PHY 5410: Homework Week 10

16.5 A harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{m}{2}\omega^2 x^2$$

is acted upon by an external force described by the potential $V = -xf(t)$, where

$$f(t) = \begin{cases} 0 & -\infty \leq t \leq t_1 \\ D\cos\,\Omega t & t_1 \leq t \leq t_2 \\ 0 & t \geq t_2 \end{cases} \quad .$$

Determine the probability for a transition from the ground state ($t < t_1$) to the nth excited state ($t > t_2$). Hint: Use the Heisenberg representation and introduce for x and p creation and annihilation operators.

16.7 An electron moves in the (one-dimensional) potential $V(x) = -\lambda\delta(x)$. At the time $t = 0$, the strength of the potential changes suddenly to the value μ ($\lambda, \mu > 0$). Using the sudden approximation, calculate the probability of a transition from the old ground state to the new ground state. Consider the special case $\mu = \lambda/2$ and discuss $\mu/\lambda \gg 1$ and $\mu/\lambda \ll 1$.

16.10 Prove for a particle in one dimension the oscillator strength sum rule (Thomas–Reiche–Kuhn sum rule)

$$\sum_n (E_n - E_a)|\langle n|x|a\rangle|^2 = \frac{\hbar^2}{2m} \quad .$$

Hint: Consider $\langle a|[\dot{x}, x]|a\rangle$ and compute this expression by direct use of $[p, x] = -\mathrm{i}\hbar$ on the one hand and by substitution of $\dot{x} = \mathrm{i}[H, x]/\hbar$ on the other hand.