

Problem 2: Transform matrices

$$a) \quad m = [0, 0, m_3]^T = [0, 0, a_3/a_2]^T = [0, 0, 2.4]^T$$

$$M_2 = I - m e_k^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2.4 & 1 \end{bmatrix}$$

$$Ma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2.4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$$

$$b) \quad v = [0 \ 5 \ 12]^T - \alpha e_2$$

$$\alpha = \pm \left\| [0 \ 5 \ 12]^T \right\| = \pm \sqrt{5^2 + 12^2} = \pm 13$$

Take negative to avoid cancellation

$$v = [0 \ 5 \ 12]^T - \alpha e_2 = [0 \ 5 \ 12]^T - [0 \ -13 \ 0]^T = [0 \ 18 \ 12]^T$$

$$H = I - 2 \frac{v v^T}{v^T v}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{2}{468} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 324 & 216 \\ 0 & 216 & 144 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.38461538 & -0.92307692 \\ 0 & -0.92307692 & 0.38461538 \end{bmatrix}$$

Let $M = H$, we have Ma :

$$Ma = \begin{bmatrix} 3.00000000e+00 \\ -1.30000000e+01 \\ -6.66133815e-16 \end{bmatrix}$$

c) The form of the Given matrix that need to be determine by us is:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix}$$

where

$$c = \frac{5}{\sqrt{5^2+12^2}} = \frac{5}{13}$$

$$s = \frac{12}{\sqrt{5^2+12^2}} = \frac{12}{13}$$

Thus, we have

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{5}{13} & \frac{12}{13} \\ 0 & -\frac{12}{13} & \frac{5}{13} \end{bmatrix}$$

Let $M = G$, we have Ma :

$$Ma = \begin{bmatrix} 3 \\ 13 \\ 0 \end{bmatrix}$$