

Graph Theory Fall 2021

Assignment 1

Due at 5:00 pm on Monday, August 30

1. Typeset the following passage using either a LaTeX editor or MSWord and the Equation Editor “alt=” shortcuts:

Let $G_{1,2}$ be the graph on two vertices with one edge.

$$L_{G_{1,2}} := \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

As we see $x^T L_{G_{1,2}} x = (x_1 - x_2)^2$, where $x = (x_1, x_2)$.

For any graph $G_{u,v}$ we define:

$$L_{G_{u,w}}(i, j) = \begin{cases} 1 & \text{if } i = j \text{ and } i \in \{u, w\} \\ -1 & \text{if } i = u \text{ and } j = w, \text{ or } i = w \text{ and } j = u \\ 0 & \text{otherwise.} \end{cases}$$

For a graph $G = (V, E)$ we define the Laplacian matrix as follows:

$$L_G = \sum_{\{u,w\} \in E} L_{G_{u,w}}.$$

2. Typeset the following passage using either a LaTeX editor or MSWord and the Equation Editor “alt=” shortcuts:

As with circulations, the set \mathcal{B} of all potential differences in D is closed under addition and scalar multiplication and, hence, is a vector space.

Analogous to the function f_C associated with a cycle C , there is a function g_B associated with a bond B . Let $B = [S, \bar{S}]$ be a bond of D . We define g_B by

$$g_B(a) = \begin{cases} 1 & \text{if } a \in (S, \bar{S}) \\ -1 & \text{if } a \in (\bar{S}, S) \\ 0 & \text{if } a \notin B \end{cases}$$

It can be verified that $g_B = \delta p$ where

$$p(v) = \begin{cases} 1 & \text{if } v \in S \\ 0 & \text{if } v \in \bar{S} \end{cases}$$