BC: Simultaneous Transmission of Info and Power

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Ex 1:

According to Nyquist Sampling theorem: a signal with bandwidth B can be completely reconstructed by sampling rate of 2B sample per second.

$f_s > 2B$

A baseband equivalent to signal of bandwidth $W/2$ can be represented as:

$$x(t) = \sum_n x\left(\frac{n}{W}\right) \operatorname{Sinc}(Wt - n) = \sum_n x[n] \operatorname{sinc}(Wt - n) \qquad (1)$$

complex baseband equivalent channel is given as:

$$y_b(t) = \sum_i a_i^b(t) x_b(t - \tau_i(t)) + W(t) \qquad (2)$$

where $a_i^b(t) = a_i(t) e^{-j2\pi f_c \tau_i(t)}$

1° Substitution of (1) in (2):

$$y_b(t) = \sum_i a_i^b(t) \sum_n x[n] \operatorname{sinc}(Wt - W\tau_i(t) - n) + w(t)$$
$$= \sum_i \sum_n a_i^b(t)\, x[n] \operatorname{sinc}\{W[t - \tau_i(t)] - n\} + w(t)$$

2° Obtain the sampled output (denoted by $y[m]$) at multiples of $1/W$ (i.e. $y[m] = y_b(m/W)$) by simply substituting $t = m/W$:

$$y[m] = y_b\left(\frac{m}{W}\right) = \sum_i \sum_n x[n]\, a_i^b\left(\frac{m}{W}\right) \operatorname{sinc}\left\{m - n - W\tau_i\left(\frac{m}{W}\right)\right\} + w[m]$$

3° using the transformation $l \triangleq m-n$: $\quad n = m - l$.

$$y[m] = \sum_i \sum_l x[m-l]\, a_i^b\left[\frac{m}{W}\right] \operatorname{sinc}\left(l - W\tau_i\left(\frac{m}{W}\right)\right) + w(m)$$
$$= \sum_l x[m-l] \sum_i a_i^b\left[\frac{m}{W}\right] \operatorname{sinc}\left(l - W\tau_i\left(\frac{m}{W}\right)\right) + w(m)$$

Define $h_l[m] = \sum_i a_i^b(m/W) \operatorname{sinc}\left(l - W\tau_i\left(\frac{m}{W}\right)\right)$

$$y[m] = \sum_l x[m-l]\, h_l[m] + w(m)$$

$h_l[m]$ is known as l^{th} complex channel filter tap at time m.

EX2:

The optimization problem:

$$\max_{P_0 \dots P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right)$$

$$\text{s.t.} \begin{cases} \sum_{n=0}^{N_c-1} P_n = P & h_i(x) = \sum_{n=0}^{N_c-1} P_n - P = 0. \\ P_n \geq 0 & f_i(x) = -P_n \leq 0. \end{cases}$$

Its Lagrangian is defined as:

$$L(\lambda, P_0, \dots P_{N_c-1}) = \sum_{n=0}^{N_c} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right) - \lambda\left(\sum_0^{N_c-1} P_n - P\right)$$

$$\frac{\partial L}{\partial P_n} = \frac{\partial L(\lambda, P_0 \dots P_{N_c-1})}{\partial P_n} = \sum_{n=0}^{N_c-1} \frac{|h_n|^2}{(N_0+P_n|h_n|^2)\ln 2} - \lambda_n$$

$$L(\lambda, \nu, P_0, \dots P_{N_c-1}) = \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right) - \sum_0^{N_c-1} \lambda_n P_n - P - \sum_0^{\nu_c-1} -P_n \cdot \nu_n$$

$$= \sum_{n=0}^{N_c-1} \frac{|h_n|^2}{(N_0+P_n|h_n|^2)\ln 2} - \lambda_n + \nu_n = \frac{\partial L}{\partial P_n} + \nu_n$$

KKT condition is:

(1) $-P_n \leq 0 \Rightarrow P_n \geq 0$

(2) $\sum_{n=0}^{N_c-1} P_n - P = 0.$

(3) $\nu_n \geq 0$

(4) $\nu_n \cdot -P_n = 0 \quad \Rightarrow \quad \nu_n \cdot P_n = 0.$

(5) $\frac{\partial(\lambda, \nu, P_0 \dots P_{N_c-1})}{\partial P_n} = 0.$

According to (5): $\frac{\partial L}{\partial P_n} = -\nu_n$

According to (4):

$$\boxed{\begin{array}{ll} 1^\circ \text{ when } P_n = 0, \nu_n \geq 0 & \frac{\partial L}{\partial P_n} \leq 0 \\ 2^\circ \text{ when } P_n > 0, \nu_n = 0 & \frac{\partial L}{\partial P_n} = 0 \end{array}}$$

According to 2°

$$\frac{\partial L}{\partial P_n} = 0 \Rightarrow \frac{|h_n|^2}{(N_0 + P_n |h_n|^2)\ln 2} = \lambda$$

$$P_n = \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2}$$

Define In Matlab:

```
%% Power allocation based on KKT Condition
P_n = 1/(lambda * log(2)) - N_0./abs_h_square;
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$$\Rightarrow \boxed{\begin{cases} \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2} & \text{if } \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|} > 0 \\ 0 & \text{if } \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|} \leq 0. \end{cases}}$$

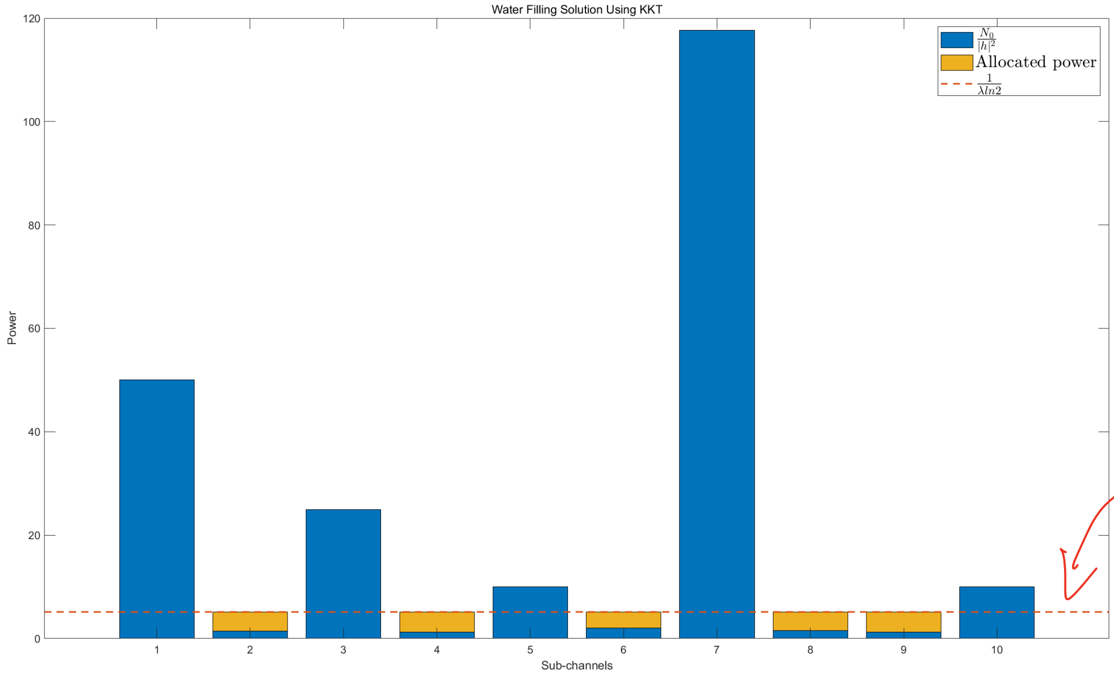
Therefore, when λ is fixed, $\frac{1}{\lambda \ln 2}$ is a threshold for each subchannel

1° $\frac{N_0}{|h_n|^2}$ indicate each subchannel quality. The higher noise to channel ratio is, the worse the channel quality is.

2° when the channel quality is worse than $\frac{1}{\lambda \ln 2}$, we would n't allocate any power and info for it to transmit

3° This threhold $\frac{1}{\lambda \ln 2}$ just like a bottle volume. when $\frac{N_0}{|h_n|^2}$ occupies the whole bottle, we could n't allocate any power to this bottle. we only fill power to those bottle which have space. In this way, we only transmit power and info in the better subchannel, we achieve the best channel capicity!

Result shown below:



water level thershold $\frac{1}{\lambda \ln 2}$

EX3: Step1,2:

The optimization problem:

$$\max_{P_0,\dots,P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right)$$

$$\text{s.t.} \quad \begin{cases} \sum_0^{N_c-1} P_n \le P \\ P_n \ge 0,\ n=0,\dots,N_c-1 \\ -\sum_{n=0}^{N_c-1} \mathbb{E}[|y_n|^2] \le -P_d \end{cases}, \qquad E[|y_n|^2] = \sum_{n=0}^{N_c-1} |h_n|^2 P_n + N_0$$

$$\Downarrow$$

$$\max_{P_0,\dots P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right)$$

$$\text{s.t.} \quad \begin{cases} \sum_0^{N_c-1} P_n = P & h_1(x) = \sum_0^{N_c-1} P_n - P = 0 \\ -\sum_{n=0}^{N_c-1} E[|y_n|]^2 = -P_d & h_2(x) = \sum_{n=0}^{N_c-1} -E[|y_n|^2] + P_d = 0 \\ P_n \ge 0 & f(x) = -P_n \le 0 \end{cases}$$

$$L(\lambda, \mu, P_0, \dots P_{N_c-1}) = \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right) - \lambda\left(\sum_0 P_n - P\right) - \mu\left(-\sum_0^{N_c-1} E|y_n|^2 + P_d\right)$$

$$L(\lambda, \mu, v, P_0, \dots P_{N_c-1}) = L(\lambda, \mu, P_0 \dots P_{N_c-1}) + \sum_0^{N_c-1} -P_n \cdot v_n$$

Its KKT Optimality condition is:

(1) $-P_n \le 0$

(2) $\sum_0^{N_c-1} P_n - P = 0$

(3) $\sum_0^{N_c-1} -E[|y_n|^2] + P_d = 0$

(4) $v_n \ge 0$

(5) $v_n \cdot -P_n = 0 \Rightarrow v_n P_n = 0.$

(6) $\nabla L(\lambda, \mu, v, P_0 \dots P_{N_c-1}) = 0$

From (6): $\frac{\partial L(\lambda, \mu, \nu, P_0 \dots P_{Nc-1})}{\partial P_n} = \nabla L_{\lambda,\mu} + \sum_0^{Nc-1} + \nu_n$

$$\nabla L_{\lambda,\mu} = \sum_0^{Nc-1} \frac{|h_n|^2}{(N_0 + P_n|h_n|^2)\ln 2} - \lambda + \mu |h_n|$$

$$\nabla L_{\lambda,\mu,\nu} = \sum_0^{Nc-1} \frac{|h_n|^2}{(N_0 + P_n|h_n|^2)\ln 2} - \lambda + \mu |h_n| + \nu_n = 0.$$

$$\nabla L_{\lambda,\mu} + \nu_n = 0$$

$$\nabla L_{\lambda,\mu} = -\nu_n$$

Similar to Ex2:

we obtain $\begin{cases} \nabla L_{\lambda,\mu} \le 0 & \text{If } P_n = 0 \\ \nabla L_{\lambda,\mu} = 0 & \text{If } P_n > 0 \end{cases}$

When $P_n > 0$, $\nabla L_{\lambda,\mu} = 0$

$$\Rightarrow \frac{|h_n|^2}{(N_0 + P_n|h_n|^2)\ln 2} - \lambda + \mu |h_n|^2 = 0$$

$$P_n = \frac{1}{\ln 2(\lambda - \mu|h_n|^2)} - \frac{N_0}{|h_n|^2}$$

$$\Rightarrow \boxed{P_n = \begin{cases} \frac{1}{\ln 2(\lambda - \mu|h_n|^2)} - \frac{N_0}{|h_n|^2} & \text{if } \frac{1}{\ln 2(\lambda - \mu|h_n|)^2} - \frac{N_0}{|h_n|^2} > 0 \\ 0 & \text{if } \frac{1}{\ln 2(\lambda - \mu|h_n|)^2} - \frac{N_0}{|h_n|^2} \le 0 \end{cases}}$$

Matlab defined below:

```
%% Power allocation based on KKT Condition
P_n = 1./((lambda - u*abs_h_square) * log(2)) - N_0./abs_h_square;

% Setting elements of P_n less than 0 to 0
P_n(P_n < 0) = 0;
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Therefore, we can conclude that:

1° when $\mu=0$, $L(\lambda, 0, P_1, \ldots P_{Nc-1}) = L(\lambda, P_1 \cdots P_{Nc-1})$

Ex3 is equal to Ex2 objective, which means $Pd=0$.

2° when $\mu>0$, we get:

$$Pd = \sum_{n=0}^{Nc-1} |h_n|^2 P_n + N_0 = \sum_{n=0}^{L} \frac{|h_n|^2}{\ln_2(\lambda - \mu|h_n|^2)} + \sum_{n=0}^{Nc-1-L} N_0.$$

where L is the number of subchannel that be allocated power. [$P_n>0$]

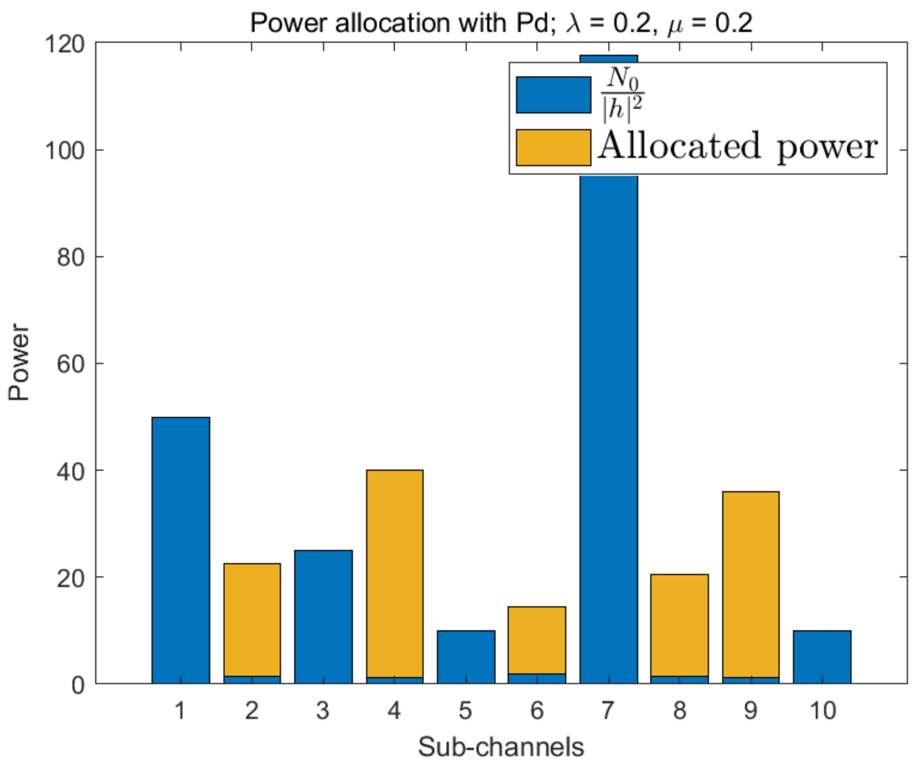
we know that: fixed λ, $\mu\uparrow$ $Pd\uparrow$

meanwhile, we should guarante $\lambda - \mu|h_n|^2 > 0$

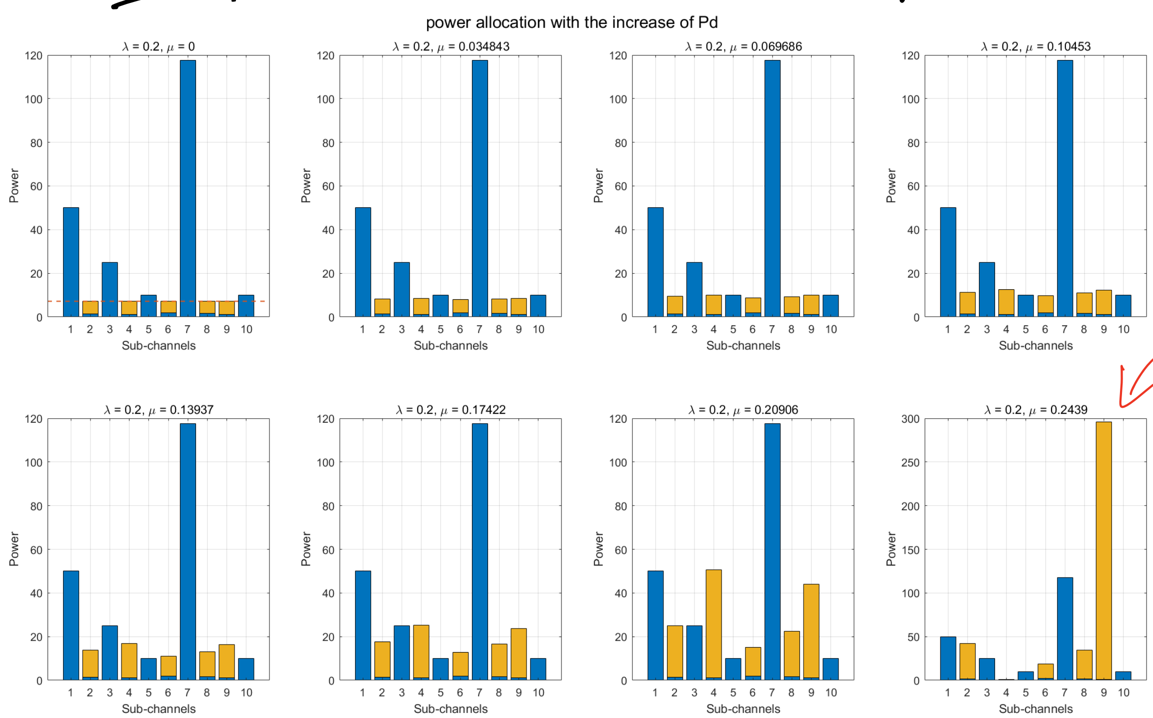
$\lambda = \mu|h_n|^2$.

Matlab Simulation result shown below:

Step3: $\lambda=0.2$, $\mu=0.2$



Step4: Increase μ to Increase Pd.



best subchannel allocated the most power

We Can find that, as the Pd increased, our algorithm will allocate power to better channel, and the better quality of the channel will be allocated more power.