2D Heat Conduction FDM Python Implementation

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1 The Equation

The heat conduction equation could be written in the form of

$$\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0 \tag{1}$$

where $u = (\mathbf{x}, t)$ is the temperature, $\mathbf{x}$ is the coordinate, and t is time. FDM could be used to solve this equation numerically by setting static boundary condition and setting grid in $\mathbf{x}$-coordinate space. For the case of 2D where $\mathbf{x} = (x, y)$, setup meshgrid like $\mathbf{x}_{ij} = (x_i, y_j)$ and $x_{i+1} - x_i = \Delta x$, $y_{j+1} - yj = \Delta y$, then by FDM

$$\frac{u_{ij}^{m+1} - u_{ij}^{m}}{\Delta t} = \alpha \Delta^2 u_{ij}^{m} \cdot \frac{1}{\Delta \mathbf{x}^2} \tag{2}$$

where

$$\Delta^2 u_{ij}^{m} \cdot \frac{1}{\Delta \mathbf{x}^2} = \left(\frac{u_{i+1j} - 2u_{ij} + u_{i-1j}}{\Delta x^2} + \frac{u_{ij+1} - 2u_{ij} + u_{ij-1}}{\Delta y^2} \right) \tag{3}$$

Note that u_{ij}^{m} is an approximation to $u(\mathbf{x}_{ij}, m\Delta t)$.

2 Result

Please refer to the notebook.