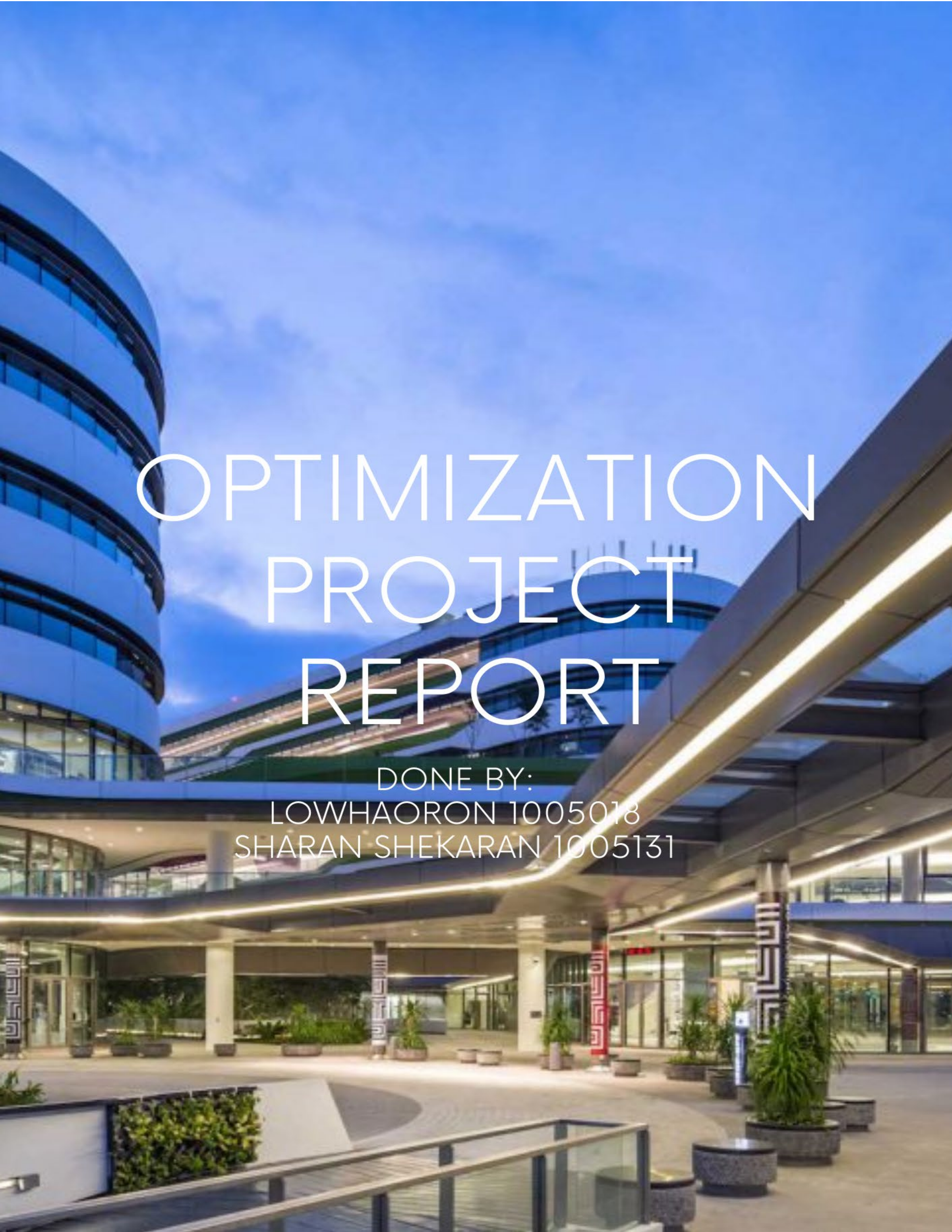

OPTIMIZATION PROJECT REPORT

DONE BY:
LOWHAORON 1005018
SHARAN SHEKARAN 1005131

Optimisation Project Report

Problem

Due to the rapid spread of Covid-19 virus, more people in Singapore have been in close contact with Covid-19 positive patients. Hence, to ensure that the rapid spread of Covid-19 virus is under control, Singapore have placed a stronger emphasis on the importance of self-testing with the use of self-test kits such as the Antigen Rapid Test Kit (ART kit). Singapore have set up 100 vending machines across Singapore for collection of Covid-19 self-test kits. The goal is to achieve the maximum value of vending machines based on the number of vending machines in each region of Singapore to cover the maximum population of Singapore given the number of dwelling units.

Model

Assume that the placement of one vending machine in a particular area does not affect the placement of a vending machine in another area (Independence). We must then solve the problem of which region of Singapore should be given priority in the allocation of the vending machines. This can be found by taking a ratio of the total number of dwelling units in a region to the total population of people living in that region. Let this ratio be C_i (unitless) where i = 1,…,5, be the coefficient of the decision variables X_i in the objective function which represents the number of vending machines to be allocated. The higher the C value, the higher the priority given to vending machine allocation. This can be modelled as an integer linear program problem. We can then solve this problem of allocating a particular number of vending machines to each specified area of Singapore by solving for the decision variable X_i.

Decision Variables:

- x_i: Number of vending machines allocated based on region of Singapore, $\mathrm{i} = 1,2,3,4,5$
- I: 1,2,3,4,5 represents the Central, East, North, Northeast, and West area respectively
- C_i: Ratio of the total number of dwelling units in a region to the total population of people
- y_i: Number of dwelling units in a region i of Singapore
- Z_i: Population size of people living in a region i of Singapore
- W_1: The number of people that 1 vending machine should at least cover
- W_2: The number of dwelling units that 1 vending machine should at least cover
- M_1: Maximum number of vending machines in each region based on population size
- M_2: Maximum number of vending machines in each region based on dwelling unit

Constraints:

One of the constraints is that the sum of the total vending machine allocated to each region of Singapore is within the current number of vending machines available in Singapore, 100.

$$X_1 + X_2 + X_3 + X_4 + X_5 \leq 100$$

Another constraint is that the value of vending machine allocated to each region must be a non-negative value.

$$X_1 , X_2 , X_3 , X_4 , X_5 \ \geq 0$$

Another constraint is that the value of vending machine allocated to each region must be of integer value. Since vending machines cannot be in the form of fractional value.

$$X_1, X_2, X_3, X_4, X_5 = Integer$$

Region	X Variable	Dwelling units	Population Size	C_i	M1	M2
Central	X1	1092816	922,580	1.184521667	22.81214069	20.08783151
East	X2	1092213	685,940	1.592286497	16.96087037	20.07674733
North	X3	1071283	582,330	1.83964934	14.39896149	19.69201805
NorthEast	X4	1094566	930,860	1.175865329	23.01687581	20.11999951
West	X5	1089311	922,540	1.180773733	22.81115164	20.0234036
SUM	NIL	5440189	4,044,250	1.345166347	100	100

Every 1 Vending machine should cover		
	40443	W1
	54402	W2

The ratio C_i, of the total number of dwelling units in a region to the total population of people is calculated by:

$$C_i = \frac{Dwelling\ units\ of\ region\ i}{Population\ size\ of\ region\ i}$$

Since currently we have 100 vending machines, hence one of our constraints is that 1 vending machine should be able to cover, W_1 , population of people. This number is derived from:

$$W_1 = \frac{Sum\ of\ population\ sizes\ of\ each\ region}{100}$$

$$W_1 = \frac{922580 + 685940 + 582330 + 930860 + 922540}{100}$$

$$W_1 = 40442.5 \approx 40443$$

Similarly, 1 vending machine must cover, W_2 , number of dwelling units. This number is derived from:

$$W_2 = \frac{Sum\ of\ dwelling\ units\ of\ each\ region}{100}$$

$$W_2 = \frac{1092816 + 1092213 + 1071283 + 1094566 + 1089311}{100}$$

$$W_2 = 54401.89 \approx 54402$$

Furthermore, to calculate M_1 , which is maximum number of vending machines in each region based on population size of region i, is calculated by:

$$M_1 = \frac{population\ size\ of\ region\ i}{W_1}$$

Similarly, to calculate M_2 , which is maximum number of vending machines in each region based on the dwelling units of region i, is calculated by:

$$M_2 = \frac{Dwelling\ units\ of\ region\ i}{W_2}$$

Region	X Variable	Dwelling units	Population Size	C_i	M1	M2
Central	X1	1092816	922,580	1.184521667	22.81214069	20.08783151
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SUM	NIL	5440189	4,044,250	1.345166347	100	100

Every 1 Vending machine should cover		
	40443	W1
	54402	W2

Another constraint is that for each region of Singapore, there will be a maximum number of vending machines installed in each region of Singapore. This is determined by the maximum of vending machines of each region based on its population size and the maximum number of vending machines of each region based on dwelling units. Taking the higher value of the two comparisons, as shown by the highlighted yellow value in the diagram below, would set the upper limit for our ILP model, so as to ensure that we do not allocate vending machines that exceeds these range in each region of Singapore.

$$X_1 \leq 23\ (rounded\ to\ nearest\ integer)$$

$$X_2 \leq 20\ (rounded\ to\ nearest\ integer)$$

$$X_3 \leq 20\ (rounded\ to\ nearest\ integer)$$

$$X_4 \leq 23\ (rounded\ to\ nearest\ integer)$$

$$X_5 \leq 23\ (rounded\ to\ nearest\ integer)$$

Objective Function: (ILP)

$$Maximise\ C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 + C_5X_5$$

Where $C_1 = 1.1845$, $C_2 = 1.5922$, $C_3 = 1.8396$, $C_4 = 1.8396$, $C_5 = 1.8396$

Correctness of model

Each feasible solution $x = x_1,\ldots,x_5$ represents the number of vending machines that should be allocated to each of the areas in order to maximise test kit distribution. The distribution that was found considers the constraints above where the sum of all the vending machines cannot exceed 100. The maximum number of vending machines has also been determined based on the C_1, which is the ratio of dwelling units and population size.

Solving an instance using JuMP

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In [1]:

```julia
using JuMP, MosekTools
```

In [2]:

```julia
m = Model(Mosek.Optimizer)

#Variable
@variable(m,x[1:5],Int);

#constraints
@constraint(m,x[1]+x[2]+x[3]+x[4]+x[5]<=100);

@constraint(m,x[1]>=0);
@constraint(m,x[1]<=23);

@constraint(m,x[2]>=0);
@constraint(m,x[2]<=20);

@constraint(m,x[3]>=0);
@constraint(m,x[3]<=20);

@constraint(m,x[4]>=0);
@constraint(m,x[4]<=23);

@constraint(m,x[5]>=0);
@constraint(m,x[5]<=23);

#objective
@objective(m,Max,1.1845*x[1]+1.5922*x[2]+1.8396*x[3]+1.1758*x[4]+1.1807*x[5]);
print(m)
```

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```julia
#objective
@objective(m,Max,1.1845*x[1]+1.5922*x[2]+1.8396*x[3]+1.1758*x[4]+1.1807*x[5]);
print(m)
```


$$
\begin{aligned}
\max \quad & 1.1845x_1 + 1.5922x_2 + 1.8396x_3 + 1.1758x_4 + 1.1807x_5 \\
\text{Subject to} \quad & x_1 \geq 0.0 \\
& x_2 \geq 0.0 \\
& x_3 \geq 0.0 \\
& x_4 \geq 0.0 \\
& x_5 \geq 0.0 \\
& x_1 + x_2 + x_3 + x_4 + x_5 \leq 100.0 \\
& x_1 \leq 23.0 \\
& x_2 \leq 20.0 \\
& x_3 \leq 20.0 \\
& x_4 \leq 23.0 \\
& x_5 \leq 23.0 \\
& x_1 \in \mathbb{Z} \\
& x_2 \in \mathbb{Z} \\
& x_3 \in \mathbb{Z} \\
& x_4 \in \mathbb{Z} \\
& x_5 \in \mathbb{Z}
\end{aligned}
$$

In [4]:
```
1 solution_summary(m)
```

```
MOSEK error 2950: No dual information is available for the integer solution.
```

Out[4]:
```
* Solver : Mosek

* Status
  Termination status : OPTIMAL
  Primal status      : FEASIBLE_POINT
  Dual status        : NO_SOLUTION
  Message from the solver:
  "Mosek.MSK_SOL_STA_INTEGER_OPTIMAL"

* Candidate solution
  Objective value      : 139.4968
  Objective bound      : 139.4968

* Work counters
  Solve time (sec)   : 0.15600
  Simplex iterations : 0
  Barrier iterations : 6
  Node count         : 0
```

In [5]:
```
1 value.(x)
```

Out[5]:
```
5-element Vector{Float64}:
 23.0
 20.0
 20.0
 14.0
 23.0
```

In [6]:
```
1 objective_value(m)
```

Out[6]:
```
139.4968
```

The Singapore government can consider deploying 23 vending machines in Central, 20 vending machines in East, 20 vending machines in North, 14 vending machines in Northeast, and 23 vending machines in West area when taking the population size and dwelling units in the area into account.

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