# PHY3110: Classical Mechanics

## Lagrangian / Hamiltonian formulation

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|---|---|
| Homework | 30% |
| Midterm | 30% |
| Final | 40% |

Tutorial

Reference book:

H. Goldstein. C. Poole. J. Safko, Classical Mechanics, 3rd Edition. Pearson.

⊛ J.R. Taylor, Classical Mechanics, University Science Books.

⊛ T.W.B. Kibble. F.H. Berkshire, Classical Mechanics, 5th Edition. Imperial College Press.

⊛ 梁昆淼. 力学(下册) 理论力学. 4th Edition, 高等教育出版社.

Jianhui Zhang. Office: RA 319.

Classical mechanics describes the motion of macroscopic objects, which are not extremely massive and not extremely fast.

Speed

$3\times10^{8}$ m/s → speed of light.

Size ↑ $10^{-9}$ m

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|---|---|
| Classical Mechanics | Relativistic Mechanics ✓ |
| Quantum Mechanics ✗ | Quantum Field Theory ✗ |

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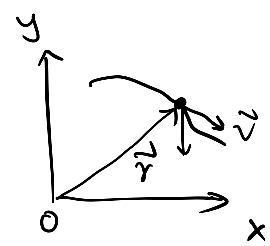
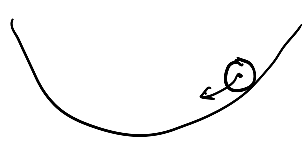
Review of Newtonian Mechanics

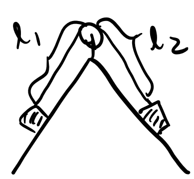
vectorial quantities of motion:

$$\vec{r},\ \vec{v},\ \vec{F},\ \vec{p}=m\vec{v},\ \vec{L}=\vec{r}\times\vec{p}=\vec{r}\times m\vec{v}$$



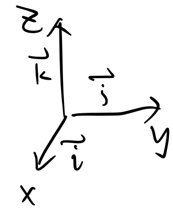



Constraint.




$l_1+l_2=\text{const}.$

Analytical mechanics:




It uses scalar quantities of motion:

kinetic energy: $T = \frac{1}{2} m \vec{v}^2$.

$\vec{v}\cdot\vec{v}$
$= (v_1\vec{i} + v_2\vec{j} + v_3\vec{k})\cdot(v_1\vec{i} + v_2\vec{j} + v_3\vec{k})$
$= v_1^2 + v_2^2 + v_3^2$

potential energy:
$V = V(\vec{r})$.

$\vec{i}\cdot\vec{i} = 1$,
$\vec{i}\cdot\vec{j} = \vec{i}\cdot\vec{k} = \vec{k}\cdot\vec{j} = 0$

Constraints:

They are used to reduce the number of dofs.

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The formalism of analytical mechanics can also be generalized to electrodynamics, statistical/quantum mechanics, relativity & QFTs.

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Newton's 2nd law:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\vec{a}, \quad \vec{a} = \frac{d\vec{v}}{dt}.$$

Valid in an inertial frame.

$$\frac{d\vec{p}}{dt} = 0 \quad \text{if } \vec{F} = 0. \Rightarrow \vec{p} = \text{const}.$$

$\boxed{\vec{p} \text{ is conserved if } \vec{F} = 0.}$

Angular momentum $\vec{L}$ & torque $\vec{N}$.

$$\vec{L} = \vec{r} \times \vec{p}.$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \underbrace{\frac{d\vec{r}}{dt}} \times (m\vec{v}) + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \underbrace{\vec{v} \times m\vec{v}}_{=0} + \underbrace{\vec{r} \times \vec{F}}_{=\vec{N}}$$

$(\vec{A} \times \vec{B})_i \quad (i=1,2,3)$

$= \sum_{j,k} \varepsilon_{ijk} A_j B_k$ summation over indices

$$\Rightarrow \frac{d\vec{L}}{dt} = \vec{N} = \vec{r} \times \vec{F}$$

$\boxed{\vec{L} \text{ is conserved if } \vec{N} = 0}$

permutation

$\varepsilon_{123} = 1,$

$\varepsilon_{132} = -1.$

$\varepsilon_{312} = +1$

$\varepsilon_{iij} = 0$

$\varepsilon_{122} = 0$

$\varepsilon_{133} = 0.$

$$(\vec{A} \times \vec{B})_1$$

$$= \varepsilon_{1jk} A_j B_k$$

$$= \varepsilon_{123} A_2 B_3 + \underbrace{\varepsilon_{132}}_{=-\varepsilon_{123}} A_3 B_2$$

$$= \varepsilon_{123} A_2 B_3 - \varepsilon_{123} A_3 B_2$$

$$= A_2 B_3 - A_3 B_2$$

$$V_2 V_3 - V_3 V_2 = 0$$

$$(\vec{A} \times \vec{B}) \times \vec{c}$$


Work done by the external force:

$$W_{12} = \int_1^2 \vec{F} \cdot d\vec{s} = \int_1^2 \frac{d\vec{p}}{\cancel{dt}} \cdot \vec{v}\, \cancel{dt}$$

$$= \int_1^2 m\vec{v} \cdot d\vec{v}$$

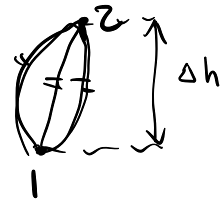
$$= \frac{1}{2} m\vec{v}_2^2 - \frac{1}{2} m\vec{v}_1^2$$

$$= T_2 - T_1$$

$W = \oint \vec{F} \cdot d\vec{s} = 0$

Define a scalar function $V(\vec{r})$.

$\vec{F} = -\nabla V(\vec{r})$.




V: potential energy,

$\vec{F}$: conservative force.

$$W_{12} = V_1 - V_2 = T_2 - T_1$$

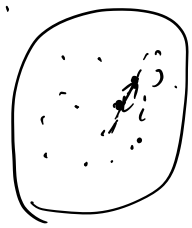
$$\Rightarrow \underbrace{V_1 + T_1}_{E_1} = \underbrace{V_2 + T_2}_{E_2}$$

$$E_1 = E_2$$

Total energy is conserved.

Consider a system of multiple particles:

$$\frac{d\vec{p}_i}{dt} = \dot{\vec{p}}_i = \vec{F}_i^{(e)} + \sum_j \vec{F}_{ji}$$




Assume Newton's 3rd law holds:

Forces that act on each other are equal & opposite.

$\vec{F}_{ji} = -\vec{F}_{ij}$

$$\sum_i \dot{\vec{p}}_i = \frac{d}{dt}\left(\sum_i \vec{p}_i\right) = \sum_i \vec{F}_i^{(e)} + \underbrace{\sum_{\substack{i,j \\ i \neq j}} \vec{F}_{ji}}_{=0}$$

$$= \sum_i \vec{F}_i^{(e)}$$

Center of mass of the system:

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \vec{r}_i}{M} \qquad M = \sum_i m_i .$$

$$\dot{\vec{P}} = \sum_i \dot{\vec{p}}_i = M\ddot{\vec{R}} = \vec{F}^{(e)} = \sum_i \vec{F}_i^{(e)}.$$

$\vec{P}$ is conserved if $\vec{F}^{(e)} = 0$

Total AM:

$$\frac{d\vec{L}}{dt} = \dot{\vec{L}} = \frac{d}{dt} \sum_i \vec{r}_i \times \vec{p}_i$$

$$= \sum_i \vec{r}_i \times \vec{F}_i^{(e)} + \sum_{\substack{i,j \\ i \neq j}} \vec{r}_i \times \vec{F}_{ji}$$

Assume the mutual force between 2 particles lie along the line between them.

$$\sum_{\substack{i,j \\ i \neq j}} \vec{r}_i \times \vec{F}_{ji} \Longrightarrow \frac{1}{2} \sum_{\substack{i,j \\ i \neq j}} \underbrace{\vec{r}_{ij}}_{(\vec{r}_i - \vec{r}_j)} \times \vec{F}_{ji} = 0$$

$$\boxed{\dot{\vec{L}} = \vec{N}^{(e)} = \sum_i \vec{r}_i \times \vec{F}_i^{(e)}.}$$

$\vec{L}$ is conserved if $\vec{N}^{(e)} = 0$.