MAT4220 FA22 HW#01

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Problem 1 (P5 Q3)**.**

(b) Second-order linear homogeneous, since the equation $\mathcal{L}u = g$ has

$$\mathcal{L} = \partial_t - \partial_{xx} + x$$

which is a linear operator and $g = 0$.

(c) Third-order nonlinear, since there is a uu_x term.

(d) Second-order linear nonhomogeneous, since the operator

$$\mathcal{L} = \partial_{tt} - \partial_{xx}$$

while $g = -x^2 \neq 0$.

(h) Forth-order nonlinear, since there is a $\sqrt{u+1}$ term.

Problem 2 (P10 Q3)**.** Note that the set of characteristic curve $h(x, y) = c$ has the properties that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+x^2}$$

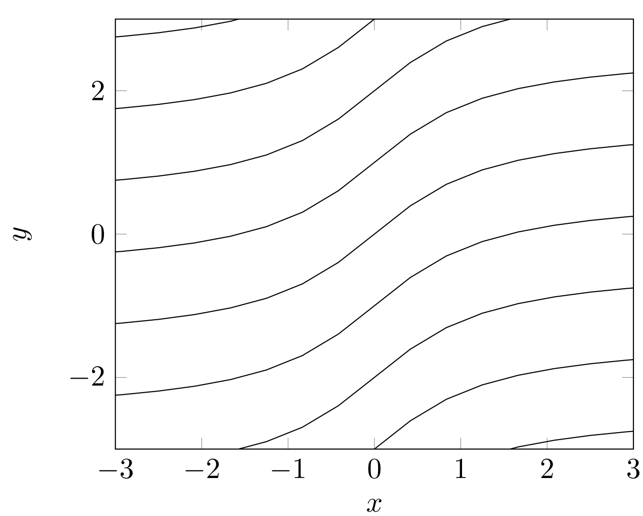
Then we have

$$y = \arctan x + C$$

Then

$$u(x, y) = g(C) = g(y - \arctan x)$$

Sketch of the characteristic curves:



Problem 3 (P10 Q7)**.**

(a) Characteristic curve satisfies

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{y} \Rightarrow x^2 - y^2 = C$$

Then

$$u(x, y) = g(x^2 - y^2)$$

Plug in $u(0, y) = g(-y^2) = e^{-y^2}$, we have $u(x, y) = e^{x^2-y^2}$.

(b) Whole xy plane.

Problem 4 (P10 Q10)**.** Change the variable by

$$x' = x + y$$
$$y' = -x + y$$

Then $u_x = u_{x'} - u_{y'}$, $u_y = u_{x'} + u_{y'}$, and therefore

$$u_x + u_y + u = 2u_{x'} + u = e^{(3x'+y')/2}$$

Solve the homogeneous case $u_{x'} + u = 0$, we get kernel $\phi(x', y')$

$$\phi(x', y') = C(y')e^{-x'/2}$$

Suppose a specific solution of the equation is in the form

$$u(x', y') = ae^{(3x'+y')/2} + bx'e^{(3x'+y')/2}$$

Easy to obtain that $a = 1/4$. Then the general solution of the equation would be

$$u(x, y) = \frac{1}{4}e^{(3x'+y')/2} + C(y')e^{-x'/2} = \frac{1}{4}e^{x+2y} + C(-x + y)e^{-(x+y)/2}$$

Apply the boundary condition $u(x, 0) = 0$, we get

$$u(x, 0) = \frac{1}{4}e^{x} + C(-x)e^{-x/2} = 0 \Rightarrow C(x) = -\frac{1}{4}e^{-3x/2}$$

Then our solution would be

$$u(x, y) = \frac{1}{4}e^{x+2y} - \frac{1}{4}e^{x-2y}$$

Problem 5 (P28 Q5)**.**

(a) Let $y = 0$, then we have

$$u_x(x, 0) = 0$$

which means along $(x, 0)$, u is a constant. This contradicts with the boundary condition $u(x, 0) = \phi(x) = x$ since $u(x, 0)$ is not a constant. Therefore, there is no solution.

(b) Applying the technique of characteristic curve, we know that $u(x, y)$ is in the form of

$$u(x, y) = f(ye^{-x})$$

Applying the boundary condition, we have

$$u(x, 0) = f(0) = 0$$

There are different $f = f(x)$ satisfies $f(0) = 0$. Then there are multiple solutions.

Problem 6 (P31 Q1)**.**

(a) The equation is $u_{xx} - 4u_{xy} + u_{yy} + 4u = 0$

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

Since $\det A < 0$, the equation is hyperbolic.

(b) Since

$$A = \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$$

and $\det A = 0$, the equation is parabolic.