**Problem 3: Bessel recurrence vs. floating point**

a) Please check problem3_a.py
   For z = 20, we have the following result
   n=2,    value=-0.160341        error=0
   n=3,    value=-0.0989014       error=1.82415e-15
   n=4,    value=0.130671         error=2.12408e-16
   n=5,    value=0.15117          error=3.67211e-16
   n=6,    value=-0.055086        error=7.55788e-16
   n=7,    value=-0.184221        error=3.01328e-16
   n=8,    value=-0.0738689       error=5.63611e-16
   n=9,    value=0.125126         error=2.21821e-16
   n=10,   value=0.186483         error=1.48837e-16
   n=11,   value=0.0613563        error=4.52367e-16
   n=12,   value=-0.118991        error=2.33259e-16
   n=13,   value=-0.204145        error=0
   n=14,   value=-0.146398        error=7.5836e-16
   n=15,   value=-0.000812069 error=1.10948e-13
   n=16,   value=0.14518          error=1.91181e-16
   n=17,   value=0.2331           error=1.19072e-16
   n=18,   value=0.25109          error=6.63242e-16
   n=19,   value=0.218862         error=1.26818e-15
   n=20,   value=0.164748         error=5.05419e-16
   n=21,   value=0.110634         error=2.2579e-15
   n=22,   value=0.0675829        error=3.69621e-15
   n=23,   value=0.0380487        error=1.82369e-16
   n=24,   value=0.0199291        error=3.13361e-15
   n=25,   value=0.00978117       error=1.95089e-15
   n=26,   value=0.00452381       error=1.91733e-15
   n=27,   value=0.00198074       error=6.56848e-16
   n=28,   value=0.000824178   error=1.31549e-16
   n=29,   value=0.000326963   error=1.65799e-15
   n=30,   value=0.000124015   error=2.40418e-15
   n=31,   value=4.50828e-05    error=1.05215e-15
   n=32,   value=1.57413e-05    error=5.81145e-15
   n=33,   value=5.28924e-06    error=6.50179e-14
   n=34,   value=1.71324e-06    error=3.26306e-14
   n=35,   value=5.35784e-07    error=4.86134e-14
   n=36,   value=1.62001e-07    error=4.06847e-14
   n=37,   value=4.74202e-08    error=2.76307e-14
   n=38,   value=1.34536e-08    error=4.67277e-14
   n=39,   value=3.70356e-09    error=4.55629e-14
   n=40,   value=9.90239e-10    error=5.53409e-14
   n=41,   value=2.57401e-10    error=3.9969e-14
   n=42,   value=6.51039e-11    error=1.07203e-14
   n=43,   value=1.60356e-11    error=5.21885e-14
   n=44,   value=3.84926e-12    error=2.64419e-14

n=45,  value=9.01145e-13  error=2.00571e-14
n=46,  value=2.05887e-13  error=3.53113e-14
n=47,  value=4.59366e-14  error=3.95662e-14
n=48,  value=1.00149e-14  error=4.25353e-14
n=49,  value=2.13469e-15  error=4.85951e-14
n=50,  value=4.45104e-16  error=2.37046e-14

b) Please check problem3_b.py
For z = 20, we have the following result
n=2,   value=-0.160341     error=0
n=3,   value=-0.0989014    error=1.82415e-15
n=4,   value=0.130671      error=2.12408e-16
n=5,   value=0.15117       error=7.34421e-16
n=6,   value=-0.055086     error=2.26736e-15
n=7,   value=-0.184221     error=4.51993e-16
n=8,   value=-0.0738689    error=1.8787e-15
n=9,   value=0.125126      error=4.43641e-16
n=10,  value=0.186483      error=5.9535e-16
n=11,  value=0.0613563     error=2.26184e-15
n=12,  value=-0.118991     error=5.83146e-16
n=13,  value=-0.204145     error=2.7192e-16
n=14,  value=-0.146398     error=1.51672e-15
n=15,  value=-0.000812069  error=1.96662e-13
n=16,  value=0.14518       error=3.82361e-16
n=17,  value=0.2331        error=2.38143e-16
n=18,  value=0.25109       error=2.21081e-16
n=19,  value=0.218862      error=5.07271e-16
n=20,  value=0.164748      error=1.17931e-15
n=21,  value=0.110634      error=2.50878e-16
n=22,  value=0.0675829     error=1.64276e-15
n=23,  value=0.0380487     error=5.65343e-15
n=24,  value=0.0199291     error=1.63644e-14
n=25,  value=0.00978117    error=6.01228e-14
n=26,  value=0.00452381    error=2.5117e-13
n=27,  value=0.00198074    error=1.19524e-12
n=28,  value=0.000824178   error=6.37739e-12
n=29,  value=0.000326963   error=3.77686e-11
n=30,  value=0.000124015   error=2.4639e-10
n=31,  value=4.50828e-05   error=1.75942e-09
n=32,  value=1.57413e-05   error=1.36796e-08
n=33,  value=5.28924e-06   error=1.15281e-07
n=34,  value=1.71324e-06   error=1.0488e-06
n=35,  value=5.3579e-07    error=1.02644e-05

n=36,  value=1.62019e-07   error=0.000107724
n=37,  value=4.74775e-08   error=0.00120889
n=38,  value=1.36483e-08   error=0.0144685
n=39,  value=4.38591e-09   error=0.184244
n=40,  value=3.45678e-09   error=2.49086
n=41,  value=9.44122e-09   error=35.6791
n=42,  value=3.52522e-08   error=540.476
n=43,  value=1.38618e-07   error=8643.39
n=44,  value=5.60806e-07   error=145691
n=45,  value=2.32893e-06   error=2.58441e+06
n=46,  value=9.91936e-06   error=4.81786e+07
n=47,  value=4.33001e-05   error=9.42606e+08
n=48,  value=0.000193591   error=1.93304e+10
n=49,  value=0.000885938   error=4.1502e+11
n=50,  value=0.00414751    error=9.31806e+12

c)  the truncation error in (1) due to loss of precision caused by cancellation cannot be bounded anymore when n reaches round 30

d)  Yes. When we calculate the result from 50, the number have less precision bits than it needs in the computer. Then we used the number with bits lost to do the calculation. The error will be accumulated during computing and then the precision would definitely lost. (Please check problem3_d.py)