

Problem 1: Develop the Cholesky factorization

a)

$$i) \quad A = BB^T = (B^T)^T B^T = (BB^T)^T = A^T$$

Hence, A is symmetric

$$ii) \quad x^T A x = x^T (BB^T) x = (x^T B)(B^T x) = (x^T (B^T)^T)(B^T x) = (B^T x)^T (B^T x)$$

Since B is nonsingular and $x \neq 0$, $(B^T x)$ cannot be 0.

Thus $(B^T x)^T (B^T x)$ is positive.

Thus A is positive definite

b)

$$i) \quad LL^T =$$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$= \begin{bmatrix} l_{11}^2 & l_{21}l_{11} & l_{31}l_{11} \\ l_{21}l_{11} & l_{21}^2 + l_{22}^2 & l_{31}l_{21} + l_{32}l_{22} \\ l_{31}l_{11} & l_{21}l_{31} + l_{22}l_{32} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

$$ii) \quad a_{11} = l_{11}^2$$

$$a_{21} = l_{21} l_{11} \quad a_{22} = l_{21}^2 + l_{22}^2$$

$$a_{31} = l_{31} l_{11} \quad a_{32} = l_{21} l_{31} + l_{22} l_{32} \quad a_{33} = l_{31}^2 + l_{32}^2 + l_{33}^2$$

$$iii) \quad l_{11} \rightarrow l_{21} \rightarrow l_{22} \rightarrow l_{31} \rightarrow l_{32} \rightarrow l_{33}$$

Thus, by the steps above, we can now derive expressions for the matrix elements of the lower triangular factor **L** very easily:

$$l_{11} = \sqrt{a_{11}}$$

$$l_{21} = a_{21} / l_{11} \quad l_{22} = \sqrt{a_{22} - l_{21}^2}$$

$$l_{31} = a_{31} / l_{11} \quad l_{32} = (a_{32} - l_{21}l_{31}) / l_{22} \quad l_{33} = \sqrt{a_{33} - (l_{31}^2 + l_{32}^2)}$$

c) (L be an $n \times n$ new matrix with all zeros)

for $i = 1$ **to** n

for $k = 1$ **to** $i + 1$

if $i = k$ **then**

$$l_{kk} = \sqrt{a_{kk} + \sum_{j=1}^k L_{ij}L_{kj}}$$

else

$$l_{ik} = \frac{1}{l_{kk}}(a_{ik} - \sum_{j=1}^{k-1} l_{ij}l_{kj})$$

end

end

d) Please check problem1_d.py

relative error: 1.23155e-16

condition number of A: 117362

e) Please check problem1_e.py

the size of the matrix n: 5 numpy.linalg.det(A): 9.32813e-05

my computed determinant: 9.32813e-05 relative error: 8.90607e-14

the size of the matrix n: 10 numpy.linalg.det(A): 0.000148628

my computed determinant: 0.000148628 relative error: -8.57131e-15

the size of the matrix n: 100 numpy.linalg.det(A): 1.75478e+49

my computed determinant: 1.75478e+49 relative error: 4.36119e-12