Graph Theory Fall 2021

Assignment 2

Due at 5:00 pm on Wednesday, September 8

1. Among all simple graphs (no loops, no parallel edges) with $n = 100$ vertices, determine (with justification) the minimum possible and the maximum possible values for m, the number of edges such a graph could have.

2. Let G be the simple graph with vertex set

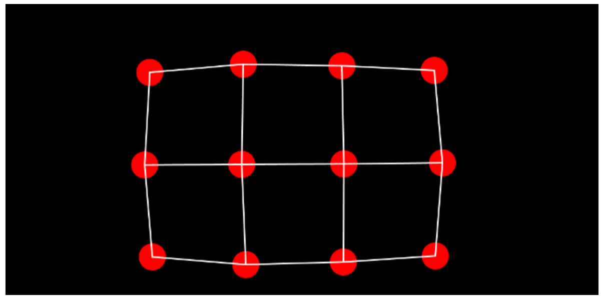
$$V = \{00, 01, 02, 10, 11, 12, 20, 21, 22\}; \quad |V| = 9$$

and where vertices ab and cd are joined by an edge when exactly one of the following conditions holds (so there are no loops):

$$a = c \text{ or } b = d.$$

A. Sketch G; you are allowed to do this by hand and then copy your sketch electronically into your PDF submission.
B. Determine m, the number of edges of G.

3. The “grid graph” $P_{r,s}$ is the cartesian product of the two paths P_r and P_s. For instance, $P_{3,4}$ is drawn below:



   A. In terms of r and s, find a formula for the number of vertices of the grid graph $P_{r,s}$.
   B. In terms of r and s, find a formula for the number of edges of the grid graph $P_{r,s}$.

4. Recall that a graph G is “cubic” if and only if it is 3-regular.
   A. Show that there exists no cubic graph with an odd number of vertices.
   B. For every integer $n \geq 3$, show that there exists a simple cubic graph (no loops, no parallel edges) with $2n$ vertices. One way to do this is to produce a construction, i.e., give a set of $2n$ vertices and a recipe for when vertices are joined by edges for constructing such graphs.