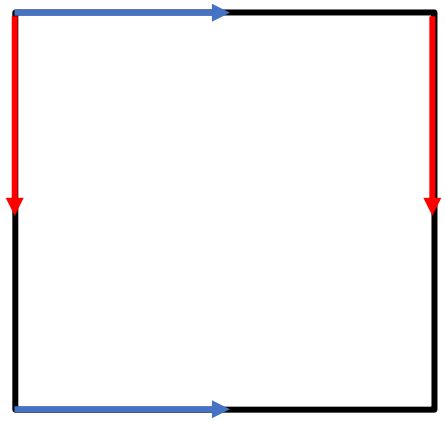
Graph Theory Fall 2021

Assignment 7

Due at 5:00 pm on Wednesday, November 17

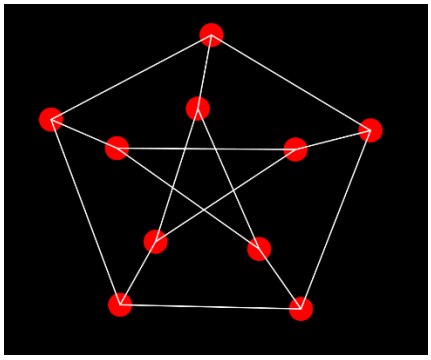
Questions with a ($\star$) are each worth 1 bonus point for 453 students.

1. Some questions about drawing Q_k graphs on surfaces.
   A. Show that Q_3 can be drawn on the plane.
   B. Use the fact that Q_4 is bipartite and a total edge count argument to show that Q_4 cannot be drawn on the plane.
   C. Draw Q_4 on the torus (use the $aba^{-1}b^{-1}$ square representation drawn below). HINT: Q_4 is isomorphic to $C_4 \times C_4$.

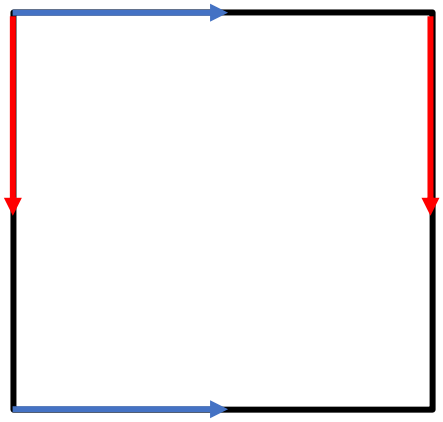


   D. Show that $g(Q_5) \geq 5$. Recall that for Q_5 we have $n = 32$ and $m = 80$. As a first step, use a total edge count argument to show that $r \leq 40$. Feed this information into Euler's formula
$$n - m + r = 2 - 2\,g(Q_5).$$
   E. ($\star$) Generalize the strategy in part D to obtain a "meaningful" lower bound for $g(Q_k)$. Here, recall that $n = 2^k$ and $m = k2^{k-1}$.

2. The Petersen graph P is depicted:



A. Use a total edge count argument to show that P is non-planar. You may use the fact that P has no triangles or 4-cycles as subgraphs.
B. Draw P on a torus without the edges crossing.



3. Draw $K_{4,4}$ on a torus without the edges crossing. Suggestion: Start with your two partite sets (every edge joins a red vertex to a blue vertex) arranged as shown: