

## PHY5410 FA22 HW#02

Haoran Sun (haoransun@link.cuhk.edu.cn)

**Problem 1** (3.10).

**Problem 2** (3.15). Since

$$\frac{(\Delta p)^2}{2m} = c(\Delta x)^4$$

$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

Then

$$\Delta x \geq \left( \frac{\hbar^2}{8mc} \right)^{1/6}$$

and

$$E_0 \approx c \left( \frac{\hbar^2}{8mc} \right)^{4/6}$$

**Problem 3** (4.2). Using the fact that the energy uncertainty of a free wave packet is  $\Delta E = p_0 \Delta p / m$  and the time uncertainty is  $\Delta t = m \Delta x / p_0$ . Then

$$\Delta E = \frac{\hbar}{2dm} p_0$$

$$\Delta E \Delta t = \sqrt{1 + \left( \frac{t \hbar}{2md^2} \right)^2} \frac{\hbar}{2}$$

**Problem 4** (5.3).

*Claim.*  $L_i = L_i^\dagger$

*Proof.* Using the fact  $[x_i, p_j] = i\hbar \delta_{ij}$  ( $x_i, p_j$  commute when  $i \neq j$ )

$$\begin{aligned} L_i^\dagger &= [\epsilon_{ijk}(x_j p_k - x_k p_j)]^\dagger \\ &= \epsilon_{ijk}(p_k^\dagger x_j^\dagger - p_j^\dagger x_k^\dagger) \\ &= \epsilon_{ijk}(p_k x_j - p_j x_k) \\ &= \epsilon_{ijk}(x_j p_k - x_k p_j) = L_i \end{aligned}$$

□

*Claim.*  $\langle \psi | L_i^2 | \psi \rangle \geq 0$ .

*Proof.* Since  $L_i = L_i^\dagger$ , we have

$$\begin{aligned} \langle \psi | L_i^2 | \psi \rangle &= \langle L_i^\dagger \psi | L_i \psi \rangle \\ &= \langle L_i \psi | L_i \psi \rangle \geq 0 \end{aligned}$$

□

By the claims above, we can derive that

$$\langle \psi | \mathbf{L}^2 | \psi \rangle = 0 \Leftrightarrow \sum_i \langle \psi | L_i^2 | \psi \rangle = 0 \Leftrightarrow \langle \psi | L_i^2 | \psi \rangle = \langle L_i \psi | L_i \psi \rangle = 0 \Leftrightarrow |L_i \psi\rangle = |0\rangle \Rightarrow \langle \psi | L_i | \psi \rangle = 0$$

note that we can derive  $|L_i \psi\rangle = |0\rangle$  if  $|L_i \psi\rangle$  is indeed a continuous function under any representations.