# PHY3110 Homework Assignment 4

1. (20 points) Use the equations of motion for a point mass $m$ moving in a central potential $V(r)$, show that ($u = 1/r$, $l$ is the angular momentum)

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2}\frac{d}{du}V\left(\frac{1}{u}\right). \tag{1}$$

2. (25 points) If the orbit of a point mass under a central force $F(r)$ is given by $r = k\theta^2$ with $k$ being a constant, try to derive the explicit form of $F(r)$.

3. (20 points) Two particles move around each other in circular orbits under gravitational forces with a period $\tau$. If they suddenly stop at a given instant and then start to fall into each other, show that they collide after a time $\tau/(4\sqrt{2})$.

4. (35 points) A particle moves in a force field described by

$$V(r) = -k\frac{e^{-ar}}{r}, \tag{2}$$

where $k, a$ are positive constants.
a) Use the effective potential to discuss the qualitative nature of the orbits for different values of energy and angular momentum.
b) What is the period of the motion when the orbit is a circle?