PHY 5410: Homework Week 1

2.1 (**a**) Show that, for complex α with $\mathrm{Re}\,\alpha > 0$,

$$\int_{-\infty}^{+\infty} dx\, \mathrm{e}^{-\alpha x^2} = \sqrt{\pi}/\alpha$$

holds.

(**b**) Compute

$$\int_{-\infty}^{+\infty} d^3k\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\, \mathrm{e}^{-k^2\alpha^2} \quad .$$

2.4 Show that

(**a**) $p^\dagger = p$, $(p^2)^\dagger = p^2$, $V(\boldsymbol{x})^\dagger = V(\boldsymbol{x})$

(**b**) $(AB)^\dagger = B^\dagger A^\dagger$

(**c**) $[AB, C] = A\,[B, C] + [A, C]\,B$.

2.5 (**a**) Show that

$$[A, B^n] = nB^{n-1}[A, B]$$

under the assumption $[[A, B], B] = 0$.

2.7 Let $\psi_a(x)$ and $\psi_b(x)$ be two orthonormal solutions of the time independent Schrödinger equation for a given potential with energy eigenvalues E_a and E_b. At the time $t = 0$, suppose that the system is in the state

$$\psi(x, t = 0) = \frac{1}{\sqrt{2}}\left(\, \psi_a(x) + \psi_b(x) \,\right) \quad .$$

Discuss the probability density at a later time t.

2.8 For the operators

$$p_i = \frac{\hbar}{\mathrm{i}}\partial_i \quad \text{and} \quad L_i = \varepsilon_{ijk}\, x_j\, p_k \quad ,$$

compute the commutators

$$[p_i^2, f(\boldsymbol{x})] \quad \text{and} \quad [L_i, L_j] \quad .$$