# PHY 5410: Homework Week 6

**8.1** For Schrödinger operators $A$, $B$, and $C$, let $[A, B] = C$. What is the commutation relation for the corresponding operators in the Heisenberg representation?

**8.2** Derive the Heisenberg equations of motion for the one-dimensional harmonic oscillator

$$H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 x^2 \quad .$$

Compare with the classical equations of motion. Calculate the time dependence of the operators $a_{\mathrm{H}}$, $a_{\mathrm{H}}^\dagger$, $p_{\mathrm{H}}$, and $x_{\mathrm{H}}$. Determine $a_{\mathrm{H}}(t)$ from the equation of motion and directly by use of the Baker–Hausdorff formula.

**8.4** Calculate the matrix representation of the angular momentum operators $L_x$, $L_y$, $L_z$, and $\boldsymbol{L}^2$ for the values $l = 1/2,\ 1,\ 3/2$, and $2$ by using the formulae

$$\langle l', m'|L^2|l, m\rangle = \hbar^2 \delta_{ll'}\, \delta_{mm'}\, l(l+1) \quad ,$$

$$\langle l', m'|L_z|l, m\rangle = \hbar \delta_{ll'}\, \delta_{mm'} m \quad ,$$

$$\langle l', m'|L_-|l, m\rangle = \hbar\sqrt{(l-m+1)(l+m)}\delta_{ll'}\, \delta_{m-1,m'} \quad ,$$

$$\langle l', m'|L_+|l, m\rangle = \hbar\sqrt{(l+m+1)(l-m)}\delta_{ll'}\, \delta_{m+1,m'} \quad ,$$

$$-l \le m \le l \quad .$$

**8.5** Show $[H, \boldsymbol{L}] = 0$, $[H, \boldsymbol{P}] = 0$, where

$$H = \sum_{n=1}^{N} \frac{\boldsymbol{p}_n^2}{2m_n} + \frac{1}{2}\sum_{n,n'} V(|\boldsymbol{x}_n - \boldsymbol{x}_{n'}|),\ \boldsymbol{L} = \sum_{n=1}^{N} \boldsymbol{x}_n \times \boldsymbol{p}_n,\ \boldsymbol{P} = \sum_{n=1}^{N} \boldsymbol{p}_n,$$

**(a)** by evaluating the commutators

**(b)** by using that $\boldsymbol{L}$ and $\boldsymbol{P}$ generate rotations and translations respectively.