PHY 5410: Homework Week 8

11.3 In dimensionless variables $\left(\hbar = 1, m = 1, \text{ and thus } p_j = \frac{1}{\mathrm{i}}\frac{\partial}{\partial x_j}\right)$ the Hamiltonian of a two-dimensional oscillator takes the form

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \frac{1}{2}(1 + \delta xy)(x^2 + y^2) \quad ,$$

where we suppose that $\delta \ll 1$. Determine the wave functions for the three lowest lying energy levels in the case $\delta = 0$. Calculate the shift of these levels for $\delta \neq 0$ in first-order perturbation theory. Note the degeneracy which occurs.

11.5 Estimate, using the variational principle, the ground state energy of the one-dimensional harmonic oscillator. Use as a test function $\psi(\mu) = N\,\mathrm{e}^{-\mu x^2}$, where the parameter $\mu > 0$.

11.7 Consider the potential $V(x) = \begin{cases} V_0 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$

(a) Find the ground state energy by the variational ansatz

$$\psi_0(x) = x\,\mathrm{e}^{-\kappa_0 x} \quad .$$

(b) Find the energy of the first excited state by the variational ansatz

$$\psi_1(x) = x(x - n)\,\mathrm{e}^{-\kappa_1 x} \quad ,$$

where the coefficients are to be chosen such that ψ_1 is orthogonal to ψ_0.

(c) Compare with the exact result.

(d) What is the result of the variational ansatz

$$\psi_0 = x\,\mathrm{e}^{-\kappa_0 x^2} \quad ?$$