HW4

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CS 453

1.

a.

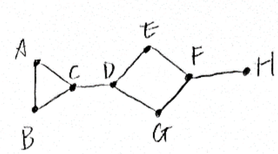
	1	2	3	4	5	6	7
1	0	1	0	1	0	1	0
2	1	0	1	0	1	0	1
3	0	1	0	1	0	1	0
4	1	0	1	0	1	0	1
5	0	1	0	1	0	1	0
6	1	0	1	0	1	0	1
7	0	1	0	1	0	1	0

b.

	1	2	3	4	5	6	7
1	0	0	0	1	1	1	1
2	0	0	0	1	1	1	1
3	0	0	0	1	1	1	1
4	1	1	1	0	0	0	0
5	1	1	1	0	0	0	0
6	1	1	1	0	0	0	0
7	1	1	1	0	0	0	0

2.

a.



ecc(A) = 5   ecc(C) = 4
ecc(B) = 5   ecc(D) = 3
ecc(E) = 3   ecc(G) = 3
ecc(F) = 4   ecc(H) = 5

b. Diameter = 5, radius = 3

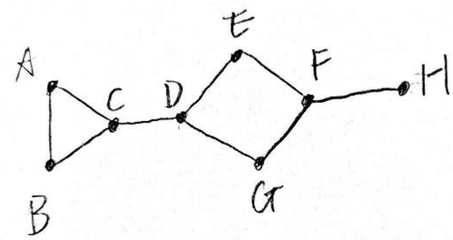
c. Central vertices are E, D, G.

d. Peripheral vertices are A, B, H.

e. With eccentricity, diameter, and radius, we can determine if this graph has only one component.

f. Since the radius is the shortest one of eccentricity in any connected graph G and the diameter is the longest one of eccentricity in G. Therefore, radius(G) $\leqslant$ diameter(G). To prove diameter(G) $\leqslant$ 2radius(G), let vertices u, v in G such that D(u,v) = diameter(G). Set w be the central vertex of G such that ecc(w) = radius(G). Since ecc(w) = radius(G), it means d(u,w) $\leqslant$rad(G) and d(v,w) $\leqslant$ rad(G). Therefore, d(u,w)+d(v,w) $\leqslant$ 2rad(G) and if we view vertices u, v, w forms a triangle, then d(u,v) = diameter(G) $\leqslant$ d(u+w)+d(v,w) $\leqslant$2rad(G). Therefore, radius(G) $\leqslant$ diameter(G) $\leqslant$ 2radius(G).

3.



Bridges: CD, FH
Rest edges are Links

a.
b. 3 components remain
c. 2

4.

a. For all vertices u, u~u. Therefore, it's reflexive
b. For all vertices u and v, if u ~ v, then v ~ u. Therefore, it's symmetric
c. For all vertices u, v, w, if u ~ v and v ~ w, then u ~ w. Therefore, it's transitive.
d. [u] is the set of all vertices connected to u. [u] can not be empty. For two equivalence classes [u] and [w], either $[u] \cap [w] = \emptyset$ or $[u] = [w]$ $as\ sets$. And if $w \in [u]$, $then$ $[u] = [w]$. In this case, [u] = x.