

# Introduction to Deep Reinforcement Learning

# **Model-based Methods**

Zhiao Huang

# Model-free Methods

- The model  $p(s'|s, a)$  is unknown
  - we solve  $Q, V$  and the policy from the sampled trajectories/transitions

# Model-based Method

- Learn the environment model directly

$$p(s' | s, a)$$

- Learn  $R(s, a, s')$  if it's unknown

# Model-based Method

- Learn the environment model directly by supervised learning

$$p(s' | s, a)$$

- **Search** the solution with the model directly

# Model-based Methods

- Model and search have broad meanings

## Model

Physics  
Geometry  
Probability model  
Inverse Dynamics  
Game Engine

....

## Search

MCTS  
CEM  
RL  
iLQR  
RRT/PRM

.....

# Model-Predictive Control

- Forward model with parameters  $\theta$

$$f_{\theta}(s, a): S \times A \rightarrow S$$

- Predicts what will happen if we execute the action  $a$  at the state  $s$

# Forward Model



- We may have a forward model in mind

# Learning the Forward Model

- Sample transitions  $(s, a, s')$  from the replay buffer and train the model with **supervised learning**

$$\min_{\theta} E[\|f_{\theta}(s, a) - s'\|^2]$$



# Rollout

- Predict a short trajectory  $s_1, s_2, \dots, s_T$  if we start at  $s_0$  and execute  $a_0, a_1, a_2, \dots, a_{T-1}$  sequentially



“rollout” the forward model

# Model-Predictive Control

- Given the forward model  $f_\theta$  and the current state  $s_0$ , find a sequence of action  $a_0, a_1, a_2, \dots, a_{T-1}$  such that has the maximum reward

$$\max_{a_{0:T-1}} \sum_{i=1}^T R(s_i, a_i, s_{i+1}) \quad \text{s.t.} \quad s_{i+1} = f_\theta(s_i, a_i)$$

# Random Shooting

- Sample  $N$  random action sequences:

$$a_0^1, a_1^1, a_2^1, \dots, a_{T-1}^1$$

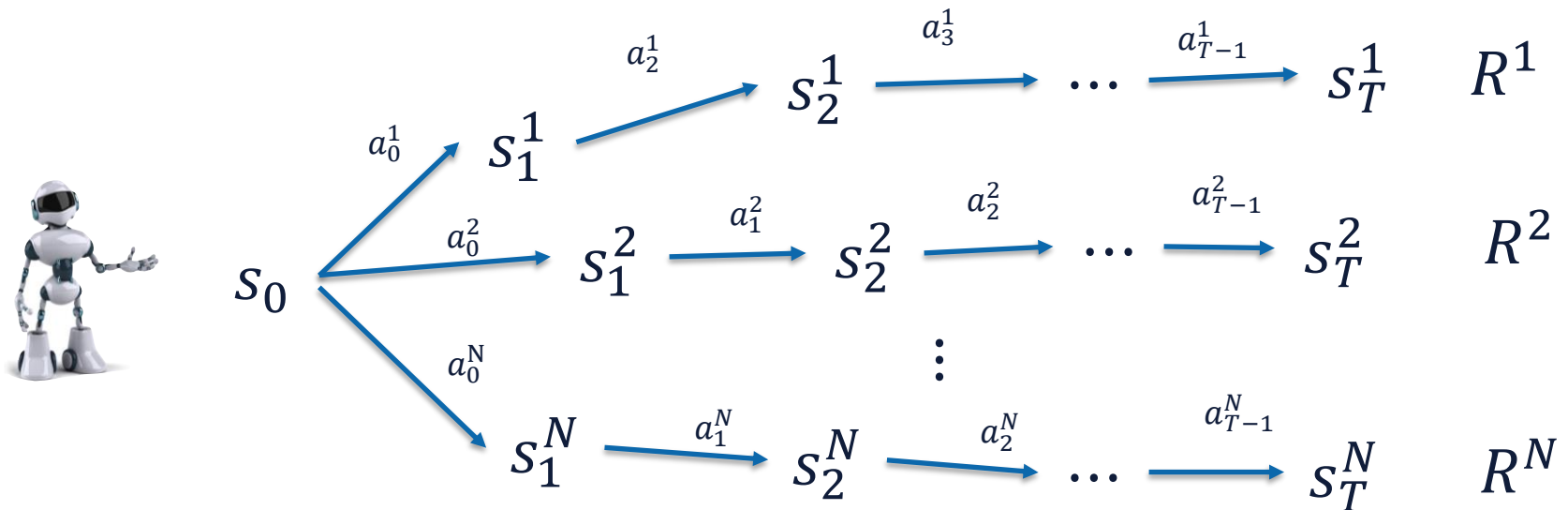
$$a_0^2, a_1^2, a_2^2, \dots, a_{T-1}^2$$

$\vdots$

$$a_0^N, a_1^N, a_2^N, \dots, a_{T-1}^N$$

# Random Shooting

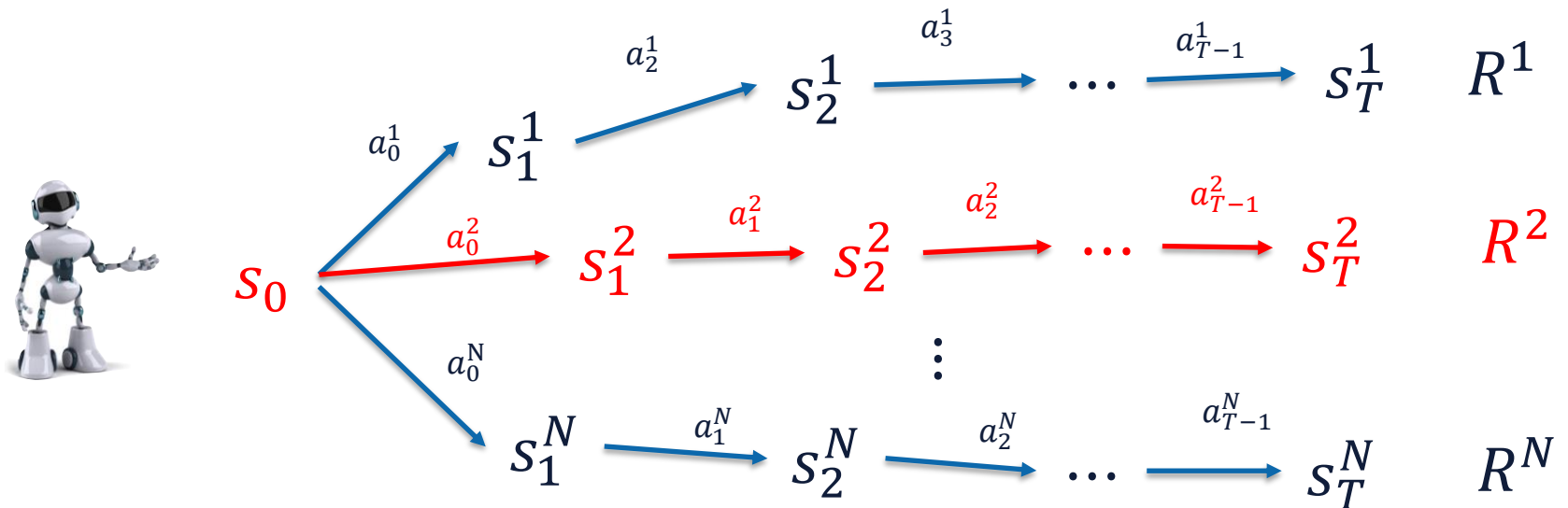
- Evaluate the reward of each action sequence by simulating the model  $f_\theta$



“rollout” the forward model  $f_\theta(s, a)$

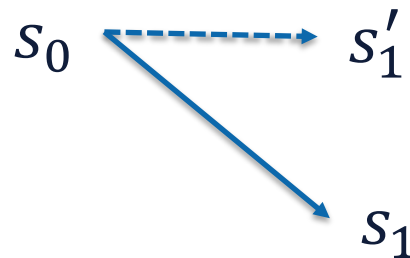
# Random Shooting

- Return the best action sequence  $a_{0:T-1}^*$  and execute in the real environment



# Planning at Each Step

If we execute the searched action sequence  $a_0, a_1, a_2, \dots$  in the environment



Model

Reality

- The action sequence  $a_1, a_2, \dots, a_{T-1}$  maximize the reward from  $s'_1 = f_\theta(s_0, a_0)$  but not  $s_1$
- Search new actions for state  $s_1$  again!

# Model Predictive Control

- Repeat
  - Observe the current state  $s$
  - Sample  $N$  random action trajectories
  - Evaluate the reward of each action sequence from  $s$  with the model  $f_\theta$ ; Find the best action sequence  $\{a_0^k, a_1^k, \dots, a_{T-1}^k\}$
  - Execute  $a_0^k$  in the environment

# Cross-Entropy Method

- Black-box Function Optimization

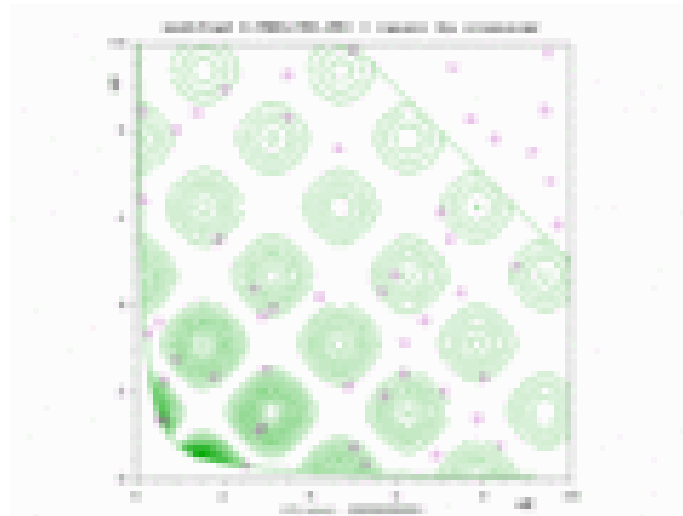
$$\max_x f(x)$$

- We have many other choices
  - Cross Entropy Method



# Cross-Entropy Method

- Basically the simplest evolutionary algorithm
- Maintain the distribution of solutions



# Cross-Entropy Method

- Initialize  $\mu \in \mathbb{R}^d, \sigma \in \mathbb{R}_{>0}^d$
- For iteration = 1, 2, ...
  - Sample  $n$  candidates  $x_i \sim N(\mu, \text{diag}(\sigma^2))$
  - For each  $x_i$  evaluate its value  $f(x_i)$
  - Select the top  $k$  of  $x$  as elites
  - Fit a new diagonal Gaussian to those samples and update  $\mu, \sigma$

# Cross-Entropy Method (in Python)

```
def cem(f, mean, std, num_iter=10, population_size=100, elite_size=20):  
    for i in range(num_iter):  
        populations = np.array([np.random.normal() * std + mean for j in range(population_size)])  
        values = np.array([f(j) for j in populations])  
        elites = populations[values.argsort()[-elite_size:]]  
        mean, std = elites.mean(), elites.std()  
    return mean
```

# Model Predictive Control

- Hyper parameters

$$\mu_a, \sigma_a, n_{\text{iter}}, n_{\text{pop}}, n_{\text{elite}}$$

- Initialize an action sequence  $\mu = \{a_i = \mu_a\}_{i < T}$
- Repeat
  - Observe the current state  $s$
  - Search the new action sequence with CEM
$$\{a'_0, a'_1, \dots, a'_{T-1}\} = \text{CEM}(\mu, \{\sigma_a\}_{i < T})$$
  - Execute  $a'_0$  in the environment
  - Update  $\mu \leftarrow \{a'_1, a'_2, \dots, a'_{T-1}, \mu_a\}$

# Model Predictive Control

- Hyper parameters

$$\mu_a, \sigma_a, n_{\text{iter}}, n_{\text{pop}}, n_{\text{elite}}$$

- Initialize

- Repeat

- Observe

- Search the new action sequence with CEM

$$\{a'_0, a'_1, \dots, a'_{T-1}\} = \text{CEM}(\mu, \{\sigma_a\}_{i < T})$$

- Execute  $a'_0$  in the environment

- Update  $\mu \leftarrow \{a'_1, a'_2, \dots, a'_{T-1}, \mu_a\}$

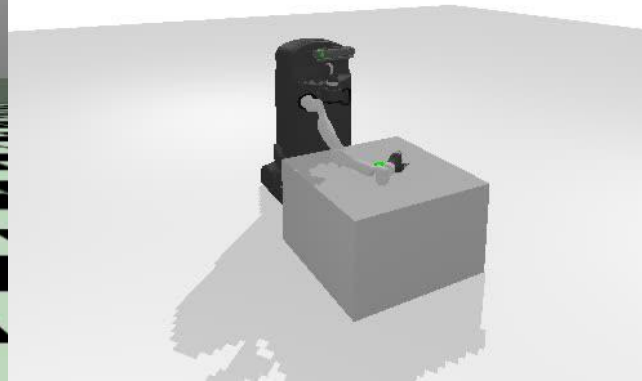
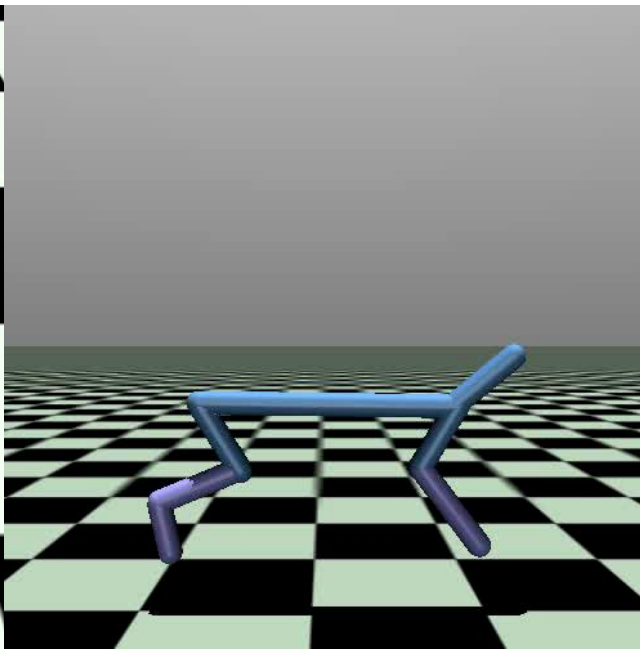
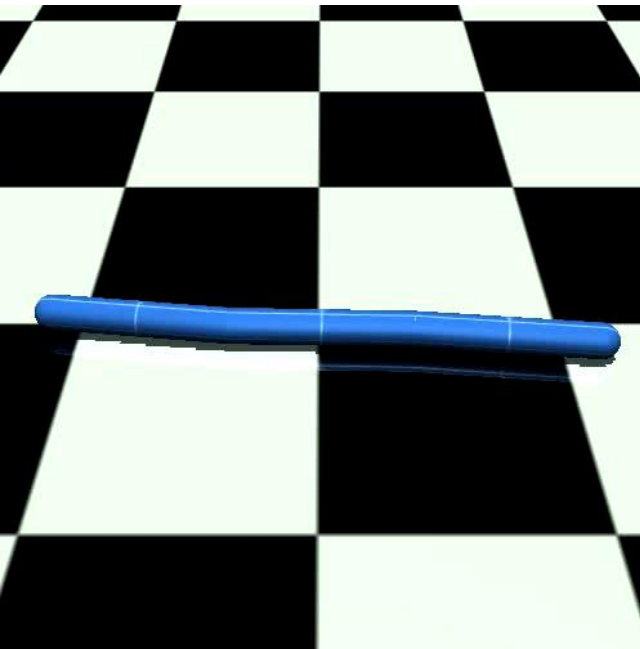
```
def cem(f, mean, std, num_iter=10, population_size=100, elite_size=20):  
    for i in range(num_iter):  
        populations = np.array([np.random.normal() * std + mean for j in range(population_size)])  
        values = np.array([f(j) for j in populations])  
        elites = populations[values.argsort()[-elite_size:]]  
        mean, std = elites.mean(), elites.std()  
    return mean
```

# Notes

- CEM performs well for most control tasks
- Instead of searching for action sequence, we can also search for the parameters of the network
- General “Gradient Descent”
- CEM and Random shooting work poorly for very long horizons  $T$  or dense reward

# Performance of CEM

- When the model is known



# Comparisons

- On-policy methods: Policy
- Off-policy methods: Value/Q
- Model-based methods: Model

Model  $\Rightarrow$  Value  $\Rightarrow$  Policy



# Comparisons

- How difficult to model it
  - $Q \text{ Value} > \text{Policy}$
  - Model depends on the priors
- Robustness
  - $\text{Model} < Q \text{ Value} < \text{Policy}$
- Time complexity
  - $\text{Model} > Q/\text{Policy}$
- Data-efficient/Generalization
  - $\text{Model} > Q \text{ Value} > \text{Policy}$

# Conclusion

- Very few data / We know the model well
  - Model-based methods
- We can't model the environment and we don't want to sample too much
  - Off-policy methods
- We have enough time/money
  - Off-policy + On-policy methods