Homework 7 - Summer 2022 MATH 104-003
Introduction to Analysis

1. Find the largest subset of $\mathbf{R}$ on which the given sequence converges pointwise, and determine the intervals on which the convergence is uniform.

   (a) $f_n(x) = x^n(1-x)^n$

   (b) $g_n(x) = \dfrac{x^{2n}}{2+x^{2n}}$

   (c) $\sum_{n=0}^{\infty} f_n(x)$, where f_n is defined from part (1a)

2. Let f_n be continuous on $[a,b]$ for each n and let f_n converge uniformly on $(a,b) \cap \mathbf{Q}$.

   Prove that f_n converges uniformly on $[a,b]$.

3. Suppose that $f_n \to f$ and $g_n \to g$ uniformly on $\mathbf{R}$ and f and g are bounded. Prove that $f_n g_n \to fg$ uniformly on $\mathbf{R}$.

4. Find the closed form for the following sum

$$f(x) = \sum_{n=1}^{\infty} (n^2+n)(x-1)^n$$

   and state its interval of convergence.

5. Find

$$\sum_{k=1}^{\infty} \frac{3^{-k}}{k}.$$

6. (a) State Dirichlet's Test for Uniform Convergence.

   (b) Determine if the following sum converges pointwise, uniformly, or absolutely for all $x \in \mathbf{R}$,

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{x^4+n}.$$

7. Show that

$$f(x) = \begin{cases} 1+\sqrt{x}\sin(1/x^6), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

   is integrable on $[0,1]$.

8. Abbott problem #7.2.4

9. Suppose $a < 0$ and $b > 0$. Show from the definition, that the function $f(x) = x^2$ is integrable over $[a,b]$ without using the uniform continuity of f.