Problem 3: Least squares fitting with Gram-Schmidt and QR

a) Check problem3_a.py
b) Check problem3_b.py
c) Check problem3_c.py

matrix shape:	(5, 5)
relative error(Gram-Schmidt):	8.00372e-17
relative error(Householder):	1.90845e-16
condition number of A:	42.4793

matrix shape:	(10, 10)
relative error(Gram-Schmidt):	1.16754e-16
relative error(Householder):	2.22547e-16
condition number of A:	234.609

matrix shape:	(100, 80)
relative error(Gram-Schmidt):	2.61334e-16
relative error(Householder):	4.89216e-16
condition number of A:	13.3714

d) Check problem3_d.py

1) result from Gram-Schmidt

a * x + b
a = 0.00175825993442
b = 0.845568857432
relative residual(1): 0.172859

a * x^2 + b * x + c
a = 2.72895864077e-06
b = 0.000814040244716
c = 0.900176229486
relative residual(2): 0.171639

a * x^3 + b * x^2 + c * x + d
a = 1.73416859413e-07
b = -8.72743913947e-05
c = 0.013288539243
d = 0.537894744924
relative residual(3): 0.128342

a * x^4 + b * x^3 + c * x^2 + d * x + e
a = -7.20779692429e-10
b = 6.72196406574e-07
c = -0.000198324609702
d = 0.0218559686433
e = 0.387748406167
relative residual(4): 0.121558

a * x^5 + b * x^4 + c * x^3 + d * x^2 + e * x + f
a = 5.92040969509e-12
b = -5.84193407868e-09
c = 2.24838379058e-06
d = -0.000403281743497
e = 0.0320456311369
f = 0.267845555304
relative residual(5): 0.11797

2) result from Householder

a * x + b
a = 0.00175825993442
b = 0.845568857432
relative residual(1): 0.172859

a * x^2 + b * x + c
a = 2.72895864077e-06
b = 0.000814040244717
c = 0.900176229486
relative residual(2): 0.171639

a * x^3 + b * x^2 + c * x + d
a = 1.73416859413e-07
b = -8.72743913947e-05
c = 0.013288539243
d = 0.537894744924
relative residual(3): 0.128342

a * x^4 + b * x^3 + c * x^2 + d * x + e
a = -7.20779692429e-10
b = 6.72196406574e-07
c = -0.000198324609702
d = 0.0218559686433
e = 0.387748406167
relative residual(4): 0.121558

a * x^5 + b * x^4 + c * x^3 + d * x^2 + e * x + f
a = 5.9204096951e-12
b = -5.84193407869e-09
c = 2.24838379059e-06
d = -0.000403281743497
e = 0.0320456311369
f = 0.267845555303
relative residual(5): 0.11797

3) result from numpy.linalg.lstsq

a * x + b
a = 0.00175825993442
b = 0.845568857432
relative residual(1): 0.172859

a * x^2 + b * x + c
a = 2.72895864077e-06
b = 0.000814040244717
c = 0.900176229486
relative residual(2): 0.171639

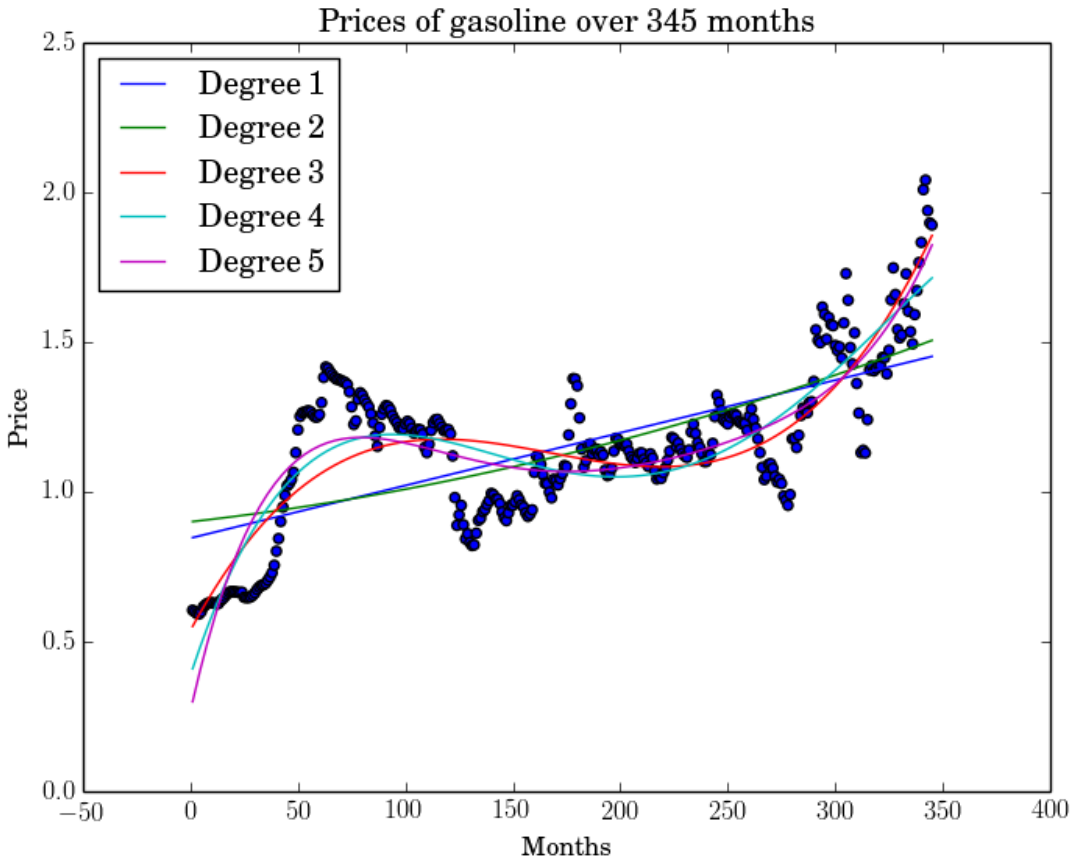
a * x^3 + b * x^2 + c * x + d
a = 1.73416859412e-07
b = -8.72743913939e-05
c = 0.0132885392428
d = 0.537894744939
relative residual(3): 0.128342

a * x^4 + b * x^3 + c * x^2 + d * x + e
a = -7.20779691366e-10
b = 6.72196404858e-07
c = -0.000198324608935
d = 0.021855968494
e = 0.387748411471
relative residual(4): 0.121558

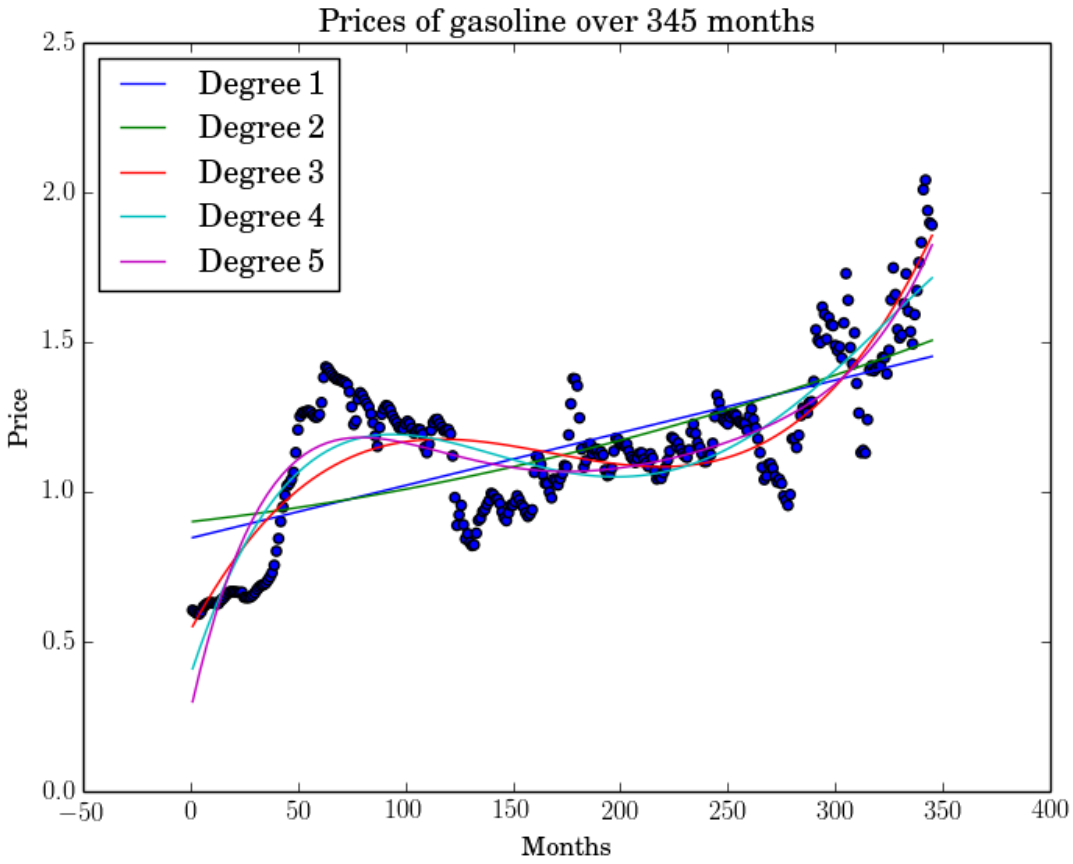
a * x^5 + b * x^4 + c * x^3 + d * x^2 + e * x + f
a = 5.92040649226e-12
b = -5.8419331224e-09
c = 2.24838318399e-06
d = -0.000403281763541
e = 0.0320457062595
f = 0.26784397237
relative residual(5): 0.11797

4) plots

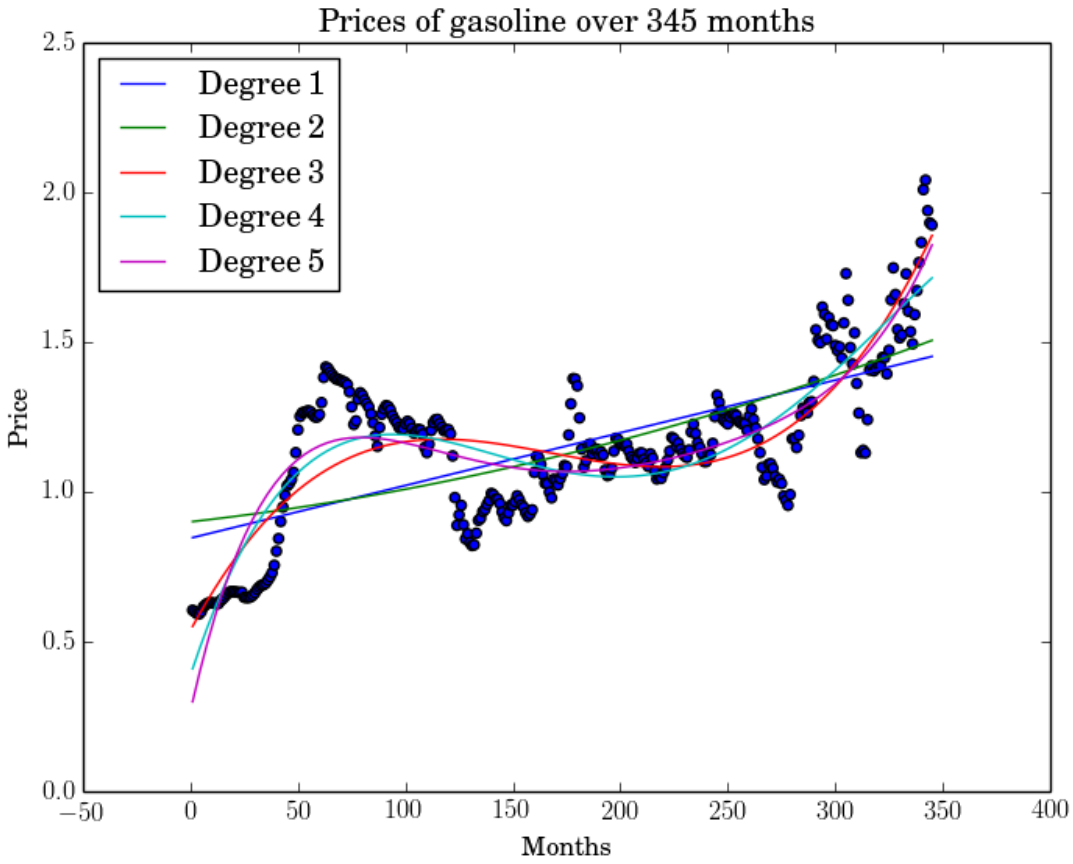
i) Plot of Gram-Schmidt



ii) Plot of House



iii) Plot of numpy.linalg.lstsq



5) Evaluation
    i) Methods differ in the relative error obtained.

        modified Gram-Schmidt procedure is more accurate than Householder reflectors

    ii) Polynomial degrees differ in the relative error obtained

        5th degree gives the best approximant since it result in the smallest relative error