Name: ____________________ SID: ____________________

MTH4220(2022 Fall): Quiz 1

- This quiz will last for 50 mins.
- No phones or discussion, book and notes closed, no calculators.

1. For a given function f, consider the first order PDE:

$$2\partial_x u + \partial_y u = f(u)$$

   (a) (10pts) What kind of coordinates (x', y') you can introduce so that the terms on the left hand side of the equation can be reduced to $C\partial_{x'} u$ for some constant C.

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   (b) (10pts) What are the characteristic lines/curves to the corresponding homogeneous equation, i.e. $f(u) \equiv 0$ in the equation? Sketch these.

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   (c) (5pts) A solution u to the equation is constant on any of these characteristic curves/lines. True or false? ____________________

   (d) (10pts) Consider $f(u) = u$. For $k, m \in \mathbb{R}$ and $g = g(x)$ given, discuss the existence and uniqueness of solutions to the equation with prescribed data: $u(x, y(x)) = g(x)$ on the line $y(x) = kx + m$.

2. Let $a, b \in \mathbb{R}$ be given constants, consider the second order equation

$$\partial_{tt}u - (a+b)\partial_{tx}u + ab\partial_{xx}u = 0.$$

(a) (10pts) Determine the equation type(elliptic, parabolic or hyperbolic).

(b) (10pts) Is it possible to introduce some suitable coordinates (ξ, η) such that the equation can be reduced to $\partial_{\xi\eta}u = 0$? If so, what are they?

(c) (10pts) Solve the equation for the case $a = 3, b = 4$.

3. Let $\Omega \subset \mathbb{R}^n$ be a smooth connected domain, $T \in (0, \infty)$ a given constant, and $\Omega_T = \Omega \times [0, T]$.

   (a) (5pts) What is the parabolic boundary, $\partial_p \Omega_T$, of Ω_T?

   (b) (10pts) If u attains a **minimum** at the point $(\vec{x}_0, t_0) \in \Omega_T \backslash \partial_p \Omega_T$, what can you say about the signs of the first and second derivatives of u there?

   (c) (10pts) Does the **Minimum Principle** holds to the equation $\partial_t u - \Delta u \geqslant 0$ on Ω_T? If so, describe it.

   (d) (10pts) Give an example of approximate function, v_ε, of u in order to show the **Minimum Principle** to the heat equation $\partial_t u - \Delta u = 0$ on Ω_T.

   Then show that

$$\lim_{\varepsilon \to 0^+} \max_{\Omega_T} v_\varepsilon = \lim_{\varepsilon \to 0^+} \max_{\Omega_T} u.$$