

PHY3110 Homework Assignment 5

1. (20 points) Find the orbits of a point mass moving in a central force field $F = -kr$, where k is a positive constant. What if k is a negative constant?

2. (25 points) A point mass m moves in a central force field with $F = -\frac{\alpha m}{r^2}$. If its orbit is an ellipse with the semi-major axis a , derive the following relation between its velocity and r , a

$$v^2 = \alpha \left(\frac{2}{r} - \frac{1}{a} \right). \quad (1)$$

What if the orbit is a hyperbola or parabola?

3. (35 points) Consider the scattering produced by a repulsive force $F = \frac{k}{r^3}$, show that the cross section takes the form

$$\sigma(\theta) = \frac{k\pi^2}{2E} \frac{\pi - \theta}{\theta^2(2\pi - \theta)^2 \sin \theta}. \quad (2)$$

4. (20 points) Show that for an antisymmetric 3×3 real matrix \mathbf{A} , the matrix $\mathbf{B} = (\mathbf{1} + \mathbf{A})(\mathbf{1} - \mathbf{A})^{-1}$ is orthogonal, where $\mathbf{1}$ is the identity matrix.