

PHY3110 Homework Assignment 10

1. (20 points) The Lagrangian for a system can be written as

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2 + y^2}, \quad (1)$$

where a, b, c, f, g, k are constants. What is the Hamiltonian? What quantities are conserved?

2. (20 points) For a given Lagrangian

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + bq_1^2} + k_1q_1^2 + k_2\dot{q}_1\dot{q}_2, \quad (2)$$

with a, b, k_1, k_2 being constants, find the equations of motion in the Hamiltonian formulation.

3. (35 points) A Hamiltonian of one degree of freedom has the form

$$H = \frac{p^2}{2a} - bqpe^{-\alpha t} + \frac{ba}{2}q^2e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{kq^2}{2}, \quad (3)$$

where a, b, α, k are constants.

- a) Find the Lagrangian corresponding to this Hamiltonian.
- b) Find an equivalent Lagrangian that is not explicitly dependent on time.
- c) What is the Hamiltonian corresponding to the second Lagrangian, and what is the relationship between the two Hamiltonians?

4. (25 points) a) The Lagrangian for a system with one degree of freedom reads

$$L = \frac{m}{2}(\dot{q}^2 \sin^2 \omega t + \dot{q}q\omega \sin 2\omega t + q^2\omega^2). \quad (4)$$

What is the corresponding Hamiltonian? Is it conserved?

- b) Introduce a new coordinate defined by $Q = q \sin \omega t$. Find the Lagrangian in terms of the new coordinate and the corresponding Hamiltonian. Is H conserved?