# PHY5410 FA22 HW08

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**Problem 1** (16.5).

**Problem 2** (16.7).

**Problem 3** (16.10). Since

$$[\dot{x}, x] = \left[\frac{1}{i\hbar}(xH - Hx), x\right] = \frac{1}{i\hbar}(2xHx - Hx^2 - x^2H)$$

Note that

$$\begin{aligned}
\langle a|xHx|a\rangle &= \sum_n \langle a|x|n\rangle \langle n|Hx|a\rangle = \sum_n E_n|\langle n|x|a\rangle|^2 \\
\langle a|Hx^2|a\rangle &= \sum_n \langle a|Hx|n\rangle \langle n|x|a\rangle = \sum_n E_a|\langle n|x|a\rangle|^2 \\
\langle a|x^2H|a\rangle &= \sum_n \langle a|x|n\rangle \langle n|xH|a\rangle = \sum_n E_a|\langle n|x|a\rangle|^2
\end{aligned}$$

Therefore

$$\langle a|[\dot{x}, x]|a\rangle = \frac{2}{i\hbar}\sum_n (E_n - E_a)|\langle n|x|a\rangle|^2$$

For $H = p^2/2m + m\omega^2 x/2m$, we have

$$\dot{x} = \frac{\partial}{\partial p}H = \frac{p}{m} \Rightarrow [\dot{x}, x] = -\frac{i\hbar}{m}$$

Therefore

$$\sum_n (E_n - E_a)|\langle n|x|a\rangle|^2 = \frac{\hbar^2}{2m}$$