L& Dynamics

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Kinematics v.s. Dynamics

- *Kinematics* describes the motion of objects. We have been talking about rigid transformation and derivatives wr.t. time.
- Dynamics describes the cause of motion. We will talk about mass, energy, momentum, and force.
- The basic law of dynamics, Newton's Law, describes the motion of a point mass:

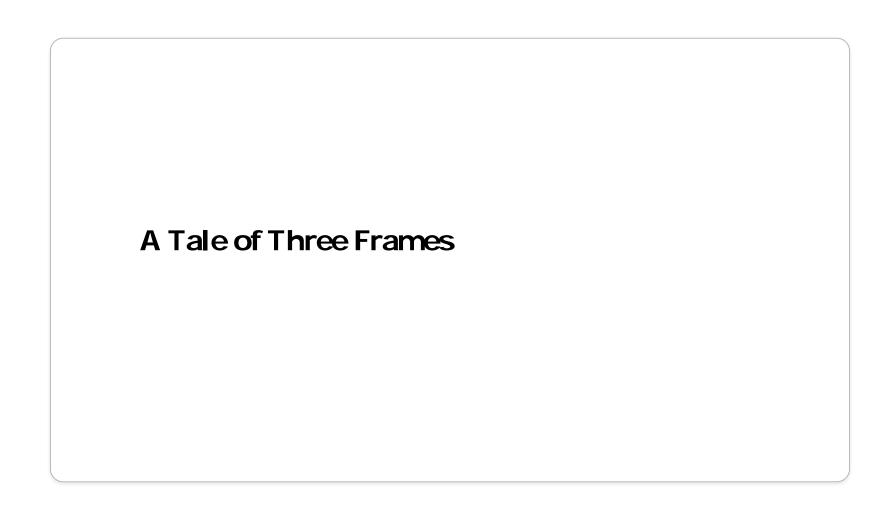
$$oldsymbol{f}=moldsymbol{a}$$

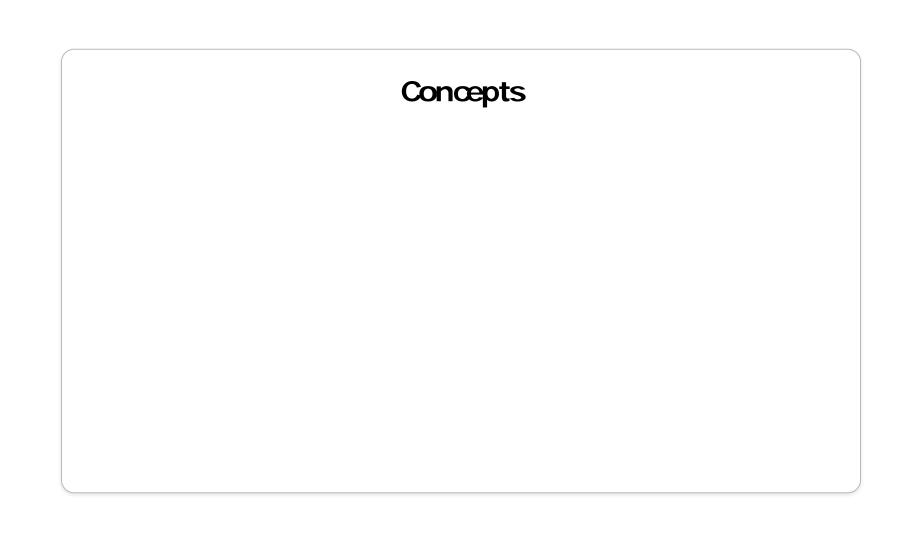
• But there are caveats that you may not be aware of.

Kinematics v.s. Dynamics

- We start from point mass dynamics and will move on to rigid body dynamics.
- We will provide certain proofs but not all (many are very tricky and lengthy).

H Show this help
Left & Right Previous & Next step
P Presenter console
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Concepts

- Observer's Frame:
 - \circ When we record any motion, we choose the observer's frame \mathcal{F}_o , so that every point would have a coordinate and every vector will have a direction and length.
 - For our symbols, this is on the superscript.
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Some Notes on Reference Frame

- Reference Frame:
 - When recording the movement of objects, we introduce a reference frame so that the notion of movement is *relative to* this frame.
- We have not discussed this frame much in developing robot kinematics theories.

In dynamics, the choice of reference frame is not arbitrary!

Recoding a Relative Velocity

- ullet We introduce s(t) to denote a reference frame which may be moving.
- Then we denote the relative velocity as below.
- Relative velocity for a point mass

$$egin{array}{l} \circ ~ oldsymbol{v}^o_{s(t)
ightarrow b(t)} = oldsymbol{v}^o_{o
ightarrow b(t)} - oldsymbol{v}^o_{o
ightarrow s(t)} \end{array}$$

• Relative velocity for rigid body

$$egin{array}{l} \circ \; oldsymbol{\xi}^o_{s(t)
ightarrow b(t)} = oldsymbol{\xi}^o_{b(t)} - oldsymbol{\xi}^o_{s(t)} \; . \end{array}$$

Consistency

$$oldsymbol{v}^o_{s(t) o b(t)} = oldsymbol{\xi}^o_{s(t) o b(t)} p^o$$

where p^o is a point observed in \mathcal{F}_o

Inertia Frame

- Inertia frame refers to the choice of the *reference frame*
- Only in an inertia frame can Newton's lawbe written as $m{f}=mm{a}$.
- Debnition of Inertia frame:
 - Where the law of inertia (Newton's First Law) is satisped.
 - o Any free motion has a constant magnitude and direction.
- A dear notion of Newton's Second Law.

$$oldsymbol{f}^o = moldsymbol{a}^o_{s(t)
ightarrow b(t)}$$

where s(t) is an inertia frame.

- What if the reference frame is not an inertia frame?
- ullet Assume we have two moving frames, $\mathcal{F}_{s(t)}$ and $\mathcal{F}_{b(t)}$
 - $\circ\,$ e.g., the earth and an object sitting on the earth
- ullet We are interested in how the force $m{f}^o$ affects the relative acceleration $m{a}^o_{s(t) o b(t)}$

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 - \circ Since $\mathcal{F}_{s(t)}$ is moving with an angular velocity, any object b(t) moving along with it must also have an acceleration to gain the same angular velocity.
 - \circ Computation shows that some additional force will be consumed to maintain the relative velocity of b(t) against s(t).

• Computing $m{f}^o=\mathrm{d}(mm{v}^o_{s' o b(t)})/\mathrm{d}t$ (note: s' is chosen to be an inertia frame), and we have

$$m{f}^o - mrac{\mathrm{d}m{\omega}^o}{\mathrm{d}t} imes r^o - 2mm{\omega}^o imes m{v}^o - mm{\omega}^o imes (m{\omega}^o imes r^o) = mm{a}^o$$

where

- \circ f^o : the physical forces acting on the object
- $\circ \; oldsymbol{\omega}^o := oldsymbol{\omega}^s_{s' o s(t)}$
- $\circ \; oldsymbol{v}^o := oldsymbol{v}^o_{s(t)
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- $egin{array}{ll} \circ ~ oldsymbol{r}^o := oldsymbol{r}^o_{s(t)
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- ullet Euler force: $-mrac{\mathrm{d} \pmb{\omega}^o}{\mathrm{d} t} imes r^o$
- ullet Centrifugal force: $-moldsymbol{\omega}^o imes(oldsymbol{\omega}^o imes r^o)$
- ullet Coriolis force: $-2moldsymbol{\omega}^o imesoldsymbol{v}^o$

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Some Notes on Reference Frame

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