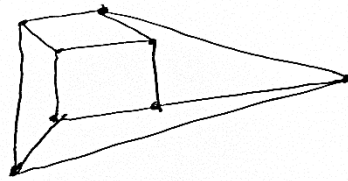Haorui Zhang

11-15-2021

CS 453

1.

    a. Plane is a drawing of a planar graph without edges crossing. If Q3 can be drawn on a plane, then it needs to be drawn without edge crossing. Q3 can be drawn without edge crossing, and therefore, it's a planar graph and can be drawn on the plane.
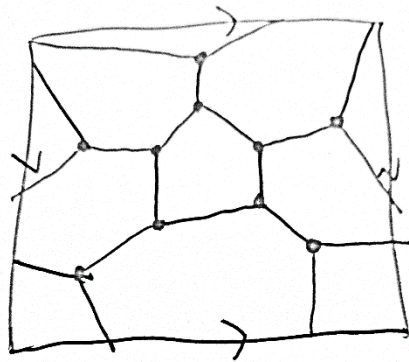

    b. Q4 is bipartite and it has 16 vertices and 32 edges. Based on Euler's formula, A bipartite planar graph should have property of $e \leq 2v-4$. But here, $2v-4 = 28 < 32$. Therefore, Q4 violates this property, and therefore, not a planar graph. This means it can not be drawn on the plane.
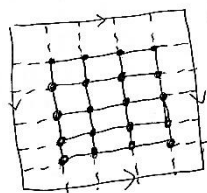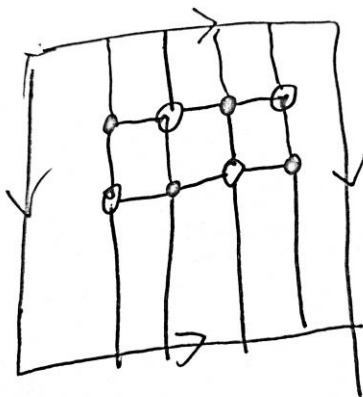

    c. Q4:

    d. For Q5, n=32, m=80, using the total edge count argument, the number of edges is 40. Since there are only 40 edges, it can not be more than 40 regions. Then using given Euler's formula: $n-m+r = 2 -2 g(Q5)$, $2g(Q5) = 2-r+m-n$. $g(Q5) = 1-16+20+r$. For r= 40, the max number of regions in Q5, $g(Q5) = 5+r =5+40=54$. Therefore, $g(Q5) \geq 5$.

2.

    a. For P, if it is planar, then $n-m+r=2$. N= 10, m=15, then r = 7. The region in P needs at least 5 edges, 4 vertices. Therefore, to have 7 regions, it needs 35 edges or more. (17 edges based on edge count), which is not 15 and therefore, P is planar.

b.

3.