MAT4220 FA22 HW10

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Problem 1 (P352 Q1)**.** Let

$$\tilde{u}(\xi, t) = \int \mathrm{d}x\ e^{-i\xi x} u(x, t)$$

Hence

$$\tilde{u}_t(\xi, t) = -\kappa\xi^2 \tilde{u}(\xi, t) + i\mu\xi\tilde{u}(\xi, t) \Rightarrow \tilde{u}(\xi, t) = e^{-\kappa\xi^2 t + i\mu\xi t} C(\xi)$$

which means

$$\begin{aligned} u(x, t) &= \frac{1}{2\pi} \int \mathrm{d}\xi\ e^{-\kappa\xi^2 t + i\mu\xi t} e^{i\xi x} C(\xi) \\ u(x, 0) &= \frac{1}{2\pi} \int \mathrm{d}\xi\ e^{i\xi x} C(\xi) = \phi(x) \\ \Rightarrow C(\xi) &= \int \mathrm{d}x\ \phi(x) e^{-i\xi x} \end{aligned}$$

then we solved the equation.

Problem 2 (P352 Q2)**.** Let

$$\tilde{u}(\mu, y) = \int \mathrm{d}x\ e^{-i\mu x} u(x, y)$$

Hence

$$\frac{\partial^2}{\partial y^2} \tilde{u}(\mu, y) = \mu^2 \tilde{u}(\mu, y) \Rightarrow \tilde{u}(\mu, y) = A(\mu) \sinh \mu y + B(\mu) \cosh \mu y$$

which means

$$\begin{aligned} u(x, y) &= \frac{1}{2\pi} \int \mathrm{d}\mu\ e^{i\mu x} [A(\mu) \sinh \mu y + B(\mu) \cosh \mu y] \\ u_y(x, y) &= \frac{1}{2\pi} \int \mathrm{d}\mu\ e^{i\mu x} [\mu A(\mu) \cosh \mu y + \mu B(\mu) \sinh \mu y] \\ u_y(x, 0) &= \frac{1}{2\pi} \int \mathrm{d}\mu\ e^{i\mu x} \mu A(\mu) = h(x) \\ \Rightarrow A(\mu) &= \frac{1}{\mu} \int \mathrm{d}x\ e^{-i\mu x} h(x) \end{aligned}$$

then we solved the equation.