Optical Communication

Notes Part A : Introduction

1. Introduction

The communication of information using light is an ancient technique; the use of signal fires and flares, smoke, and mirrors having been among the first long distance communication means. On the other hand, the possibility of guiding light within channels of transparent material, such as glass rods and tubes, and even jets of water, has also been long known. It is the coming together of these two things that has made optical communication grow into such an important technique over the last three decades. In 1966, Kao and Hockham proposed the use of glass fibre as a medium for transmission of optical signals. It would be some years before the technique became practical, but with the development of high quality fibre, along with high speed detectors and, more importantly, diode lasers, the application of fibre optic communication increased rapidly.

The technologies that fibre has increasingly supplanted are all based on the modulation and propagation of electromagnetic waves, in cables or through the atmosphere. Optical communication is also based on electromagnetic radiation propagating in cables. Why then has it been found to offer such advantages, and what are these?

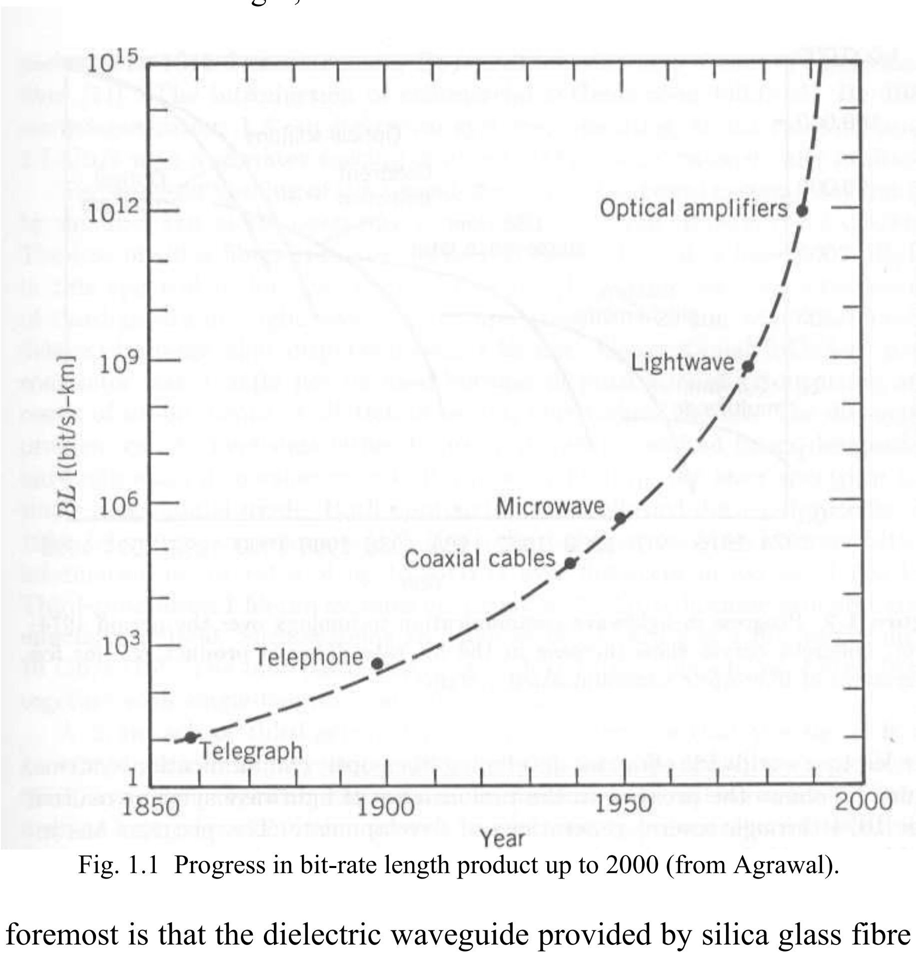

Fig. 1.1 Progress in bit-rate length product up to 2000 (from Agrawal).

The first and foremost is that the dielectric waveguide provided by silica glass fibre can transport radiation with far less attenuation and distortion (particularly dispersion) than can any cable based on conductive media (e.g. copper). However, dielectric waveguides are only practical for the very short wavelengths of optical frequencies, as the lateral dimensions of the guides are directly determined by wavelength, as we shall see. Once the data rates on telephone trunk lines reached about 100 Mbit/s, in the early eighties, coaxial copper cable could only manage about 2 km between repeaters, and the possibility for further increase in bandwidth was minimal. Now, optical systems in place carry 100's of Gbit/s over 100's of km without repeaters, and laboratory demonstrations show that several orders of magnitude further improvement is possible.

Another important reason for this great capacity in optical communications is that the carrier frequencies are extremely high. Visible light has wavelengths from ~ 0.4 to 0.7 μm, with the corresponding frequencies being in the vicinity of 10^{15} Hz (the wavelengths used for optical communication are actually in the near infra-red, at 0.8 to 1.5 μm, for reasons we will discuss). This suggests that with only a small fractional frequency spread, a vast amount of signal bandwidth could be accommodated. On the other hand, the wavelengths are much smaller than the dimensions of circuit and system components, whereas in radio they are typically much larger (metres to kilometres). Electronic circuits cannot be constructed for optical frequencies, and spectrally narrow sources and detectors are very difficult to achieve. It also proves to be significant that the quanta of energy in optics (photon energies) are much larger than both those of electronic systems and the thermal quantum, kT, as we shall see.

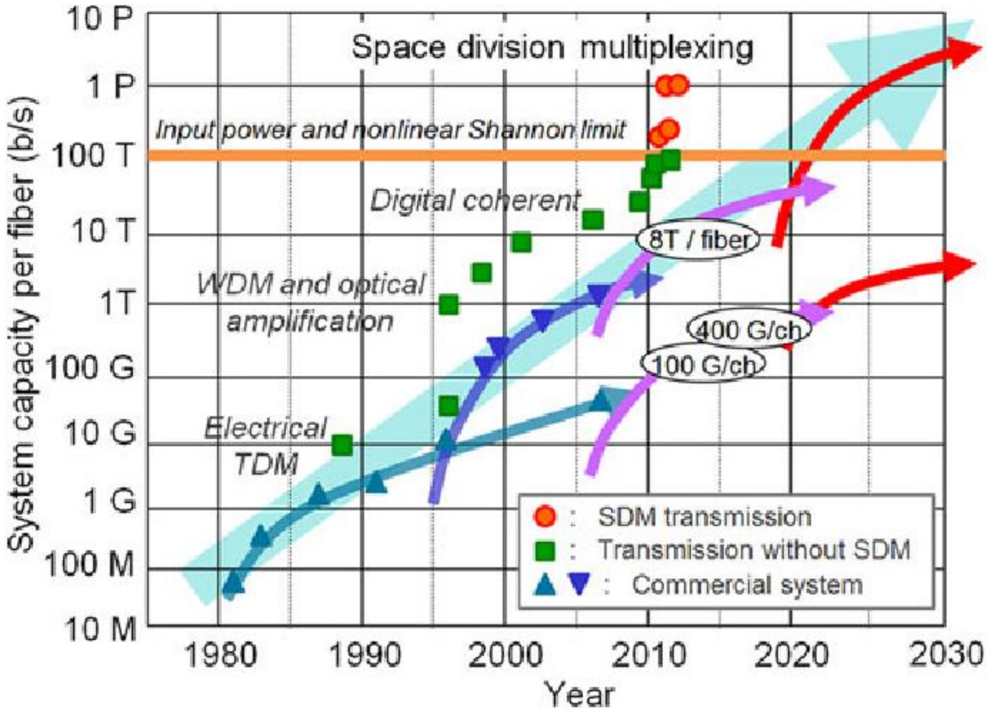

Fig. 1.2 Progress in optical bandwidth (from Mizuno et al, J Lightwave Tech, 2015).

This quite extraordinary capacity for high data rate, long distance digital links has justified huge investment in fibre optic technology, particularly for critical undersea links. This investment has resulted in dramatic increases in component and fibre quality, and greatly reduced costs, making fibre practical for an increasing range of applications. Today, it is the method of choice for virtually all high bandwidth digital links over anything but the shortest distances, and increasingly for low bandwidth and analogue links. Fibre already carries data directly into homes and offices, and even between cabinets and boards in major data facilities.

The practicality of fibre in such a range of applications allows other advantages to be exploited. Fibre cable is, for a typical application, substantially smaller and lighter than the equivalent copper cable, reducing costs and easing deployment. Cross-talk between fibres, and interference from or to external sources, is negligible. The lack of conductors in the cable is a safety advantage in flammable or explosive environments, or in the vicinity of high voltages. The isolation of the signals makes surreptitious monitoring (spying) more difficult, and easier to detect. Optical fibre is rugged, has high reliability and long lifetime, uses plentiful materials, and by eliminating repeaters can greatly reduce maintenance costs.

In this course we shall examine the properties of optical fibre leading to its extraordinary performance, the sources and detectors that make a communication link possible, and the development of fibre optics into complete communication systems.

2. Electromagnetic Radiation

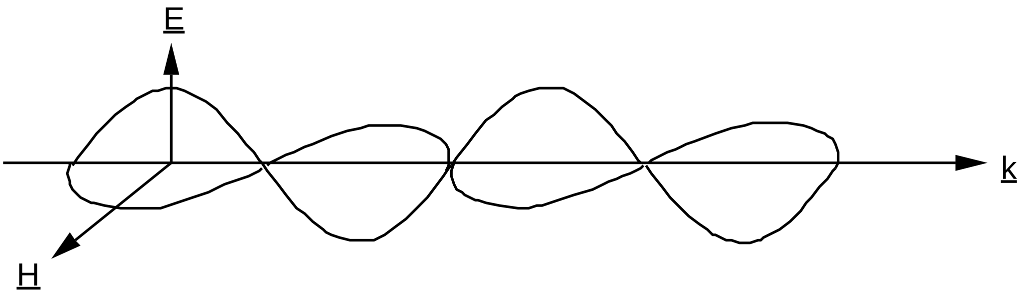

Figure 2.1 Geometric form of propagating electro-magnetic wave.

Electromagnetic radiation consists of, in most circumstances, oscillating electric and magnetic fields, perpendicular to each other and perpendicular to the direction of propagation. A field of arbitrary form, such as a beam, can be described as a superposition of plane waves. A plane wave is a wave of infinite lateral extent, with no variation in amplitude or phase of E & H in the directions perpendicular to that of propagation, and is thus characterised by planes of constant phase.

The electric and magnetic fields associated with such a wave can be conveniently written in the following form:

$$E = E_o \exp j\,(\underline{k} \cdot \underline{r} - \omega t) \qquad H = H_o \exp j\,(\underline{k} \cdot \underline{r} - \omega t)$$

Here ω and $\underline{k}$ are the angular frequencies in time and space respectively. Because space is multidimensional, $\underline{k}$ is a vector, and its direction is the direction of propagation of the wave. The temporal and spatial frequencies are related to the period and wavelength by:

$$\omega = 2\pi/T \qquad |\underline{k}| = 2\pi/\lambda$$

$\underline{k}$ has components in the x,y, and z directions, and since $\underline{r}$ is the position vector $\underline{r} = (x, y, z)$, we can write:

$$\underline{k} \cdot \underline{r} = k_x x + k_y y + k_z z$$

The relations between the magnitudes of E & H, and of $\underline{k}$ and ω, are determined by the medium through which the wave propagates. The most important medium properties for E-M radiation are the relative permittivity and permeability ε_r and μ_r. In general, these are functions of direction, but we will mainly be concerned with isotropic materials (where ε_r and μ_r are scalars);

only certain crystalline materials have significant directional dependence of ε_r and μ_r. Furthermore, for all but ferromagnetic materials (with which we are not concerned), $\mu_r \cong 1$, so that we are really only concerned with one quantity, ε_r.

The relation between ω and k is simply:

$$v_p = \omega/k \qquad \text{where } k = |\underline{k}|$$

Here v_p is the phase velocity - the speed at which a particular phase appears to move forward in space. This quantity is determined by the material, according to:

$$v_p = \frac{c}{\sqrt{\varepsilon_r}}$$

Here c is the speed of light in vacuum, $c = 3 \times 10^8$ m/s. The quantity $\sqrt{\varepsilon_r}$ is such an important quantity in optics that it has its own name:

$n = \sqrt{\varepsilon_r}$ is the index of refraction.

It will also prove very convenient to define a free-space wave vector:

$$k_o = \omega/c$$

so that in an arbitrary medium, the magnitude of $\underline{k}$ is given by:

$$|\underline{k}| = nk_o$$

In free space, n=1, and in air this is also approximately true. In typical glasses, $n \cong 1.5$; in water n = 1.33, and in semiconductors n is typically in the range 3 – 4.

In general, n is a function of the frequency; this property is called material dispersion, of which more later. It is also in general a complex quantity, the imaginary part being related to absorption.

Let us consider a plane wave travelling in the z direction, through an isotropic medium of refractive index n_1. Then $|\underline{k}| = n_1 k_o$, and:

$$E(z,t) = E_o \exp j\,(n_1 k_o z - \omega t)$$

If the material is a dielectric with some absorption losses, then n_1 will have a small imaginary part, i.e. $n_1 = n_R + j\,n_x$, with $n_x << n_R$. Then k ($= n_1 k_o$) will also have an imaginary part, and so

$$E(z,t) = E_o \exp j(n_R k_o z - \omega t)\,\exp(-n_x k_o z)$$

This is the equation of a decaying travelling wave, with spatial period $2\pi/n_R k_o = \lambda_o/n_R$, and with 1/e decay distance $\lambda_o/(2\pi n_x)$. (λ_o is the free space wavelength, see below.) Since $n_x << n_R$, we can relate the real and imaginary parts of n and ε_r by $\varepsilon_r \cong n_R^2 + j2n_R n_x$.

There is no fundamental difference between radio frequency waves and light; they are simply different aspects of the same phenomenon. There is of course a continuous spectrum of EM radiation from radio frequency (up to ~ 1 GHz) through microwaves, to infra-red, visible and ultraviolet light to X-rays and gamma rays. However, as the wavelengths shorten, the fundamental units (quanta) of radiation, photons, become more physically significant.

The energy of a single photon is equal to Planck's constant times the frequency, usually written as:

$$\mathcal{E} = h\nu \ (= hc/\lambda_o)$$

where $h = 6.63\times10^{-34}$ J·s, or 4.14×10^{-15} eV·s. We often use units of electron-volts (eV) for photon energies. For example, red light at a wavelength of 650 nm has a photon energy of about 3×10^{-19} J, or 1.9 eV. At room temperature, the thermal quantum kT is about 26 meV.

In optics, it is more usual to refer to the wavelength of radiation than to the frequency of radiation. Naturally, the two are related, according to $\lambda\nu = v_p$, where $\lambda = 2\pi/k$, and $\nu = \omega/2\pi$. (At conventional frequencies we usually use f rather than ν). Note that this can cause confusion, as while the frequency of a particular photon is constant, its wavelength can change as it passes through various media. When we speak about a 0.85 μm wavelength semiconductor laser, we mean that the wavelength of the emitted light in air is 0.85 μm, although λ will be less in the semiconductor itself, or in a fibre, due to the higher indices of these materials. Here we have used λ_o to imply this free-space wavelength $\lambda_o = 2\pi/k_o$.

While <u>E</u> & <u>H</u> will be perpendicular to each other, their directions may be fixed in space (linear polarisation), or may rotate together as the wave propagates (circular or elliptical polarisation). The latter case is equivalent to a superposition of two linear polarisations of different phase, so we will concern ourselves primarily with linear polarisation.

The relation between the E & H magnitudes is:

$$\frac{H_o}{E_o} = \sqrt{\frac{\varepsilon}{\mu}} = n\sqrt{\frac{\varepsilon_o}{\mu_o}}$$ for $\mu_r = 1$, where $\sqrt{\frac{\mu_o}{\varepsilon_o}}$ is the impedance of free space, 375 Ω.

Note also that $c = \sqrt{\dfrac{1}{\mu_o\varepsilon_o}}$.

3. The Electromagnetic Waveguide

We can create a guide or channel for some type of energy by making a path it can travel down but not get out of. Electrical current is channelled in conductors surrounded by insulators. However, although the current cannot leak out of a wire, an EM field is created outside the wire by the varying current. At high signal frequencies, this external field becomes important, as the interactions with other conductors and even dielectrics become stronger, resulting in dispersion, reflections and loss of energy. It becomes important to contain these fields as well, and this is what is done in coax cable. The EM field here is almost entirely contained in the very uniform material and geometry of the dielectric between the inner and outer conductors. Very little of this field leaks outside the cable, because the outer metal is a highly effective reflector for EM radiation.

In general, to trap EM fields we need to make them bounce back and forth between "walls" of some sort. To examine how these walls can be made, let us look at the reflection of plane waves at plane boundaries.

In figure 3.1, the y-z plane is arbitrarily chosen as the interface between media of index n_1 and n_2. The propagation vectors of the incident, reflected and transmitted waves, and their associated angles, are drawn. On the right side of the figure the effect of phase matching on the boundary can be seen. We have chosen an example where $n_2 < n_1$, so that the wave is faster in the upper medium, and the phase fronts are consequently farther apart (the separation of lines of equal phase is simply λ_o/n). Keeping this spacing while matching the phases along the interface with those of the incident (and reflected) waves determines the angle of propagation in medium 2, i.e. Snell's Law (see below).

We wish to determine the relative amplitudes of the three waves - either the $\underline{E}$ amplitudes or the $\underline{H}$ amplitudes, but we will usually concern ourselves with the former.

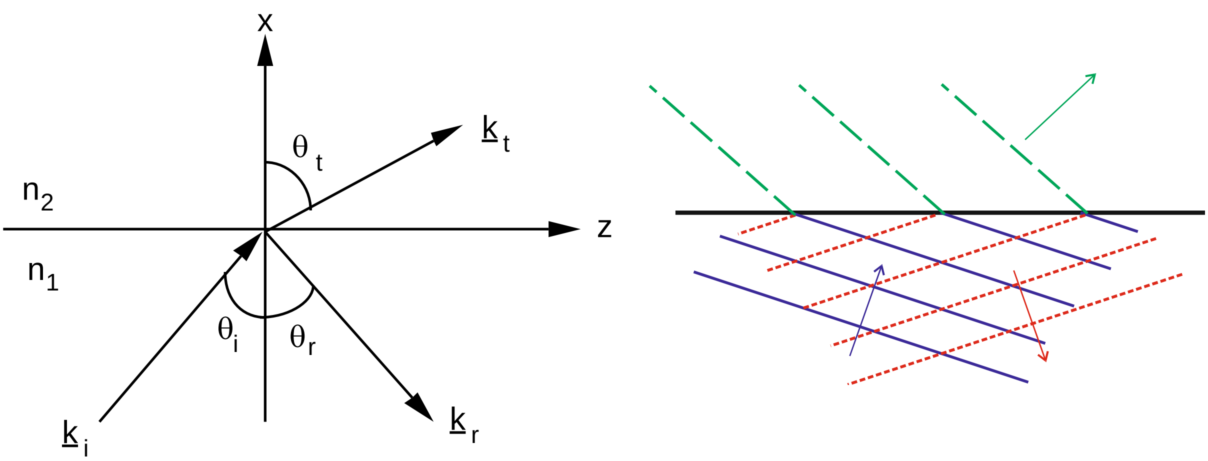

Figure 3.1 Reflection from a plane interface shown as rays (left), and as phase fronts (right).

Firstly, $\underline{k}_i$ $\underline{k}_r$ and $\underline{k}_t$ will be coplanar, and this plane (x-z) we call the *plane of incidence*. The $\underline{k}$ vectors have no y components in this case, and thus geometry gives us:

$$|k_i|^2 = n_1^2 k_o^2 = k_{ix}^2 + k_{iz}^2$$

and $$k_{ix} = |k_i| \cos\theta_i = n_1 k_o \cos\theta_i$$

$$k_{iz} = |k_i| \sin\theta_i = n_1 k_o \sin\theta_i$$

and equivalently for $\underline{k}_r$ and $\underline{k}_t$.

The expression for the reflection coefficient depends on the polarisation of the incident wave; in general, the $\underline{E}$ field could be in any direction perpendicular to $\underline{k}_i$, but we will define two normal polarisations:

TE (transverse electric): $\underline{E}$ field is transverse to the boundary, ie in the y direction. $\underline{H}$ has x & z components.

TM (transverse magnetic) : H is the in the y direction; E has x & z components.

Any arbitrary polarisation can be divided into TE and TM components. Note that it is only the presence of the interface plane which gives a reference by which TE and TM can be defined.

For TE polarisation, Maxwell's equations solved at the boundary result in:

$$\frac{E_r}{E_i} = \frac{k_{ix} - k_{tx}}{k_{ix} + k_{tx}}$$

Also, all the waves must phase-match along the boundary, so that

$$k_{iz} = k_{rz} = k_{tz} .$$

From this we conclude that $\theta_r = \theta_i$, and we can derive Snell's law:

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

For normal incidence, $\frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$. The power reflection coefficient will be the square of this, since the optical power is proportional to the square of the field amplitude:

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

For most dielectric materials, the indices are in the range 1 to 2, for which the maximum value of R is 0.11, so clearly we cannot get strong reflection this way (glass/air interfaces, for example, reflect about 4% of incident energy).

Metals, however, behave rather differently; their permittivities are dominated by ohmic conductivity for frequencies up to about the infra-red range, and can approximately be given by:

$$\varepsilon_m = j\sigma/\omega$$

Taking the √ of the relative permittivity we get:

$$n_m = \frac{1+j}{\sqrt{2}}\sqrt{\frac{\sigma}{\varepsilon_0\omega}}$$

where σ is the conductivity.

At radio frequencies, $\sigma \gg \varepsilon_0\omega$, so that $| n_m | \gg 1$. This means that even at normal incidence, the reflection coefficient is very large. Also, the propagation constant in the metal now has a large imaginary part, given by:

$$\kappa = \mathrm{Im}\{n_m\}\, k_o = k_o \sqrt{\frac{\sigma}{2\varepsilon_o\omega}}$$

The field in the metal is:

$$\underline{E}_t = E_{to} \exp j\underline{k}_m\underline{r} = E_t \exp(j\underline{k}_R\cdot\underline{r}) \exp(-\underline{\kappa}\cdot\underline{r})$$

where $|\, \underline{k}_R \,| \equiv \mathrm{Re}\{n_m\}\, k_o$. Thus the field decays rapidly as it propagates, and its extent into the metal is very limited. We can define a characteristic distance that a field can penetrate a metal (the 1/e decay distance) as the skin depth:

$$\gamma = \frac{1}{\kappa} = \frac{1}{k_o}\sqrt{\frac{2\varepsilon_o\omega}{\sigma}}$$

Then for normal incidence:

$$\frac{E_r}{E_i} = \frac{\gamma k_o - (1+j)}{\gamma k_o + (1+j)}$$

For $\gamma k_o \ll 1$, $R = |E_r/E_i|^2 \cong 1 - 2\gamma k_o$

Example:

Copper: $\sigma = 6 \times 10^7$ S/m

At 1MHz : $\omega = 2\pi \times 10^6\ \mathrm{s}^{-1}$

$k_o = \omega/c$, $\varepsilon_o = 8.85 \times 10^{-12}$ F/m

Giving: $\gamma k_o = 1.3 \times 10^{-6}$, skin depth $\gamma = 62\ \mu\mathrm{m}$, $R = .9999974$

For radio frequencies, R will be very close to unity, but for optical frequencies, the losses begin to get quite substantial. For example, a conventional mirror uses a metal (typically aluminium) surface as the reflector, and in the visible wavelength range the absorption on reflection is about 5%. This is acceptable for the single reflection of a mirror, but not for the large number of reflections needed to guide waves down an extended path. Thus a different method must be found to guide light, one which does not involve inherently lossy materials like metals, but which provides very high reflection coefficients.

This other way to achieve strong reflection is the phenomenon of total internal reflection. Referring again to figure 3.1, if $n_2 < n_1$, there comes an angle called the critical angle:

$$\theta_c = \sin^{-1}(n_2/n_1)$$

beyond which Snell's law gives an imaginary value for $\sin\theta_t$. While an imaginary angle is a difficult physical concept, we can instead consider what happens to k_{tx}.

In general, $k_x^2 + k_z^2 = nk_o^2$ (if $k_y = 0$). And here, $k_{iz} = k_{tz}$. We usually call this common z component β. Then:

$$\beta^2 = n_1^2 k_o^2 - k_{ix}^2 = n_2^2 k_o^2 - k_{tx}^2$$

$$k_{tx} = \{k_{ix}^2 - (n_1^2 - n_2^2)\, k_o^2\}^{1/2}$$

So if $k_{ix} < \sqrt{n_1^2 - n_2^2}\, k_o$, then k_{tx} is wholly imaginary, and the field in medium 2 is again exponentially decaying, although this time with no real component. We call such a wave an evanescent wave.

Again using $\kappa = \mathrm{Im}\,\{k_{tx}\}$, we get:

$$\frac{E_r}{E_i} = \frac{k_{ix} - j\kappa}{k_{ix} + j\kappa}\,, \qquad \text{yielding } R = 1.$$

Total internal reflection can give perfect reflection without loss at any frequency. The only drawback is that this time, κ is not particularly large. In fact, from above we see that the maximum κ, at $\sin\theta_t \cong 1$, is $\kappa = (n_1^2 - n_2^2)^{1/2}\, k_o$. The penetration distance into the second medium is thus $\gamma = \lambda / 2\pi(n_1^2 - n_2^2)^{1/2}$. For dielectric media, this will not be substantially less than a wavelength.

In the case of coax cable the conductors are much thicker than γ, so the field is effectively contained. If a dielectric interface is used, the outer medium must still be much thicker than γ so that the evanescent field does not leak out beyond it, causing attenuation and interference. For example, at 1 MHz the wavelength is 300m; in this case we would need a guide thickness of more than a kilometre!! But for optical wavelengths the requirement is easily satisfied.

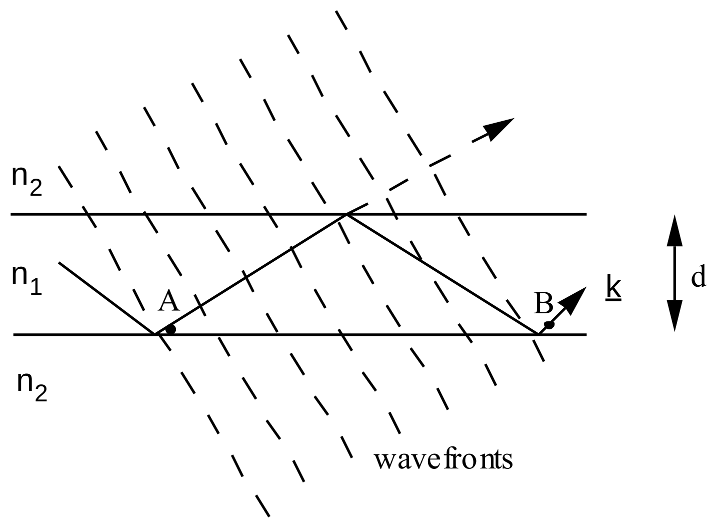

Figure 3.2 Ray propagation in a symmetric slab waveguide.

When we introduce two parallel boundaries, we have a planar or "slab" waveguide (Fig. 3.2). A ray will now propagate infinitely in the central slab (or 'core') if it has a direction beyond the critical angle. There is a second requirement, however. The adjacent reflections must be in phase with each other, otherwise destructive interference will cancel them out.

This means that the phase at B, after two reflections, must be the same as that of the adjacent unreflected wave at B. For the former, the total phase change from A to B is due to the linear path, with $\Delta\varphi = \underline{k}\Delta\underline{r} = k_x\Delta x + k_z\Delta z$, and two reflection phase shifts φ_r. Therefore we require:

$$2k_{1x}d + \beta\Delta z + 2\varphi_r = \beta\Delta z + m\,(2\pi)$$

where m is an arbitrary integer. Here we have used $k_{1x} = n_1 k_o \cos\theta_i$; within medium 1 the upwards and downwards rays will have $k_x = +k_{1x}$ and $-k_{1x}$ respectively (equivalent to k_{ix} and k_{rx} for the single interface reflection). We now obtain:

$$\varphi_r + k_{1x}d = m\pi$$

For TE waves: $\dfrac{E_r}{E_i} = \dfrac{k_{ix} - j\kappa}{k_{ix} + j\kappa}$ from which:

$$\varphi_r = 2\tan^{-1}(-\kappa/k_{ix})$$

We can rewrite the phase condition:

$$\tan -\frac{\varphi_r}{2} = \tan\left(\frac{k_{1x}d}{2} + \frac{m\pi}{2}\right) = \kappa/k_{1x}$$

(Note that the sign of m is arbitrary.)

For even values of m, $\tan(x + m\pi/2) = \tan x$, and for odd m, $\tan(x + m\pi/2) = -\cot x$. (These are just trigonometric identities.) This gives us the two so-called eigenvalue equations:

$$\kappa = k_{1x}\tan\frac{k_{1x}d}{2} \qquad\qquad \kappa = -k_{1x}\cot\frac{k_{1x}d}{2}$$

To get solutions to these equations, we need another relation between κ and k_{1x}. This is provided by the phase matching along the boundaries between the fields in the two media, i.e.:

$$\beta^2 = n_1^2 k_0^2 - k_{1x}^2 = n_2^2 k_0^2 + \kappa^2$$

$$\therefore\ \kappa^2 + k_{1x}^2 = (n_1^2 - n_2^2)\,k_o^2$$

The last relation plots circular arcs in the $k_{1x} - \kappa$ diagram, and where they cross the lines of the eigenvalue equations is where the valid solutions are. Thus the guide can only support waves at a certain discrete set of angles. These are called <u>modes</u> of the waveguide.

As the semicircular arcs get bigger, more and more modes are supported. However, the lowest order mode m=0 is always supported. It can be seen from the graph that in order for a particular mode to exist, we need:

$$(n_1^2 - n_2^2)\,k_o^2 \geq \left(\frac{m\pi}{d}\right)^2$$

giving: $d \geq \frac{1}{2} \quad \frac{m\lambda}{(n_1^2 - n_2^2)^{1/2}}$

This is called the cutoff condition for a mode; if the guide thickness drops below this value for a given m, the mode of order m is prevented from propagating, or 'cut off'. Thus the number of modes goes up with guide thickness and with index difference. The cutoff equation tells us how many modes exist for TE polarised light in a certain guide. However, if randomly polarised light is launched into the guide, there will be light propagated in both TM and TE modes, the former having slightly different β values from the latter. Therefore, the minimum number of modes is two: the m=0 mode for each polarisation.

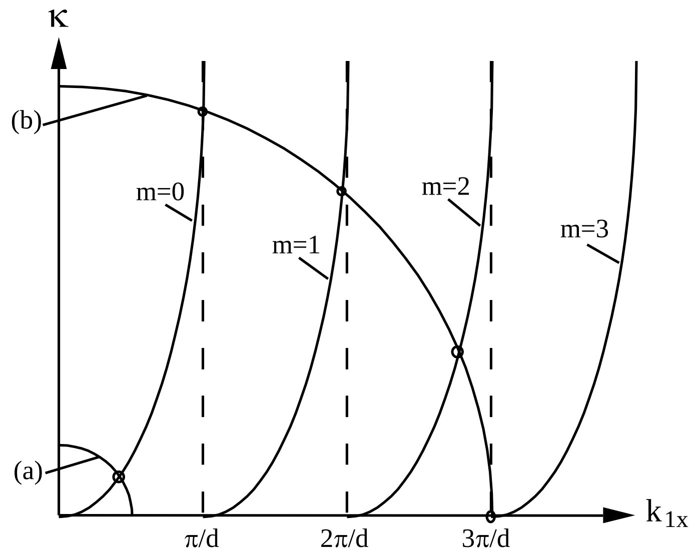

Figure 3.3 Graphical solution for the modes of a slab waveguide for TE polarised light. The lines labelled m=0 etc. are the eigenvalue equations, and the circular arcs are for the particular cases (a) : $(n_1^2-n_2^2)k_o^2= (\pi/2d)^2$ and (b) : $(n_1^2-n_2^2)k_o^2 = (3\pi/d)^2$.

Note that where more than one mode exists, the k_{1x} values are all different, and thus so are the β values. But $k_{1x} > 0$, and according to the arc:

$$k_{1x} \leq (n_1^2 - n_2^2)^{1/2} \; k_o$$

Since $\beta^2 = n_1^2k_o^2 - k_{1x}^2$, we have:

$$n_2k_0 \leq \beta \leq n_1k_o$$

β is the propagation constant of the mode, and it is useful to define a mode "effective index":

$$n' = \beta/k_0$$

This effective index gives the phase velocity for that mode, according to $v_p = c/n'$. We also have: $n_2 \leq n' \leq n_1$, i.e. the effective index of a mode is always equal to some weighted mean of the core and cladding indices. Note: higher m means higher k_{1x}, lower β, therefore lower n', because more of the field energy is in the cladding for the higher order mode.

We can find the modal solutions by another approach. The ray formulation is not the most realistic for light that is so closely confined; instead, we should consider the distribution of the E and H fields, which will also indicate the shape of the modes. For a particular mode, there will be a single value of k_z, and therefore of k_x in each medium. In the central medium there are two sign values of k_x , while in the outer media the sign is determined by the physical requirement that the wave decays as x goes to ± infinity. We can then write:

$E_1(x) = A' \exp j(k_{1x}x) + A'' \exp j(-k_{1x}x)$ $\qquad$ ($-d/2 < x < d/2$)
$E_2(x) = B \exp(-\kappa x)$ $\qquad$ ($x > d/2$)

We can greatly simplify the further analysis by arguing that since the guide is symmetric, the field must be symmetric or antisymmetric, giving respectively:

$E_S(x) = A \cos(k_{1x}x)$ or $E_{AS}(x) = A \sin(-k_{1x}x)$ $\qquad$ ($-d/2 < x < d/2$)

the mode shape is now found by two conditions, the first of which is simply that E(x) should be continuous at the boundary, giving for the symmetric case :

$A \cos(k_{1x}d/2) = B \exp(-\kappa d/2)$

the second condition is true only for TE polarisation, and this is that the slope of the E-field (dE/dx) must be continuous at the boundary. This gives :

$k_{1x}A \sin(k_{1x}d/2) = \kappa B \exp(-\kappa d/2)$

dividing this by the previous equation gives :

$k_{1x} \tan(k_{1x}d/2) = \kappa$

which is the same as the eigenvalue equation obtained earlier. The antisymmetric case gives the other eigenvalue equation in the same way. We can now also plot the field and intensity distributions for each mode. This is done in figure 3.4, which shows that the mode order also indicates the number of nulls in intensity.

When searching for a solution to the modal values numerically, it is convenient to note that the eigenvalue equations can be combined with the phase matching condition (circular arcs equation) and simplified to give the following transcendental equations:

$$\frac{\cos x}{x} = \pm \frac{1}{R} \quad \text{and} \quad \frac{\sin x}{x} = \pm \frac{1}{R}$$

where $R^2 = (n_1^2 - n_2^2)(k_o d/2)^2$, and $x = k_{1x}d/2$. These can be solved straightforwardly to give even and odd modes respectively. However, not all solutions to these equations correspond to valid modes, since only the (+,+) quadrant of Fig. 3.3 gives legitimate solutions. Approximate values should be found graphically first.

For TM modes, the second boundary condition becomes continuity of dH/dx. This gives slightly different solutions; the same number of modes is obtained, but the β values are slightly different. For the cylindrical geometry of fibre, the polarisation considerations are quite different.

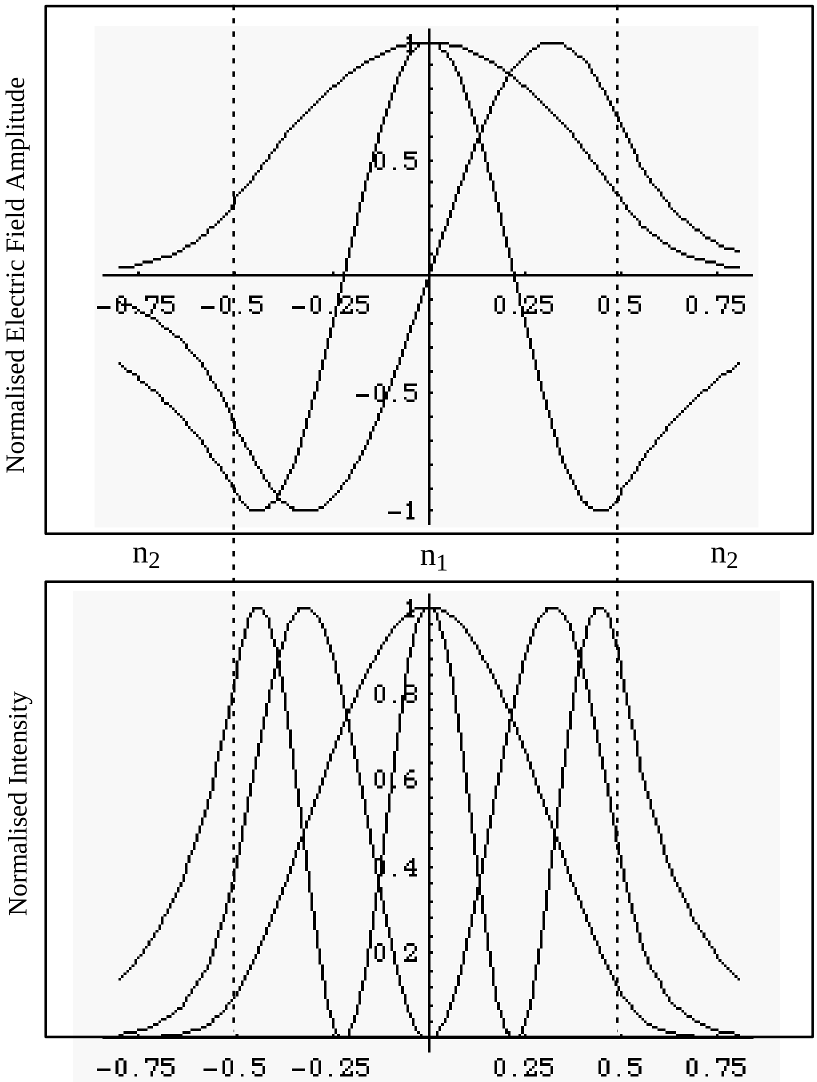

Figure 3.4 Plot of normalised electric field strength, and corresponding intensity distribution, for the three TE modes of a symmetric slab waveguide 15 µm thick, with $n_1 = 1.505$, $n_2 = 1.500$ and $\lambda = 1.5$ µm. The horizontal axis gives displacement from the centre of the guide, normalised to the guide thickness. Dashed lines indicate core/cladding boundaries.

Part A: Summary

Introduction

Why optics, especially optical fibre, is useful for communications: high bandwidth, low attenuation and dispersion.

How optics differs from other electromagnetic propagation of signalling: short wavelengths make dielectric waveguiding possible, photon energy is significant, e.g. compared to kT.

Electromagnetic Radiation

The geometric form of the e-m plane wave: orthogonality of E and H fields and direction of propagation. The k vector and its relation to frequency and wavelength.

Definition of phase velocity, $v_p = \omega/k$. Relation between v_p and c, index of refraction. Wave vector in a real medium.

Photon energy, $\varepsilon = hc/\lambda_o$, comparison to kT.

Electromagnetic Waveguide

Reflection at a plane boundary, boundary conditions, reflection coefficients. Polarisation, definition of TE and TM. Reflection from conductors, skin depth.

Total internal reflection, critical angle, meaning and extent of evanescent field.

The dielectric slab waveguide. Phase matching for multiple reflections; discrete modes of propagation.

The eigenvalue equations, cut-off conditions. Finding k values for modes graphically; effective index.

Solving modes by field profiles.

Reading

Keiser, Ch. 1.1 – 1.3, 2.1 – 2.3
Gowar, Ch 1, 2.5

Part A: Problems

1) Find the photon energies for the 3 main optical communications wavelengths of 850, 1330 and 1550 nm. At what temperatures is the thermal quantum of equal magnitude for each of these? At what frequency is the photon energy equal to kT at room temperature?

2) For each of the 3 wavelengths of (1), what bandwidth is provided (in GHz) by a 1% frequency deviation?

3) For Fig 3.1, take n_1=3.46 (silicon), n_2 = 1 and θ_i = 10°. Find θ_r and θ_t. For a free space wavelength of 1.3 μm, what would be the values of k in the two media? What will the spatial period of oscillation be along each of k_i, k_t, the z axis? Find the critical angle for this system.

4) The reflection coefficient for TM polarised waves is given by:

$$\frac{Er}{Ei} = \frac{n_1^2 k_{2x} - n_2^2 k_{1x}}{n_1^2 k_{2x} + n_2^2 k_{1x}}$$

Show that there is an angle of incidence for which the reflection coefficient for TM waves is zero, and that this angle can be given by $\theta_B = \tan^{-1}(n_2/n_1)$. (This is called Brewster's angle).

5) Show that at normal incidence the TM and TE reflection coefficients are the same, and explain by a symmetry argument why this must be the case.

6) A symmetric glass slab waveguide has an index difference of 0.005. Find the thickness d for which the 3rd (m=2) TE mode is just cut off. Estimate the k_{1x} values, and hence the effective indices, of the m = 0 and 1 modes. What will the effective index be for any mode which is just cut off?

7) Find the eigenvalue equations (k_{1x} vs K) for TM modes in a symmetric dielectric slab waveguide, and draw the corresponding graph, showing the TE and TM lines superimposed.

Do the cutoff conditions differ?
How do the β values of the TE and TM modes compare for a given m number (which is bigger)? Which of TE and TM has the bigger mode field?