Haorui Zhang

8/26/2021

Assignment 1

1. Let $G_{1,2}$ be the graph on two vertices with one edge.

$$L_{G_{1,2}} := \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

As we see $x^T L_{G_{1,2}} x = (x_1 - x_2)^2$, where $x = (x_1, x_2)$.

For any graph $G_{u,v}$, we define:

$$L_{G_{u,w}}(i,j) = \{ \begin{matrix} 1 & if\ i = j\ and\ i \in \{u, w\} \\ -1 & if\ i = u\ and\ j = w, or\ j = w\ and\ j = u \\ 0 & otherwise \end{matrix}$$

For a graph $G = (V, E)$ we define the Laplacian matrix as follows:

$$L_G = \sum_{\{u,w\} \in E} L_{G_{u,w}}$$

2. As with circulations, the set $\mathfrak{B}$ of all potential differences in D is closed under addition and scalar multiplication and, hence, is a vector space.

Analogous to the function f_c associated with a cycle C, there is a function g_B associated with a bond B. Let B = [S, $\bar{S}$] be a bound of D. We define g_B by

$$g_n(a) = \{ \begin{matrix} 1 & if & a \in (S, \bar{S}) \\ -1 & if & a \in (\bar{S}, S) \\ 0 & if & a \notin B \end{matrix}$$

It can be verified that $g_B = \delta_p\ where$

$$p(v) = \{ \begin{matrix} 1 & if & v \in S \\ 0 & if & v \in \bar{S} \end{matrix}$$