

PHY 5410: Homework Week 2

3.10 For a general, one-dimensional wave packet

$$\psi(x, t) = \int \frac{dp}{2\pi\hbar} g(p) \exp\{i[px - E(p)t + \hbar\alpha(p)]/\hbar\} \quad ,$$

whose real weighting function $g(p)$ is symmetric about its maximum at p_0 and which differs substantially from zero only in a region $|p - p_0| \leq \Delta p$, calculate the expectation values $\langle p \rangle$, $\langle p^2 \rangle$, $\langle x \rangle$, and $\langle x^2 \rangle$. It is advisable to carry out the computation of the positional uncertainty in the momentum representation.

3.15 Using the uncertainty relation, find an estimate of the ground state energy of a particle (mass m) located in a potential $V(x) = cx^4$.

4.2 For a Gaussian wave packet, calculate the energy uncertainty ΔE and $\Delta E \Delta t$, where Δt is the characteristic time for the wave packet to pass through the point x .

5.3 Show that $(\psi, \mathbf{L}^2 \psi) = 0$ also implies $(\psi, L_k \psi) = 0$.