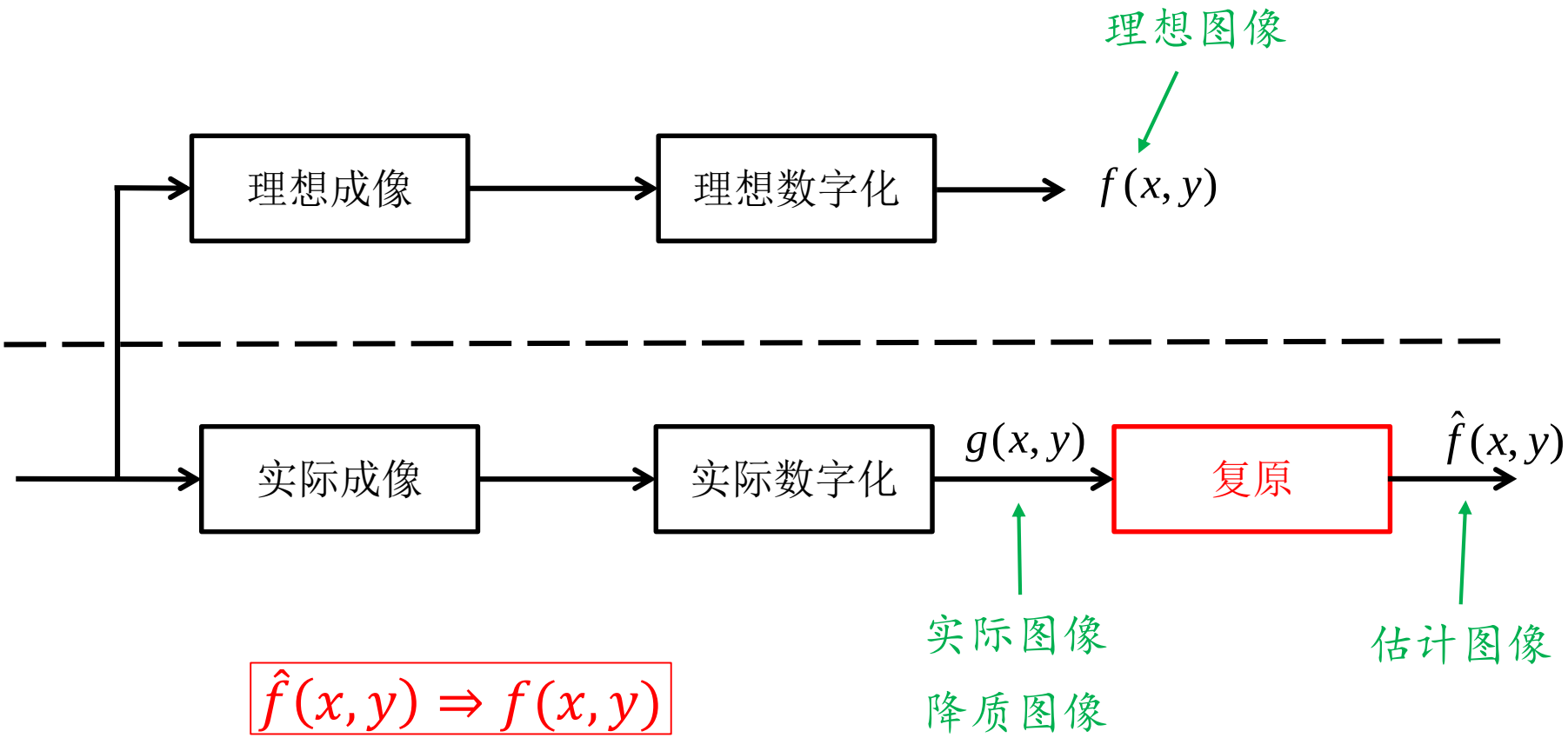
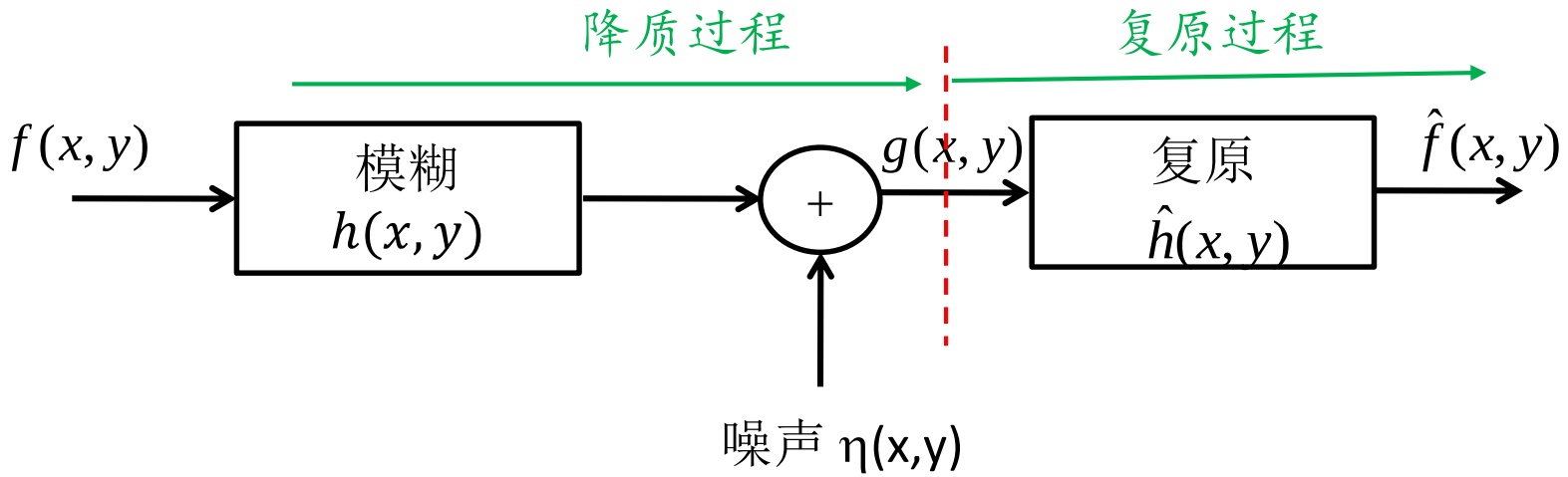
图像复原
Image Restoration

成像降质模型



$$\hat{f}(x,y) \Rightarrow f(x,y)$$

线性模糊模型



$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$$

$$G(u,v)=H(u,v)\cdot F(u,v)+N(u,v)$$

$$\hat{f}(x,y)=\hat{h}(x,y)*g(x,y)=\hat{h}(x,y)*[h(x,y)*f(x,y)+\eta(x,y)]$$

$$\hat{F}(u,v)=\hat{H}(u,v)\cdot G(u,v)=\hat{H}(u,v)\cdot[H(u,v)\cdot F(u,v)+N(u,v)]$$

逆滤波（Inverse Filter）

$$\hat{F}(u,v)=\hat{H}(u,v)\cdot G(u,v)=\hat{H}(u,v)\cdot[H(u,v)\cdot F(u,v)+N(u,v)]$$

如果 $N(u,v)=0$，并且$\hat{F}(u,v)=F(u,v)$

$$\hat{H}(u,v)\cdot H(u,v)=1 \Rightarrow \hat{H}(u,v)=\frac{1}{H(u,v)}$$

考虑噪声存在时，逆滤波：

$$\hat{F}(u,v)=F(u,v)+\frac{N(u,v)}{H(u,v)}$$

$H(u,v)\to 0$ 噪声严重放大

Wiener Filter

$$E\{|f(x,y)-\hat{f}(x,y)|^2\}\rightarrow \min$$

$$\hat{F}(u,v)=\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2+S_n(u,v)}G(u,v)$$

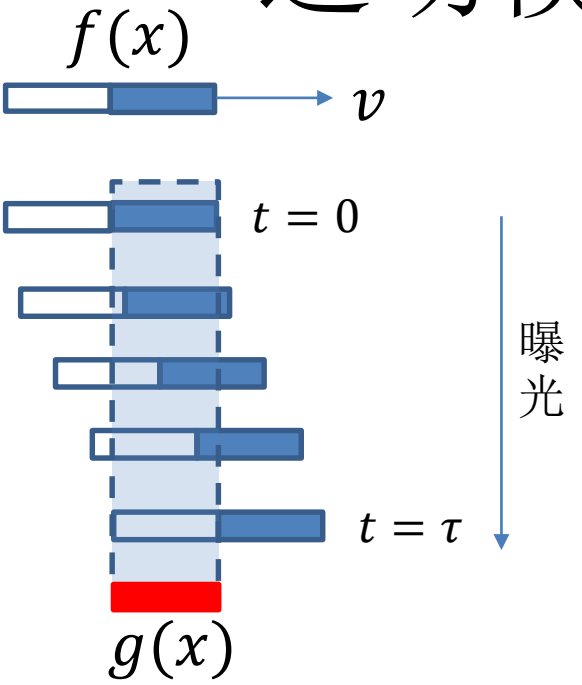
$$\hat{F}(u,v)=\frac{H^*(u,v)}{|H(u,v)|^2+S_n(u,v)/S_f(u,v)}G(u,v)$$

$$S_n(u,v)=|N(u,v)|^2$$

$$S_f(u,v)=|F(u,v)|^2$$

简化 $K=S_n(u,v)/S_f(u,v)$

运动模糊（匀速直线运动）



$$g(x)=\int_0^{\tau} f(x-vt)dt$$

τ: 曝光时间

v: 运动速度

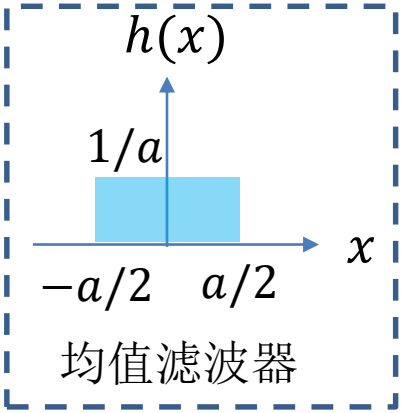
$$g(x)=\int_0^{a} f(x-s)ds$$

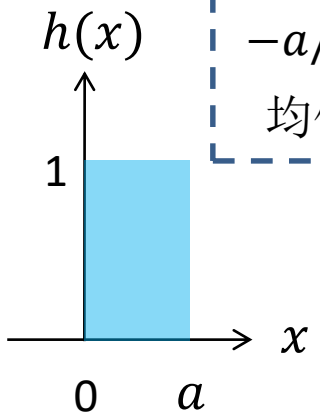
$a=v\tau$: 位移

$$g(x)=\int_{-\infty}^{\infty} h(s)f(x-s)ds$$

$$h(s)=\begin{cases}1 & (0\le s\le a)\\ 0 & otherwise\end{cases}$$

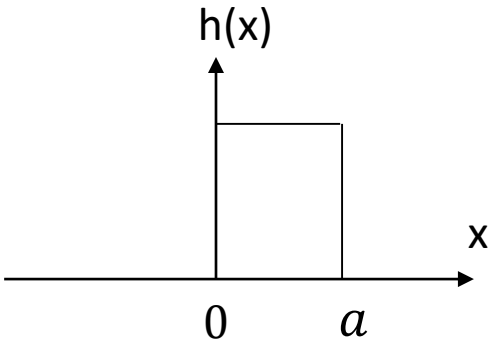
$$g(x)=h(x)*f(x)$$

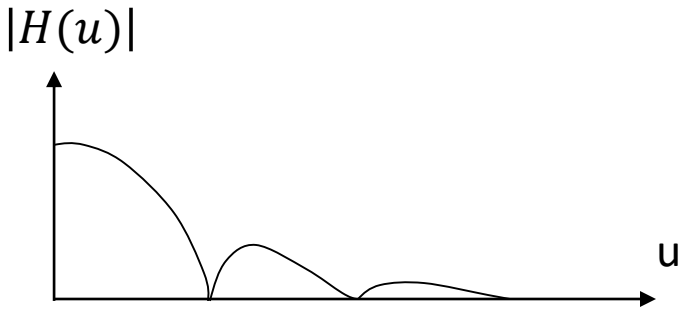




运动参数估计

Cannon Method





$$H(u)=\int_0^a e^{-j2\pi ux}dx=\frac{e^{-j2\pi au}-1}{-j2\pi u}=e^{-j\pi au}\frac{e^{j\pi au}-e^{-j\pi au}}{2j\pi u}=e^{-j\pi au}\frac{\sin(au\pi)}{\pi u}$$

$$|H(u)|=\frac{|\sin(au\pi)|}{\pi u}$$

Cannon方法

假设运动发生在x方向上

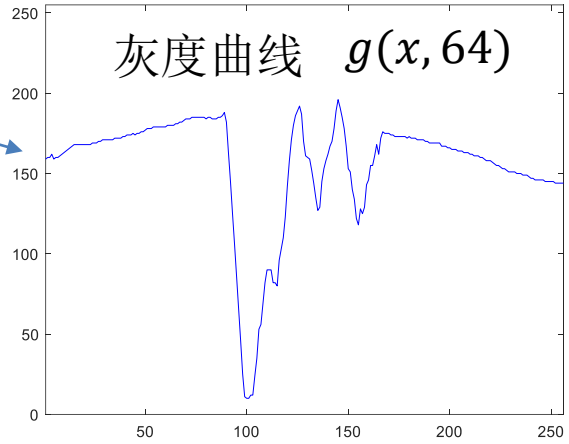
理想图像 $f(x,y)$　第y行的频谱记为　$\mathrm{F}(u,y)$

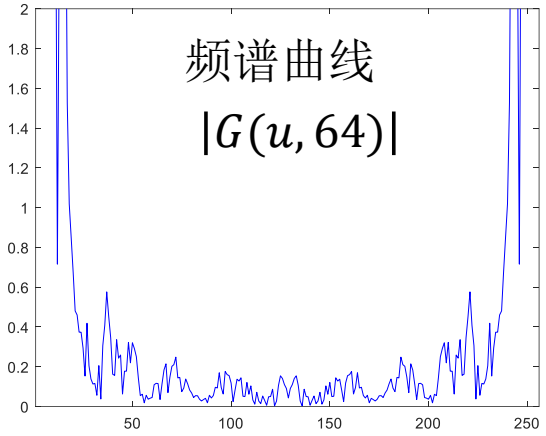
实际图像 $g(x,y)$　第y行的频谱记为　$G(u,y)$

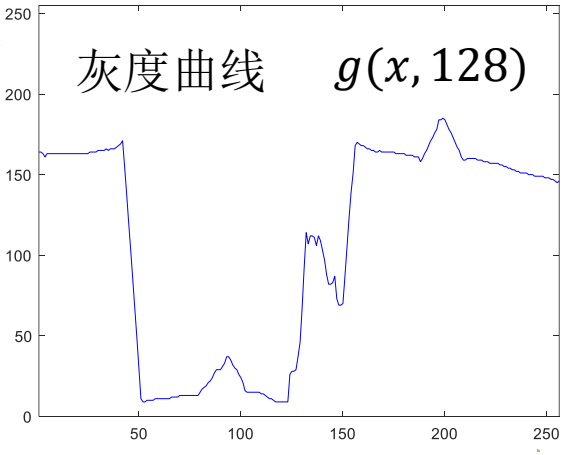
$$|G(u,y)| = |H(u)F(u,y)| = |H(u)||F(u,y)|$$

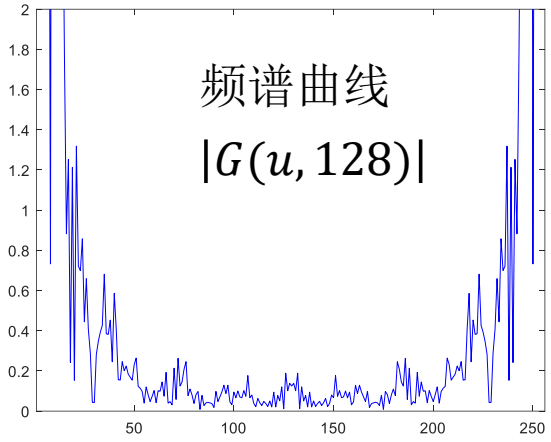
$$|G(u)| = \sum_y |G(u,y)| = |H(u)| \sum_y |F(u,y)|$$

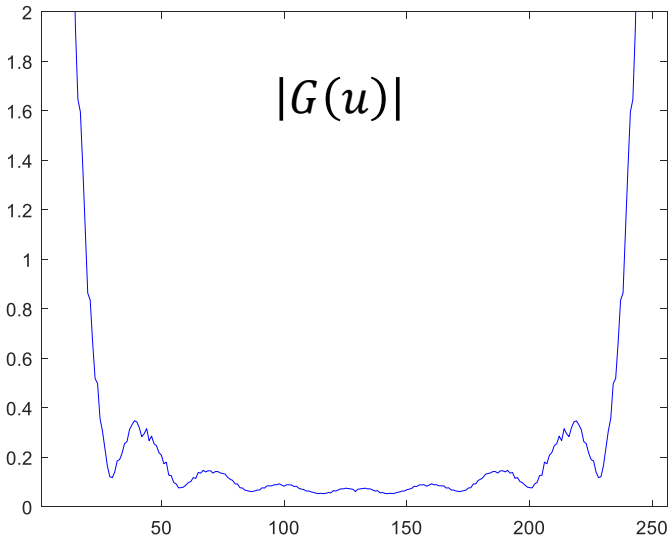
$|G(u)|$的周期性0点由 $|H(u)|$ 决定


灰度曲线 $g(x,64)$


频谱曲线
$|G(u,64)|$


灰度曲线 $g(x,128)$


频谱曲线
$|G(u,128)|$


$|G(u)|$

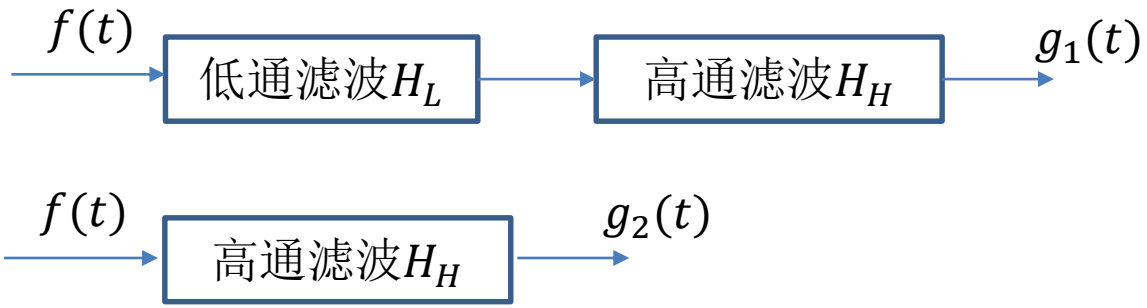
Identification of blur parameters from motion blurred images

Y. Yitzhaky and N.S. Kopeika

Graphical Models and Image Processing, 59(5), 1997:310-320

判断运动方向方法

算法依据：

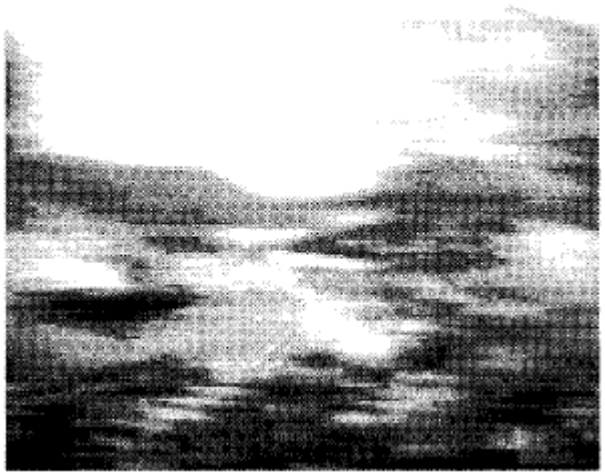


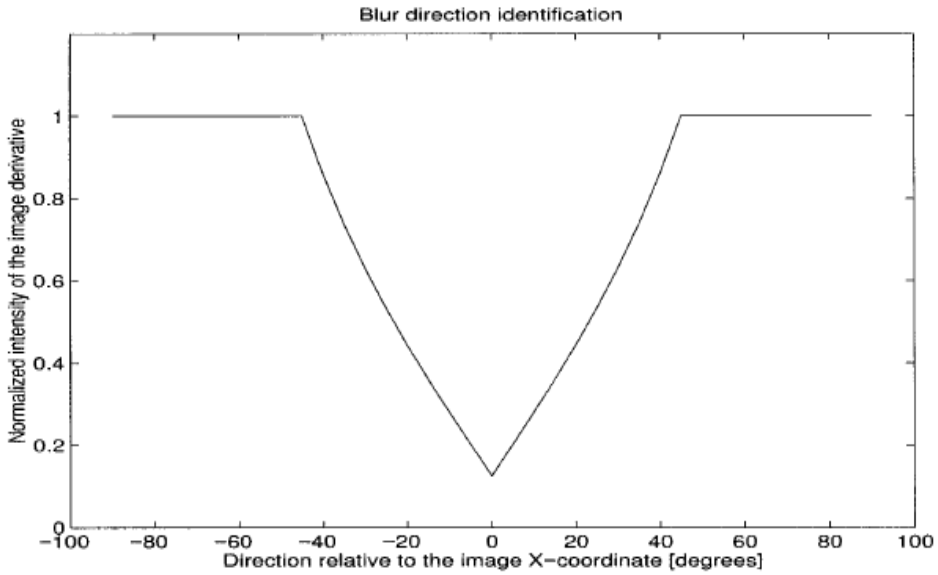
$$\int |g_1(t)|^2 dt < \int |g_2(t)|^2 dt$$

运动模糊等同于均值滤波 -> 低通滤波

在所有方向上做微分（高通滤波）并计算结果图像的能量
在运动方向上获得能量极小值

水平方向运动





$$I(\Delta f)_{[k^\circ]} = \sum_i \sum_j \left| \Delta f(i,j)_{[k^\circ]} \right|$$