PHY 5410: Homework Week 3

5.1 (**a**) Show that for the eigenstates of L_z the expectation values of L_+, L_-, L_x, and L_y vanish.

(**b**) Determine the average square deviation $(\Delta L_i)^2$ for the components of the angular momentum operator in the states Y_{ll} and check the uncertainty relation.

(**c**) Show that for the eigenstates of $\boldsymbol{L}^2$ and L_z the expression $(\Delta L_x)^2 + (\Delta L_y)^2$ is minimal if $m = \pm l$.

5.6 (**a**) Find the eigenfunction ψ of $\boldsymbol{L}^2$ and L_x with eigenvalues $2\hbar^2$ and $\hbar$, respectively. (Hint: Represent the eigenfunctions of $\boldsymbol{L}^2$ and L_z of interest in terms of Cartesian coordinates, and determine ψ by a rotation through $\pi/2$.)

(**b**) Express ψ as a linear combination of eigenfunctions of $\boldsymbol{L}^2$ and L_z.

6.2 Show that for the associated Laguerre polynomials

$$L_r^s(x) = (d/dx)^s \, L_r(x) \quad ,$$
$$L_r(x) = \mathrm{e}^x \, (d/dx)^r \, \mathrm{e}^{-x} x^r \quad ,$$

the following relations hold:

(**a**) $$L_r^s(x) = \sum_{k=0}^{r-s} (-1)^{k+s} \frac{[r!]^2 x^k}{k! \, (k+s)! \, (r-k-s)!} \quad ,$$

(**b**) $$\frac{(-1)^m \, \mathrm{e}^{-xt/(1-t)}}{(1-t)^{m+1}} = \sum_{n=0}^{\infty} \frac{t^r}{(r+m)!} L_{r+m}^m(x) \quad .$$