Haorui Zhang

CS 453

Assignment 5

* 1. Each vertex in G has degree of 4, and G is an even graph. Also, since every vertex has a path connect to every other vertex, it is a connected graph. Based on the theorem, G admits a Euler circuit. This means G is Eulerian.
  2. To prove that G is Hamiltonian, we need to show that there is a Hamiltonian cycle in G. Here is the order of vertices in cycle : [00, 01, 02, 12, 11, 21, 22, 20, 10, 00]. Therefore, G is Hamiltonian.
  3. If is not guaranteed to be a connected even graph when some vertices have odd degree. Since only the connected even graph can admit the Euler circuit, does not contain the Euler circuit, and therefore, not Eulerian graph. For instance, both and contains two odd vertices.
  4. To be a Hamiltonian graph, the graph must contain a Hamiltonian cycle. For , q≥2, there is at least one Hamiltonian cycle. For instance:

A graph with numbers and symbols

Description automatically generated with low confidence

Then a valid Hamiltonian cycle would be: [01,11, 21, 31, … n1, n2, … 32, 22, 12, 02, 01], and this kind of cycle exists in every graph. Therefore, is a Hamiltonian graph.

* 1. Graph looks like this: A picture containing wire

     Description automatically generated A Hamilton cycle would be [01, 11, 21, 31, 32, 22, 12, 02,01]. Therefore, is a Hamiltonian graph.
  2. Based on the given graph, red vertices have no direct edge connection to blue nodes. This means is a bipartite graph. Based on the property of the bipartite graph, the graph can not contain cycles with odd number of vertices. Since has odd number of vertices. It means there is no cycle that covers all vertices in , and therefore, has no Hamilton cycle. So is not Hamiltonian.
  3. If then can only contain odd vertices. By using prove from the previous problem, is also a bipartite graph contains odd number of vertices and therefore, is not Hamiltonian.
  4. K’ = K, M’ = M -1, N’ = N

K’ = N’ – M’

Replace K’, N’ and M’ with K, N, and M:

K = N – M +1

* 1. K’ = K, M’ = M-ℓ, N’ = N.

N’ = M’ + K’

Replace K’, N’ and M’ with K, N and M:

N = (M- ℓ) + K

* 1. Skipped
  2. 2
  3. 6