BIOS731 Advanced Statistical Computing
Fall 2018
Homework 3

Due 10/29/2018 Monday midnight

Instruction: Please submit both write-ups and programs. The programs need to be written in a high-level language (no compilation required), and R is highly recommended. The codes for all problems need to be saved in a single file named NAME_hw3.EXT. Replace NAME by your name, and EXT by proper extension name, e.g., R, sas, etc. Provide adequate comments in the codes to clearly mark the section for different questions. The codes should generate all results and figures in the homework. Please make sure the codes are "self-contained", e.g., does not depend on platform, can be run at any other machine in any subdirectory, and does not require user input.

Problem 1. Evaluate integral $E(X^3 I(0 < X < 5))$ in which X follows Gamma distribution with mean 0.5, variance 0.25, using vanilla Monte Carlo and importance sampling approaches. With the same number of samples, compare their estimation accuracy.

Problem 2. Inference for a mixture Normal distribution $X \sim \pi N(\theta_1, 1) + (1-\pi)N(\theta_2, 1)$. Assume non-informative prior for all parameters involved. Write down the joint posterior distribution. Using the data shown below, design and implement an MCMC algorithm to sample the two mean parameters θ_1 and θ_2 from their posterior distribution. Obtain trace plots and autocorrelation plots to examine the convergence of your Markov chains.

2.2656547718607
0.0475116560589399
1.67796814317884
4.76848366876514
-0.322178469606521
0.813992315659205
2.51993952700739
-0.230085009350269
3.80998228337117
2.92711708313745
0.00698193539200804
0.301471700015266
2.74769842182939
1.9772827221674
-0.687953509535934
3.37823628789538
2.56751523647356
4.22533486072877
3.16661615698266
-1.06294197108741
0.187601065761359
0.684337641788993
3.37427374759632
0.712107534848925
3.32300778310447

2.43255005083827
0.336805356623137
-0.988243597352675
4.14385052334378
3.78143250161659