mathbb{E}[|\mathcal{N}_s| \sum_{i \in \mathcal{N}_s} (\mathbb{1}_i^{s,\tau} P_{i,s}^\tau f_{i,s}(w_s^\tau) - d_{i,s} f_{i,s}(w_s^\tau))^2], \text{ where } R = \frac{2K^3 \bar{\sigma}^2}{e_w^2 e_f^2 \theta},$$

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$\mathbb{E}[Z_g^\tau] \rightarrow$ Sampled update variance (GVR)

In the proof: <https://tinyurl.com/mmflos>

From the upper bound to variance term:

$$\left\| \sum_{t=1}^K \nabla f_{i,s} \right\|^2 \leq K \sum_{t=1}^K \|\nabla f_{i,s}\|^2 \text{ (GM-HM inequality)}$$

$$= \sum_{s=1}^S \left[\mathbb{E} \left[\left\| \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^\tau} U_{i,s}^\tau \right\|^2 \right] - \left\| \sum_{i=1}^N d_{i,s} U_{i,s}^\tau \right\|^2 \right] \quad (9)$$

$$= \sum_{s=1}^S \left[\mathbb{E} \left[\sum_{i,j} \frac{d_{i,s} (U_{i,s}^\tau)^\top d_{j,s} U_{j,s}^\tau}{p_{s|i}^\tau p_{s|j}^\tau} \mathbb{1}_{i,j \in \mathcal{A}_{\tau,s}} \right] - \sum_{i,j} d_{i,s} d_{j,s} (U_{i,s}^\tau)^\top U_{j,s}^\tau \right] \quad (10)$$

$$= \sum_{s=1}^S \left[\sum_{i \neq j} d_{i,s} (U_{i,s}^\tau)^\top d_{j,s} U_{j,s}^\tau + \sum_{i=1}^N \frac{d_{i,s}^2 (U_{i,s}^\tau)^\top U_{i,s}^\tau}{p_{s|i}^\tau} - \sum_{i,j} d_{i,s} d_{j,s} (U_{i,s}^\tau)^\top U_{j,s}^\tau \right] \quad (11)$$

$$= \sum_{s=1}^S \left(\sum_{i=1}^N \left(\frac{\|d_{i,s} U_{i,s}^\tau\|^2}{p_{s|i}^\tau} - \|d_{i,s} U_{i,s}^\tau\|^2 \right) \right) \quad (12)$$

$$= \sum_{s=1}^S \sum_{i=1}^N \frac{\|d_{i,s} U_{i,s}^\tau\|^2}{p_{s|i}^\tau} - \sum_{s=1}^S \sum_{i=1}^N \|d_{i,s} U_{i,s}^\tau\|^2 \quad (13)$$

Convergence proof

Based on some common assumptions (L-smoothness, μ -strongly convex, etc.)

We modified and adapted the proof from [2].

$$\mathbb{E}[Z_g^\tau] = K \sum_{i \in \mathcal{N}_s} \frac{(d_{i,s} \sigma_{i,s})^2}{p_{s|i}^\tau} + 4LK \sum_{i \in \mathcal{N}_s} d_{i,s} \Gamma_{i,s} + \max\left(\frac{1}{d_{i,s}}\right) \mathbb{E}\left[\sum_{i \in \mathcal{N}_s} \frac{(d_{i,s})^2 \sum_{t=1}^K \|\nabla f_{i,s}(w_{i,s}^{t,\tau})\|^2}{p_{s|i}^\tau}\right],$$

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$$\mathbb{E}[Z_p^\tau] = \left(\frac{2}{\theta} + K\left(2 + \frac{\mu}{2L}\right)\right) K^2 \bar{\sigma}^2 + \frac{2K^3 \bar{\sigma}^2}{\theta} \mathbb{E}[(\sum_{i \in \mathcal{N}_s} \mathbb{1}_i^{s,\tau} P_{i,s}^\tau - 1)^2].$$

$\mathbb{E}[Z_l^\tau] \rightarrow$ Sampled loss variance (LVR), with similar GM-HM inequality.

Client i loss value

$$\min_{\{p_{s|i}^\tau\}} \sum_{s=1}^S \mathbb{E}_{\mathcal{A}_{\tau,s}} \left[\left\| \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^\tau} U_{i,s}^\tau - \sum_{i=1}^N d_{i,s} U_{i,s}^\tau \right\|^2 \right]$$

$$\text{s.t. } p_{s|i}^\tau \geq 0, \sum_{s=1}^S p_{s|i}^\tau \leq 1, \sum_{s=1}^S \sum_{i=1}^N p_{s|i}^\tau = m \quad \forall i, s$$

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Based on some common assumptions (L-smoothness, μ -strongly convex, etc.)

We modified and adapted the proof from [2].

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$$\mathbb{E}[Z_l^\tau] = R \mathbb{E}[|\mathcal{N}_s| \sum_{i \in \mathcal{N}_s} (\mathbb{1}_i^{s,\tau} P_{i,s}^\tau f_{i,s}(w_s^\tau) - d_{i,s} f_{i,s}(w_s^\tau))^2], \text{ where } R = \frac{2K^3 \bar{\sigma}^2}{e_w^2 e_f^2 \theta},$$

$$\mathbb{E}[Z_p^\tau] = \left(\frac{2}{\theta} + K\left(2 + \frac{\mu}{2L}\right)\right) K^2 \bar{\sigma}^2 + \frac{2K^3 \bar{\sigma}^2}{\theta} \mathbb{E}\left[\left(\sum_{i \in \mathcal{N}_s} \mathbb{1}_i^{s,\tau} P_{i,s}^\tau - 1\right)^2\right].$$

$$P_{i,s}^\tau = \frac{d_{i,s}}{p_{s|i}^\tau}$$

$\mathbb{E}[Z_p^\tau]$ -> Participation heterogeneity (or variance).

The red term is only related to dataset distribution and sampling distribution.

What is the meaning of this term?

Convergence proof

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$$P_{i,s}^\tau = \frac{d_{i,s}}{p_{s|i}^\tau}$$

$\mathbb{E}[Z_p^\tau]$ -> Participation heterogeneity (or variance)

Recall our global aggregation rule:

$$w_s^{\tau+1} = w_s^\tau - \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s}}{p_{s|i}^\tau} U_{i,s}^\tau$$

Can be rewritten as:

$$w_s^{\tau+1} = w_s^\tau - (H_s^\tau)^\top U_s^\tau$$

$$H_s^\tau = [\dots, \mathbb{1}_i^{s,\tau} P_{i,s}^\tau, \dots]^\top, U_s^\tau = [\dots, U_{i,s}^\tau, \dots]$$

Convergence proof

Based on some common assumptions (L-smoothness, μ -strongly convex, etc.)

We modified and adapted the proof from [2].

$$\mathbb{E}[Z_p^\tau] = \left(\frac{2}{\theta} + K\left(2 + \frac{\mu}{2L}\right)\right)K^2\bar{\sigma}^2 + \frac{2K^3\bar{\sigma}^2}{\theta} \mathbb{E}\left[\left(\sum_{i \in \mathcal{N}_s} \mathbb{1}_i^{s,\tau} P_{i,s}^\tau - 1\right)^2\right].$$

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Can be rewritten as:

$$w_s^{\tau+1} = w_s^\tau - (H_s^\tau)^\top U_s^\tau$$

$$H_s^\tau = [\dots, \mathbb{1}_i^{s,\tau} P_{i,s}^\tau, \dots]^\top, U_s^\tau = [\dots, U_{i,s}^\tau, \dots]$$

$$|H_s^\tau|_1 = \sum_{i=1}^N \mathbb{1}_i^{s,\tau} P_{i,s}^\tau = \sum_{i=1}^N \mathbb{1}_i^{s,\tau} \frac{d_{i,s}}{p_{s|i}^\tau}$$

Notice $\mathbb{E}[|H_s^\tau|_1] = 1$, therefore

$$\text{red term} = \mathbb{E}[(|H_s^\tau|_1 - 1)^2]$$

This is also a variance!

How does this variance influence the training?

The influence of participation heterogeneity

$$|H_s^\tau|_1 = \sum_{i=1}^N 1_i^{s,\tau} P_{i,s}^\tau = \sum_{i=1}^N 1_i^{s,\tau} \frac{d_{i,s}}{p_{s|i}^\tau}$$

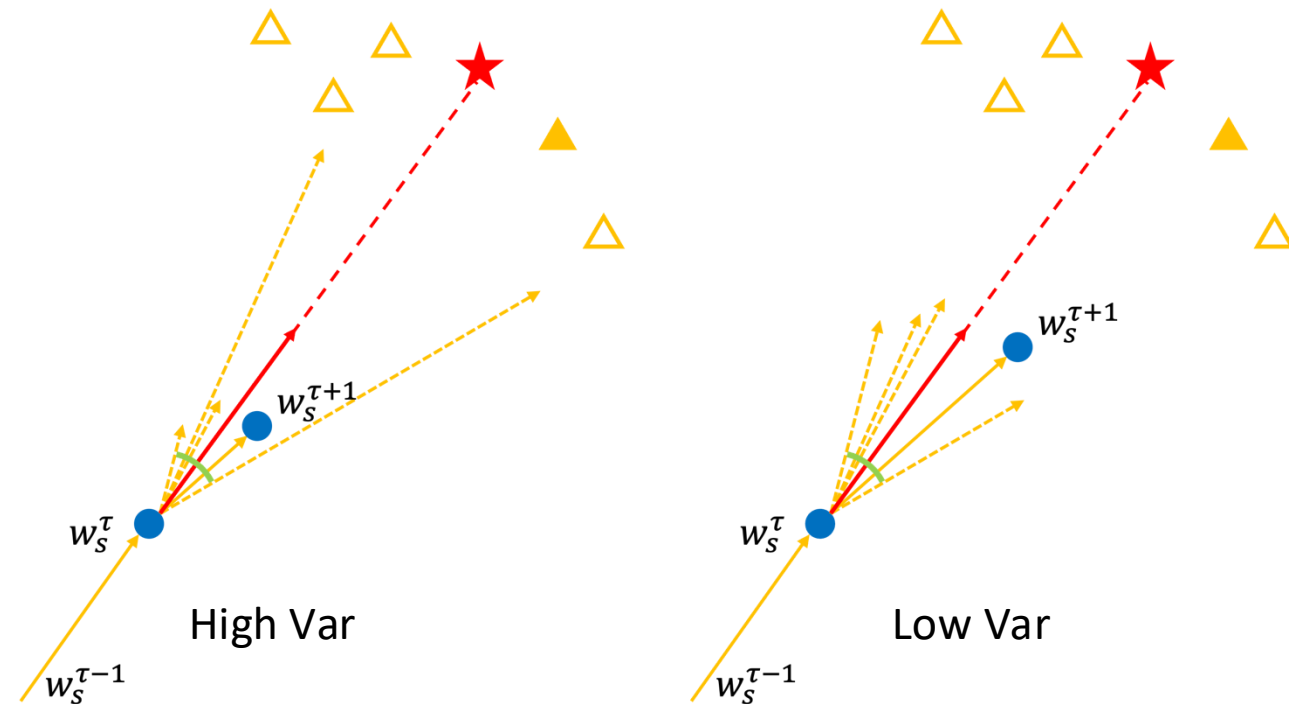
$$\text{Var}_H = \mathbb{E}[(|H_s^\tau|_1 - 1)^2]$$

High Var_H : $|H_s^\tau|_1$ may change a lot across rounds.

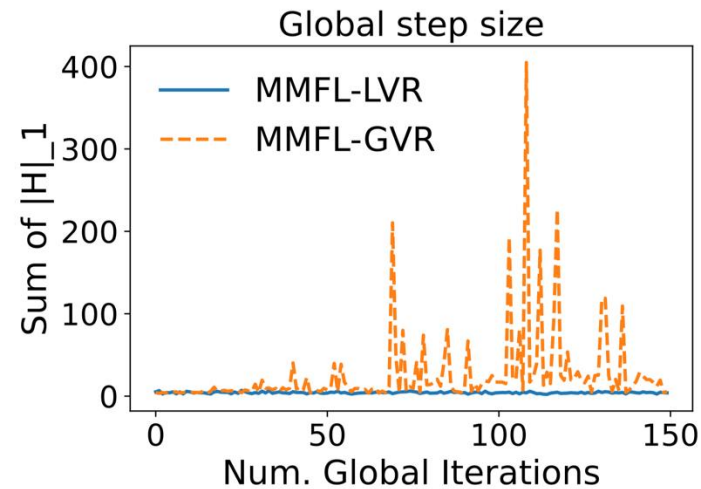
Lead to unstable “global step.”

$$w_s^{\tau+1} = w_s^\tau - (H_s^\tau)^\top U_s^\tau$$

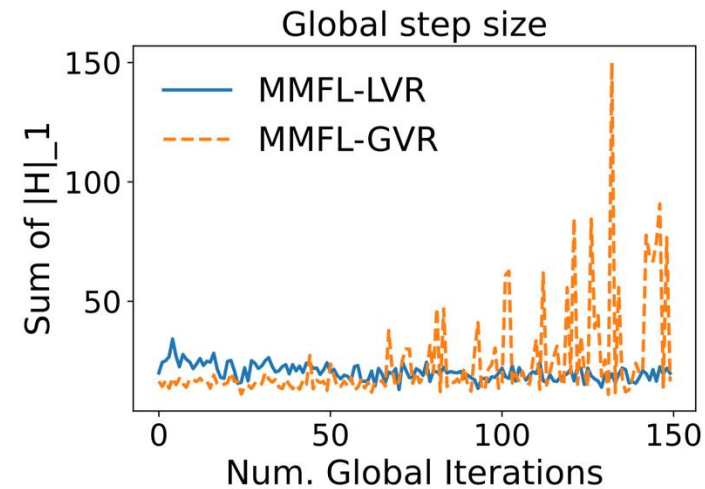
Impact the training especially at the end stage of the training.



Compare GVR and LVR



3 models



5 models

$$w_s^{\tau+1} = w_s^\tau - (H_s^\tau)^\top U_s^\tau$$

How to mitigate the impact of unstable “global step?”

Mitigate the impact of participation heterogeneity

Previous Aggregation Rule:

$$|H_S^\tau|_1 = \sum_{i=1}^N 1_i^{s,\tau} P_{i,s}^\tau = \sum_{i=1}^N 1_i^{s,\tau} \frac{d_{i,s}}{p_{s|i}^\tau}$$

$$w_S^{\tau+1} = w_S^\tau - (H_S^\tau)^\top U_S^\tau$$

New Aggregation Rule [3]:

$$w_S^{\tau+1} = w_S^\tau - \left(\sum_{i=1}^N d_{i,s} h_{i,s}^\tau + \sum_{i \in \mathcal{A}_{\tau,s}} \frac{d_{i,s} (U_{i,s}^\tau - h_{i,s}^\tau)}{p_{s|i}^\tau} \right)$$

$$h_{i,s}^\tau = \begin{cases} U_{i,s}^{\tau-1}, & \text{if } i \in \mathcal{A}_{\tau-1,s} \\ h_{i,s}^{\tau-1}, & \text{if } i \in \mathcal{A}_{\tau-1,s} \end{cases}$$

$U_{i,s}^\tau - h_{i,s}^\tau$ should be small.
Even though $|H_S^\tau|_1$ has a high variance, the impact is small.

Server stores stale updates from clients, and use stale updates to stabilize the training. **GVR***

Outline

- Introduction

 - Federated learning (FL) ✓

 - Multi-model federated learning (MMFL) ✓

- Variance-reduced client sampling in a simple MMFL system ✓

- Modeling computational heterogeneity in MMFL

- Experiments

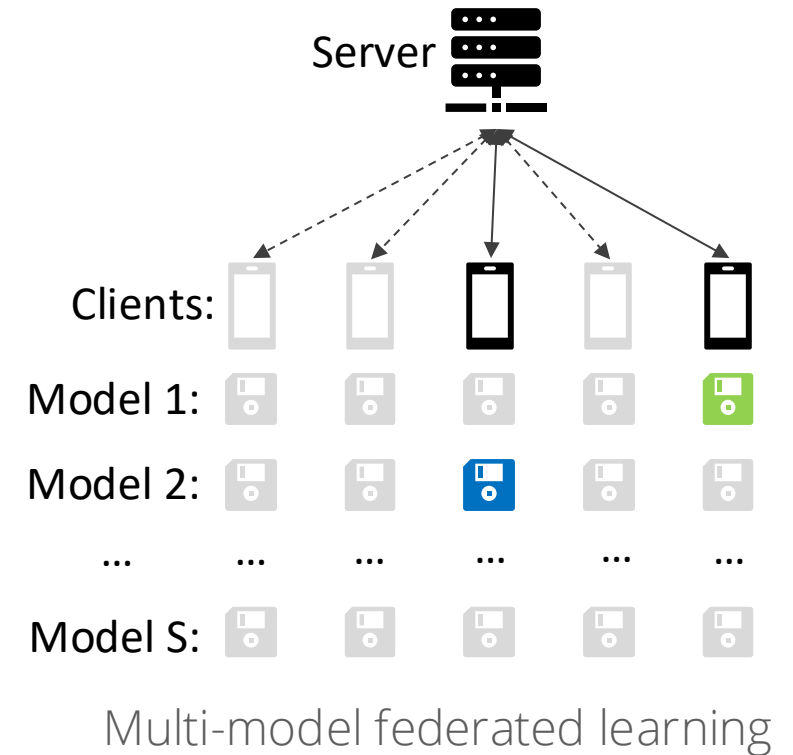
Recall

Key assumptions from previous work [1]

In each round, the server only allows partial participation, and each active client can only train one model.

- 1) Partial Participation: reduce communication cost
- 2) Only train one model: computational constraints

“Only train one model” is too ideal, without considering heterogeneity of computational abilities.



Multi-Model Federated Learning

Make more realistic assumptions

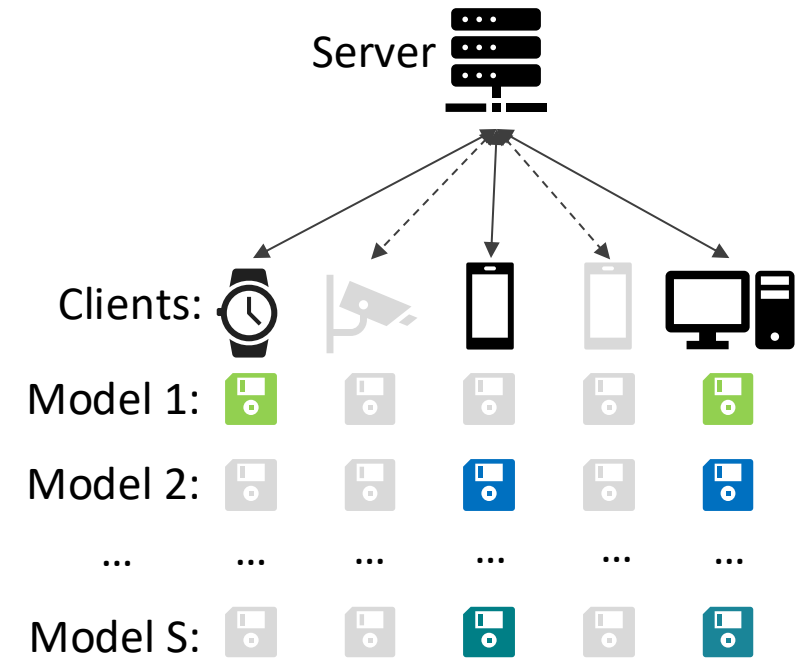
In each round, the server only allows partial participation, and each active client i can train B_i models in parallel.

1) Partial Participation: reduce communication cost

2) Client i can train B_i models ($B_i \leq S$):

Computational constraint & heterogeneity

“Powerful” clients train more models, leading to biased convergence. How to achieve unbiased training?



Multi-model federated learning

System model for heterogeneous MMFL

For ease of description, assume client i has B_i processors, each processor (i, b) can train one model independently.

- 1) Adjust the aggregation rule to ensure unbiased training

$$w_s^{\tau+1} = w_s^\tau - \sum_{(i,b) \in \mathcal{A}_{\tau,s}} P_{(i,b),s}^\tau G_{(i,b),s}^\tau$$

$$P_{(i,b),s}^\tau = \frac{d_{i,s}}{\textcolor{red}{B_i} p_{s|(i,b)}^\tau}, \quad G_{(i,b),s}^\tau = \eta_\tau \sum_{t=1}^K \nabla f_{i,s}^{t,\tau}$$

Notations:

w_s^τ : global model parameters

$\mathcal{A}_{\tau,s}$: set of active “processors”

$d_{i,s}$: dataset size ratio

$p_{s|(i,b)}^\tau$: the probability of having processor (i, b) to train model s

τ : global round index

t : local epoch index

System model for heterogeneous MMFL

For ease of description, assume client i has B_i processors, each processor (i, b) can train one model independently.

- 1) Adjust the aggregation rule to ensure unbiased training

$$w_s^{\tau+1} = w_s^\tau - \sum_{(i,b) \in \mathcal{A}_{\tau,s}} P_{(i,b),s}^\tau G_{(i,b),s}^\tau$$

$$\mathbb{E} \left[\sum_{i=1}^N \sum_{b=1}^{B_i} 1_{(i,b),s}^\tau \frac{d_{i,s}}{B_i p_{s|(i,b)}^\tau} G_{(i,b),s}^\tau \right] = \sum_{i=1}^N d_{i,s} \mathbb{E}[G_{(i,b),s}^\tau]$$

Sampling at the "processor-level"

Notations:

w_s^τ : global model parameters

$\mathcal{A}_{\tau,s}$: set of active "processors"

$d_{i,s}$: dataset size ratio

$p_{s|(i,b)}^\tau$: the probability of having processor (i, b) to train model s

τ : global round index

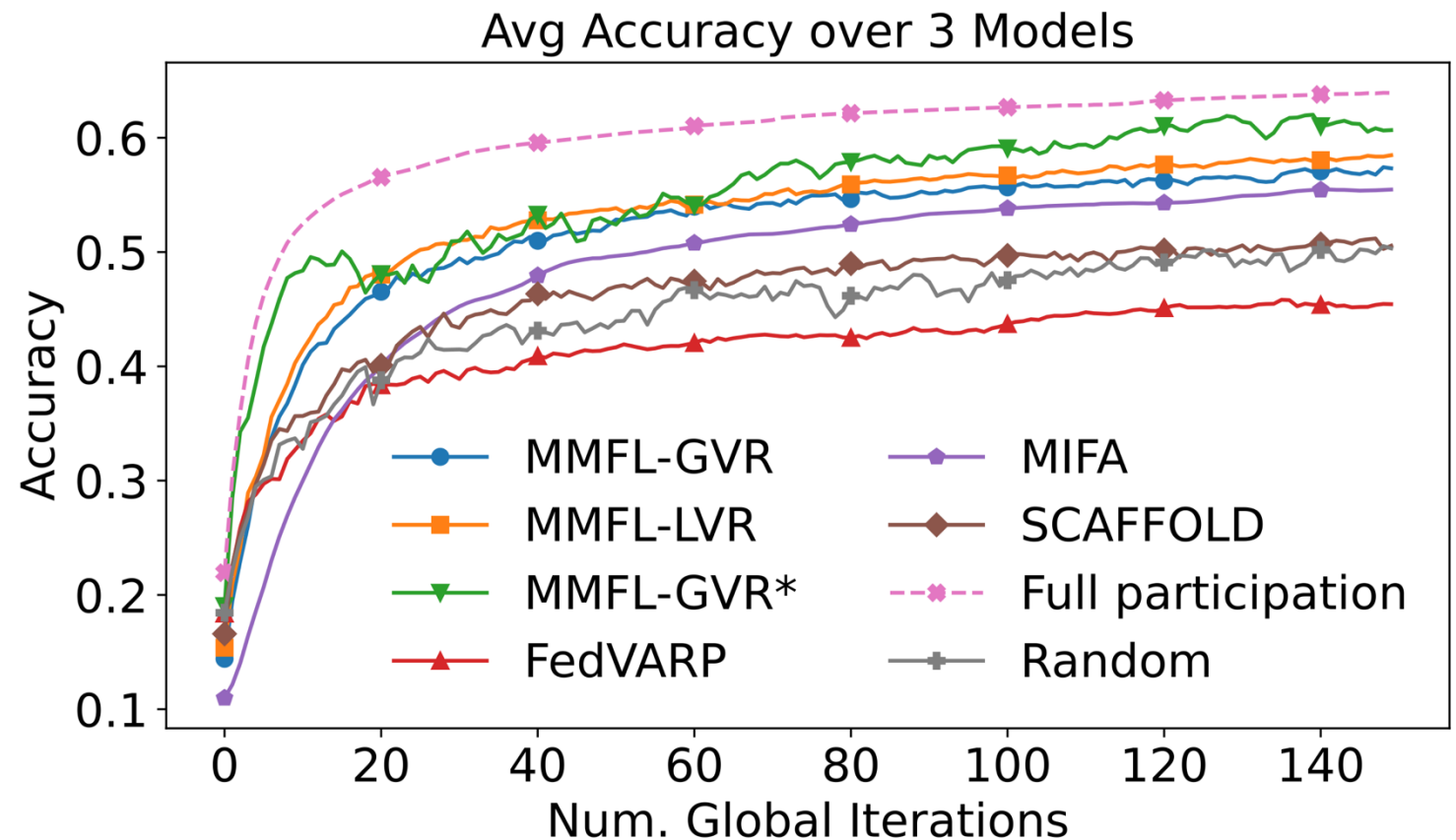
t : local epoch index

Experiments

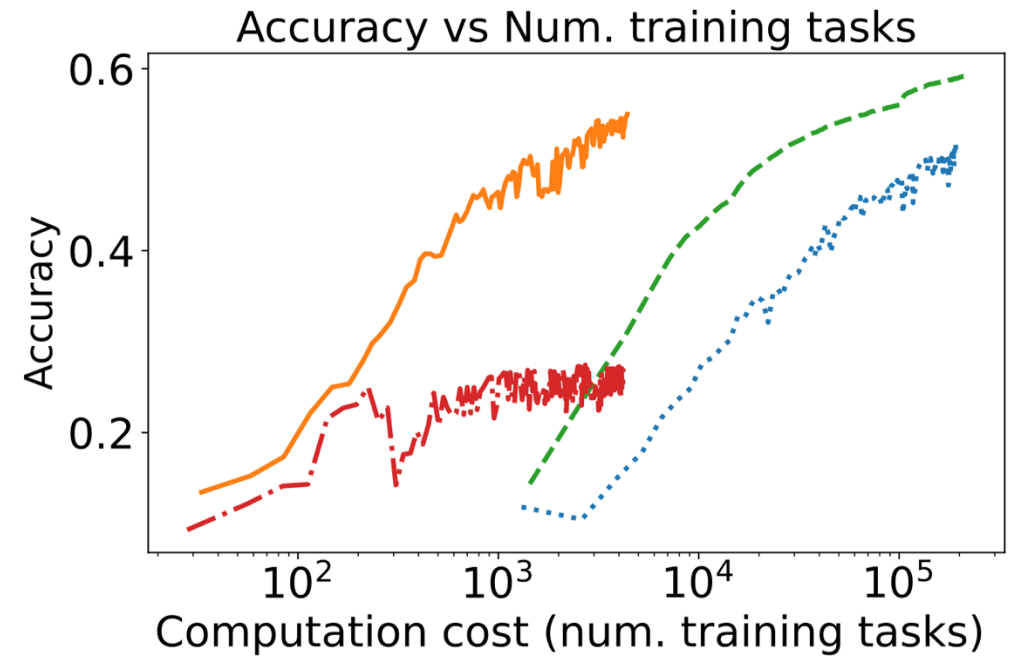
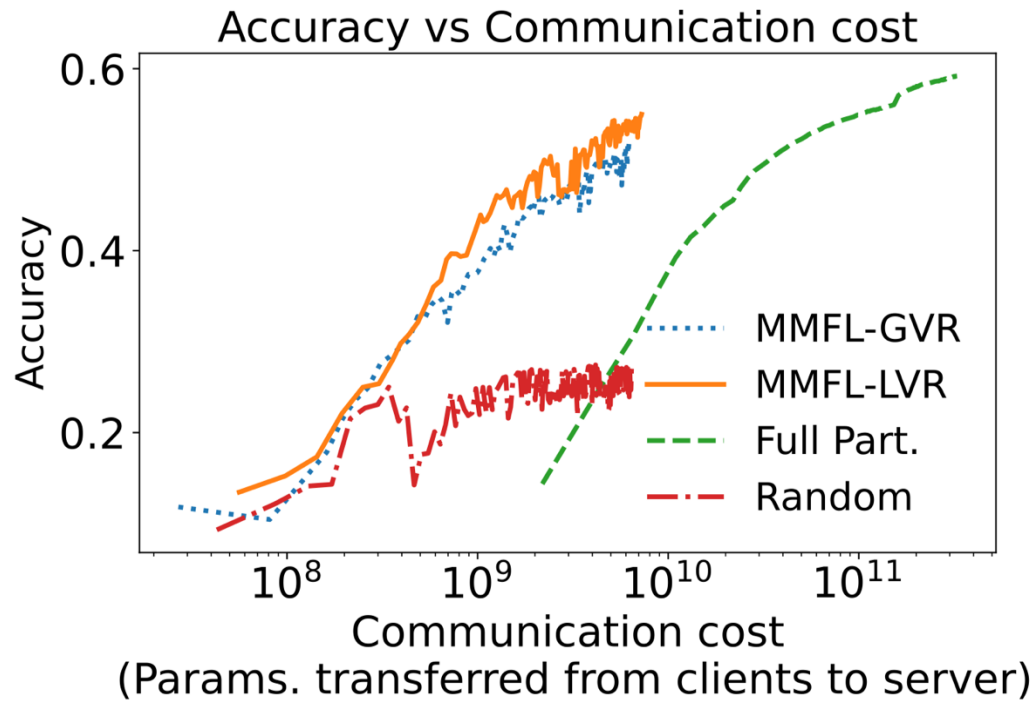
3 Models: all Fashion-MNIST.
N=120 clients
m=12 (active rate=0.1)
Each client: 30% labels.

For each model: 10% high-data
clients, 90% low-data clients.
10% clients hold 52.6% data of
each task.

25% clients: $B_i = 3$
50% clients: $B_i = 2$
25% clients: $B_i = 1$



Experiments



Experiments

3 Models: all Fashion-MNIST.

5 Models: two Fashion-MNIST, one CIFAR-10, one EMNIST, one Shakespeare.

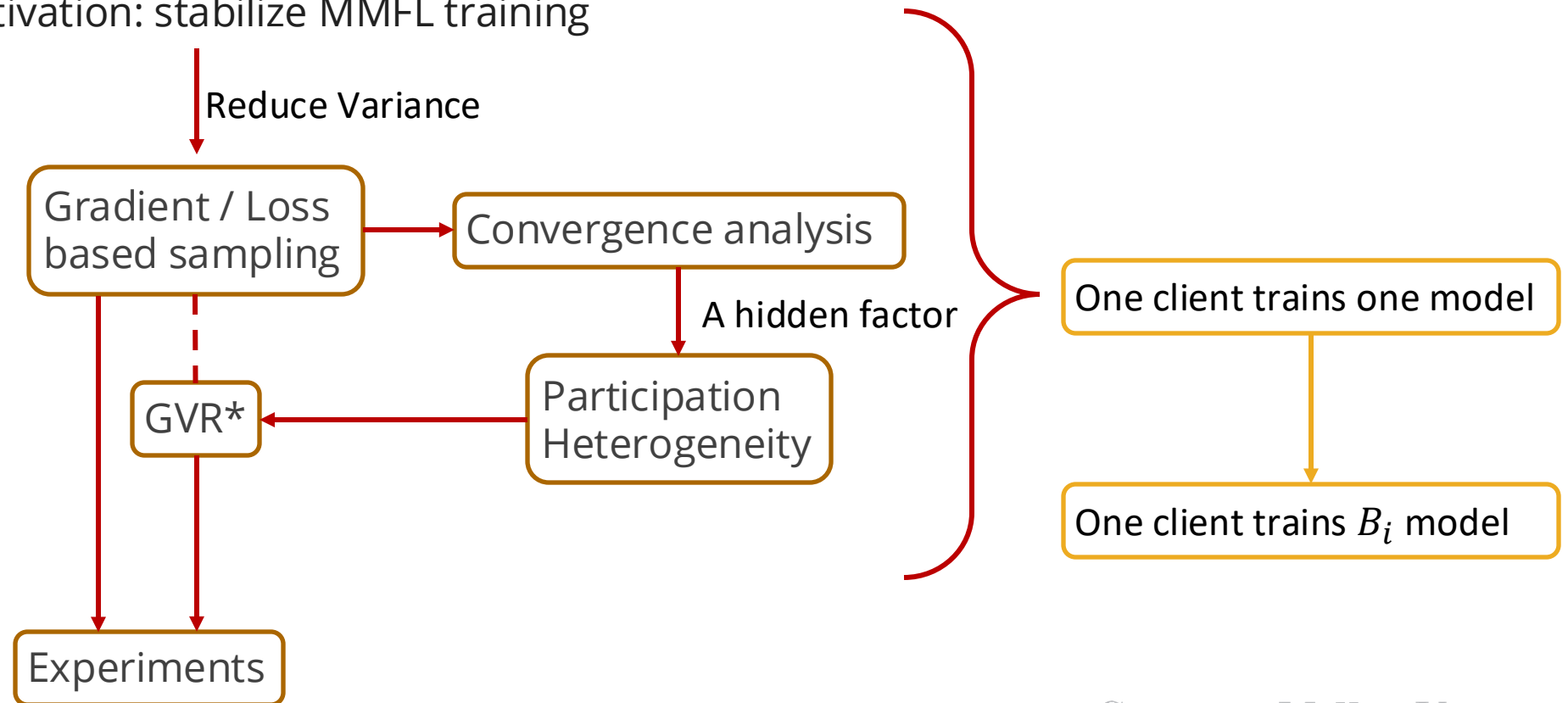
10% clients only have data for S-1 models.

TABLE I
FINAL AVERAGE MODEL ACCURACY RELATIVE TO THAT FROM FULL PARTICIPATION (THEORETICALLY THE BEST UNDER THE SAME LOCAL TRAINING SETTINGS).

Methods	3 tasks	5 tasks	Comm. Cost	Comp. Cost	Mem. Cost
FedVARP [30]	$0.712 \pm .14$	$0.690 \pm .19$	Low	Low	High
MIFA [31]	$0.868 \pm .18$	$0.835 \pm .18$	Low	Low	High
SCAFFOLD [32]	$0.794 \pm .14$	$0.650 \pm .24$	Low	Low	Low
Random	$0.778 \pm .19$	$0.749 \pm .23$	Low	Low	Low
Full Participation	$1.000 \pm .13$	$1.000 \pm .14$	High	High	Low
MMFL-GVR	$0.893 \pm .14$	$0.842 \pm .20$	Low	High	Low
MMFL-LVR	$0.912 \pm .15$	$0.849 \pm .16$	<u>Low</u>	<u>Low</u>	<u>Low</u>
MMFL-GVR*	$0.960 \pm .15$	$0.869 \pm .18$	Low	High	High

Summary

Motivation: stabilize MMFL training



Multi-Model Federated Learning

Make more realistic assumptions

In each round, the server only allows partial participation, and each active client i can train B_i models in parallel.

Other ways to model computational heterogeneity:

- 1) Asynchronous training [4]
- 2) Flexible local epochs number [5]
- 3) Flexible model architectures [6]

