PHY3110 Homework Assignment 11

1. (20 points) Verify the Jacobi identity for the Poisson brackets.

2. (20 points) Show by the use of Poisson brackets that for a one-dimensional harmonic oscillator there is a constant of motion u defined as

$$u(q, p, t) = \ln(p + im\omega q) - i\omega t, \qquad \omega = \sqrt{\frac{k}{m}}. \tag{1}$$

What is the physical significance of it?

3. (35 points) Show that the following transformation is canonical (α is a fixed parameter):

$$\begin{aligned} x &= \frac{1}{\alpha}(\sqrt{2P_1}\sin Q_1 + P_2), \qquad & p_x &= \frac{\alpha}{2}(\sqrt{2P_1}\cos Q_1 - Q_2), \\ y &= \frac{1}{\alpha}(\sqrt{2P_1}\cos Q_1 + Q_2), \qquad & p_y &= -\frac{\alpha}{2}(\sqrt{2P_1}\sin Q_1 - P_2), \end{aligned} \tag{2}$$

Apply this transformation to the problem of a particle of charge q moving in a plane that is perpendicular to a constant magnetic field $\vec{B}$. Express the Hamiltonian for this problem in the (Q_i, P_i) coordinates letting the parameter α take the form

$$\alpha^2 = \frac{qB}{c}. \tag{3}$$

From this Hamiltonian, obtain the motion of the particle as a function of time.

4. (25 points) Use the method of infinitesimal canonical transformations to solve the motion of a one-dimensional harmonic oscillator as a function of time.