Practical 3: Maximum Likelihood

This work should be submitted to NESS before ***4pm Friday 19th October 2018****.*

The number of text messages I receive per hour follows a Poisson distribution with rate θ. The probability I receive x texts per hour is given by the probability mass function,

$$\Pr(X = x) = \frac{\theta^x}{x!}\mathrm{e}^{-\theta}, \qquad x = 0, 1, 2, \ldots.$$

Observing how many texts I receive in each hour, over a period of n hours gives a sample of observations $x_1, x_2, \ldots, x_n$.

Question 1:

Write an R function which generates the log-likelihood value for a sample of size n from a Poisson distribution, with a given value of θ. The function should take the sample and a value of θ as inputs, and return the corresponding log-likelihood value.

[5 marks]

Question 2:

Suppose we observe the following sample of size $n = 10$:

12 8 14 8 11 6 13 9 9 10

Use your function from Question 1 to produce a plot of log-likelihood against θ for this sample. From your plot what would be a sensible estimate for $\hat{\theta}$?

[5 marks]

Question 3:

Modify your function from Question 1 so that it now only takes the sample of data as an input, and returns the maximum likelihood estimate $\hat{\theta}$. By using the sample of data from Question 2, does the estimate of $\hat{\theta}$ from your function agree with your estimate by inspecting the log-likelihood plot?

[5 marks]

Question 4:

Write an additional function which takes input n, and generates $N = 1000$ samples of size n from a $Po(\theta = 10)$ distribution and computes the maximum likelihood estimate of θ for each sample. Your function should then return the mean and variance of the n maximum likelihood estimates.

[5 marks]

Question 5:

Call your function from Question 4 for values of $n = 5, 10, 25, 50, 100, 1000$ and record the resulting means and variances either in a table or graphically. From this output is your maximum likelihood estimator unbiased and/or consistent?

[5 marks]