

Problem 1

$$\begin{aligned}
\text{cond}(f) &= \frac{|\text{relative change in solution}|}{|\text{relative change in input data}|} = \frac{[f(x+\Delta x, y+\Delta y) - f(x, y)]/f(x, y)}{[(x+\Delta x, y+\Delta y) - (x, y)]/(x, y)} \\
&= \frac{|(x+\Delta x) - (y+\Delta y) - (x-y)|/(x-y)}{(\Delta x, \Delta y)/(x, y)} = \frac{(\Delta x - \Delta y)(|x| + |y|)}{(x-y)(|\Delta x| + |\Delta y|)} \\
&\geq \frac{\Delta x - \Delta y}{(|\Delta x| + |\Delta y|)\varepsilon} \geq \frac{1}{\varepsilon}
\end{aligned}$$

we can conclude that subtraction is sensitive when ε is small