PHY 5410: Homework Week 7

9.5 Consider the precession of the spin of an electron in a homogeneous magnetic field $(0, 0, B)$.

(**a**) Using the Hamiltonian $H = (e_0/mc)\boldsymbol{S} \cdot \boldsymbol{B}$, write down the equation of motion for the spin operator in the Heisenberg picture and give the solution for the initial condition $\boldsymbol{S}(t = 0) = \boldsymbol{S}(0)$. The solution is

$$
\begin{aligned}
S_z(t) &= S_z(0), \\
S_x(t) &= \cos\,\omega t\, S_x(0) - \sin\,\omega t\, S_y(0), \\
S_y(t) &= \sin\,\omega t\, S_x(0) + \cos\,\omega t\, S_y(0).
\end{aligned}
$$

(**b**) Given $\Psi(0) = \binom{a}{b}$, determine the state $\Psi(t)$ at the time t.

(**c**) What is the probability of obtaining the value $\hbar/2$ in a measurement of S_z at the time t if at the time $t = 0$ the spin was oriented in the x-direction?

(**d**) The same as in problem (c) for a measurement of S_x.

(**e**) How can part (b) of this problem be derived from the Pauli equation?

10.2 Suppose that a system consists of two distinguishable particles each with spin $S = 1/2$ and is described by the Hamiltonian

$$
H = -\frac{a+b}{2}\left(S_{1,z} + S_{2,z}\right) B - \frac{a-b}{2}\left(S_{1,z} - S_{2,z}\right) B + J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \quad ,
$$

where a, b, and J are constants, and B represents the external magnetic field. Determine the energy eigenvalues.

11.4 A one-dimensional harmonic oscillator carrying charge e is located in an external electric field of strength E pointing in the positive x-direction:

$$
H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2}{2}x^2 - eEx \quad .
$$

Calculate the energy levels in second-order and the wave function in first-order perturbation theory and compare with the exact result.