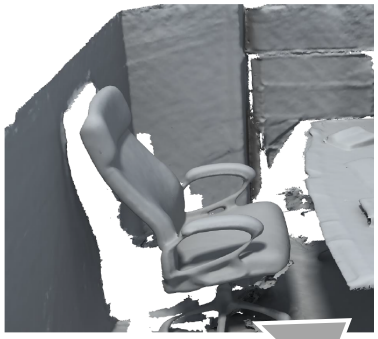
L1: Introduction

Hao Su

Agenda

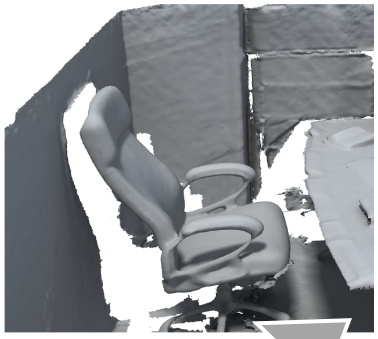
- Syllabus
- Logistics
- Curve Theory

Geometry Understanding is Important



Robotics

Geometry Understanding is Important



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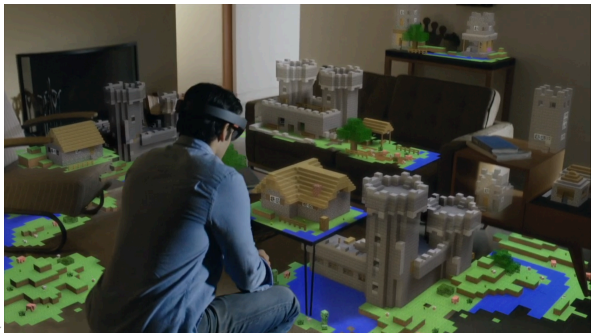
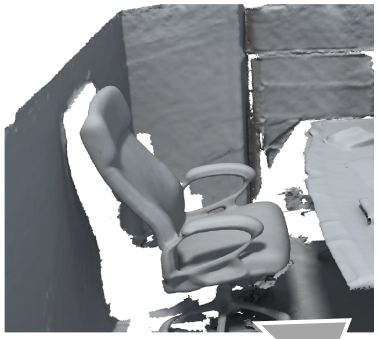
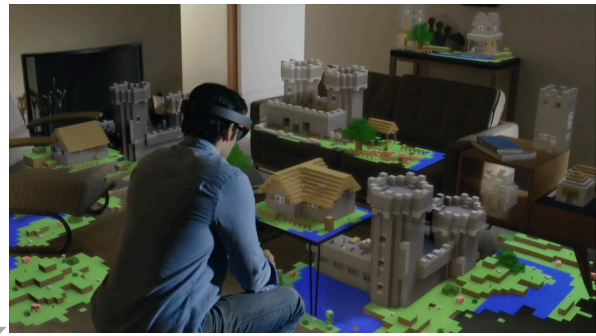


Augmented Reality

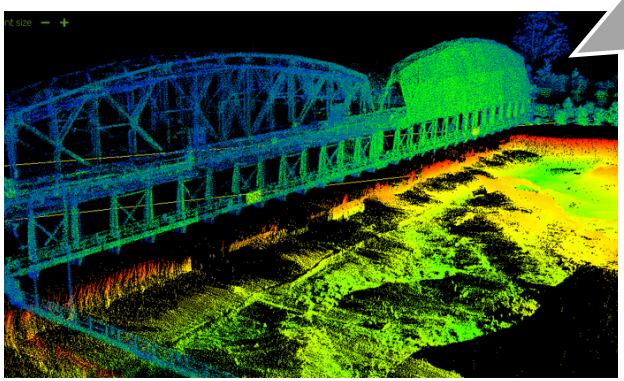
Geometry Understanding is Important



Robotics

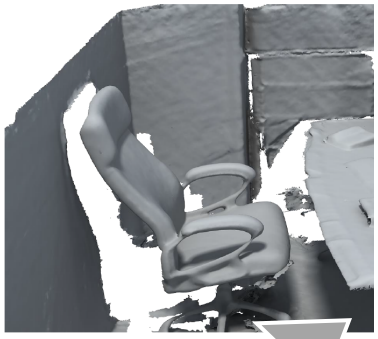


Augmented Reality

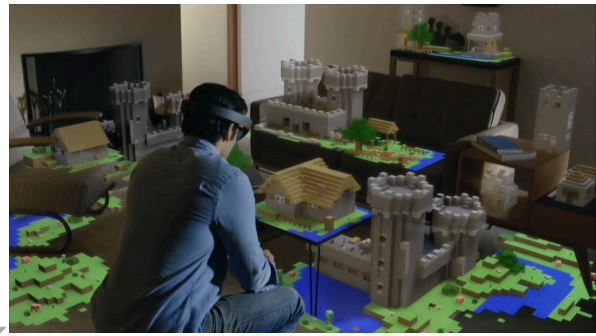


Autonomous driving

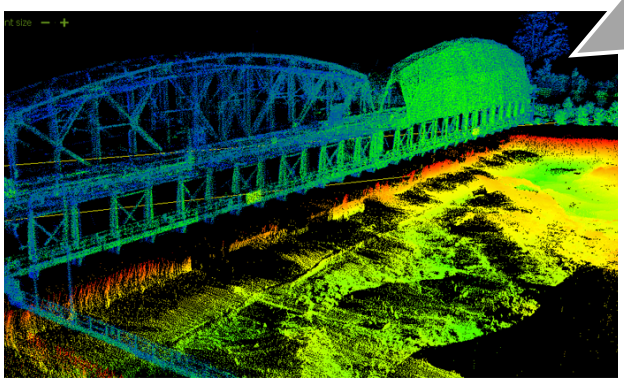
Geometry Understanding is Important



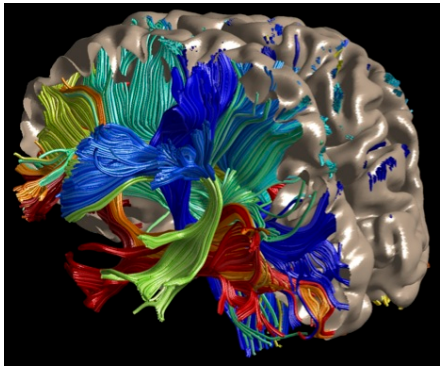
Robotics



Augmented Reality



Autonomous driving



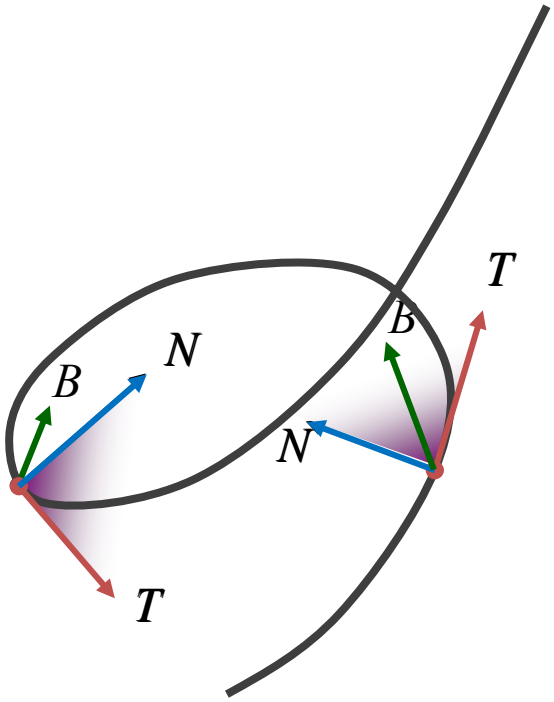
Medical Image Processing

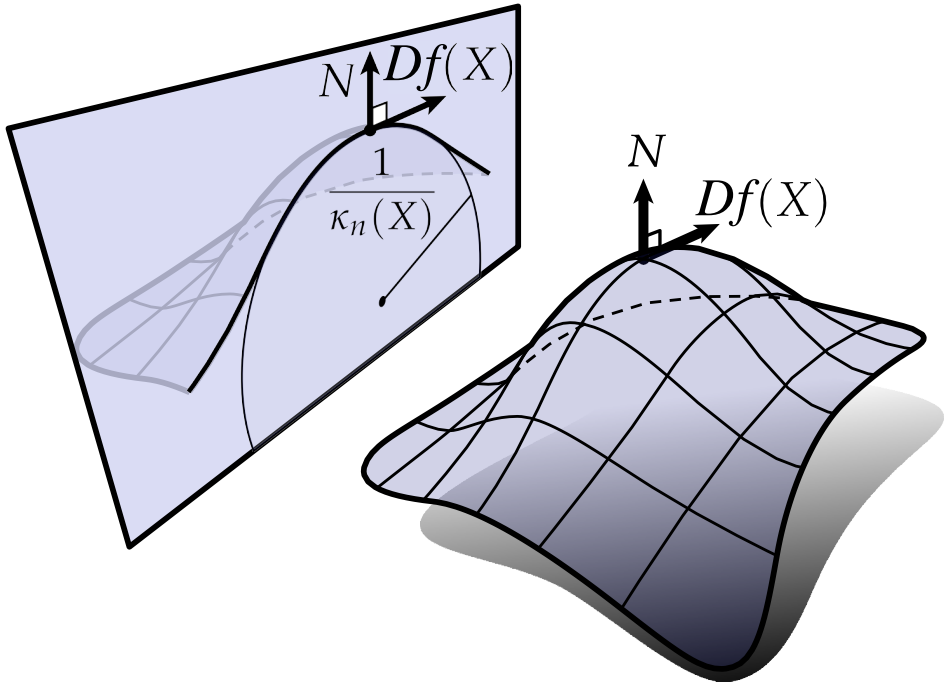
Use Machine Learning to Understand Geometries

A priori knowledge of the 3D world

Topics Covered in This Course

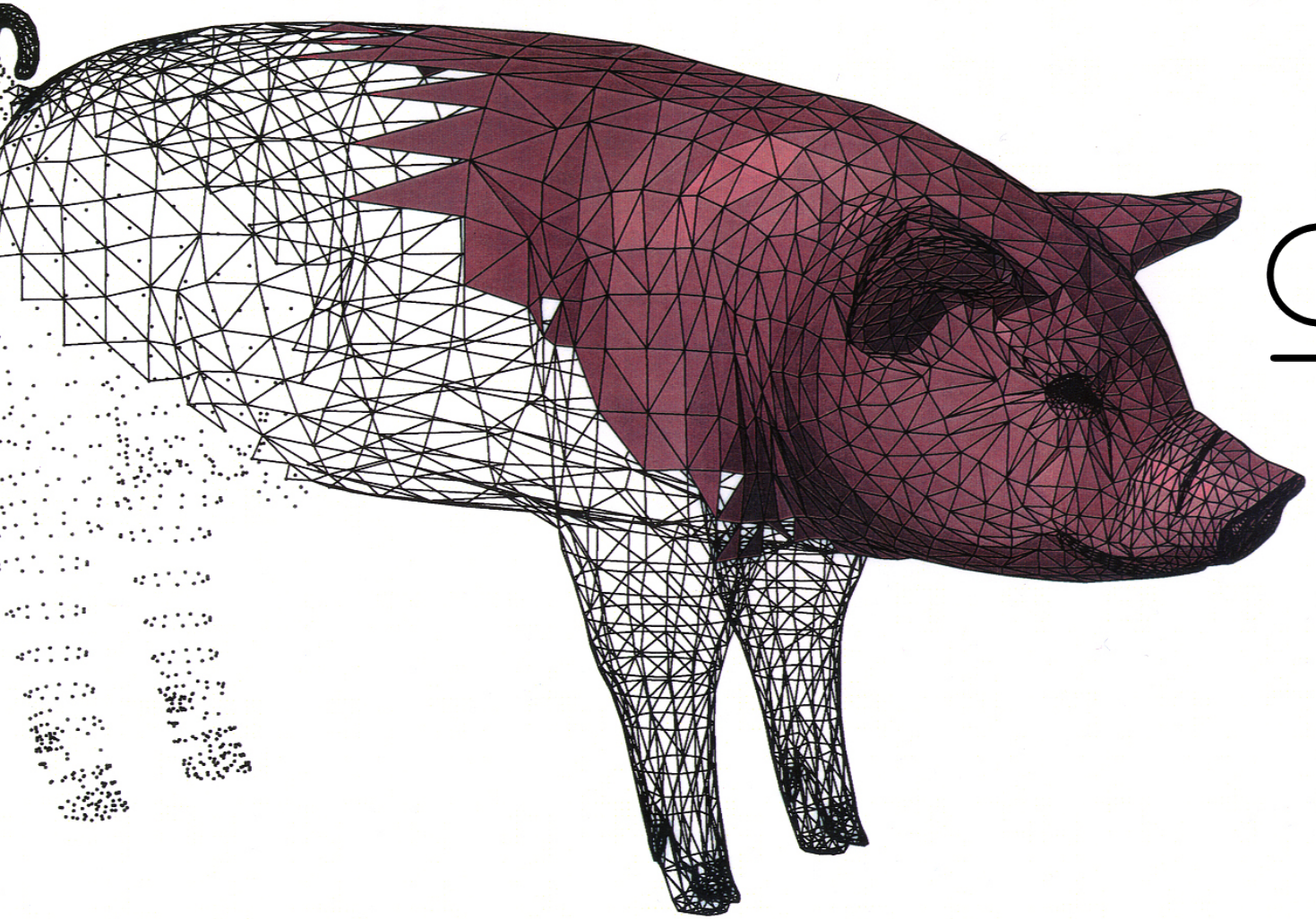
- Geometry theories





Topics Covered in This Course

- Computer Representation of Geometries



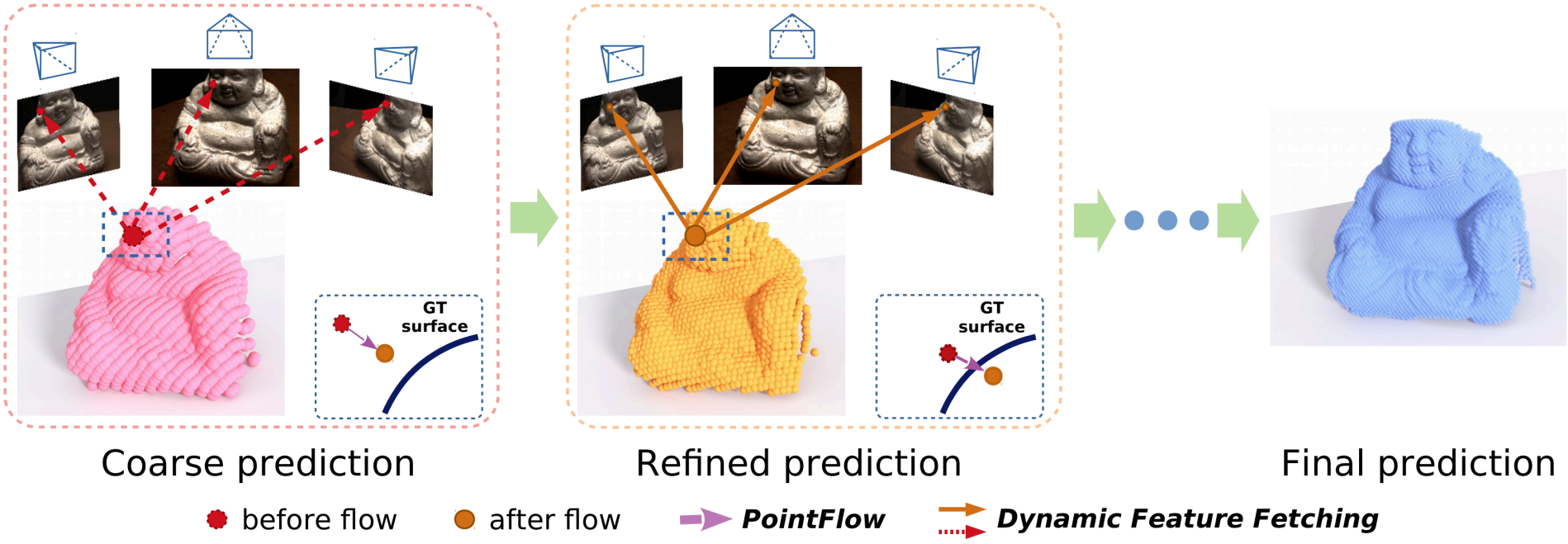
$$\subseteq \mathbb{R}^3$$

Topics Covered in This Course

- Sensing: 3D reconstruction from a single image

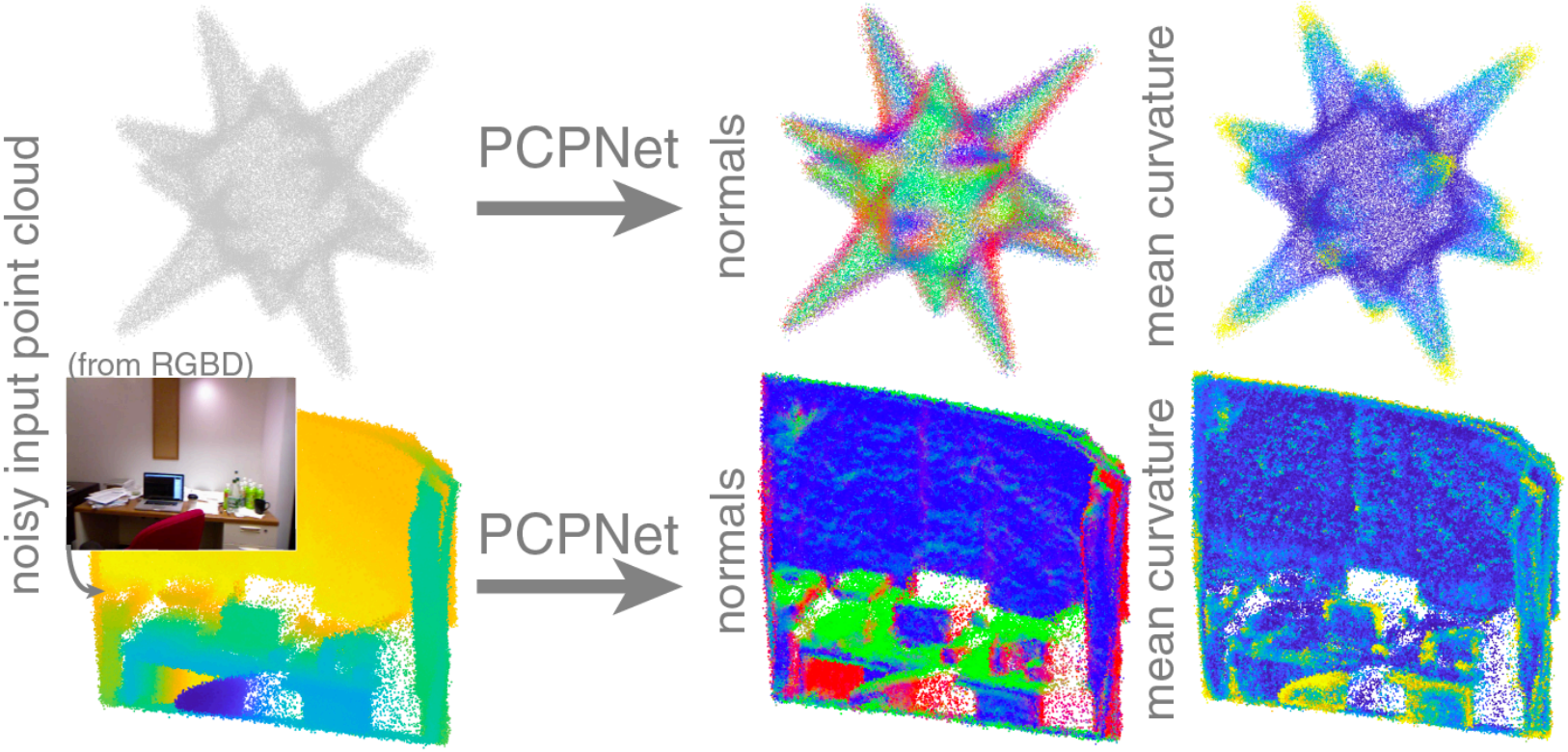
Topics Covered in This Course

- Sensing: 3D reconstruction from multiple views



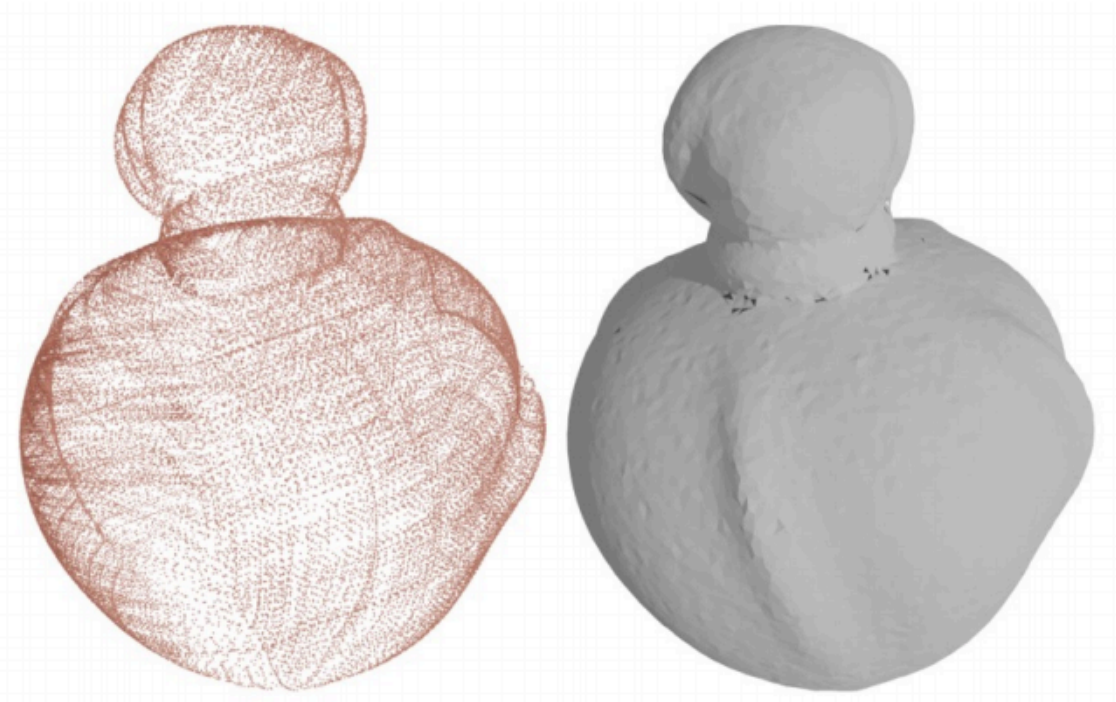
Topics Covered in This Course

- Geometry Processing: Local geometric property estimation



Topics Covered in This Course

- Geometry Processing: Surface reconstruction

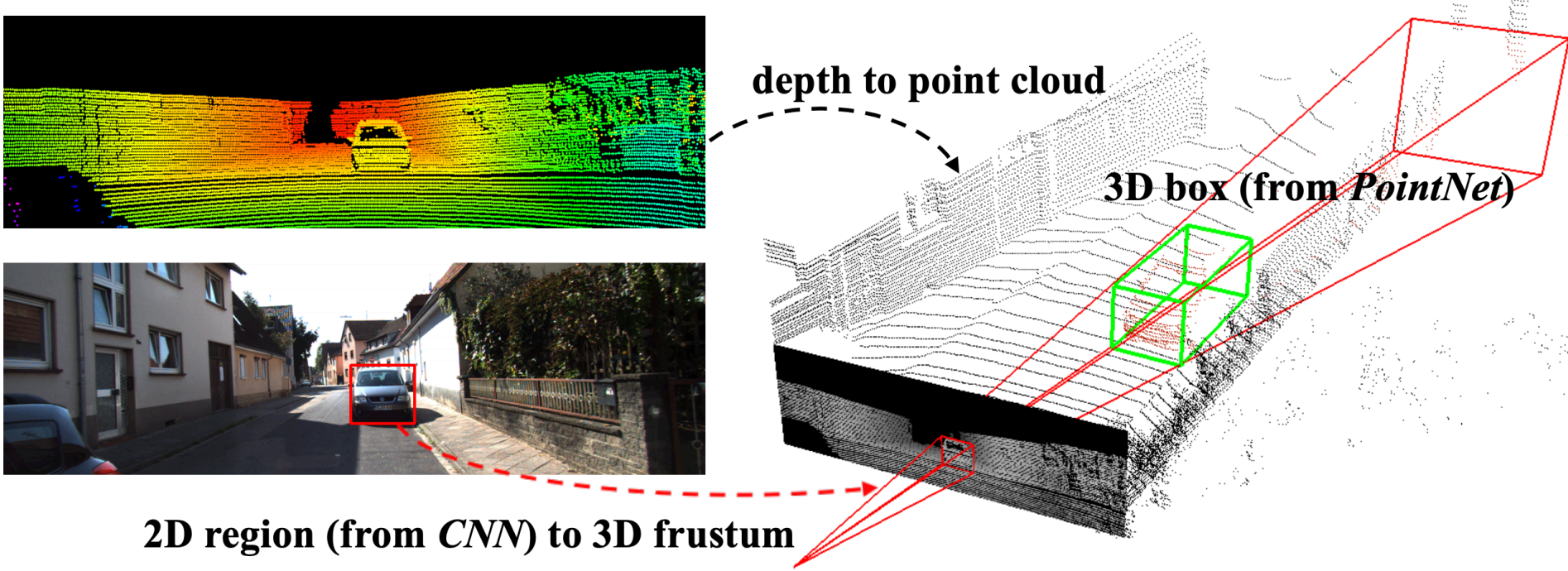


Topics Covered in This Course

- Recognition: Object classification

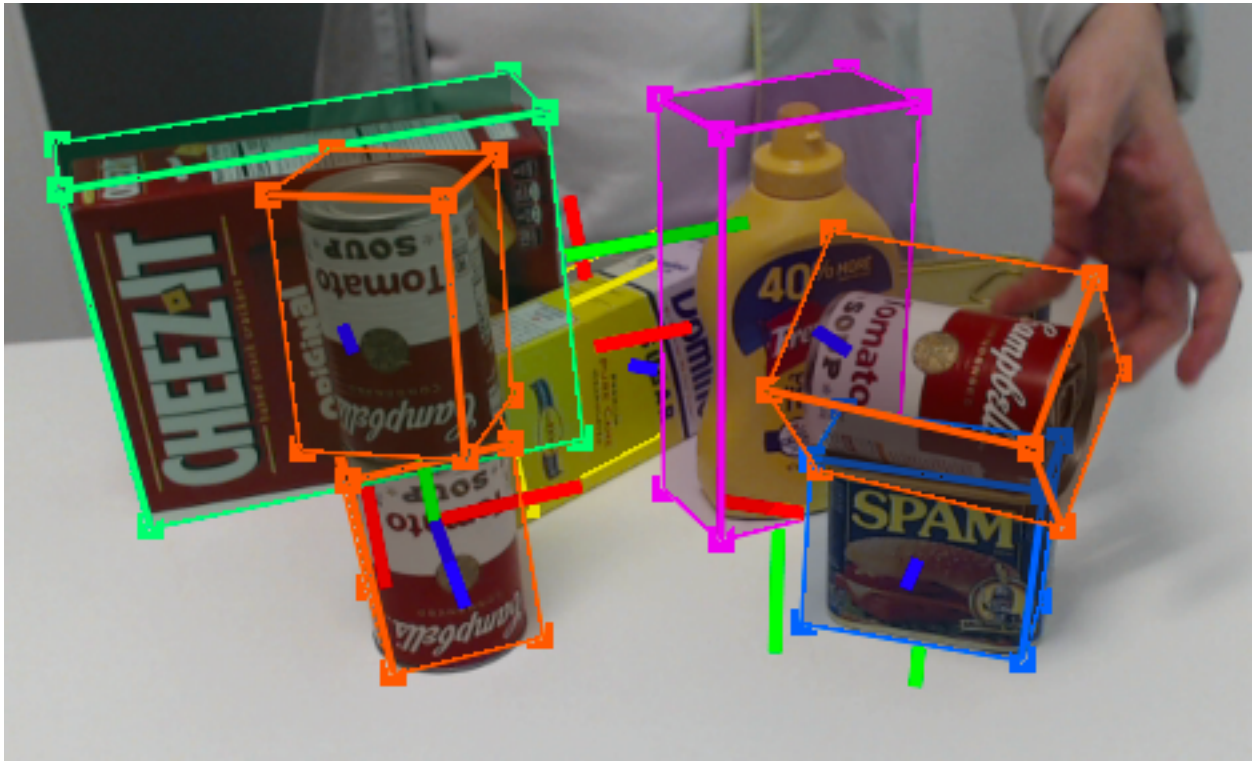
Topics Covered in This Course

- Recognition: Object detection



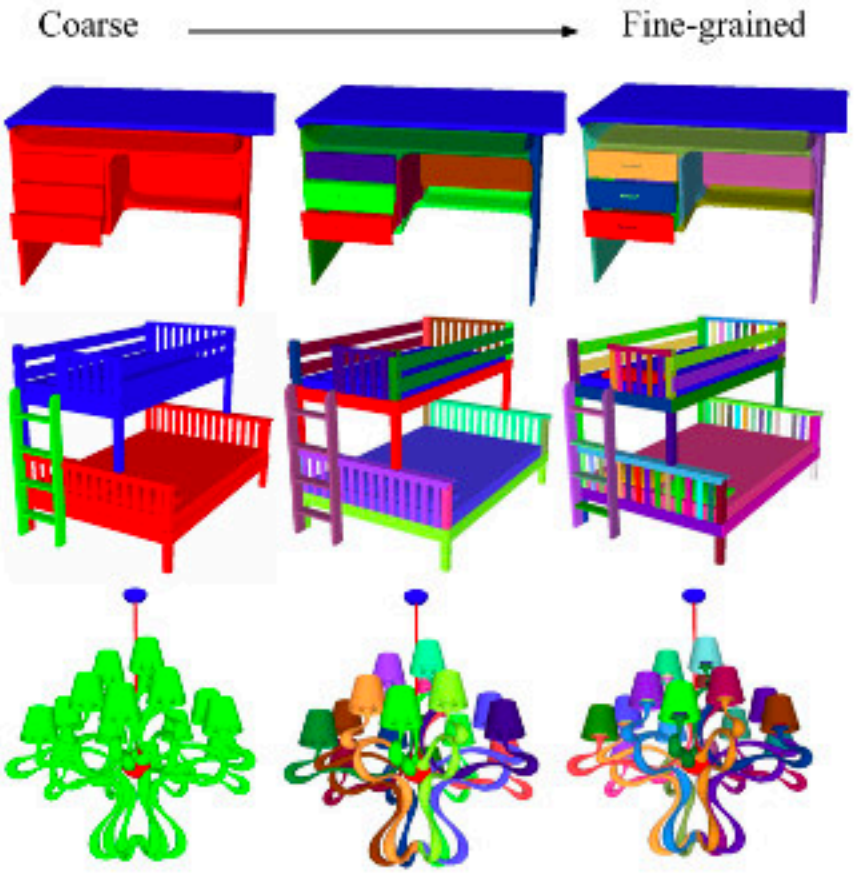
Topics Covered in This Course

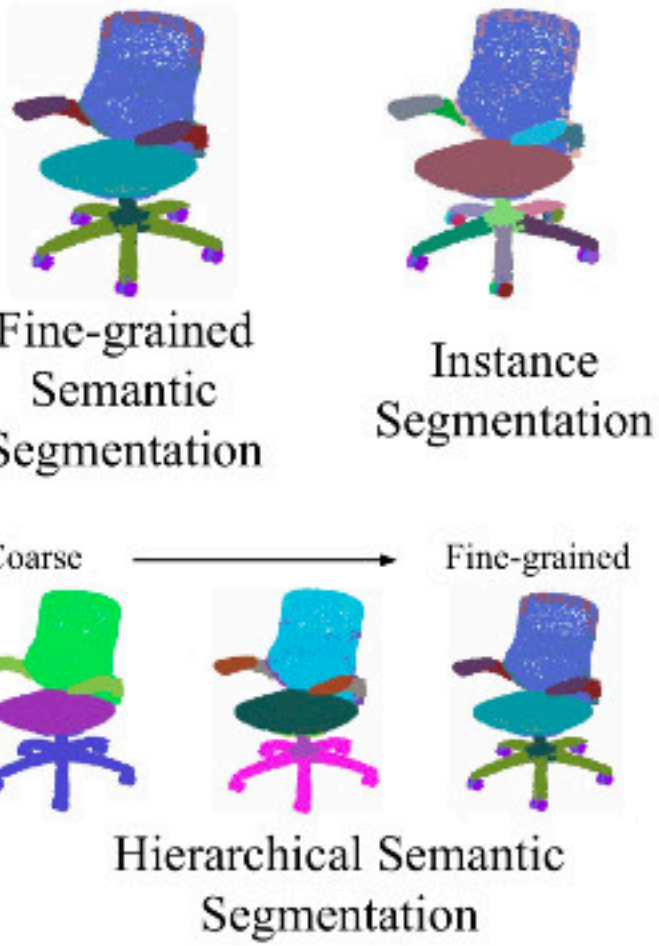
- Recognition: 6D pose estimation



Topics Covered in This Course

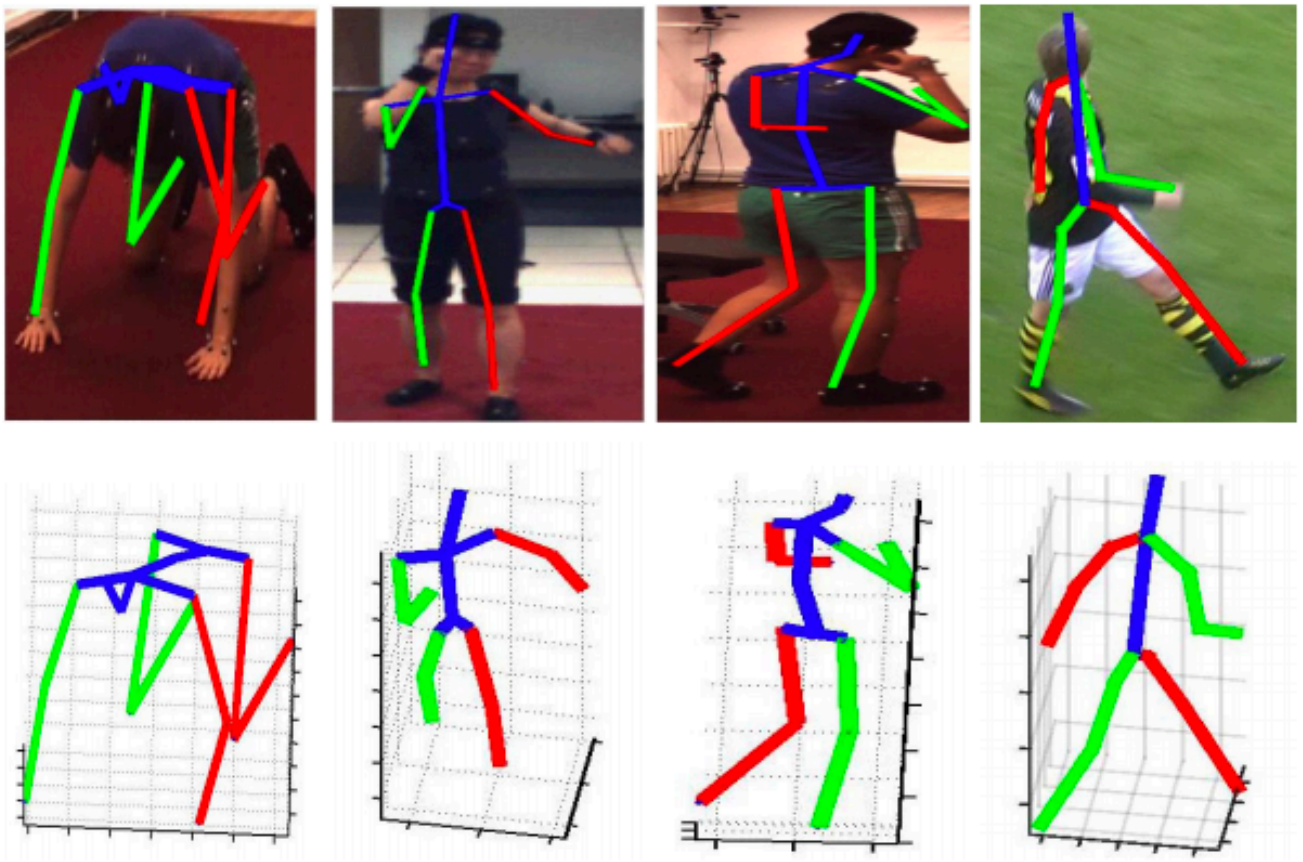
- Recognition: Segmentation





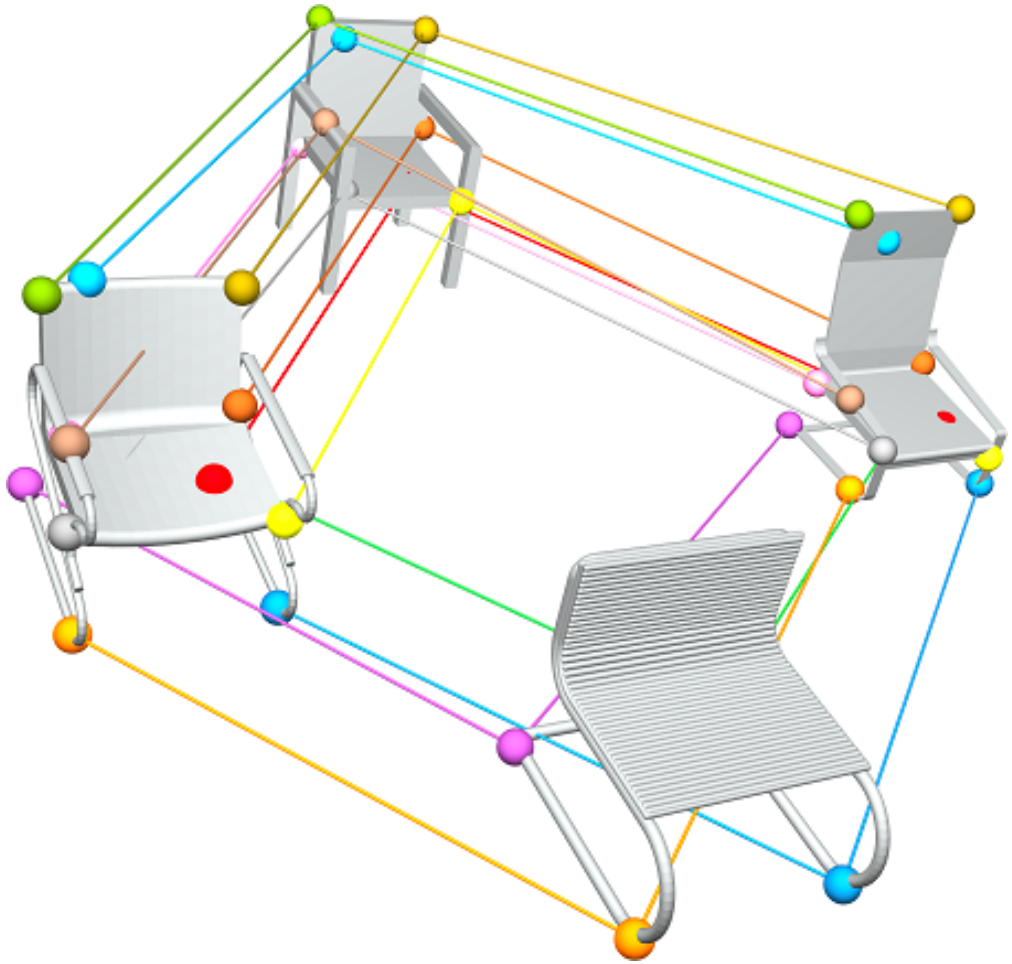
Topics Covered in This Course

- Recognition: Human pose estimation

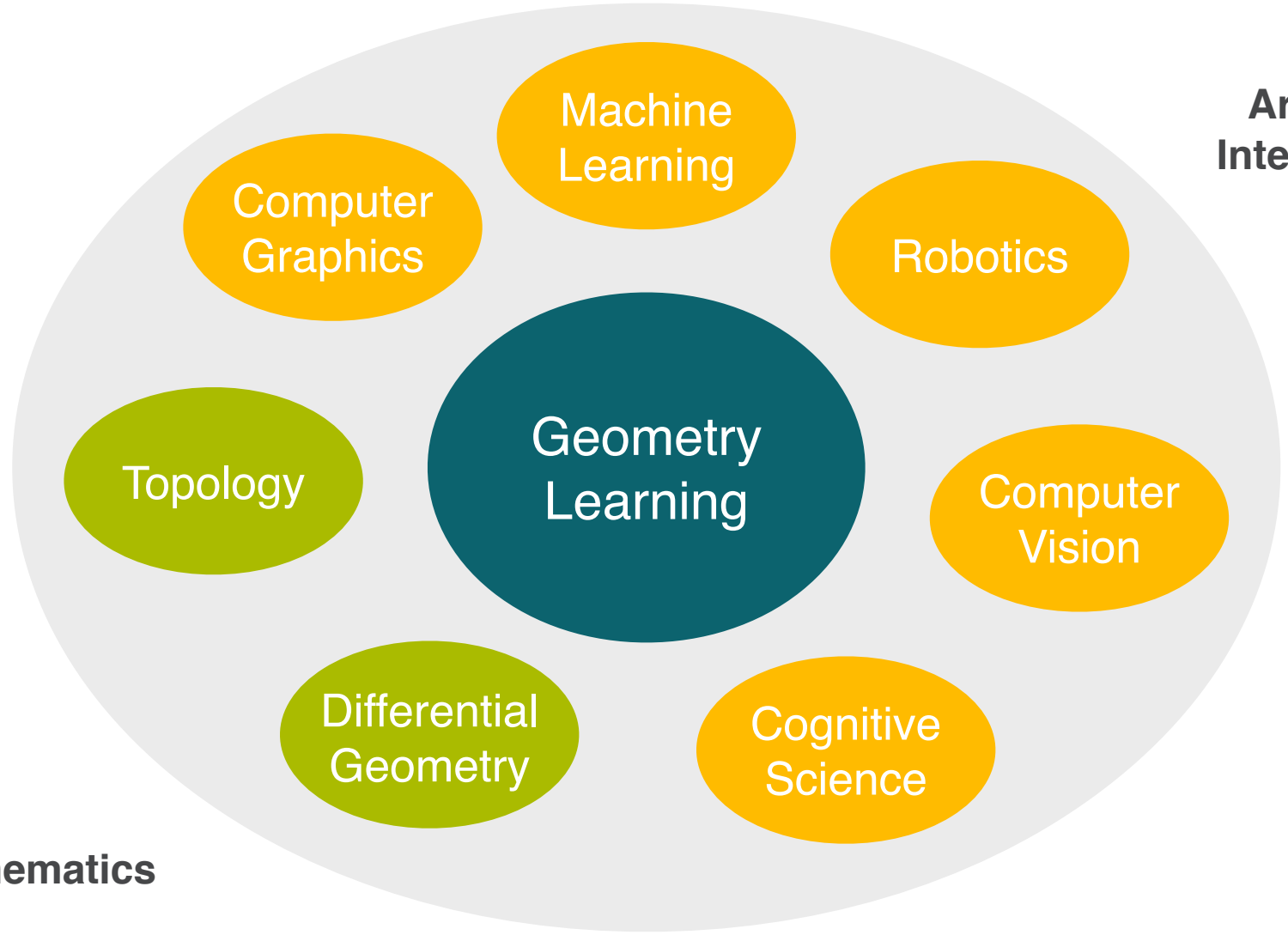


Topics Covered in This Course

- Relationship Analysis: Shape correspondences



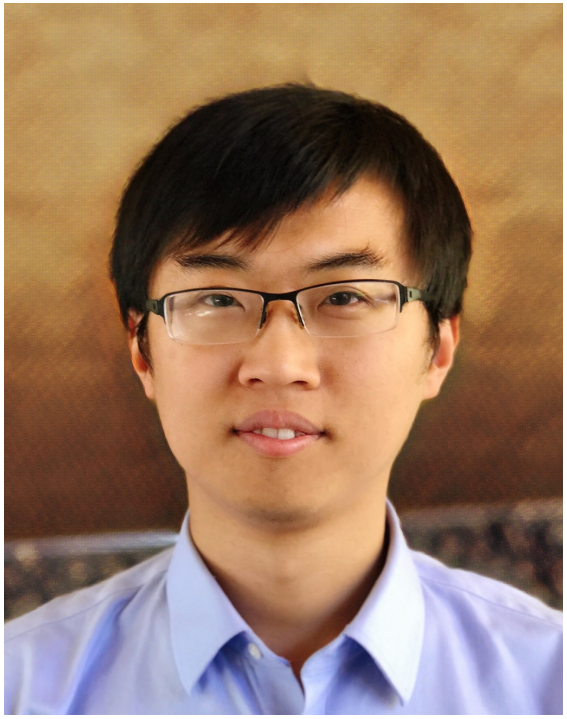
Highly Interdisciplinary Field



Course Logistic

Instructors

Instructor: Hao Su



TA: Fanbo Xiang

Teaching Goal

- State-of-the-art
  - **Enable** you to read and replicate recent 3D papers in top CV/CG conferences (not industry job oriented)
- Hands-on
  - **Heavy** programming assignments to exercise what are taught in class
- Foundational
  - Theory problems are **proof based**
  - Programming problems ask you to **implement low-level modules from scratch**

Pre-requisite: Technique

- **Skilled** in Linear Algebra
- **Familiar** with Multi-variable Calculus
- **Familiar** with Probability and Numerical Methods
- **Strong** programming skills
  - Familiar with Linux Toolchain
  - Familiar with python, numpy, and pytorch
- Course/project experiences in computer vision or deep learning

Background Check

- On Piazza now (HW0)
  - Visible to enrolled and waitlist students
- 5 points in your final grade
- **Mandatory!** We will not grade your subsequent homeworks without seeing your HW0.
- If you are in the waitlist and intend to enroll, you need to submit HW0 by this deadline
- **Due: 1/12/2021**

Pre-requisite: Resources

- This course requires deep learning resources (to run a 3D recognition challenge)

- Unfortunately, we do not have computational resources to support ~50 students

- Please find the server with the following configuration:
  >= 50G disk space
  >= 1 GPU with 10G memory

Assignments

- 4 assignments and 1 final project
  - HW0: due week 2 (5 points)
  - HW1: due week 4 (20 points)
  - HW2: due week 6 (20 points)
  - HW3: due week 8 (20 points)
  - Final project: final week (35 points)
  - No mid-term/final exams
- Extra credit for participation 5% (ask/answer questions in class, attend office hours)
- HW0-HW3: theory problems + programming
- Late policy: 15% grade reduction for each 12 hours late. No acceptance 72 hours after the due time.

3D Recognition Competition

- HW0-HW3: build individual modules
- Final project: integrate modules and test new ideas. Score by performance ranking. Online evaluation system will be set up
- We estimate **>=15 hrs per week** (out of class) solid time commitment
- We allow you to see homework (through Piazza) and attend the competition *even if you **audit** the course*

Course Resources

- Course website: https://haosulab.github.io/ml-meets-geometry/WI21/index.html (Google “Hao Su” -> Prof. Homepage -> Teaching -> this link)
  - Collaboration policy
  - Lecture slides
  - Office hour and location

- Piazza
  - Homework/Solution release
  - Discussions

Questions?

Curve

- Definition of curve
- Describing the shape of curves by calculus

Parameterized Curves

Intuition:

- A particle is moving in space
- At time t its position is given by

$$\gamma(t) = (x(t), y(t))$$

Example

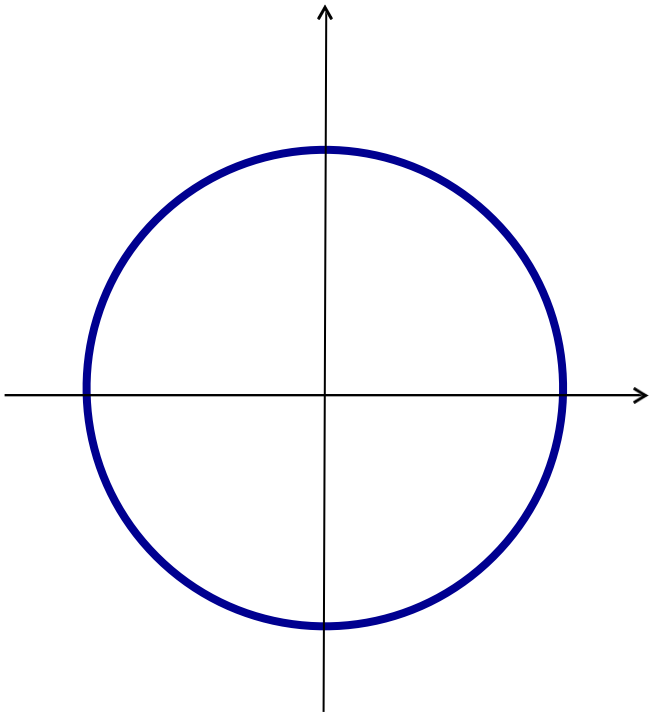
Explicit curve/circle in 2D

$$\mathbf{p} : \mathbb{R} \to \mathbb{R}^2$$

$$t \mapsto \mathbf{p}(t) = (x(t), y(t))$$

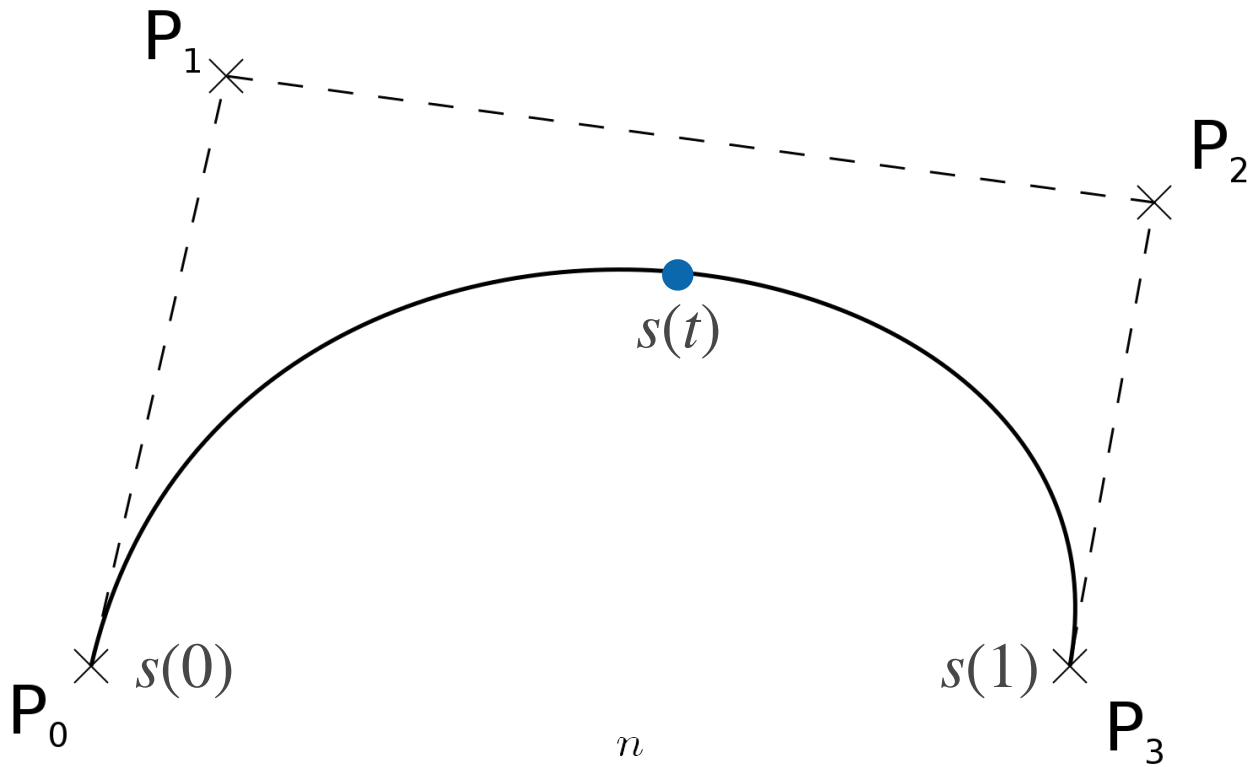
$$\mathbf{p}(t) = r\,(\cos(t), \sin(t))$$

$$t \in [0, 2\pi)$$



Application: Bezier Curves, Splines

- Smoothly "interpolate" between *a set of* points P_i
- Widely used in design (e.g., in your Powerpoint)

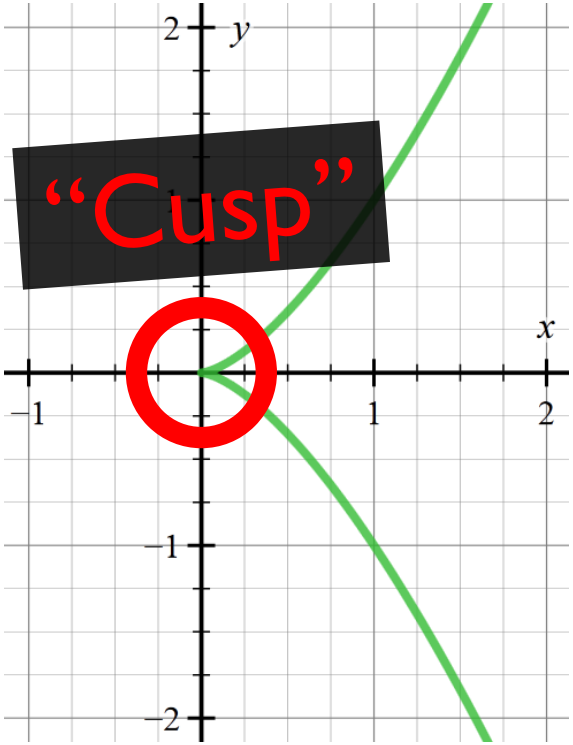


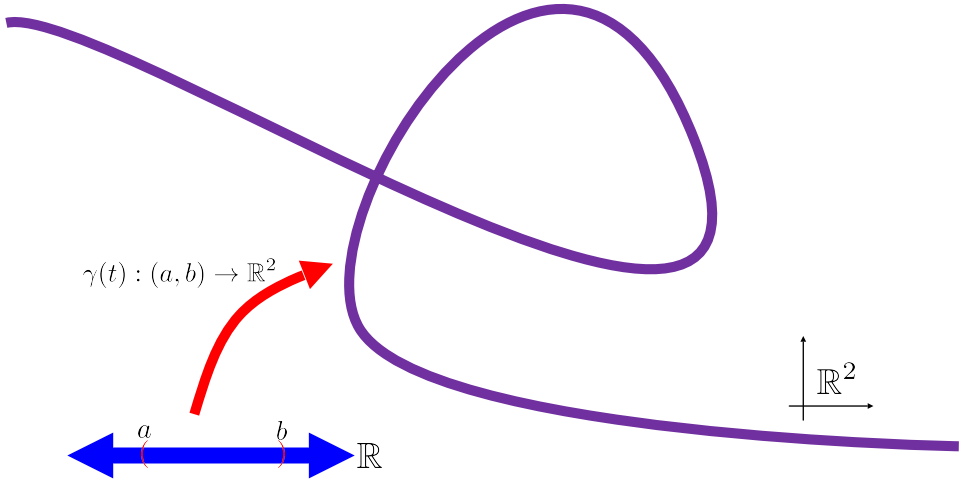
$$s(t) = \sum_{i=0}^{n} \mathbf{p}_i B_i^n(t)$$

One-dimensional “Manifold”

Set of points that locally looks like a line.

Negative Examples of Manifolds





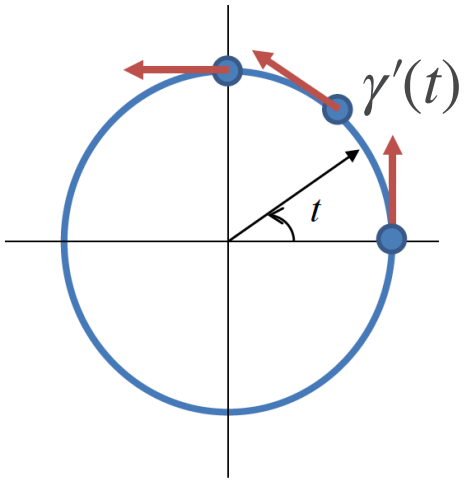
$$f(t) = (t^2, t^3)$$

Tangent

- $\gamma'(t) = (x'(t), y'(t)) \in \mathbb{R}^2$ is the tangent vector of the curve at t

Quiz: Tangent of a Circle

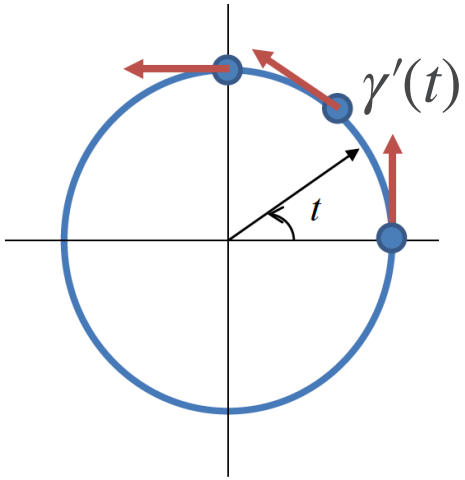
- $\gamma'(t) = (x'(t), y'(t)) \in \mathbb{R}^2$ is the tangent vector of the curve at t



$$\gamma(t) = (\cos(t), \sin(t))$$

Quiz: Tangent of a Circle

- $\gamma'(t) = (x'(t), y'(t)) \in \mathbb{R}^2$ is the tangent vector of the curve at t



$$\gamma(t) = (\cos(t), \sin(t))$$

$$\gamma'(t) = (-\sin(t), \cos(t))$$

$\gamma'(t)$ - direction of movement

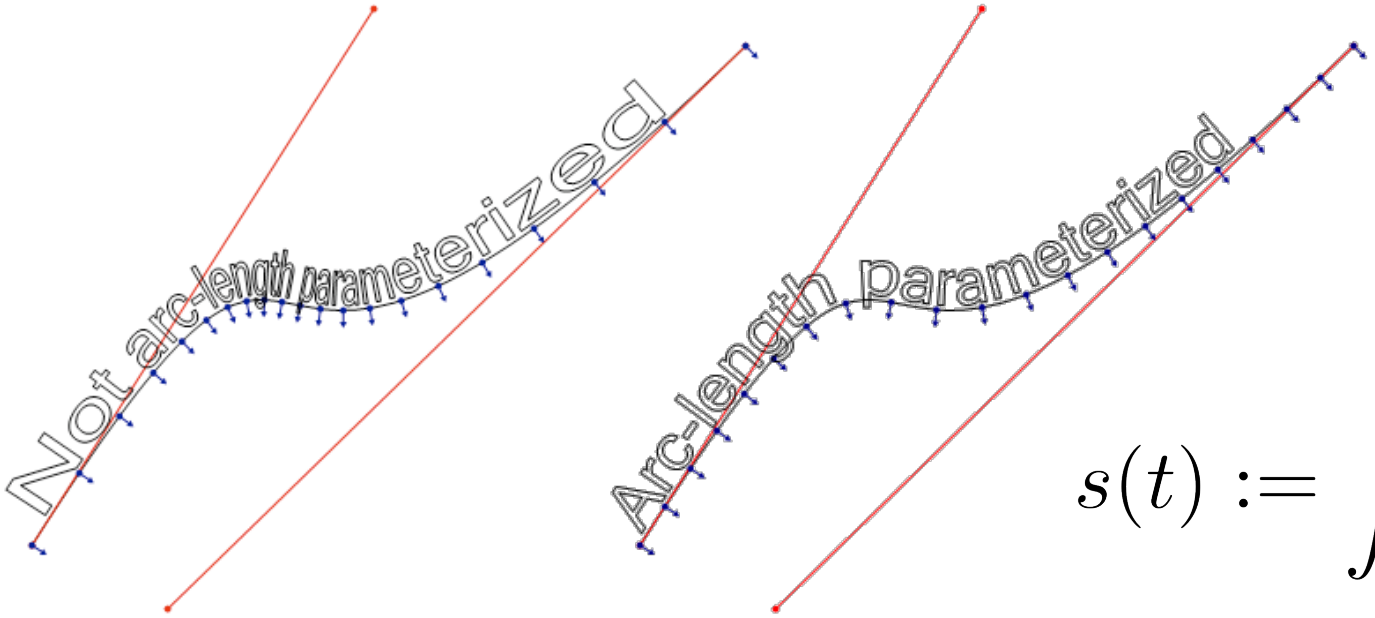
$\|\gamma'(t)\|$ - speed of movement

Arc Length

$$\int_a^b \|\gamma'(t)\| \, dt$$

Parameterization by Arc Length

http://www.planetclegg.com/projects/WarpingTextToSplines.html



$$s(t) := \int_{t_0}^{t} \|\gamma'(t)\|\, dt$$

$$t(s) := \text{inverse of } s(t)$$

$$\bar{\gamma}(s) = \gamma(t(s))$$

Constant-speed parameterization

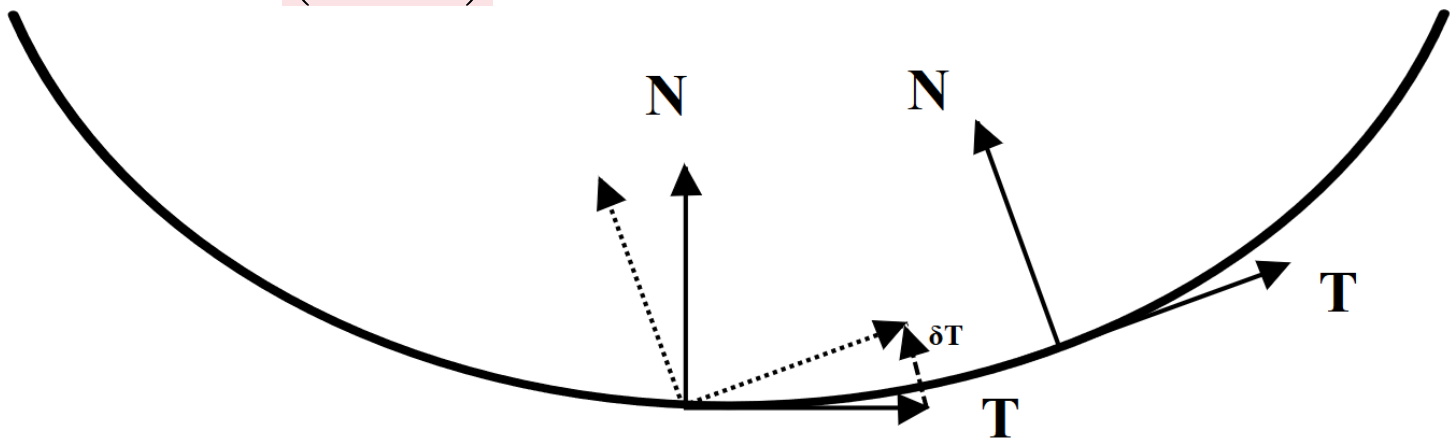
Moving Frame in 2D

$$T(s) := \gamma'(s)$$

$$\Longrightarrow \quad (\text{on board}) \ \|T(s)\| \equiv 1$$

$$N(s) := JT(s)$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



https://en.wikipedia.org/wiki/Frenet%E2%80%93Serret_formulas

Lemma

$$\frac{d}{ds}\langle u(s), v(s)\rangle = \langle \frac{du}{ds}, v\rangle + \langle u, \frac{dv}{ds}\rangle$$

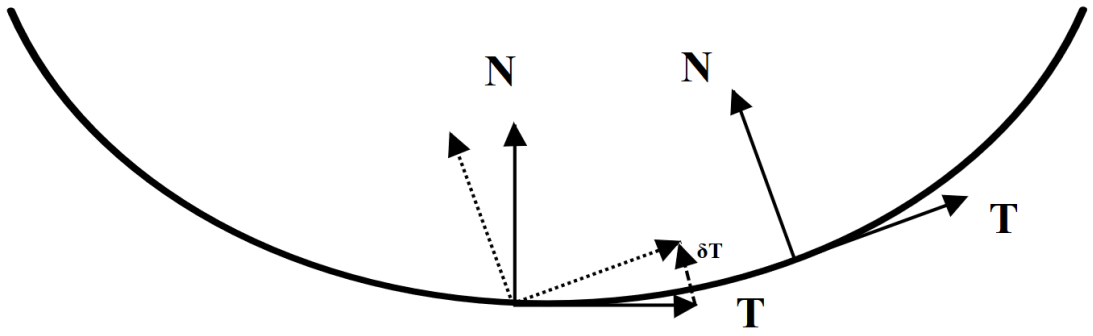
Derivation of $\|T(s)\| \equiv 1$

(See notes)

Turtles All The Way Down

On the board:

$$\frac{d}{ds}\begin{pmatrix} T(s) \\ N(s) \end{pmatrix} := \begin{pmatrix} 0 & k(s) \\ -k(s) & 0 \end{pmatrix}\begin{pmatrix} T(s) \\ N(s) \end{pmatrix}$$



https://en.wikipedia.org/wiki/Frenet%E2%80%93Serret_formulas

Use coordinates *from* the curve to express its shape!

$$\frac{d}{ds}\begin{pmatrix} T(s) \\ N(s) \end{pmatrix} := \begin{pmatrix} 0 & k(s) \\ -k(s) & 0 \end{pmatrix}\begin{pmatrix} T(s) \\ N(s) \end{pmatrix}$$

(See notes)

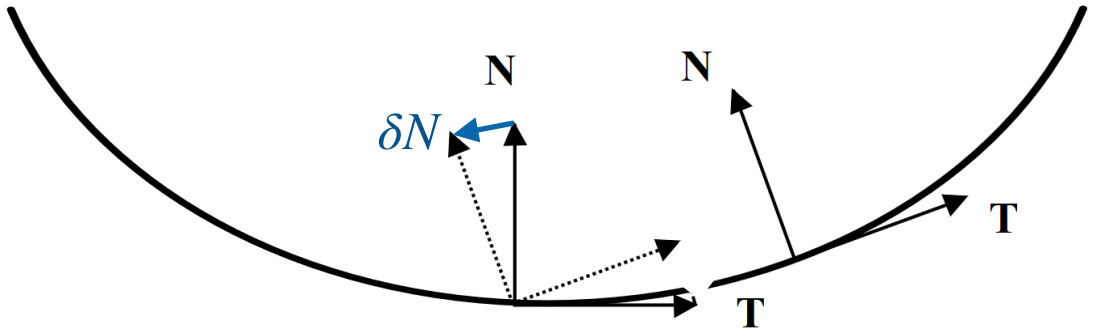
$$\frac{d}{ds}\begin{pmatrix} T(s) \\ N(s) \end{pmatrix} := \begin{pmatrix} 0 & k(s) \\ -k(s) & 0 \end{pmatrix}\begin{pmatrix} T(s) \\ N(s) \end{pmatrix}$$

(See notes)

Perspective of Normal Change

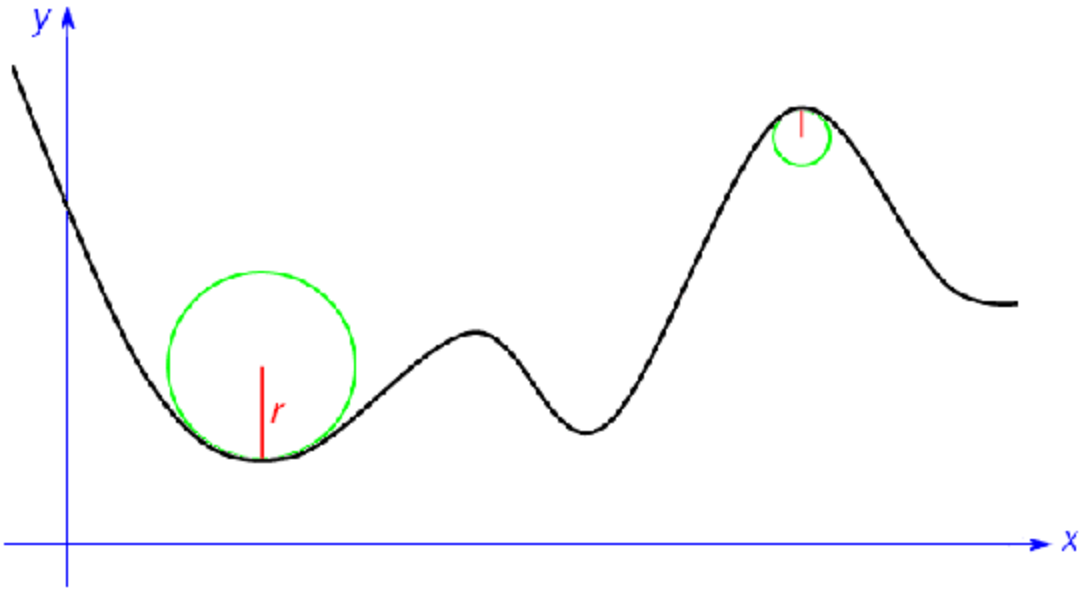
$$\mathbf{N}'(s) = -\kappa(s)\mathbf{T}(s)$$

- Curvature indicates how much the normal changes in the direction tangent to the curve



- Curvature is always positive

Radius of Curvature



$$r(s) := \frac{1}{k(s)}$$

https://www.quora.com/What-is-the-base-difference-between-radius-of-curvature-and-radius-of-gyration

Invariance is Important

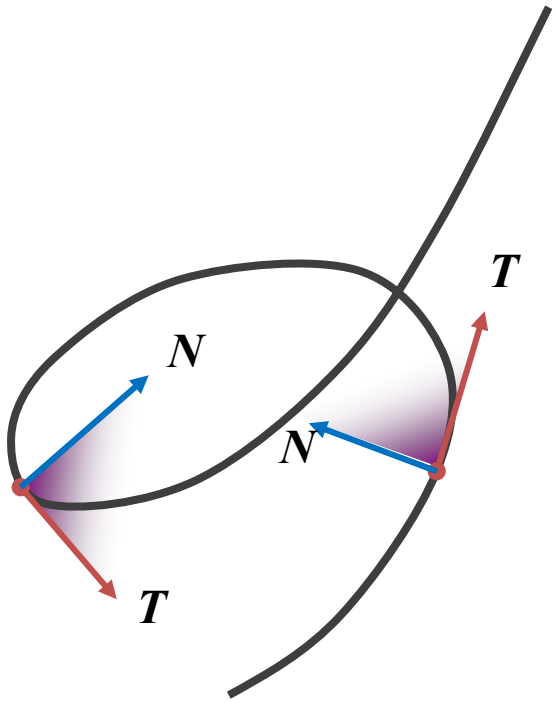
Fundamental theorem of the local theory of plane curves:

$\kappa(s)$ characterizes a **planar curve** up to rigid motion.

3D Curves

- Osculating Plane

The plane determined by the unit tangent and normal vectors $T(s)$ and $N(s)$ is called the *osculating plane* at s

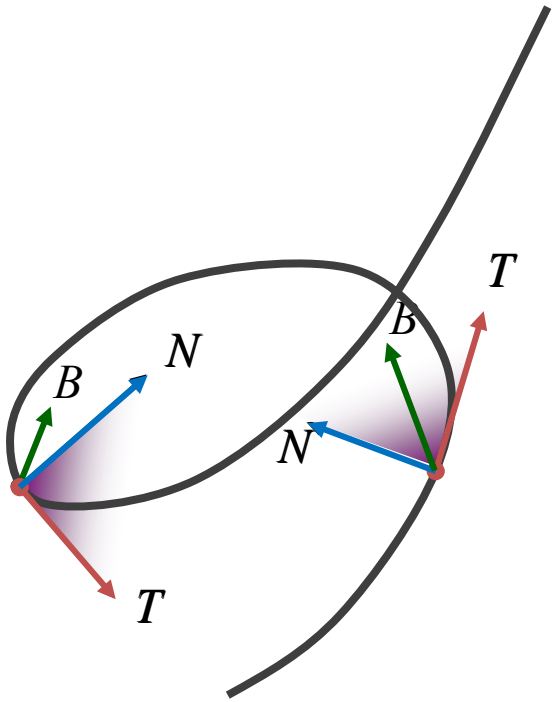


Self-reading

The Binormal Vector

For points s, s.t. $\kappa(s) \neq 0$, the *binormal vector* $\boldsymbol{B}(s)$ is defined as:

$$\boldsymbol{B}(s) = \boldsymbol{T}(s) \times \boldsymbol{N}(s)$$



The binormal vector defines the osculating plane

Self-reading

$\mathbf{T}'(s)$

- Already used it to define the curvature:

$$\mathbf{T}'(s) = \kappa(s)\mathbf{N}(s)$$

Unit vector

- Orthogonal to $\mathbf{T}(s)$ (the same derivation as 2D curve)

- Since along the direction of $\mathbf{N}(s)$, also orthogonal to $\mathbf{B}(s)$

Self-reading

$\mathbf{N}'(s)$

We know: $\langle \mathbf{N}(s), \mathbf{N}(s) \rangle$=1

From the lemma $\longrightarrow \langle \mathbf{N}'(s), \mathbf{N}(s) \rangle = 0$

(Derivative orthogonal to itself)

We know: $\langle \mathbf{N}(s), \mathbf{T}(s) \rangle = 0$

From the lemma $\longrightarrow \langle \mathbf{N}'(s), \mathbf{T}(s) \rangle = \langle -\mathbf{N}(s), \mathbf{T}'(s) \rangle$

From the definition $\longrightarrow \kappa(s) = \langle \mathbf{N}(s), \mathbf{T}'(s) \rangle$

$\longrightarrow \langle \mathbf{N}'(s), \mathbf{T}(s) \rangle = -\kappa(s)$

Self-reading

The Torsion

- From previous slide:

$$\langle \mathbf{N}'(s), \mathbf{N}(s) \rangle = 0$$
$$\langle \mathbf{N}'(s), \mathbf{T}(s) \rangle = -\kappa(s)$$

The remaining component of $\mathbf{N}'(s)$ is along $\mathbf{B}(s)$ direction:

$$\langle \mathbf{N}'(s), \mathbf{B}(s) \rangle = \tau(s)$$

Now we can express $N'(s)$ as

$$\mathbf{N}'(s) = -\kappa(s)\mathbf{T}(s) + \tau(s)\mathbf{B}(s)$$

Self-reading

Perspective of Normal Change

$$\mathbf{N}'(s) = -\kappa(s)\mathbf{T}(s) + \tau(s)\mathbf{B}(s)$$

- Curvature indicates how much the normal changes in the direction tangent to the curve
- Torsion indicates how much normal changes in the direction orthogonal to the osculating plane of the curve
- Curvature is always positive but torsion can be negative

Self-reading

$\mathbf{B}'(s)$

We know: $\langle \mathbf{B}(s), \mathbf{B}(s) \rangle = 1$

From the lemma $\longrightarrow$ $\langle \mathbf{B}'(s), \mathbf{B}(s) \rangle = 0$

We know: $\langle \mathbf{B}(s), \mathbf{T}(s) \rangle = 0, \langle \mathbf{B}(s), \mathbf{N}(s) \rangle = 0$

From the lemma $\longrightarrow$

$\langle \mathbf{B}'(s), \mathbf{T}(s) \rangle = \langle -\mathbf{B}(s), \mathbf{T}'(s) \rangle = \langle -\mathbf{B}(s), \kappa(s)\mathbf{N}(s) \rangle = 0$

From the lemma $\longrightarrow$

$\langle \mathbf{B}'(s), \mathbf{N}(s) \rangle = \langle -\mathbf{B}(s), \mathbf{N}'(s) \rangle = -\tau(s)$

Now we express $\mathbf{B}'(s)$ as:

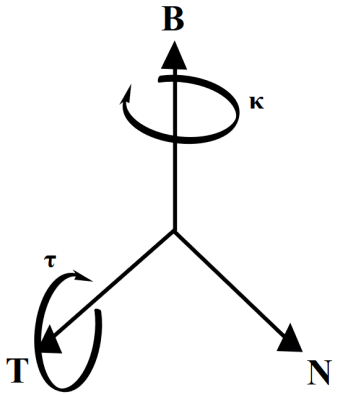
$$\mathbf{B}'(s) = -\tau(s)\mathbf{N}(s)$$

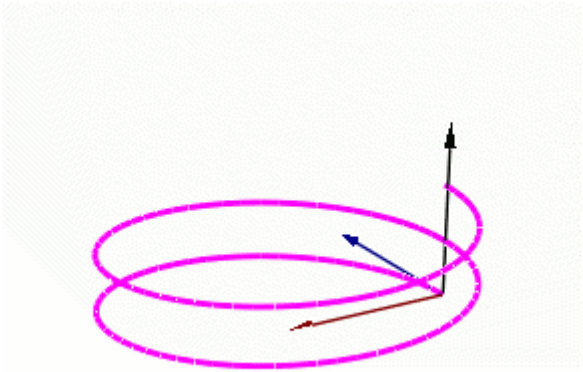
Self-reading

Frenet Frame: Curves in $\mathbb{R}^3$

- Binormal:
  - Curvature: In-plane motion
  - Torsion: Out-of-plane motion

$$\frac{d}{ds}\begin{pmatrix} T \\ N \\ B \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}\begin{pmatrix} T \\ N \\ B \end{pmatrix}$$





Self-reading

Fundamental theorem of the local theory of space curves:

Curvature and torsion characterize a 3D curve up to rigid motion.

Summary

- Curve is a map from an interval to $\mathbb{R}^n$
- Tangent describes the moving direction
- The derivative of tangent under arc-length parameterization is normal
- Curvature (and torsion) both characterize the change of normal direction, uniquely describing the shape of a curve (up to rigid transformation)
- Tangent, normal, and binormal form a moving frame (Frenet frame)