Contact Model for Grasping

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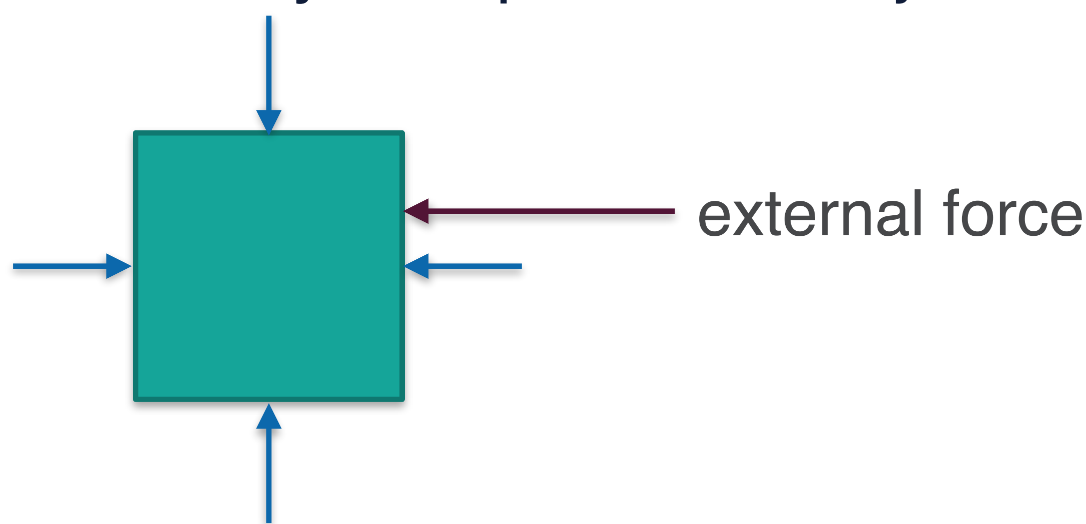
Grasping

- Robot manipulates objects with robot hands.

- The grasp planning problem is to determine a set of contact locations for the object and the fingers.

- Desirable properties:
  - **Force-closure**: Resist external forces.
  - **Manipulable**: Dexterously manipulate the object.



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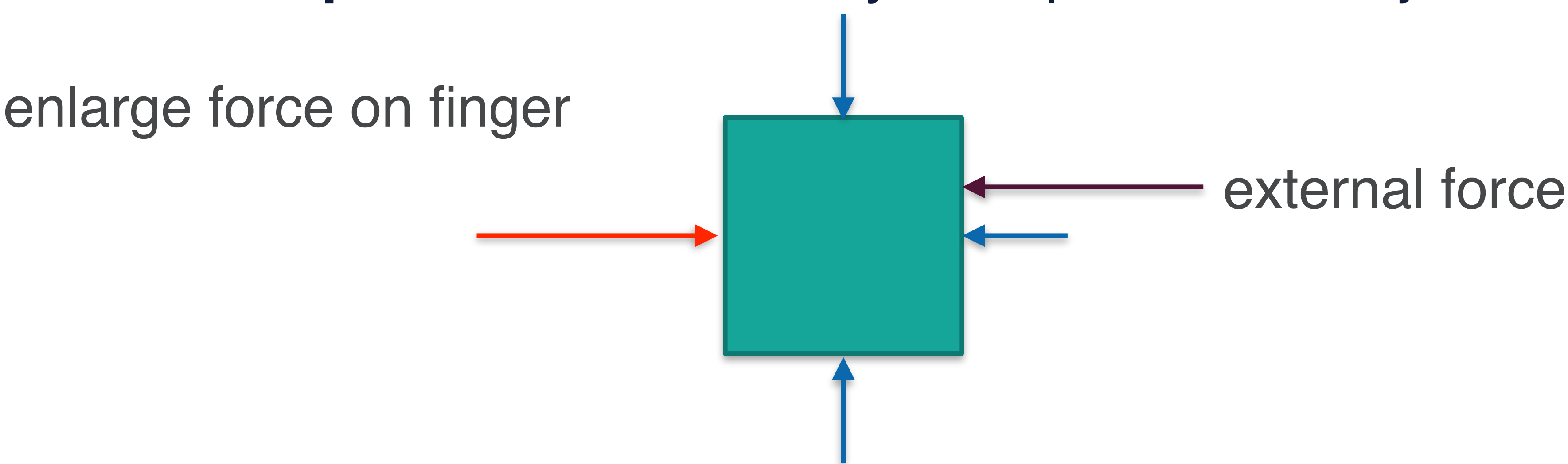


Grasp Model

- **Friction Model at a Single Contact**

- Multi-Contact Configuration

Contact

- A **contact** between a finger and an object can be described as a mapping between forces exerted by the finger at the point of contact and the resultant wrenches at some reference point on the object.

Contact Coordinate Frame

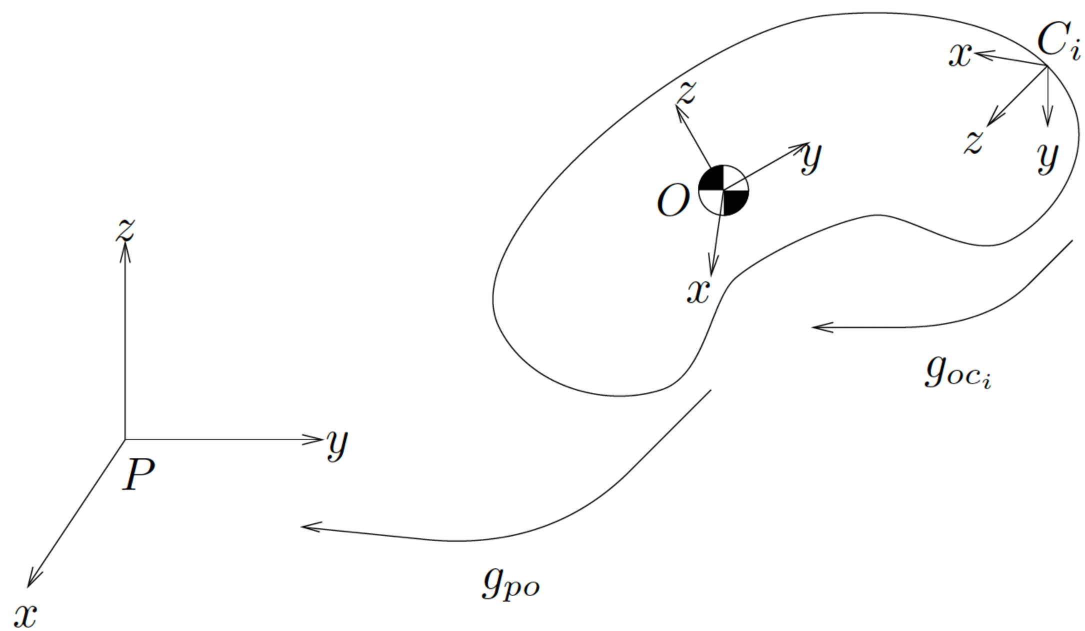
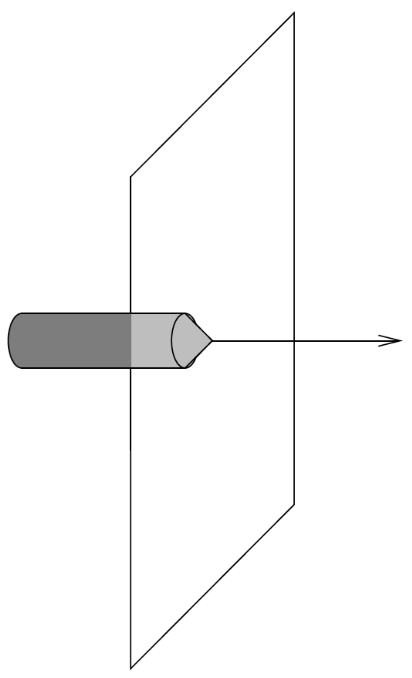
- Location of the contact point on the object is fixed
- The **contact coordinate frame** C_i satisfies that its z -axis points in the direction of the inward surface **normal** at the point of contact.
- 

Figure 5.2: Coordinate frames for contact and object forces.

Frictionless point contact

- A **frictionless point contact** is obtained when there is no friction between the fingertip and the object.



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Frictionless point contact

- Forces can only be applied in the direction that is the orthogonal to the surface of the object:

$$F_{c_i} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} f_{c_i}, f_{c_i} \geq 0$$

- , where f_{c_i} is the magnitude of the force.

Coulomb friction model

- **Coulomb friction model** is a simple contact model which deals with the friction.

- $f_t \in R$ denote the magnitude of the tangential force
- $f_n \in R$ denote the magnitude of the normal force.

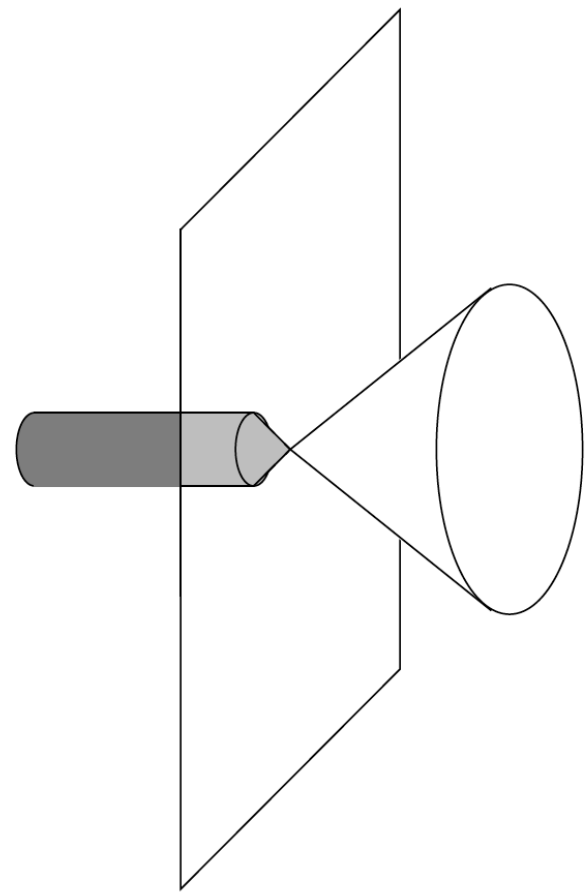
Coulomb friction model

- **Coulomb's law:** Slipping begins when $|f^t| > \mu f^n$, where $\mu > 0$ is the **static coefficient of friction**.

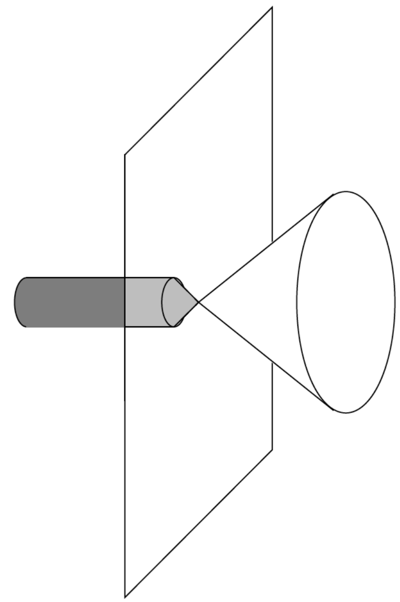
- Constraints: $|f^t| \leq \mu f^n, f_n > 0$.

Coulomb friction model

$$F_{c_i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} f_{c_i}, f_{c_i} \in FC_{c_i}$$

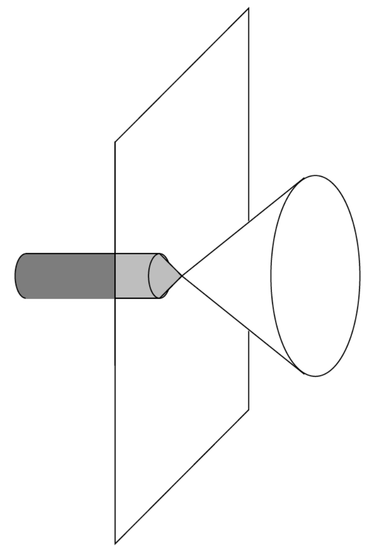


Coulomb friction model



- $FC_{c_i} = \left\{ f \in R^3 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3, f_3 \geq 0 \right\}$ is the **friction cone.**

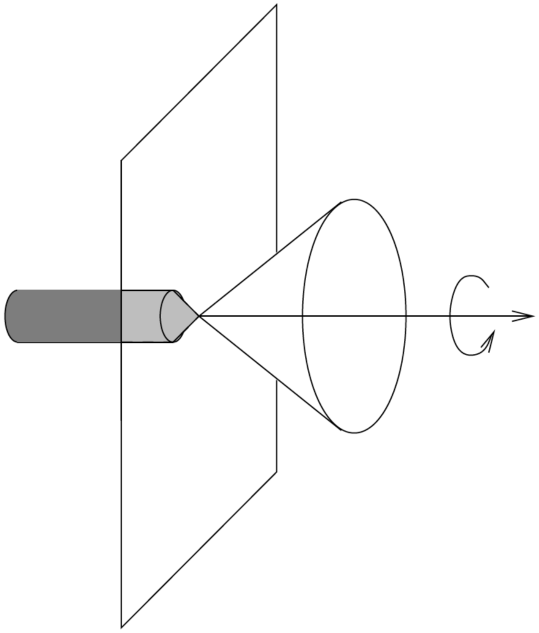
Coulomb friction model



- $FC_{c_i} = \left\{ f \in R^3 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3, f_3 \geq 0 \right\}$ is the **friction cone.**
- The angle of the cone with respect to the normal is given by $\alpha = \tan^{-1} \mu$.

Limitation of Point-contact Finger

- Point-finger contact cannot model
- A more realistic contact model is the **soft-finger contact**



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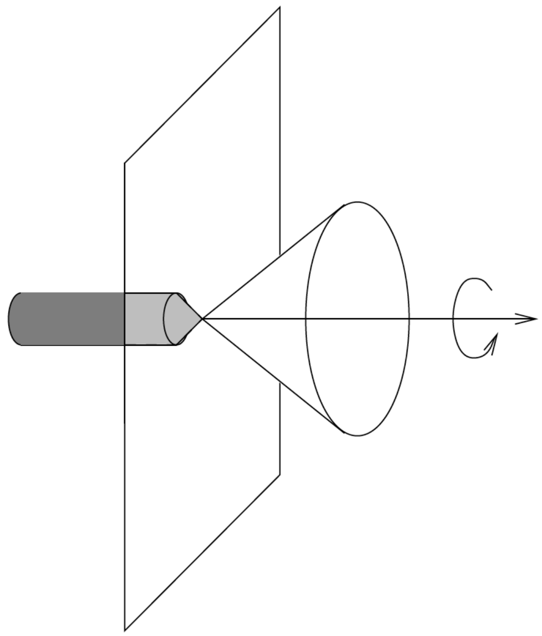
Soft-finger contact

- A more realistic contact model is the **soft-finger contact**



- Frictional forces and **torques** about that normal are allowed

Soft-finger contact

- The applied contact wrench is

$$F_{c_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} f_{c_i}, f_{c_i} \in FC_{c_i},$$

Note that the torque is only around the z-axis

Soft-finger contact

- The applied contact wrench is

$$F_{c_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} f_{c_i}, f_{c_i} \in FC_{c_i},$$

If constraints “violated”, there is slippery.

$$FC_{c_i} = \{f \in R^4 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3, f_3 \geq 0, |f_4| \leq \gamma f_3\}$$

Torsional friction coefficient

General contact model

- A contact model can be represented by using a **wrench basis** $B_{c_i} \in R^{p \times m_i}$.

- e.g. $p = 6, m_i = 4$ for $$F_{c_i} = \boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} f_{c_i}, f_{c_i} \in FC_{c_i}$$

- m_i indicates the number of independent forces that can be applied by the contact.

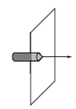
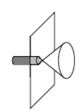
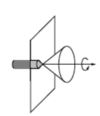
General contact model

- Friction cone FC_{c_i}:
  - The set of all possible wrench at a certain contact (determined by the local geometry and material).
  - FC_{c_i} is a closed subset of R^{m_i} with non-empty interior.
  - $f_1, f_2 \in FC_{c_i} \Rightarrow \alpha f_1 + \beta f_2 \in FC_{c_i}$ for $\alpha, \beta > 0$.

- **Contact wrench:** $F_{c_i} = B_{c_i} f_{c_i}, f_{c_i} \in FC_{c_i}$.

Summary of three kinds of grasp

Table 5.2: Common contact types.

Contact type	Picture	Wrench basis	FC
Frictionless point contact		$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$f_1 \geq 0$
Point contact with friction		$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\sqrt{f_1^2 + f_2^2} \leq \mu f_3$ $f_3 \geq 0$
Soft-finger		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\sqrt{f_1^2 + f_2^2} \leq \mu f_3$ $f_3 \geq 0$ $\lvert f_4 \rvert \leq \gamma f_3$

Grasp Model

- Friction Model at a Single Contact
- **Multi-Contact Configuration**

Multiple fingers

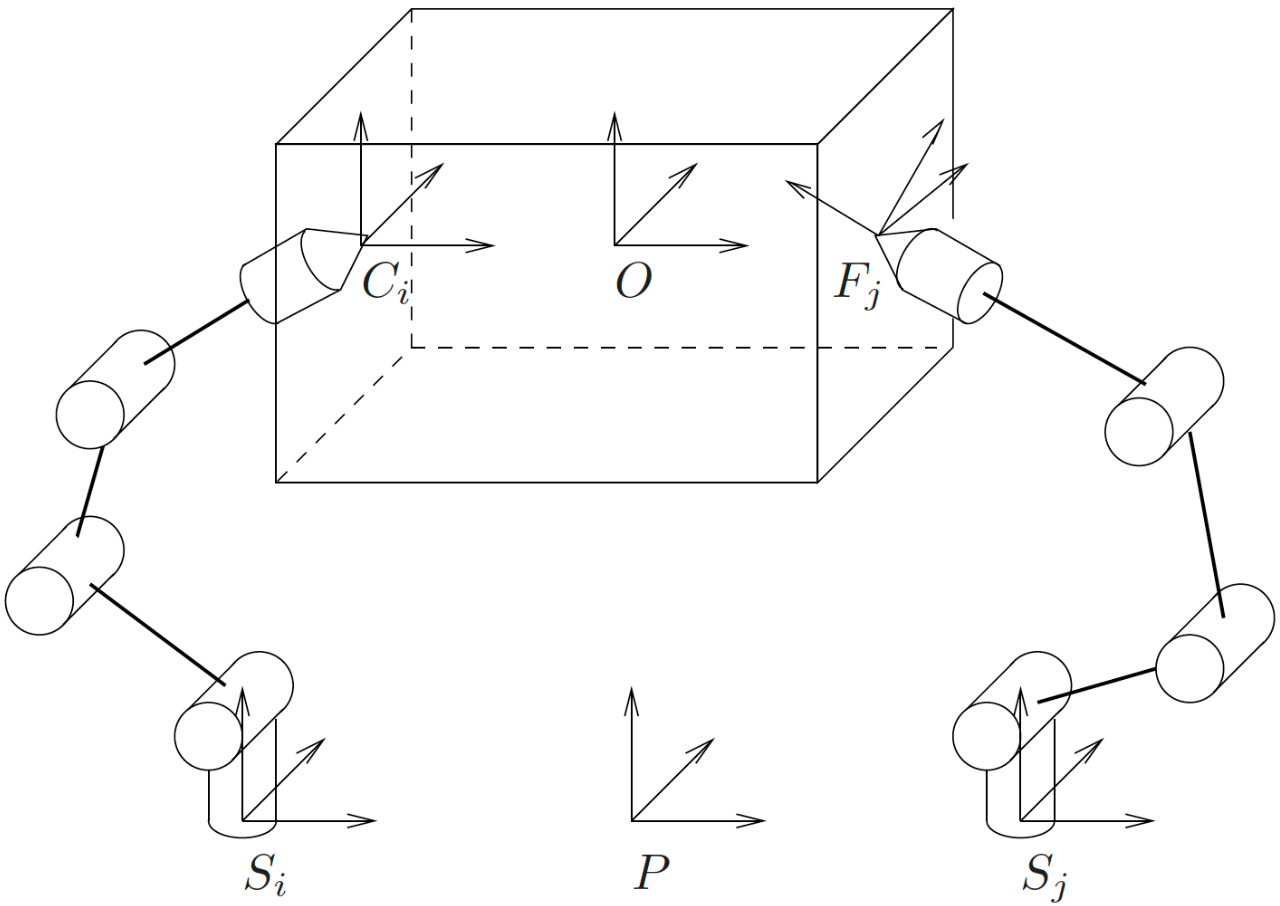
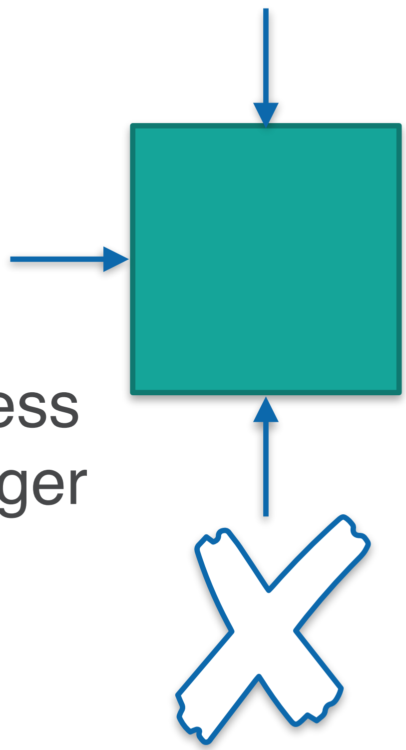


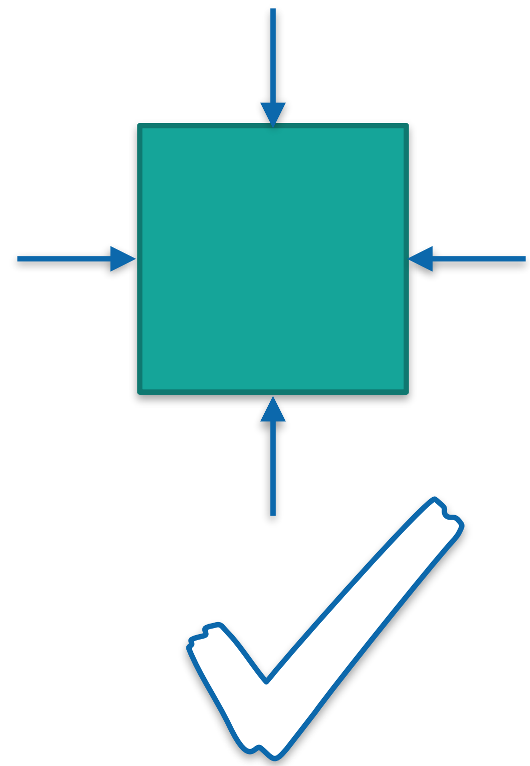
Figure 5.14: Grasp coordinate frames.

Force-closure

- **Question: How to set the contact positions so that any external wrench on the object can be resisted?**

frictionless
point finger





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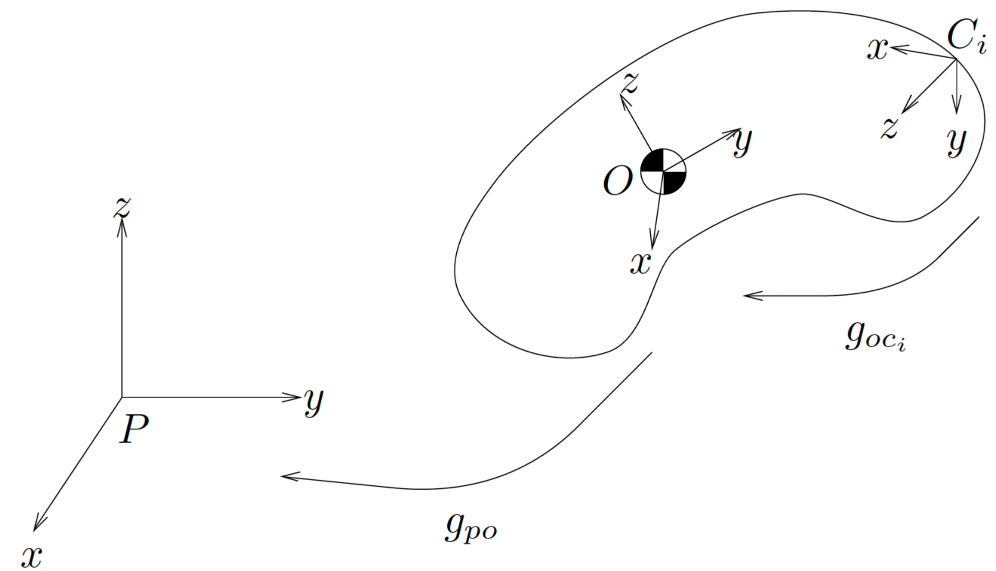
Net object wrench

- After having contact forces of each finger, we can calculate the net object force by summation of

$$F_o = \sum_{i=1}^{k} Ad^T_{g^{-1}_{oc_i}} F_{c_i} = \sum_{i=1}^{k} Ad^T_{g^{-1}_{oc_i}} B_{c_i} f_{c_i}$$



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Force closure

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$$F_o = \sum_{i=1}^{k} Ad^T_{g^{-1}_{oc_i}} F_{c_i} = \sum_{i=1}^{k} Ad^T_{g^{-1}_{oc_i}} B_{c_i} f_{c_i}$$

- Equivalent question: Given any external wrench F, can I find f_{c_i} so that $F_o = -F$

Grasp Map

- **Grasp map:** map between the contact wrench and the net wrench: $F_o = Gf_c, f_c \in FC = FC_1 \times \cdots \times FC_n$.

- $G = \left[Ad^T_{g^{-1}_{oc_1}} B_{c_1}, \cdots, Ad^T_{g^{-1}_{oc_k}} B_{c_k} \right]$, determined by contact positions and friction model types

- $f_c = \begin{bmatrix} f_{c_1} \\ \vdots \\ f_{c_k} \end{bmatrix}$, the set of wrench from all contact points

Force Closure

- Two equivalent conditions:

  - Given any external wrench $F_e \in R^p$ applied to the object, there exist contact forces $f_c \in FC$ such that $Gf_c = -F_e$.

  - $G(FC) = R^p$.

- If satisfied, the grasp (contact configuration) can resist any external wrench, which is a grasp with **force closure**.

Cases When It is Impossible to Satisfy Force Closure

- Example:
  - A frictionless sphere in R^3. It can rotate, even we have forces over the whole surface.

- **Exceptional surface** theory