

## PHY5410 FA22 HW#02

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**Problem 1** (3.10).

**Problem 2** (3.15). The energy of the system is

$$E = T + V = \frac{p^2}{2m} + cx^4$$

Substitute  $p$  by  $px = \hbar/2$

$$E = \frac{\hbar^2}{8mx^2} + cx^4$$

Find the minimum of  $E$  by letting  $dE/dx = 0$

$$\frac{dE}{dx} = 4cx^3 - \frac{\hbar^2}{4mx^3} = 0 \Rightarrow x_0 = \left( \frac{\hbar^2}{16mc} \right)^{1/6}$$

Then the ground state energy is approximately equals to

$$E_0 = cx_0^4 = c \left( \frac{\hbar^2}{16mc} \right)^{4/6}$$

**Problem 3** (4.2). Using the fact that the energy uncertainty of a free wave packet is  $\Delta E = p_0 \Delta p / m$  and the time uncertainty is  $\Delta t = m \Delta x / p_0$ . Then

$$\Delta E = \frac{\hbar}{2dm} p_0$$

$$\Delta E \Delta t = \sqrt{1 + \left( \frac{t\hbar}{2md^2} \right)^2} \frac{\hbar}{2}$$

**Problem 4** (5.3).

*Claim.*  $L_i = L_i^\dagger$

*Proof.* Using the fact  $[x_i, p_j] = i\hbar\delta_{ij}$  ( $x_i, p_j$  commute when  $i \neq j$ )

$$\begin{aligned} L_i^\dagger &= [\epsilon_{ijk}(x_j p_k - x_k p_j)]^\dagger \\ &= \epsilon_{ijk}(p_k^\dagger x_j^\dagger - p_j^\dagger x_k^\dagger) \\ &= \epsilon_{ijk}(p_k x_j - p_j x_k) \\ &= \epsilon_{ijk}(x_j p_k - x_k p_j) = L_i \end{aligned}$$

□

*Claim.*  $\langle \psi | L_i^2 | \psi \rangle \geq 0$ .

*Proof.* Since  $L_i = L_i^\dagger$ , we have

$$\begin{aligned} \langle \psi | L_i^2 | \psi \rangle &= \langle L_i^\dagger \psi | L_i \psi \rangle \\ &= \langle L_i \psi | L_i \psi \rangle \geq 0 \end{aligned}$$

□

By the claims above, we can derive that

$$\langle \psi | L_i^2 | \psi \rangle = 0 \Leftrightarrow \sum_i \langle \psi | L_i^2 | \psi \rangle = 0 \Leftrightarrow \langle \psi | L_i^2 | \psi \rangle = \langle L_i \psi | L_i \psi \rangle = 0 \Leftrightarrow |L_i \psi\rangle = |0\rangle \Rightarrow \langle \psi | L_i | \psi \rangle = 0$$

note that we can derive  $|L_i \psi\rangle = |0\rangle$  if  $|L_i \psi\rangle$  is indeed a continuous function under any representations.