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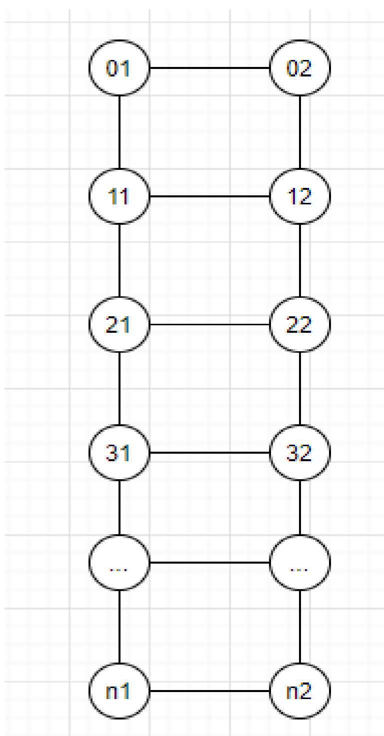
CS 453

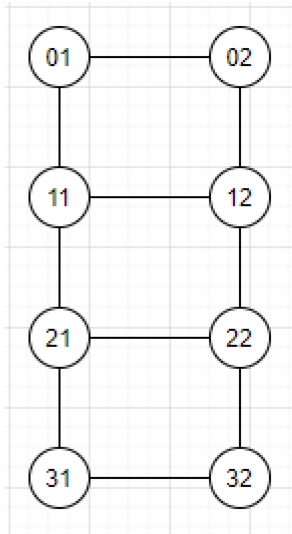
Assignment 5

1.
   a. Each vertex in G has degree of 4, and G is an even graph. Also, since every vertex has a path connect to every other vertex, it is a connected graph. Based on the theorem, G admits a Euler circuit. This means G is Eulerian.
   b. To prove that G is Hamiltonian, we need to show that there is a Hamiltonian cycle in G. Here is the order of vertices in cycle V_{ham} : [00, 01, 02, 12, 11, 21, 22, 20, 10, 00]. Therefore, G is Hamiltonian.

2.
   a. If $n \geq 3\ or\ q \geq 3, then\ each\ vertex\ in\ P_{n,q}$ is not guaranteed to be a connected even graph when some vertices have odd degree. Since only the connected even graph can admit the Euler circuit, $P_{n,q}$ does not contain the Euler circuit, and therefore, not Eulerian graph. For instance, both $P_{3,2}$ and $P_{2,3}$ contains two odd vertices.
   b. To be a Hamiltonian graph, the graph must contain a Hamiltonian cycle. For $P_{2,q}$, q⩾2, there is at least one Hamiltonian cycle. For instance:



   Then a valid Hamiltonian cycle would be: [01,11, 21, 31, … n1, n2, … 32, 22, 12, 02, 01], and this kind of cycle exists in every $P_{2,q}$ graph. Therefore, $P_{2,q}$, $q \geq 2$ is a Hamiltonian graph.

c. Graph $P_{4,4}$ looks like this: 

A Hamilton cycle would be [01, 11, 21, 31, 32, 22, 12, 02,01]. Therefore, $P_{4,4}$ is a Hamiltonian graph.

d. Based on the given graph, red vertices have no direct edge connection to blue nodes. This means $P_{3,3}$ is a bipartite graph. Based on the property of the bipartite graph, the graph can not contain cycles with odd number of vertices. Since $P_{3,3}$ has odd number of vertices. It means there is no cycle that covers all vertices in $P_{3,3}$, and therefore, $P_{3,3}$ has no Hamilton cycle. So $P_{3,3}$ is not Hamiltonian.

e. If $n\ and\ q\ are\ both\ odd$, then $P_{n,q}$ can only contain odd vertices. By using prove from the previous problem, $P_{n,q}$ is also a bipartite graph contains odd number of vertices and therefore, is not Hamiltonian.

3.

a. K' = K, M' = M -1, N' = N
K' = N' – M'
Replace K', N' and M' with K, N, and M:
K = N – M +1

b. K' = K, M' = M-ℓ, N' = N.
N' = M' + K'
Replace K', N' and M' with K, N and M:
N = (M- ℓ) + K

c. Skipped

d. 2

e. 6