PHY5410 FA22 HW#02

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Problem 1 (3.10).

Problem 2 (3.15). Since

$$\frac{(\Delta p)^2}{2m} = c(\Delta x)^4$$
$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

Then it is easy to solve that

$$\Delta x \geq \left(\frac{\hbar^2}{8mc}\right)^{1/6}$$

Problem 3 (4.2). Using the fact that the energy uncertainty of a free wave packet is $\Delta E = p_0 \Delta p/m$ and the time uncertainty is $\Delta t = m\Delta x/p_0$. Then

$$\Delta E = \frac{\hbar}{2dm} p_0$$
$$\Delta E \Delta t = \sqrt{1 + \left(\frac{t\hbar}{2md^2}\right)^2} \frac{\hbar}{2}$$

Problem 4 (5.3).

Claim. $L_i = L_i^\dagger$

Proof. Using the fact $[x_i, p_j] = i\hbar\delta_{ij}$

$$\begin{aligned} L_i^\dagger &= [\epsilon_{ijk}(x_j p_k - x_k p_j)]^\dagger \\ &= \epsilon_{ijk}(p_k^\dagger x_j^\dagger - p_j^\dagger x_k^\dagger) \\ &= \epsilon_{ijk}(p_k^\dagger x_j^\dagger - p_j^\dagger x_k^\dagger) \\ &= \epsilon_{ijk}(x_j p_k - x_k p_j) = L_i \end{aligned}$$ □

Claim. $\langle \psi | L_i^2 \psi \rangle \geq 0$.

Proof. Since $L_i = L_i^\dagger$, we have

$$\begin{aligned} \langle \psi | L_i^2 \psi \rangle &= \langle L_i^\dagger \psi | L_i \psi \rangle \\ &= \langle L_i \psi | L_i \psi \rangle \geq 0 \end{aligned}$$ □

By the claims above, it is easy to derive that

$$\langle \psi | \mathbf{L}^2 \psi \rangle = 0 \Leftrightarrow \sum_i \langle \psi | L_i^2 \psi \rangle = 0 \Leftrightarrow \langle \psi | L_i^2 \psi \rangle = \langle L_i \psi | L_i \psi \rangle = 0 \Leftrightarrow |L_i \psi\rangle = |0\rangle \Rightarrow \langle \psi | L_i \psi \rangle = 0$$

if $|L_i \psi\rangle$ is indeed a continuous function under any representations.