BIOS731 Advanced Statistical Computing
Fall 2020
Homework 3

Due 10/16/2020 Friday at 11:59PM

Instruction: Please submit both write-ups and programs. The programs need to be written in a high-level language (no compilation required). The codes for all problems need to be saved in a single file named NAME_hw3.EXT. Replace NAME by your name, and EXT by proper extension name, e.g., R, sas, py, etc. Provide adequate comments in the codes to clearly mark the section for different questions. The codes should generate all results and figures in the homework. Please make sure the codes are "self-contained", e.g., does not depend on platform, can be run at any other machine in any subdirectory, and does not require user input.

Total is 100 points. Partial credit will be given.

Problem 1 (20 pts). Jack performed an independent student t test on 5 cases and 5 controls. The test statistics is 10. What is the corresponding p-value? Design a Monte Carlo method to accurately estimate it with 3 significant digits.

Problem 2 (20 pts). Evaluate integral $E(X^3 I(0 < X < 5))$ in which X follows Gamma distribution with mean 0.5, variance 0.25, using vanilla Monte Carlo and importance sampling approaches. With the same number of samples, compare their estimation accuracy.

Problem 3 (60 pts). Two loaded coins are used to produced 10 different series of heads and tails. It is known that one coin produces more heads and the other more tails. Each series contains 50 results. Both coins are used in every one of the 10 series, and one and only one switch occurs within each series. The sequences are shown below (1-head,0-tail). Design a Gibbs sampler to estimate the switch point for each series.

  a. (10 pts) Write down your statistical model for the data, using proper probability distribution(s).
  b. (10pts) What are the model parameters? Write down the likelihood function.
  c. (10 pts) Assuming noninformative priors. Write down the joint posterior distribution and all the conditional distributions for unknown parameters.
  d. (25 pts) Implement the Gibbs sampler and check convergence.
  e. (5 pts) What are your estimate of the switching points?

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