PHY3110 SP23 Notes

Haoran Sun (haoransun@link.cuhk.edu.cn)

0 Introduction

Grading: 30% homework, 30% midterm, 40% final.

Textbooks:

- H. Goldstein, C. Poole, J. Safko, Classical Mechanics, 3rd Edition, Pearson.
- J.R. Taylor, Classical Mechanics, University Science Books.
- T.W.B. Kibble, F.H. Berkshire, Classical Mechanics, 5th Edition, Imperial College Press.
- 梁昆淼，力学（下册）理论力学，4th Edition，高等教育出版社.

Classical mechanics describe the motion of macroscopic objects, which are not extremely massive and not extremely fast.

1 Newtonian Mechanics

Vectorial quantities of motion: position $\mathbf{r}$, velocity $\mathbf{v}$, force $\mathbf{F}$, momentum $\mathbf{p} = m\mathbf{v}$, angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. Equations of motion are derived from those vector quantities.

Analytical mechanics uses scalar quantities of motion

- Kinetic energy $T = \frac{1}{2}m\mathbf{v}^2$
- Potential energy $V = V(\mathbf{r})$

Equations of motion are derived from those scalar quantities.

Newton's 2^{nd} law

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = m\mathbf{a} \tag{1}$$

valid in an inertial frame. Angular momentum $\mathbf{L}$ and torque $\mathbf{N}$ are also related

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{r} \times \mathbf{p}) = \mathbf{r} \times \mathbf{F} = \mathbf{N} \tag{2}$$

Work done by external forces

$$W_{12} = \int_1^2 \mathbf{F}\,\mathrm{d}\mathbf{s} = \int_1^2 m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\,\mathrm{d}\mathbf{s} = \int_1^2 m\mathbf{v}\,\mathrm{d}\mathbf{v} = \frac{1}{2}m\mathbf{v}^2\Big|_1^2 \tag{3}$$

Define a scalar function $V(\mathbf{r})$, then $F = -\nabla V(\mathbf{r})$ is a conservative force.

$$\oint \mathbf{F}\,\mathrm{d}\mathbf{s} = 0 \tag{4}$$

Center of mass of the system

$$\mathbf{R} = \frac{\sum_i m_i\mathbf{r}_i}{\sum_i m_i} = \frac{\sum_i m_i\mathbf{r}_i}{M} \tag{5}$$

Total momentum

$$\mathbf{P} = \sum_i m_i \mathbf{p}_i = M\dot{\mathbf{R}} \tag{6}$$

Hence $\mathbf{P}$ is conserved if external force $\mathbf{F}^{(e)}$ is zero.

Total angular momentum

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\sum_i \mathbf{r}_i \times \mathbf{p}_i = \sum_i \mathbf{r}_i \times \left(\mathbf{F}_i^{(e)} + \sum_j \mathbf{F}_{ij}\right) = \sum_i \mathbf{r}_i \times \mathbf{F}_i^{(e)} + \sum_{ij} \mathbf{r}_i \times \mathbf{F}_{ij}$$

Since $\mathbb{r}_{ij}$ parallel to $\mathbf{F}_i j$, then

$$\sum_{ij} \mathbf{r}_i \mathbf{F}_{ij} = \frac{1}{2}\sum_{ij} \mathbf{r}_{ij} \times \mathbf{F}_{ji} = 0 \tag{7}$$

Therefore

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \mathbf{N}^{(e)} \tag{8}$$