PHY3110 Homework Assignment 8

1. (20 points) Find the principal moments of inertia about the center of mass of a flat rigid body in the shape of an isosceles triangle with a uniform mass density. What are the principal axes?

2. (20 points) Consider the torque-free motion of an asymmetric rigid body with one point fixed, show from Euler equations that L^2 and T (L and T are the angular momentum and kinetic energy) are conserved.

3. (30 points) 1) Express in terms of Euler's angles the constraint equations for a uniform sphere rolling without slipping on a flat horizontal surface. Show that they are nonholonomic.
2) Set up the Lagrangian equations for this problem by the method of Lagrange multipliers. Show that the translational and rotational parts of the kinetic energy are separately conserved. Are there any other constants of motion?.

4. (30 points) A bead of mass m is constrained to move on a hoop of radius R. The hoop rotates with constant angular velocity ω around a diameter of the hoop, which is a vertical axis (line along which gravity acts).
a) Set up the Lagrangian and obtain the equations of motion of the bead.
b) Find the critical angular velocity Ω below which the bottom of the hoop provides a stable equilibrium for the bead.
c) Find the stable equilibrium position for $\omega > \Omega$.