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**DEPARTMENT of ELECTRICAL & ELECTRONIC ENGINEERING**


MSc in Communications and Signal Processing

# Formal Report No. 1

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| Name | Haoxiang Huang |
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**Communications, Control and Signal Processing Laboratory**

# Simultaneous Transmission of Information and Power

* Submission for M.Sc. C&SP 1st Formal Lab Report at Imperial College London

Haoxiang Huang
*Department of Electrical and Electronic Engineering*
*Imperial College London*
SW7 2AZ, London, U.K.
haoxiang.huang23@imperial.ac.uk

***Abstract*—The problem considered here is the simultaneous transmission of wireless information and power transfer (SWIPT) across a noisy frequency-selective channel, underpinned by the assumption of a point-to-point OFDM communication link with Additive White Gaussian Noise (AWGN). The primary solution of this problem is to optimize the simultaneous wireless information and power transfer, along with the inherent trade-off between information rate and transferred power. To achieve this, our lab adopts a Lagrange multiplier method with Karush-Kuhn-Tucker (KKT) conditions to formulate an optimal power allocation strategy. This strategy is designed to enhance the information rate while adhering to power constraints on the transmitter and receiver sides, thereby achieving SWIPT in the wireless communication system.**

***Index Terms*—Simultaneous transmission of wireless information and power transfer (SWIPT), Power allocation optimization, Karush-Kuhn-Tucker (KKT) conditions, Lagrange multiplier method, Orthogonal Frequency Division Multiplexing (OFDM).**

## I. INTRODUCTION

HISTORICALLY, the communications was usually simply defined as the process of delivering a message from one location to another. The power in the communications system was only regarded as the carrier of information, not a source of power. This separation of information and power hinder the possible new perspectives of communications development and hence led to the division of electrical engineering into two distinct sub-fields, electric power engineering and communication engineering [1].

In practice, most of electrical engineers frequently deal with both power and information at the same time. The distinction between the two is not always necessary [3]. With the rapid advancement of the communications technology, 5G wireless communications have integrated new application scenarios such as Internet of Things (IoT). IoT devices like RFID and wireless sensor are power constraints and smaller in size. Recently, several attempts have made to achieve self-sustainable in power constrained scenarios by using simultaneous wireless information and power transfer (SWIPT) [8] [9]. For examples, the SWIPT prolongs the lift-time of wireless sensor networks (WSNs) ,which is limited energy budget and computing capacity, to receive the information data from the sink nodes and harvest energy from radio frequency (RF) signal [8].

When wireless information and power simultaneous across a frequency-selective channel with AWGN, the information rate and the transferred power would have arrived at a compromise. This trade-off problem can be mathematically convert into a convex optimization problem to the maximum information rate and the transferred power subject to the required trade-off constraint [6].

In our lab, it is assumed that the communication link is point-to-point with frequency-selective AWGN channel. By employing Orthogonal Frequency Division Multiplexing (OFDM), the frequency-selective channel is partitioned into multiple orthogonal sub-channels to against the frequency selective fading. Based on those assumptions and operations, this trade-off convex optimization problem can be solved by Lagrangian method with Karush-Kuhn-Tucker (KKT) optimality conditions [7]. This method yields two distinct solutions, each corresponding to different power constraint scenarios at the transmitter and receiver sides. When the information rate is maximized without a power constraint at the receiver, the resulting solution is known as the 'water-filling' solution [1]. Conversely, adding a power constraint at the receiver leads to an alternative solution.

## II. BACKGROUND

### A. A discrete-time baseband model

a discrete-time baseband model is a widely used wireless communication channel model. This model can be obtained by sampling the continuous-time baseband model $y_b(t)$ according to the Nyquist sampling theorem. The Nyquist theorem states that any signals with bandwidth $W$ can be can be perfectly reconstructed if sampled at a rate greater than $2W$. Assume that the band-limit of input waveform is $W$. According to the Nyquist sampling theorem, a equivalent baseband input signal $x_b(t)$ of bandwidth $\frac{W}{2}$ can be represented as:

$$\begin{aligned} x_b(t) &= \sum_n x\left(\frac{W}{n}\right) \cdot \mathrm{sinc}(Wt-n) \\ &= \sum_n x[n] \cdot \mathrm{sinc}(Wt-n) \end{aligned} \quad (1)$$

Taking multi-path propagation into account (see Fig.1), the model of the received signal $y_b(t)$ can be expressed as follows:

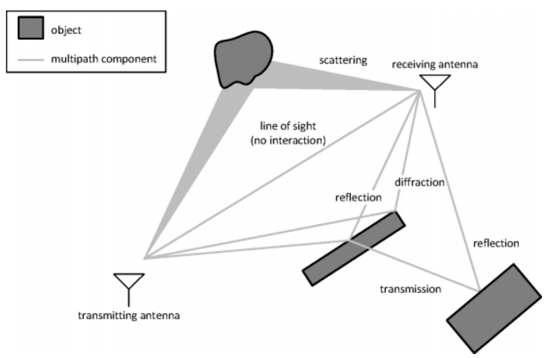



Fig. 1: Example of a muti-path channel [2].

$$y_b(t) = \sum_i a_i^b(t) x\left(t - \tau_i(t)\right) + w(t) \quad (2)$$

$$= \sum_i a_i(t) x\left(t - \tau_i(t)\right) e^{-j2\pi f_c \tau_i(t)} + w(t) \quad (3)$$

where $i$ is path index, $a_i(t)$ is time-varying attenuation of path $i$, $\tau_i(t)$ is time-varying delay of path $i$, and $\omega(t)$ is AWGN. By substituting equation (1) into equation (2), $y_b(t)$ can be represented as:

$$y_b(t) = \sum_n x[n] \sum_i a_i^b(t) \text{sinc}(Wt - W\tau_i(t) - n) + w(t) \quad (4)$$

Obtain the sampled output discrete-time baseband model $y[m]$ at multiples of $\frac{1}{W}$ (i.e. $y[m] = y_b[\frac{m}{W}]$) by simply substituting $t = \frac{m}{W}$:

$$\begin{aligned} y[m] = &\sum_n x[n] \sum_i a_i^b\left(\frac{m}{W}\right) \\ &\times \text{sinc}\left(m - n - W\tau_i\left(\frac{m}{W}\right)\right) + w[m] \end{aligned} \quad (5)$$

Using the transformation $l \triangleq m - n$ :

$$\begin{aligned} y[m] = &\sum_l x[m-l] \sum_i a_i^b\left(\frac{m}{W}\right) \\ &\times \text{sinc}\left(l - W\tau_i\left(\frac{m}{W}\right)\right) + w[m] \end{aligned} \quad (6)$$

Defining:

$$h_l[m] = \sum_i a_i^b\left(\frac{m}{W}\right) \text{sinc}\left(l - W\tau_i\left(\frac{m}{W}\right)\right) \quad (7)$$

By substituting equation (7) into equation (6), the discrete-time baseband model $y[m]$ can be written as:

$$y[m] = \sum_l h_l[m] x[m-l] + w[m] \quad (8)$$

where $h_l[m]$ is known as l[st] complex channel filter tap at the time $m$.

### B. Frequency selective channel

In wideband channels, the transmitted signal arrives at the multipaths and different times when it reaches the receiver. Unlike narrowband channels, where the channel response is typically flat, the response in wideband systems exhibits diversity across the frequency. This channel with frequency diversity is called a frequency-selective channel [6].

In frequency selective channel, an important general parameter is the multipath delay spread $T_d$, defined as:

$$T_d \triangleq \max_{i,j} |\tau_i(t) - \tau_j(t)| \quad (9)$$

The coherence bandwidth, $W_c$, is defined as:

$$W_c \triangleq \frac{1}{2T_d} \quad (10)$$

When the bandwidth $W$ of the transmitted signal is less than coherence bandwidth $W_c$, it will experience flat fading. Conversely, its fading channel is known as frequency selective channel (see equation (8)).

### C. Orthogonal Frequency Division Multiplexing (OFDM)

Orthogonal Frequency-Division Multiplexing (OFDM) is a digital multi-carrier modulation method that efficiently divides the overall frequency-selective channel into several orthogonal sub-channels with independent AWGN [2].

We allocated $P_n$ to each n $^{\text{th}}$ OFDM sub-channel with the total transmitted power $P$ constraint, which is mathematically formulated as:

$$\sum_n P_n \leq P, \quad P_n \geq 0, \quad \forall n \quad (11)$$

The channel capacity (bits per OFDM symbol) of OFDM subband is:

$$\begin{aligned} C &= \sum_{n=0}^{N_c-1} \log\left(1 + SNR_n\right) \\ &= \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \end{aligned} \quad (12)$$

where $N_c$ is the number of OFDM sub-channels, $N_0$ is the average noise power, $|h_n|$ is the amplitude of impulse response of each sub-channel, $P_n$ is the power allocation on subcarrier.

### D. Convex optimization, Lagrange multiplier method and KKT condition

In the convex optimization, the Lagrange multiplier method with KKT conditions are a powerful tool for finding optimal solutions. KKT conditions can be the necessary condition to guarantee local optimality of solvable problems. When a convex optimization problem satisfies the KKT conditions, these conditions are also the sufficient condition of the global optimality.

The standard form of a convex optimization problem is typically presented as follows:

$$\begin{aligned} \min_x \quad & f_0(x) \\ \text{s.t.} \quad & \begin{cases} f_i(x) \leq 0, & i = 1, \ldots, m \\ h_i(x) = 0, & i = 1, \ldots, p \end{cases} \end{aligned} \quad (13)$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ should be convex and $h : \mathbb{R}^n \rightarrow \mathbb{R}$ should be linear.

This problem can be formulated with the Lagrange multiplier method as follows:

$$L(x, \lambda, \nu) = f_0(x) - \sum_{i=1}^{m} \lambda_i f_i(x) - \sum_{i=1}^{p} \nu_i h_i(x) \tag{14}$$

where $\lambda$ and $\nu$ are Lagrange multiplier.

The Karush-Kuhn-Tucker (KKT) conditions of this Lagrange function is defined as follows:

$$\begin{cases} f_i(x) \leq 0, i = 1, \ldots, m & \text{(15a)} \\ h_i(x) = 0, i = 1, \ldots, p & \text{(15b)} \\ \lambda_i \geq 0, i = 1, \ldots, m & \text{(15c)} \\ \lambda_i f_i(x) = 0, i = 1, \ldots, m & \text{(15d)} \\ \nabla f_0(x) + \sum_{i=1}^{m} \lambda_i \nabla f_i(x) + \sum_{i=1}^{p} \nu_i \nabla h_i(x) = 0 & \text{(15e)} \end{cases}$$

## III. WATER FILLING SOLUTION USING KKT OPTIMALITY CONDITIONS

The power allocation optimization problem at transmitter is fundamentally assigning appropriate values of $P_n$ to each OFDM sub-channels, so as to maximize $\sum_n P_n$ in equation (11) and maximize the channel capacity in equation (12). This power allocation problem ,thus, can be is mathematically formulated as following optimization problem:

$$\begin{aligned} \max_{n=0,\ldots,N_c-1} \quad & \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_o}\right) \\ \text{s.t.} \quad & \begin{cases} \sum_{n=0}^{N_c-1} P_n = P \\ -P_n \leq 0, \quad n = 0, \ldots, N_c - 1 \end{cases} \end{aligned} \tag{16}$$

The objective function and the constraints here are either convex or linear. Therefore, this optimization problem in equation (16) is a convex optimization problem and can be reformulated using the method of Lagrange multipliers:

$$\begin{aligned} L(\lambda, P_1, \ldots, P_{N_c-1}) = & \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n}{|h_n|^2}\right) \\ & - \lambda \left(\sum_{n=0}^{N_c-1} P_n - P\right) \end{aligned} \tag{17}$$

where $\lambda$ is the Lagrange multiplier. The KKT conditions of this Lagrange function (see equation (17)) can be derived as follows (See Appendix for details):

$$\frac{\partial L}{\partial P_n} = \begin{cases} 0, & \text{if } P_n > 0, \\ \leq 0, & \text{if } P_n = 0. \end{cases} \tag{18}$$

According to equation (18), the optimal power allocation of $P_n$ to the $n^{\text{th}}$ sub-channel for maximizing channel capacity within the constraints of transmitted power can be derived as follows (see Appendix for details):

$$P_n = \begin{cases} \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2}, & \text{if } \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2} > 0, \\ 0, & \text{if } \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2} \leq 0. \end{cases} \tag{19}$$

According to equation (19), the Lagrange multiplier $\lambda$ can be derived as follows:

$$\lambda = \frac{N_c}{ln2(P + N_0 \sum_{n \in L} |h_n|^2)} \tag{20}$$

where $L$ represents the number of non-zero $P_n$. Accordingly, once the desired transmitted power $P$ is given, Equation (19) enables the determination of $\lambda$. This, in turn, allows for the derivation of the optimal power allocation $P_n$.

When desired transmitted Power $P$ is selected, the Lagrange multiplier $\lambda$ is determined by equation (20). Thus, $\frac{1}{\lambda ln2}$ is a fixed threshold for each sub-channel according to equation (19). Based on these, we can obtain following conclusions in equation (19):

**Remark 1.** *The noise to channel ratio $\frac{N_0}{|h_n|^2}$ indicate each sub-channel quality. The higher noise to channel ratio, the worse the channel quality and lower power allocation.*

**Remark 2.** *When the channel quality is worse than the threshold $\frac{1}{\lambda ln2}$, this solution wouldn't allocate any power and transmitted information for this sub-channel to transmit.*

**Remark 3.** *When the channel quality is better than the threshold $\frac{1}{\lambda ln2}$, this solution would allocate $\frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2}$ transmitted information for this sub-channel to transmit.*

This threshold $\frac{1}{\lambda \ln 2}$ can be likened to the volume of a bottle. When the noise-to-channel ratio fills the entire bottle, no power can be allocated to it. Conversely, when the noise-to-channel ratio is less than the bottle's volume, power is allocated to fill the remaining space. In this way, power and information are transmitted only through the better sub-channels, thereby achieving optimal channel capacity. This strategy without a power constraint at the receiver is known as the water-filling solution.

## IV. WIRELESS INFORMATION AND POWER TRANSFER

In this section, we consider a problem that adds a power constraint at the receiver to the power allocation problem in Section III. According to equation (8), The observed output $y_n$ for the discrete model can be written as:

$$y_n = h_n x_n + w_n, \quad n = 0, \ldots, N_c - 1. \tag{21}$$

Similar to equation (11), the transmitter has a fixed transmission power constraint denoted by $P$. Simultaneously, the total power received at the receiver is also restricted by a given threshold, $P_d$. We therefore have:

$$\sum_{n=0}^{N_c-1} E[|y_n|^2] = \sum_{n=0}^{N_c-1} |h_n|^2 P_n + N_0 \geq P_d \tag{22}$$

The objective of power allocation optimization in here becomes to maximize the channel capacity (see equation (12)), while ensuring power constraints at the transmitter (see equation (11)) and the receiver (see equation (22)) are satisfied. This power allocation optimization with power constraints at both transmitter and receiver can be mathematically formulated as:

$$
\begin{aligned}
&\max_{P_0,\dots,P_{N_c-1}} \quad \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right) \\
&\text{s.t.} \quad \begin{cases} \sum_{n=0}^{N_c-1} P_n = P, \\ \sum_{n=0}^{N_c-1} -E[|y_n|^2] + P_d = 0, \\ -P_n \le 0, \quad n = 0,\dots,N_c-1. \end{cases}
\end{aligned} \tag{23}
$$

Similar to equation (16), this optimization problem in equation (23) is a convex optimization problem and can be reformulated using the method of Lagrange multipliers:

$$
\begin{aligned}
L(\lambda,\mu,P_1,\dots,P_{N_c-1}) = &\sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right) \\
&- \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right) \\
&- \mu\left(\sum_{n=0}^{N_c-1} E[|y_n|^2] - P_d\right)
\end{aligned} \tag{24}
$$

where $\lambda$ and $\mu$ is the Lagrange multiplier. Similar to equation (20), the Lagrange multiplier $\lambda$ and $\mu$ can be calculated by predefined $P$ and $P_d$ at transmitted and receiver. The KKT conditions of this Lagrange function (see equation (24)) can be derived as follows(see Appendix for details):

$$
\frac{\partial L}{\partial P_n} = \begin{cases} 0, & \text{if } P_n > 0, \\ \le 0, & \text{if } P_n = 0. \end{cases} \tag{25}
$$

According to equation (26), the optimal power allocation of $p_n$ to the nth sub-channel for maximizing channel capacity within the constraints of transmitted and received power can be derived as follows(Proof is similar to the equation (19)):

$$
P_n = \begin{cases} \frac{1}{(\lambda-\mu|h_n|^2)\ln 2} - \frac{N_0}{|h_n|^2}, & \text{if } P_n > 0, \\ 0, & \text{if } P_n \le 0. \end{cases} \tag{26}
$$

**Remark 4.** *When $\mu = 0$, equation (24) reduces to equation (17), implying that $P_d = 0$. Under these conditions, the solution is same as the water-filling solution in Section III.*

By substituting equation (26) into equation equation (22):

$$
P_d = \sum_{n=0}^{L} \frac{|h_n|^2}{ln2(\lambda-\mu|h_n|^2)} + \sum_{n=0}^{N_c-L-1} N_0 \tag{27}
$$

where $L$ is the number of sub-channel that be allocated power ($P_n \neq 0$).

**Remark 5.** *Equation (27) suggests that for a given total transmission power $P$ , the $\lambda$ becomes fixed. Consequently, incleasing $\mu$ leads to an increase in $P_d$. However, $\lambda - \mu|h_n|^2 > 0$ should be guaranteed.*

Interestingly, a closer analysis of equation (22) reveals that:

$$
P_d = \sum_{n=0}^{N_c-1} |h_n|^2 P_n + N_0 \le |h|^2 \sum_{n=0}^{N_c-1} P_n + N_0 = |h|^2 P + N_0 \tag{28}
$$

where $|h|$ is the best channel gain in the all sub-channels.

**Remark 6.** *Equation (28) implies that increasing $P_d$ to the highest available value, all the power needs to allocated to the best sub-channel.*

## V. SIMULATION RESULTS

In this section, I implement some simulation results to evaluate the two power allocation solutions discussed in III and IV. I assume the number of sub-channels $N_c = 10$ , the noise power $N_0 = 1$, and the OFDM sub-channel vector $\mathbf{h}$ is defined as follows [1]:

$$
\mathbf{h} = \begin{pmatrix} 0.1+0.1i \\ 0.2+0.8i \\ 0.01+0.2i \\ 0.1+0.9i \\ 0.3+0.1i \\ 0.1+0.7i \\ 0.09+0.02i \\ 0.1+0.8i \\ 0.4+0.8i \\ 0.1+0.3i \end{pmatrix}^T
$$

### A. Water filling solution

Assuming a value of $\lambda$ = 0.28, we can refer to equation (19) for the simulation process. Utilizing Matlab for the simulation allows us to visualize the power allocation across each sub-channel without power constraint at receiver (as shown in Fig. 2).

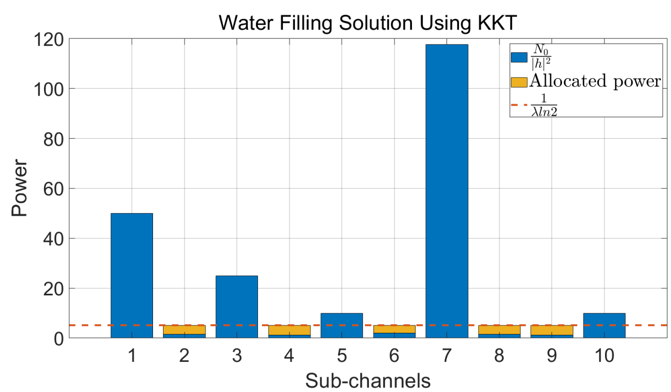



Fig. 2: Water filling solution.

In Fig.2, the yellow and blue bars represent allocated power $P_n$ for each sub-channel and the noise to signal ratio $\frac{N_0}{|h_n|^2}$. By observing the relationship of blue and yellow bars in Fig.2, it confirms remark 1 that the lower noise to channel ratio, the better the channel quality and higher power allocation. However, the dot line meaning the value of $\frac{1}{\lambda ln2}$ indicates that threshold of sum of and noise to signal ratio $\frac{N_0}{|h_n|^2}$ and allocated power $P_n$. It confirms the remark 2 and remark 3 that this threshold $\frac{1}{\lambda \ln 2}$ can be likened to the volume of a bottle, only when noise to signal ratio $\frac{N_0}{|h_n|^2}$ is less than this threshold, it can allocate power to fill the remaining space. This validate all the water filling solution analysis in section III.

### B. The solution with received power constraint

Assuming values of $\lambda = 0.2$ and $\mu = 0.2$. Based on Equation (27), the power allocation is plotted in Fig.3. In Fig.3, the yellow and blue bars represent allocated power $P_n$ for each sub-channel and the noise to signal ratio $\frac{N_0}{|h_n|^2}$. As depicted in Figure 3, setting $\mu$ larger than zero to introduce a receiver power constraint, the power allocation is not restricted to the fixed threshold discussed in the water-filling solution.

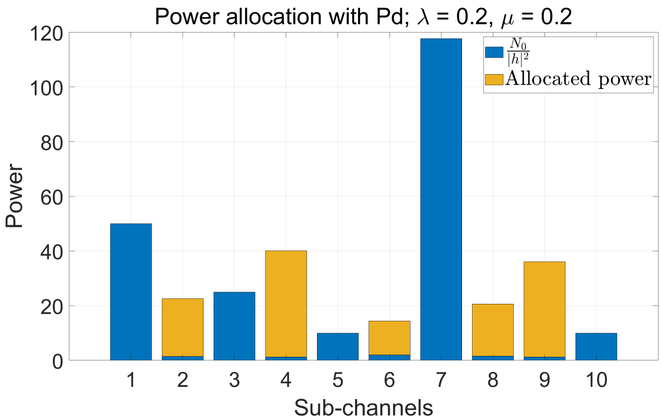



Fig. 3: Example of Power allocation with Receiver Power Constraint

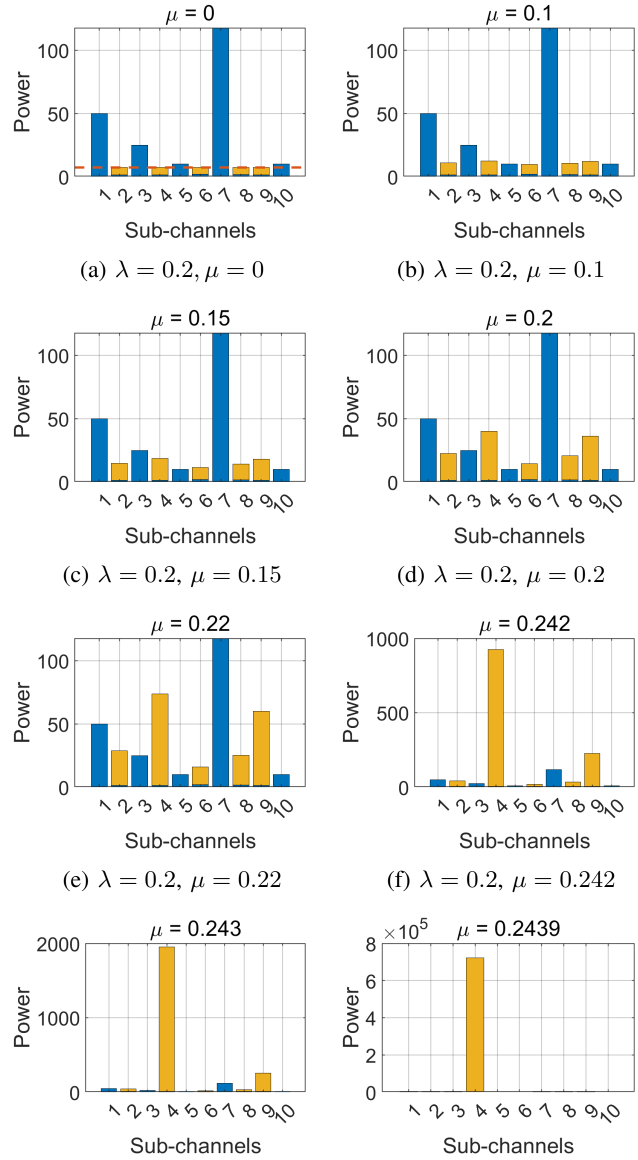
Next, I set $\lambda$ to 0.2 and progressively increased $\mu$ to increase $P_d$ according to equation (27). As $\mu$ is increased, it should ensure that the inequality $\lambda - \mu|h_n|^2 > 0$ always holds true. Consequently, this allows us to determine the maximum value of $\mu$:

$$\mu_{max} = \frac{\lambda}{\max(|h_n|^2)} \tag{29}$$

Setting $\lambda$ to 0.2, we calculate the maximum value of $\mu$ to be $\mu_{max} = 0.2439$. With this constraint, I increment $\mu$ from 0 to 0.2439 and plot 8 distinct power allocation cases using different $\mu$ as the Fig.4 shown. In Fig.4, the yellow and blue bars represent allocated power $P_n$ for each sub-channel and the noise to signal ratio $\frac{N_0}{|h_n|^2}$.

According to Fig.4, We can find that:

1) When the Lagrange multiplier $\mu$ is set to 0, there is no power constraint at the receiver. Consequently, Fig. 4a is equivalent to Fig. 2. This confirms the assertion made in Remark 4: the solution under these conditions is identical to the water-filling solution.
2) As the Lagrange multiplier $\mu$ increases, the cumulative sum represented by the yellow blocks also increases. This indicates that increasing $\mu$ augments the power constraint $P_d$. Such an observation support Remark 5.
3) As the power constraint at receiver, $P_d$ increases, a noticeable trend emerges where a progressively larger portion of power is allocated to the $4^{\text{th}}$ sub-channel with the increase of $P_d$. Ultimately, Figure 4h demonstrates that nearly the entire power budget is allocated to the $4^{\text{th}}$ sub-channel, which validate Remark 6.




(a) $\lambda = 0.2, \mu = 0$ (b) $\lambda = 0.2,\ \mu = 0.1$ (c) $\lambda = 0.2,\ \mu = 0.15$ (d) $\lambda = 0.2,\ \mu = 0.2$ (e) $\lambda = 0.2,\ \mu = 0.22$ (f) $\lambda = 0.2,\ \mu = 0.242$ (g) $\lambda = 0.2,\ \mu = 0.243$ (h) $\lambda = 0.2,\ \mu = 0.249$

Fig. 4: Power Allocation with the Increase of $P_d$

## VI. Conclusion

In this lab, it examines the optimization of power allocation for the simultaneous transmission of wireless information and power transfer (SWIPT) in OFDM communication systems under additive white Gaussian noise conditions. Our lab highlights the critical trade-off between information rate and power transfer, adding the power constraints on both the transmitter and receiver sides. By implementing the Lagrange multiplier method with KKT conditions, I implemented an optimal power allocation strategy that maximizes the information rate while maintaining power constraints, thus facilitating efficient SWIPT. The experiment shows that power distribution across all sub-channels is even, similar to water-filling method in a bottle, when there are no constraints on the received power.

Conversely, when there is a maximum received power limit, all the power gets allocated to the best sub-channel.The outcomes of this study contribute to the growing body of knowledge in electrical engineering, bridging the gap between electric power and communication engineering, and paving the way for advancements in self-sustainable communication technologies.

## APPENDIX: EXTENDED DERIVATIONS

### A. Extended Proof of Equation (18)

The Lagrange function of optimization problem in equation (16) can be represented as:

$$\begin{aligned} L(\lambda, \mu, P_1, \ldots, P_{N_c-1}) = & \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \\ & - \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right) \\ & - \sum_{n=0}^{N_c-1} \mu_n(-P_n). \end{aligned} \tag{30}$$

The KKT conditions of equation (30) are:

$$\begin{cases} -P_n \leq 0, \quad n = 0, \ldots, N_c - 1 & \text{(31a)} \\ \sum_{n=0}^{N_c-1} P_n - P = 0, \quad n = 0, \ldots, N_c - 1 & \text{(31b)} \\ \mu_n \geq 0, \quad n = 0, \ldots, N_c - 1 & \text{(31c)} \\ \mu_n(-P_n) = 0, \quad n = 0, \ldots, N_c - 1 & \text{(31d)} \\ \nabla L(\lambda, \mu, P_1, \ldots, P_{N_c-1}) = 0 & \text{(31e)} \end{cases}$$

According to equation (17), the equation (30) can be rewritten as:

$$\begin{aligned} L_{\lambda\mu}(\lambda, \mu, \ldots, P_{N_c-1}) = & L_\lambda(\lambda, P_1, \ldots, P_{N_c-1}) \\ & - \sum_{n=0}^{N_c-1} \mu_n(-P_n) \end{aligned} \tag{32}$$

Therefore, equation (31e) can be represented as:

$$\frac{\partial L_{\lambda\mu}}{\partial P_n} = \frac{\partial L_\lambda}{\partial P_n} + \mu_n = 0 \tag{33}$$

$$\frac{\partial L_\lambda}{\partial P_n} = -\mu_n \tag{34}$$

According to equation (31d) and equation (34), we can obtain as follows:

- When $P_n = 0$ and $\mu_n \geq 0$ $\quad \frac{\partial L_\lambda}{\partial P_n} \leq 0$
- When $P_n > 0$ and $\mu_n = 0$ $\quad \frac{\partial L_\lambda}{\partial P_n} = 0$

This statement is equal to equation (18). The proof is complete.

### B. Extended Proof of Equation (19)

When $P_n > 0$ and $\mu_n = 0$ $\quad \frac{\partial L_\lambda}{\partial P_n} = 0$, we have:

$$\frac{\partial L_\lambda}{\partial P_n} = \frac{|h_n|^2}{(N_0 + P_n |h_n|^2) ln2} - \lambda = 0 \tag{35}$$

$P_n$ can be represented as:

$$P_n = \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2} \tag{36}$$

Due to $P_n \geq 0$, therefore $P_n$ can be represented as equation (19):

$$P_n = \begin{cases} \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2}, & \text{if } \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2} > 0, \\ 0, & \text{if } \frac{1}{\lambda \ln 2} - \frac{N_0}{|h_n|^2} \leq 0. \end{cases}$$

The proof is complete.

### C. Extended Proof of Equation (26)

The Lagrange function of optimization problem in equation (23) can be represented as:

$$\begin{aligned} L(\lambda, \mu, \nu, P_1, \ldots, P_{N_c-1}) = & \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \\ & - \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right) \\ & - \mu\left(\sum_{n=0}^{N_c-1} E[|y_n|^2] - P_d\right) \\ & - \sum_{n=0}^{N_c-1} \nu_n(-P_n). \end{aligned} \tag{37}$$

Where $\lambda$, $\mu$ and $\nu$ are Lagrange multiplier. The KKT conditions of equation (37) are:

$$\begin{cases} -P_n \leq 0, \quad n = 0, \ldots, N_c - 1 & \text{(38a)} \\ \sum_{n=0}^{N_c-1} P_n - P = 0, \quad n = 0, \ldots, N_c - 1 & \text{(38b)} \\ \nu_n \geq 0, \quad n = 0, \ldots, N_c - 1 & \text{(38c)} \\ \nu_n(-P_n) = 0, \quad n = 0, \ldots, N_c - 1 & \text{(38d)} \\ \nabla L(\lambda, \mu, \nu, P_1, \ldots, P_{N_c-1}) = 0 & \text{(38e)} \end{cases}$$

According to equation (24), the equation (37) can be rewritten as:

$$\begin{aligned} L_{\lambda\mu\nu}(\lambda, \mu, \nu, \ldots, P_{N_c-1}) = & L_{\lambda\mu}(\lambda, \mu, P_1, \ldots, P_{N_c-1}) \\ & - \sum_{n=0}^{N_c-1} \nu_n(-P_n) \end{aligned} \tag{39}$$

Therefore, equation (38e) can be represented as:

$$\frac{\partial L_{\lambda\mu\nu}}{\partial P_n} = \frac{\partial L_{\lambda\mu}}{\partial P_n} + \nu_n = 0 \tag{40}$$

$$\frac{\partial L_{\lambda\mu}}{\partial P_n} = -\nu_n \tag{41}$$

According to equation (38d) and equation (41), we can obtain as follows:

- When $P_n = 0$ and $\nu_n \geq 0$ $\quad \frac{\partial L_{\lambda\mu}}{\partial P_n} \leq 0$
- When $P_n > 0$ and $\nu_n = 0$ $\quad \frac{\partial L_{\lambda\mu}}{\partial P_n} = 0$

This statement is equal to equation (26). The proof is complete.

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