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Assignment 6

CS 453

1. We can prove that for any k$\geqslant$ 3, if a tree T has fewer than k leaves, then the max degree $\Delta(T)$ among the vertices of T must satisfy $\Delta(T) < k$. For n = number of vertices, $and\ n_i = number\ of\ vertices\ with\ degree\ of\ i$. If a tree T has fewer than k leaves, then $n_1 < k$ and n_1 is the number of leaves in the T.
   Let for all j $\geqslant$ k, $n_j = 0$. This means n = $\sum_{j=1}^{\infty} n_j = 0$. Then $\sum_{j=1}^{n}(2-j)n_j = 0$. This means that the total degree with vertices with degrees j $\geqslant$ k doesn't exist. Therefore, if a tree T has fewer than k leaves, then the max degree $\Delta(T)$ among the vertices of T must satisfy $\Delta(T) < k$.
2.
   a. Since r is the root, $L(u) = D(r,u)\ and\ L(v) = D(r,v)$. $D(u,v)$ is the length of the unique u-v path. If r is on the unique u, v path, then r is an ancestor of v. Therefore, $D(u,v) = D(u,r) + D(r,v)$. Since r is root, $D(u,r)\ is\ essentailly\ same\ as\ D(r,u)\ and\ D(u,v) = D(r,u) + D(r,v) = L(u) + L(v)$
   b. If $L(u) + L(v) = D(u,v)$, then $D(u,v) = D(r,u) + D(r,v)$. Since both $D(r,u)$ and $D(r,v)$ passes through r, r must be on the unique u,v-path.
   c. If D(u,v) = 2H, then it means u and v have the maximum of levels of tree. It means they are both the end of the tree. Therefore, they are leaves and they must be non-parents.
   d. If $D(u,v) = 2H, then\ u, and\ v$ must be two leaves with the maximum of levels of vertices. If either of u, v is a parent, then one of they have at least one child and can't has level of less than H. Therefore, $D(u,v) < 2H\ if\ either\ u, or\ v\ is\ a\ parent$. So if $D(u,v) = 2H$, then u and v must be non-parents.
3.
   a. If b is the number of parents in the tree, then for each parent node in this tree, there are q edges and q children nodes connect to this node. Each children node, if they are not parent, have no edge connect to them. Therefore, there are $q * b + 0 * n$ edges in this tree, for n leaf nodes.
   b. For q-ary tree and b parents, there are $(q-1) * b + 1$ non-parents node. Therefore, there are $b + (q-1) * b + 1$ vertices.
   c. For q-ary tree and b parents, there are $(q-1) * b + 1\ non-parents\ nodes$.
4. There are $10^{12} + 1\ nodes$
   a. The height of the T:
      i. Lower bound: 1
      ii. Upper bound: 10^{12}
   b. The height of the saturated tree T:
      i. Lower bound: $floor(\log_2(10^{12}))$
      ii. Upper bound: $(10^{12} + 1)/2$