Simultaneous Transmission of Information and Power

Readings

Simultaneous Transmission of Information and Power.pdf

Transporting Information and Energy Simultaneously.pdf

Signal and Channel Model

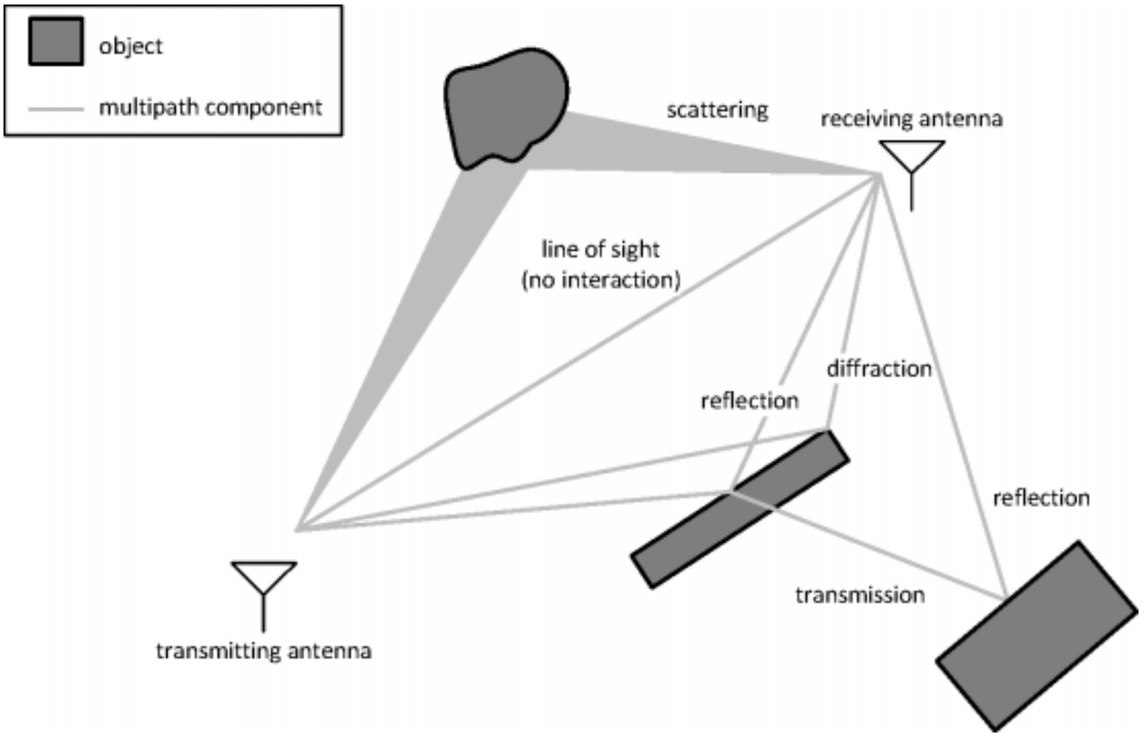
Nyquist sampling theorem: a signal with bandwidth B can be completely reconstructed by a sampling rate of $2B$ samples per second.

A baseband equivalent signal of bandwidth $W/2$ can be represented as

$$x(t) = \sum_n x\left(\frac{n}{W}\right)\mathrm{sinc}(Wt-n) = \sum_n x[n]\mathrm{sinc}(Wt-n)$$

- $x[n]$: signal value at the n-th sample
- $\{\mathrm{sinc}(Wt-n)\}_n$: sampling kernels

Multipath channel: the receive signal is the superposition of the transmit signal propagating through different paths.



Multipath channel can be modelled as tapped delay line (TDL) [src]

$$y(t) = \sum_i a_i(t) x(t - \tau_i(t)) e^{-j2\pi f_c \tau_i(t)} + \underline{w(t)}$$

function of t

AWGN

channel response

- i: path index
- $a_i(t)$: time-varying attenuation of path i
- $\tau_i(t)$: time-varying relative delay of path i

Channel model:

May 2004 **doc.: IEEE 802.11-03/940r4**

Appendix C – Model D

	Tap index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	Excess delay [ns]	0	10	20	30	40	50	60	70	80	90	110	140	170	200	240	290	340	390
Cluster 1	Power [dB]	0	-0.9	-1.7	-2.6	-3.5	-4.3	-5.2	-6.1	-6.9	-7.8	-9.0	-11.1	-13.7	-16.3	-19.3	-23.2		
AoA	AoA [°]	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9	158.9		
AS (receiver)	AS [°]	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7	27.7		
AoD	AoD [°]	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1	332.1		
AS (transmitter)	AS [°]	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4	27.4		
Cluster 2	Power [dB]											-6.6	-9.5	-12.1	-14.7	-17.4	-21.9	-25.5	
AoA	AoA [°]											320.2	320.2	320.2	320.2	320.2	320.2	320.2	
AS	AS [°]											31.4	31.4	31.4	31.4	31.4	31.4	31.4	
AoD	AoD [°]											49.3	49.3	49.3	49.3	49.3	49.3	49.3	
AS	AS [°]											32.1	32.1	32.1	32.1	32.1	32.1	32.1	
Cluster 3	Power [dB]															-18.8	-23.2	-25.2	-26.7
AoA	AoA [°]															276.1	276.1	276.1	276.1
AS	AS [°]															37.4	37.4	37.4	37.4
AoD	AoD [°]															275.9	275.9	275.9	275.9
AS	AS [°]															36.8	36.8	36.8	36.8

Submission page 37 Vinko Erceg, Zyray Wireless; et al.

IEEE TGn channel model D, attenuation (power) and delay are time-invariant [src]

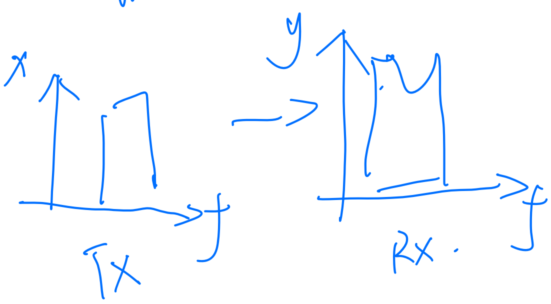
Delay spread: the span of signal arrival. *(definition not unique)*

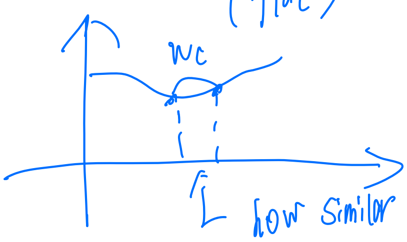
$|t_n - t_k| = t_d$.
delay spread.

$t_d \longrightarrow W_c$
delay spread — coherence bandwidth.

$W_c \propto \frac{1}{t_d}$

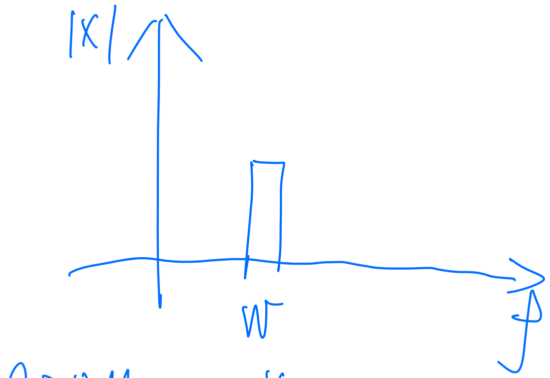
$W > W_c$: selective freq



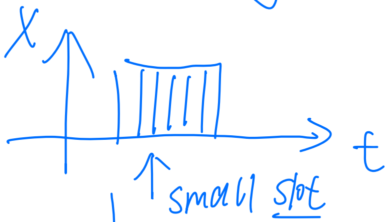


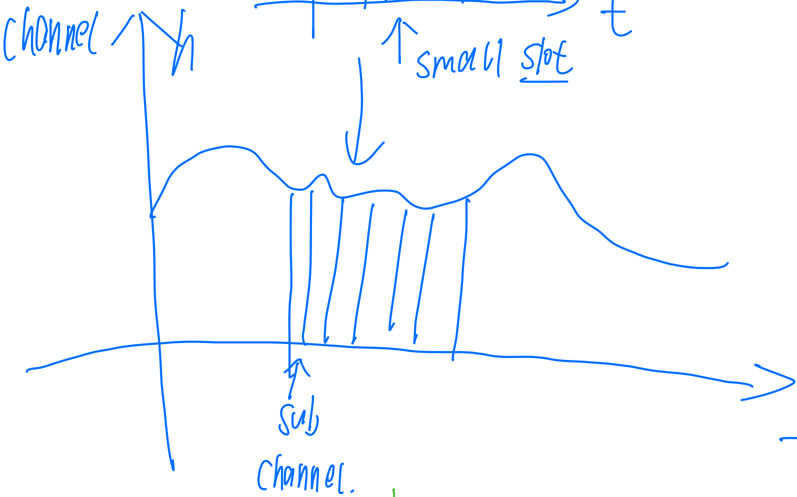
property of channel

$W < W_c$



OFDM





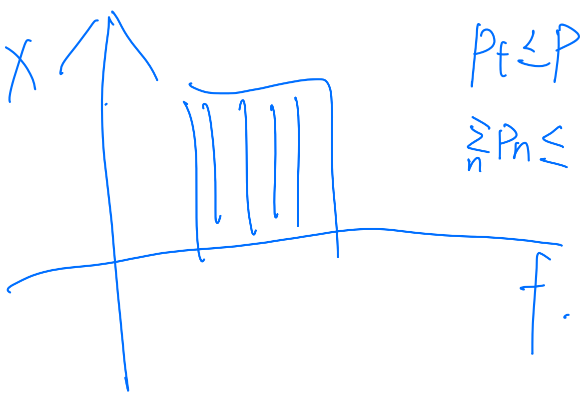
$y_1 \to h_1, x_1$

different noise.

$y_1 = h_1 x_1 + n_1$ $\quad$ $y_2 = h_2 x_2 + n_2$

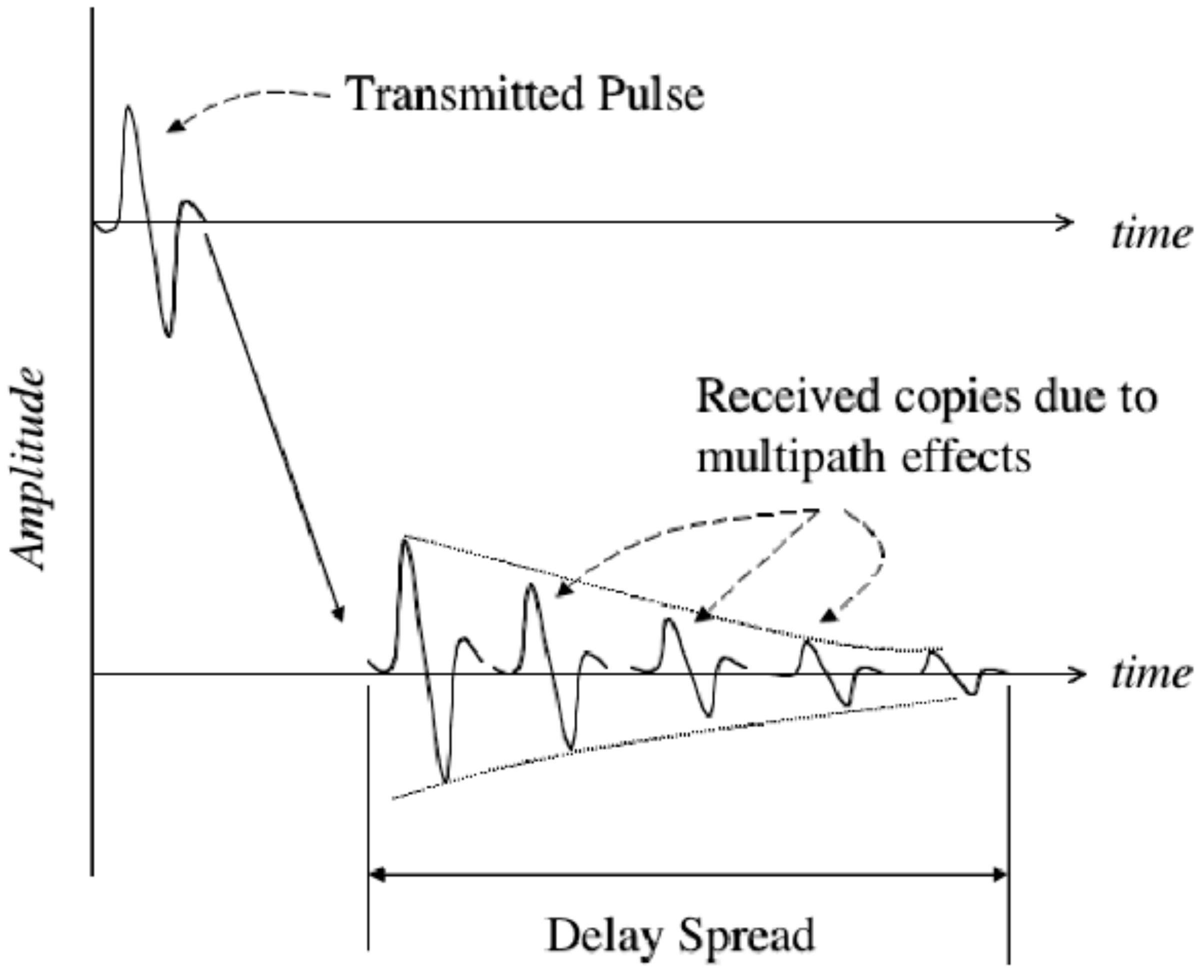
① flat fading for each compoent.

② . subchannel are parallel.



$P_t \le P$

$\sum_n P_n \le P$ $\quad$ $P_n \ge 0.$



Delay spread and multipath effect [src]

Delay spread T_d → coherence bandwidth W_c (channel characteristics)

- $W < W_c$: **flat fading** (different frequency components see the same channel)
- $W > W_c$: **frequecy-selective fading** (different frequency components see different channels)

OFDM: divide frequency-selective channel into multiple orthogonal subchannels, with independent AWGN. $\log_2(1+SNR)$ (bps)

- power allocation on subcarrier n: $\sum_n P_n = P, P_n \geq 0, \forall n$
- capacity of subband n: $\log_2\left(1+\frac{P_n|h_n|^2}{N_0}\right)$
  - $|h_n|$: amplitude response at subchannel n
  - N_0: average noise power

- if use $\log$ instead of $\log_2$, the unit is nats/s per symbol rather than bits/s per symbol

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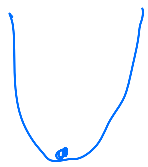
Optimization

min ←

Consider a sum-rate maximization problem subject to average transmit power constraint:

① local = global optim
② KKT conditions
⇓
global optim

$y=x^2$



$$\max_{P_0,\ldots,P_{N_c-1}} \sum_{n=0}^{N_c-1} \log_2\left(1+\frac{P_n|h_n|}{N_0}\right)$$

$$\text{s.t.} \quad \begin{cases} \sum_{n=0}^{N_c-1} P_n = P \\ P_n \geq 0, \qquad \forall n \end{cases}$$

This is a convex problem (rest assured), which enables optimization tools such as KKT optimality conditions.

i. objective is convex.
ii. constraints is convex. → min convex / max. concave.

The **standard form** of an optimization problem is

$$\min_x \quad f_0(x)$$

$$\text{s.t.} \quad \begin{cases} f_i(x) \leq 0, & i=1,\ldots,m \\ h_i(x) = 0, & i=1,\ldots,p \end{cases}$$

(FYI only, to be a convex problem, f's should be convex while h's should be linear)

It's **Lagrangian** is defined as

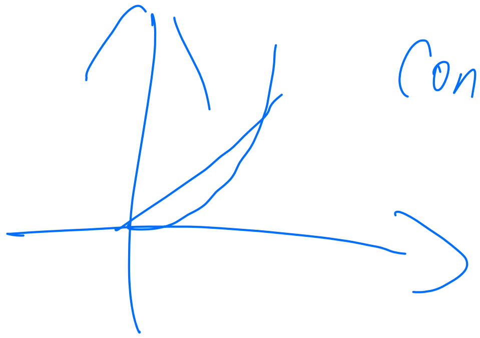
just define make KKT easy to present.

$$L(x,\lambda,\nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x)$$

Lagrangian multipter

The **KKT conditions** are

- $f_i(x) \leq 0, \quad i=1,\ldots,m$
- $h_i(x) = 0, \quad i=1,\ldots,p$
- $\lambda_i \geq 0, \quad i=1,\ldots,m$



$\sum p_n$

$\downarrow$

linear. (both concave & Convex)

standard convex.

$\min \ f_0(x)$

s.t. $f_i(x) \leq 0.$

$h_j(x) = 0 \quad j = 1, \cdots, n$

$$\begin{cases} f_0 : \text{convex.} \\ f_i(x) : \text{convex} \end{cases}$$

$h_j(x)$: linear

- $\lambda_i f_i(x) = 0, \quad i = 1, \ldots, m$
- $\nabla f_0(x) + \sum_{i=1}^{m} \lambda_i \nabla f_i(x) + \sum_{i=1}^{p} \nu_i \nabla h_i(x) = 0$

Why KKT conditions?

- it is necessary for local optimality, for solvable problem
- it is sufficient for global optimality, for convex problem

Hence, if the problem is convex, we can solve KKT conditions for the optimal solution.

Tips for Exercise 2, 3

- write the optimization problem in the standard form
- use $\log$ for simplicity
- use `bar([carrierNoise; carrierPower], 'stacked')` for a stacked view

Comments

Now it's your turn! Few suggestions:

- for $P_d = 0$ (wireless communication only), the solution is the famous "water-filling" power allocation, could you explain the intuition behind?
- ~~how do λ and μ balance the information and power transfer?~~
- ask questions and discuss together
- don't worry if you find it hard, you don't need to understand everything - this lab is just an introduction, more details in module *Wireless Communications* and *Topics in Large Dimensional Data Processing*

Q&A

- [ ] Post any public question in Teams channel! You can also email Yang (yang.zhao18@imperial.ac.uk) or Prof Bruno Clerckx (b.clerckx@imperial.ac.uk) for questions.

(Last updated by @Yang Zhao at @September 30, 2020)