

MAT4220 FA22 HW10

Haoran Sun (haoransun@link.cuhk.edu.cn)

Problem 1 (P352 Q1). Let

$$\tilde{u}(\xi, t) = \int dx e^{-i\xi x} u(x, t)$$

Hence

$$\tilde{u}_t(\xi, t) = -\kappa\xi^2\tilde{u}(\xi, t) + i\mu\xi\tilde{u}(\xi, t) \Rightarrow \tilde{u}(\xi, t) = e^{-\kappa\xi^2t+i\mu\xi t}C(\xi)$$

which means

$$\begin{aligned} u(x, t) &= \frac{1}{2\pi} \int d\xi e^{-\kappa\xi^2t+i\mu\xi t} e^{i\xi x} C(\xi) \\ u(x, 0) &= \frac{1}{2\pi} \int d\xi e^{i\xi x} C(\xi) = \phi(x) \\ \Rightarrow C(\xi) &= \int dx \phi(x) e^{-i\xi x} \end{aligned}$$

then we solve the equation.

Problem 2 (P352 Q2). Let

$$\tilde{u}(\mu, y) = \int dx e^{-i\mu x} u(x, y)$$

Hence

$$\frac{\partial^2}{\partial y^2} \tilde{u}(\mu, y) = \mu^2 \tilde{u}(\mu, y) \Rightarrow (\tilde{u})(\mu, y) = A(\mu) \sinh \mu y + B(\mu) \cosh \mu y$$

which mean

$$\begin{aligned} u(x, y) &= \frac{1}{2\pi} \int d\mu e^{i\mu x} [A(\mu) \sinh \mu y + B(\mu) \cosh \mu y] \\ u_y(x, y) &= \frac{1}{2\pi} \int d\mu e^{i\mu x} [\mu A(\mu) \cosh \mu y + \mu B(\mu) \sinh \mu y] \\ u_y(x, 0) &= \frac{1}{2\pi} \int d\mu e^{i\mu x} \mu A(\mu) = h(x) \\ \Rightarrow A(\mu) &= \frac{1}{\mu} \int dx e^{-i\mu x} h(x) \end{aligned}$$

then we solve the equation.