Haorui Zhang

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CS 453

1.

a. The number of $V(C_n * C_n) = n * n$

b. To prove that $C_n * C_n$ is a 4-regular graph, we need to show that for any vertex (a, b) of the graph $C_n * C_n$, (a, b) is adjacent to exactly other four vertices. For two vertices (a, b) and (c, d) be adjacent in $C_n * C_n$, either $a = c \text{ and } b \text{ is adjacent to } d$, or $b = d \text{ and } a \text{ is adjacent to } c$. Since the $C_n$is a cycle, it is a 2-regular graph, and each vertex is adjacent to vertices next to it. For $C_n * C_n$, each vertex in $G_1$is adjacent to the vertices which the corresponding vertex in $G_2$is adjacent to, which is another 2 vertices. Therefore, $C_n * C_n$is a 4-regular graph for n⩾3

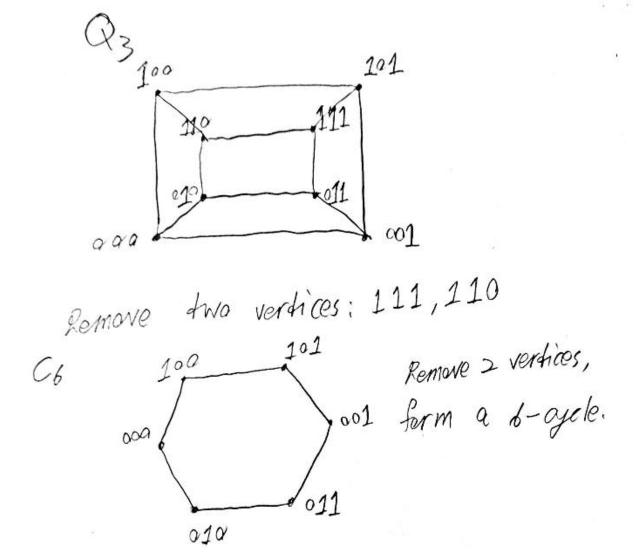
c. Number of edges in $C_n = n$, and number of edges of $C_n * C_n = n * n + n * n = 2n^2$.

2.

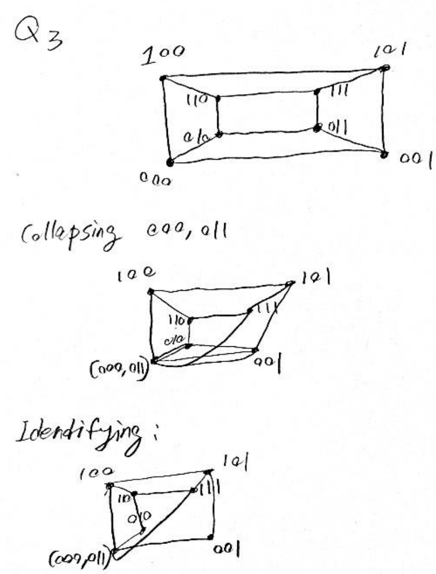
a. Assume $G = (V_1, E_1)$ is a $n - regular\ graph$ and $H = (V_2, E_2)$ is a $m - regular\ graph$, then $G * H = (V_1 * V_2, E)$. $E = \{(v_1, v_2)(u_1, u_2) |\ (v_1 = u_1 \text{ and } v_2u_2 \in E_2) or (v_2 = u_2 \text{ and } v_1u_1 \in E_1)\}$. So for (v_1, v_2), there are m edges from E_2 and n edges from E_1. Therefore, G*H is a (n+m)-regular graph.

3.

a.



b.



W has 4-degree.

4.

a. Since W has a length of k and W' has a length of k+1, for any k ⩾ 1, it means that W' always has one more vertex connects in the walk than W, which means the graph contains loop or cycle. Therefore, G can not have a longest walk.

b. A path is a walk does not repeat a vertex. Therefore, if W is a walk in the graph G has length of k, which is larger than b the upper bound length of path in G, it means W must contains one or more repeated nodes since the path does not contain repeat nodes and b is the length of walk that does not contain repeat nodes.