PHY3110 Homework Assignment 1

1. (15 points) Show that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. What if $\vec{b}$ is a differential operation ∇?

2. (20 points) Show that spherical coordinates are orthogonal coordinates. In Cartesian coordinates the line element is defined as $ds^2 = dx^2 + dy^2 + dz^2$, derive its expression in spherical coordinates.

3. (15 points) Show that Lagrange's equations

$$\frac{d}{dt}\Big(\frac{\partial T}{\partial \dot{q}_i}\Big) - \frac{\partial T}{\partial q_i} = Q_i \tag{1}$$

can also be written as the following form (known as the Nielsen form)

$$\frac{\partial \dot{T}}{\partial \dot{q}_i} - 2\frac{\partial T}{\partial q_i} = Q_i. \tag{2}$$

4. (30 points) A constraint of the form

$$\sum_{i=1}^{n} g_i(x_1, x_2, \ldots, x_n)dx_i = 0 \tag{3}$$

is holonomic only if an integrating function $f(x_1, x_2, \ldots, x_n)$ can be found that turns it into an exact differential.
a) What condition shall f fulfill to turn Eq. (3) to a holonomic constraint?
b) Are the constraints $(2x+y+z)dx+(x+2y+z)dy+(x+y+2z)dz = 0$ and $(x^2+y^2+z^2)dx + 2(xdx + ydy + zdz) = 0$ holonomic?

5. (20 points) Consider a pendulum made of a spring with a mass m on the end. The spring is arranged to lie in a straight line with the equilibrium length of the spring being l. Let the spring have length $l + x(t)$, and its angle with the vertical be $\theta(t)$. Assuming that the motion takes place in a vertical plane, find the equations of motion for x and θ.