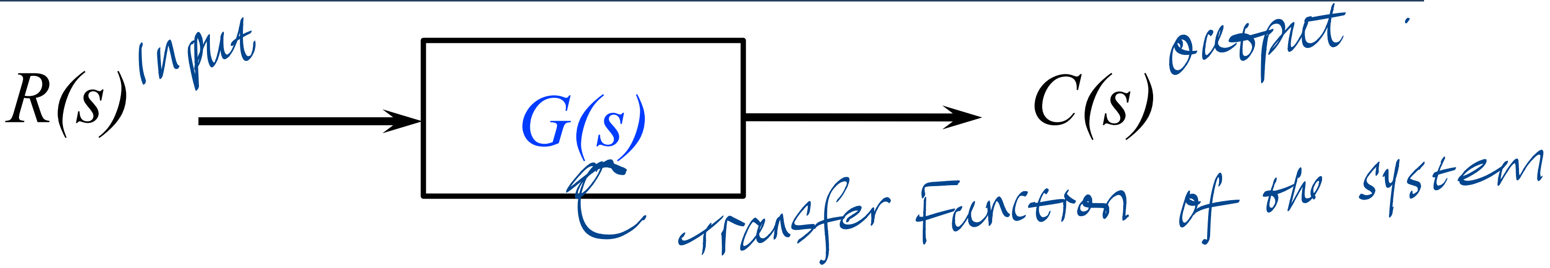
GRASP Laboratory
General Robotics & Autonomous Systems and Processes

Mechatronic Modeling and Design with Applications in Robotics

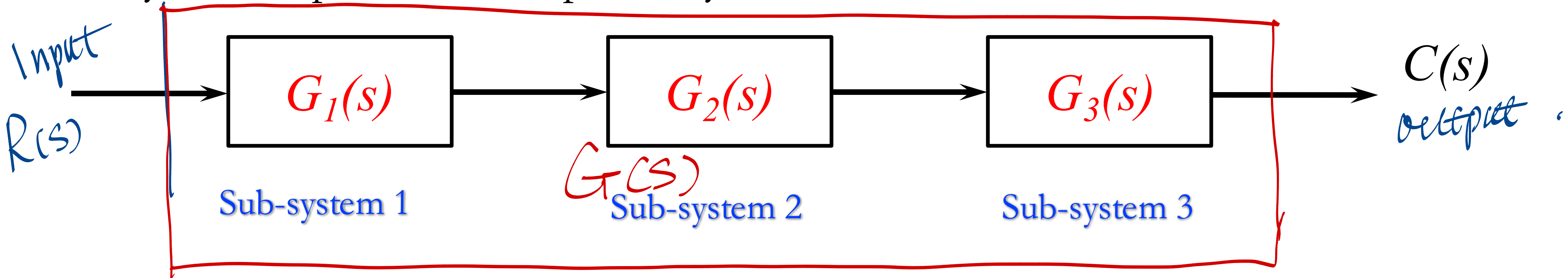
Graphical Models

Block Diagram

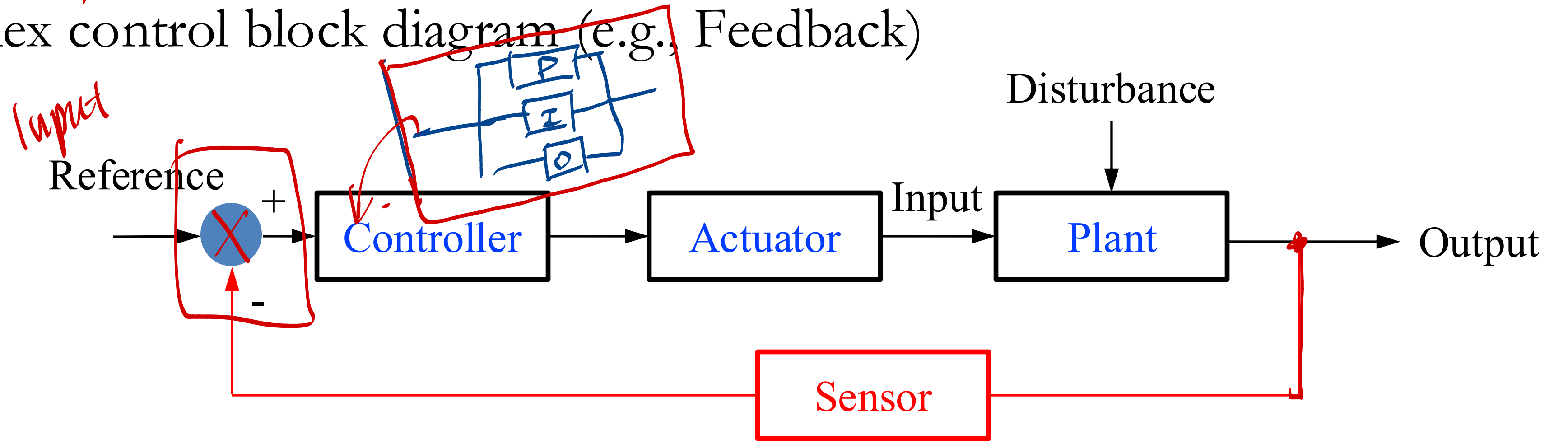
Block Diagram



Systems usually are composed of multiple subsystems:

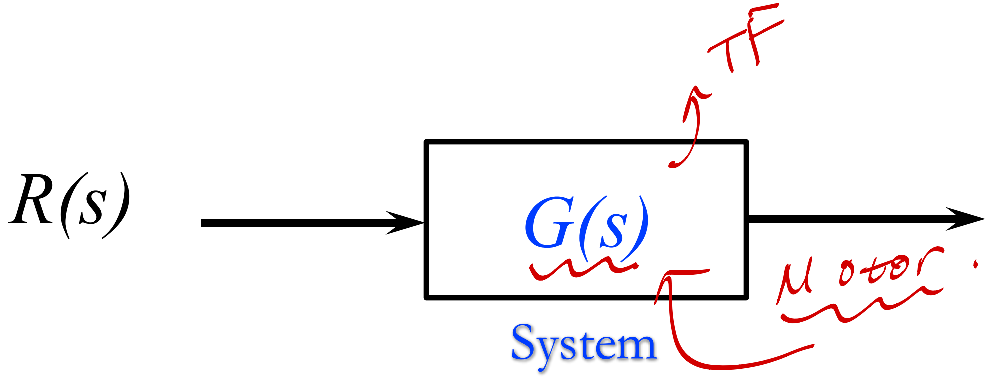


More complex control block diagram (e.g., Feedback)



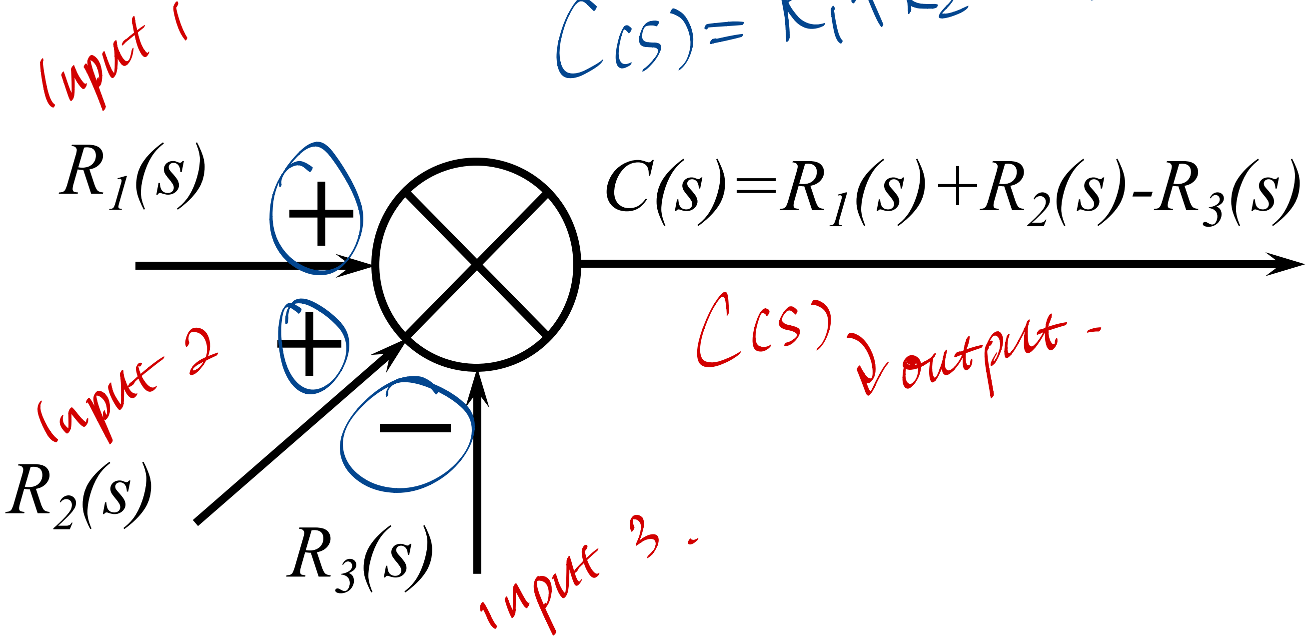
Basic Components of Block Diagrams

Signals

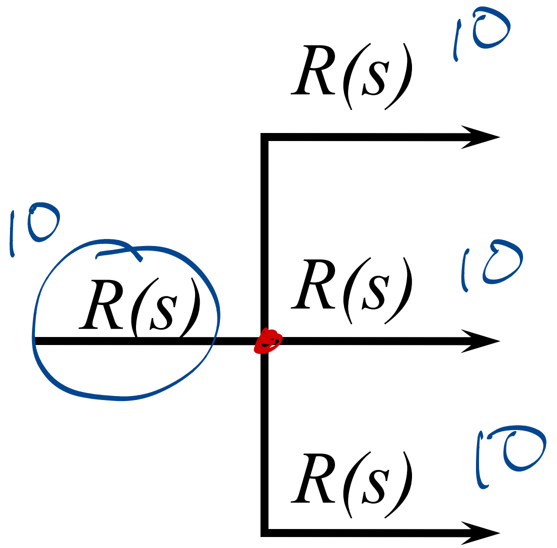


System

$C(s) = R_1 + R_2 - R_3.$

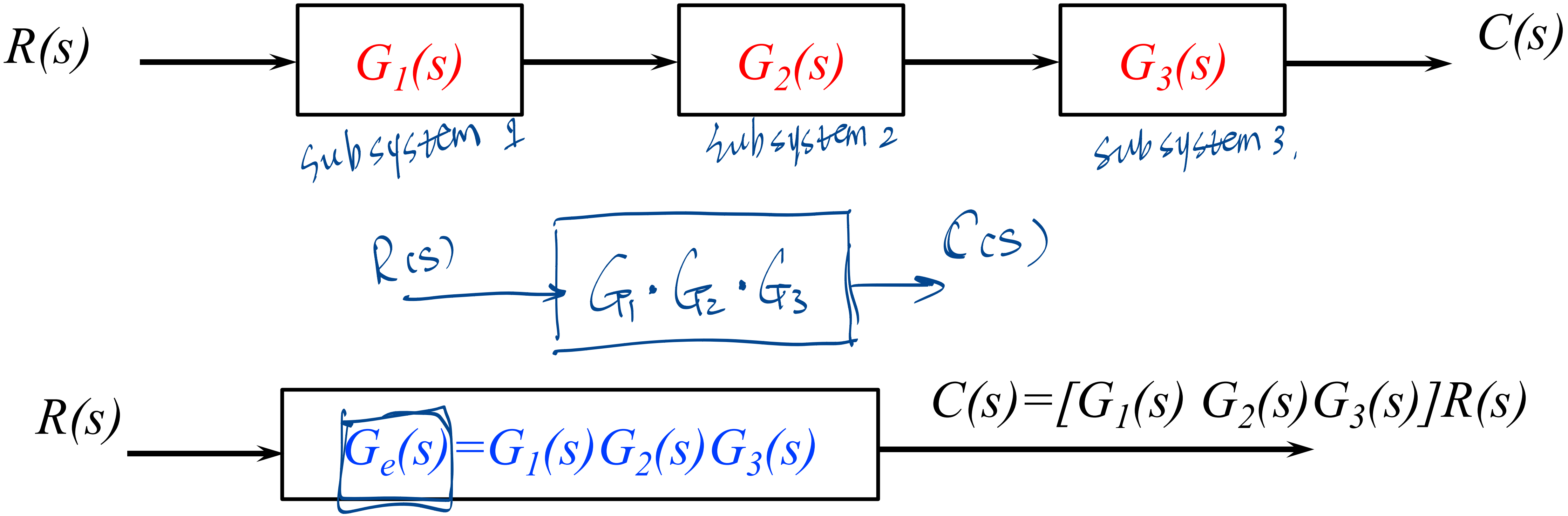


Summing Junction

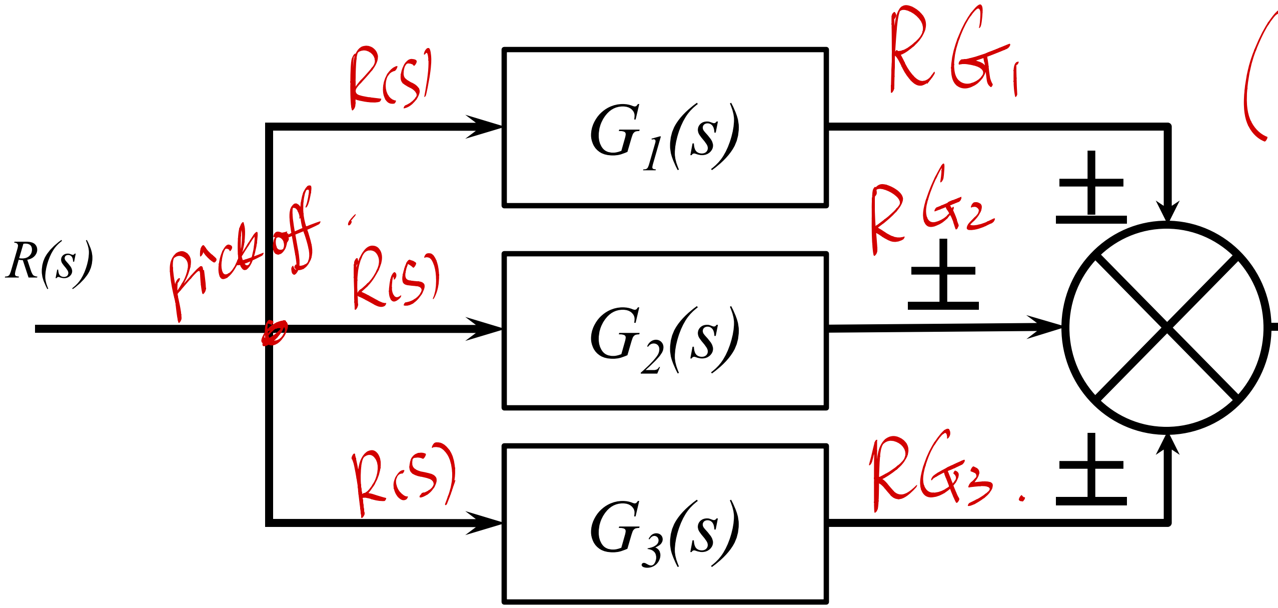


Pickoff Point

Cascade Form



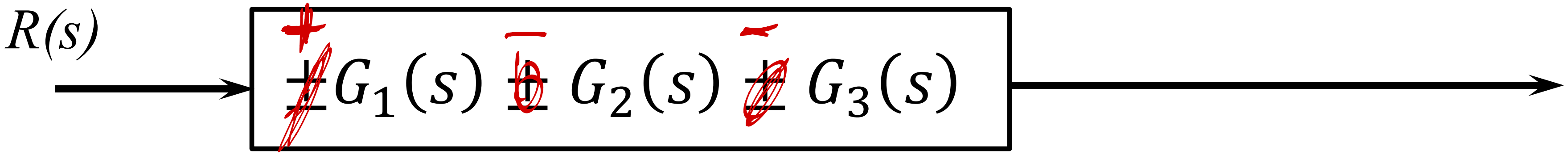
Parallel Form



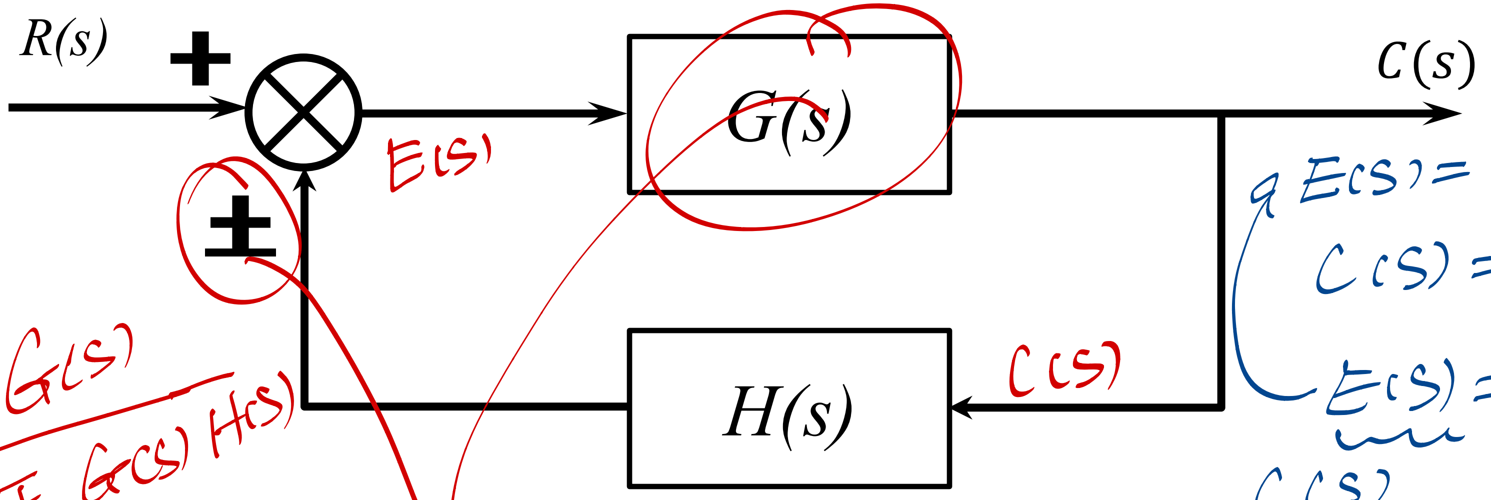
$C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]R(s)$

$(\pm RG_1 \pm RG_2 \pm RG_3)$ output.

$C(s) = R(G_1 - G_2 - G_3)$.



Feedback Form: Eliminating a Feedback Loop



Feedback (Sensor)

$$E(s) = R(s) \pm H(s)\cdot C(s)$$

$$C(s) = E(s)\cdot G(s)$$

$$E(s) = \frac{C(s)}{G(s)}$$

$$\frac{C(s)}{G(s)} = R(s) \pm H(s)\cdot C(s)$$

$$C(s) = G(s)R(s) \pm G(s)H(s)C(s)$$

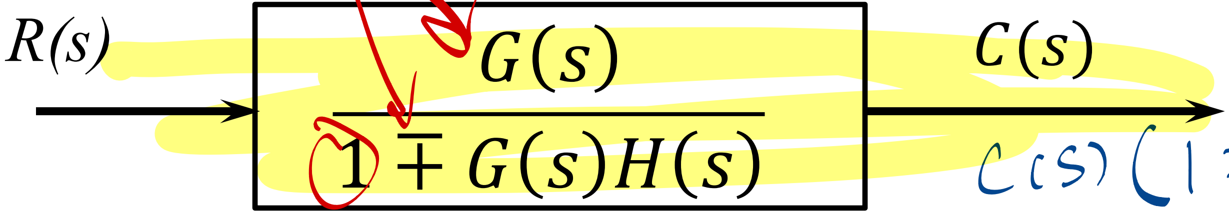
$$C(s) \mp G(s)H(s)C(s) = G(s)R(s)$$

$$C(s)\left(1 \mp G(s)H(s)\right) = G(s)R(s)$$

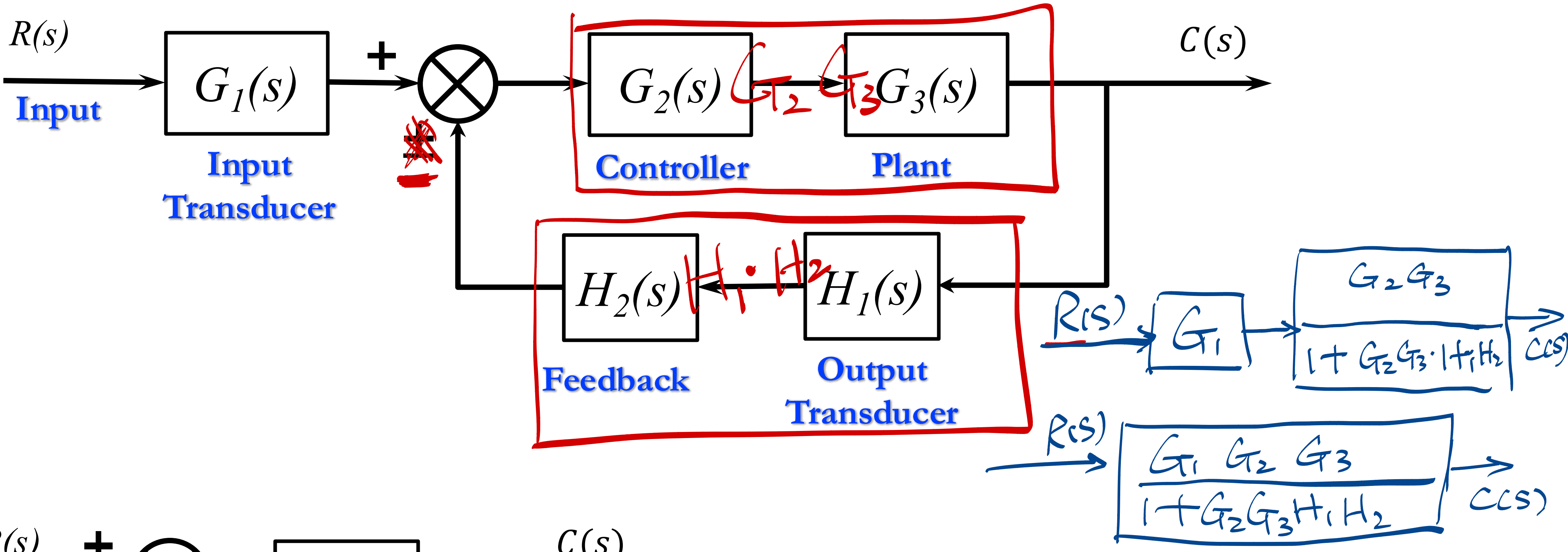
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$

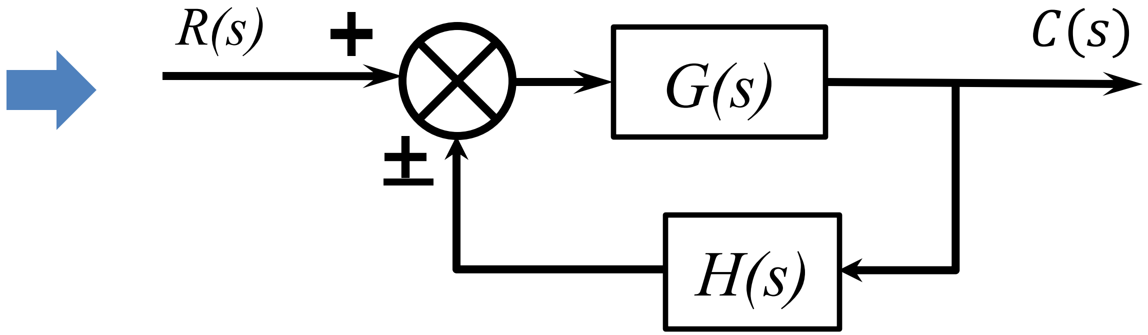
$$G_e(s) = \frac{G(s)}{1 \mp G(s)H(s)}$$

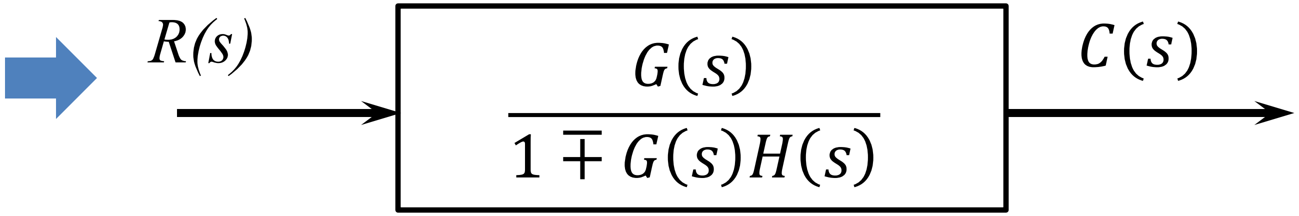
$$C(s) = \frac{G(s)}{1 \mp G(s)H(s)}$$



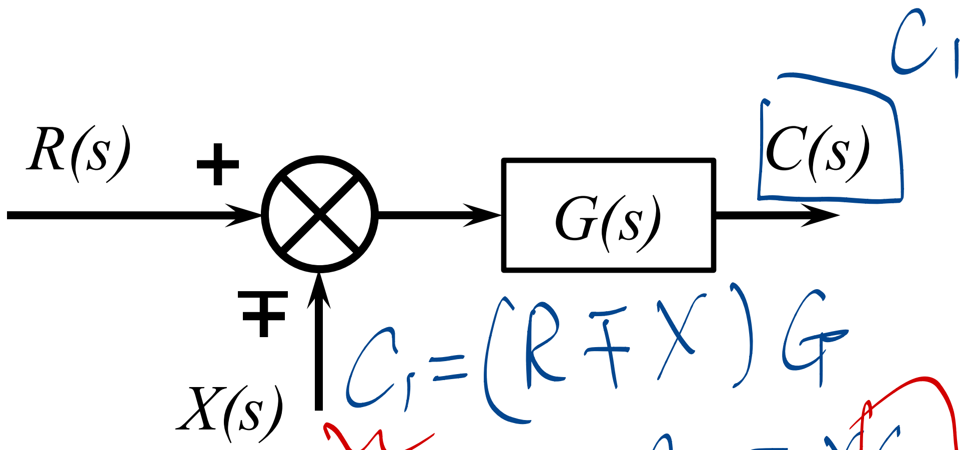
Moving Blocks to Create Familiar Forms







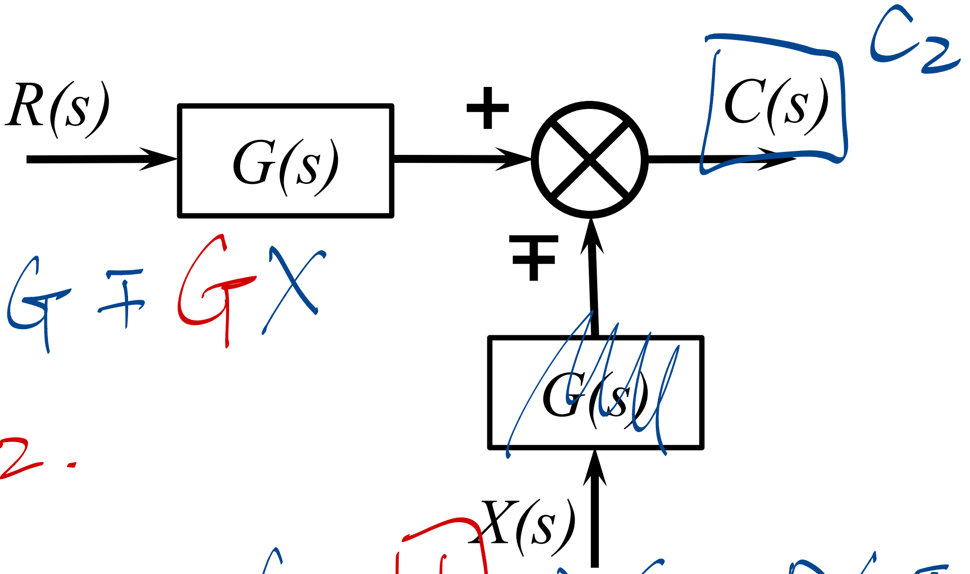
Moving a Summing Junction



$C_1 = (R \mp X)G$

$= RG \mp XG$

≡

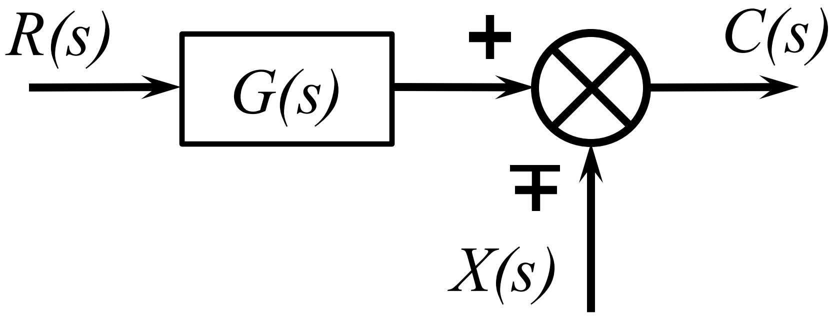


$C_2 = RG \mp GX$

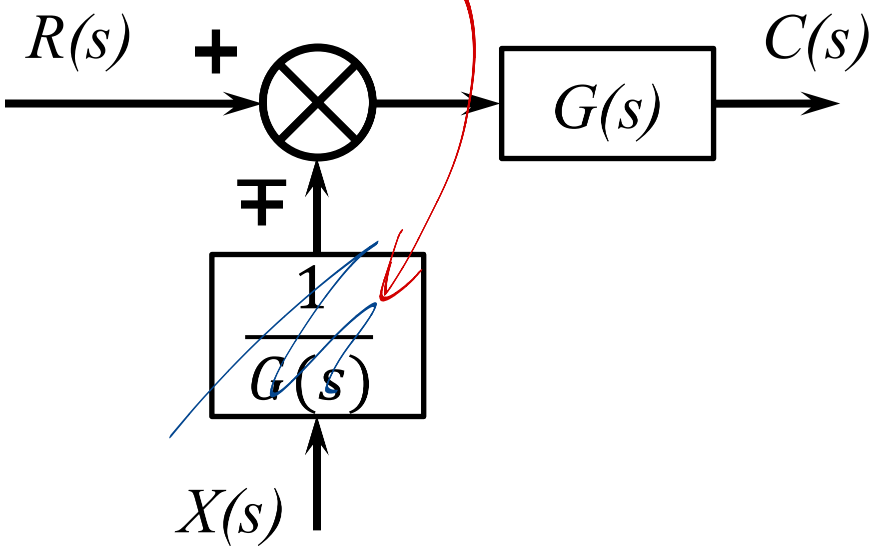
$C_1 = C_2.$

$C_2 = (R \mp \frac{1}{G}X)G = RG \mp \cancel{G}X$

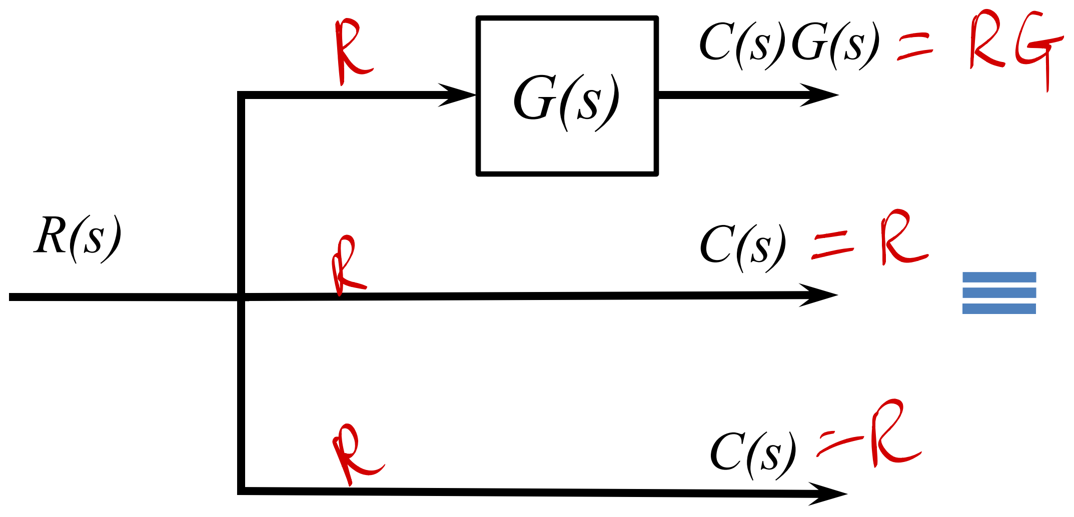
$C = RG \mp X$



≡



Moving a Pickoff Point



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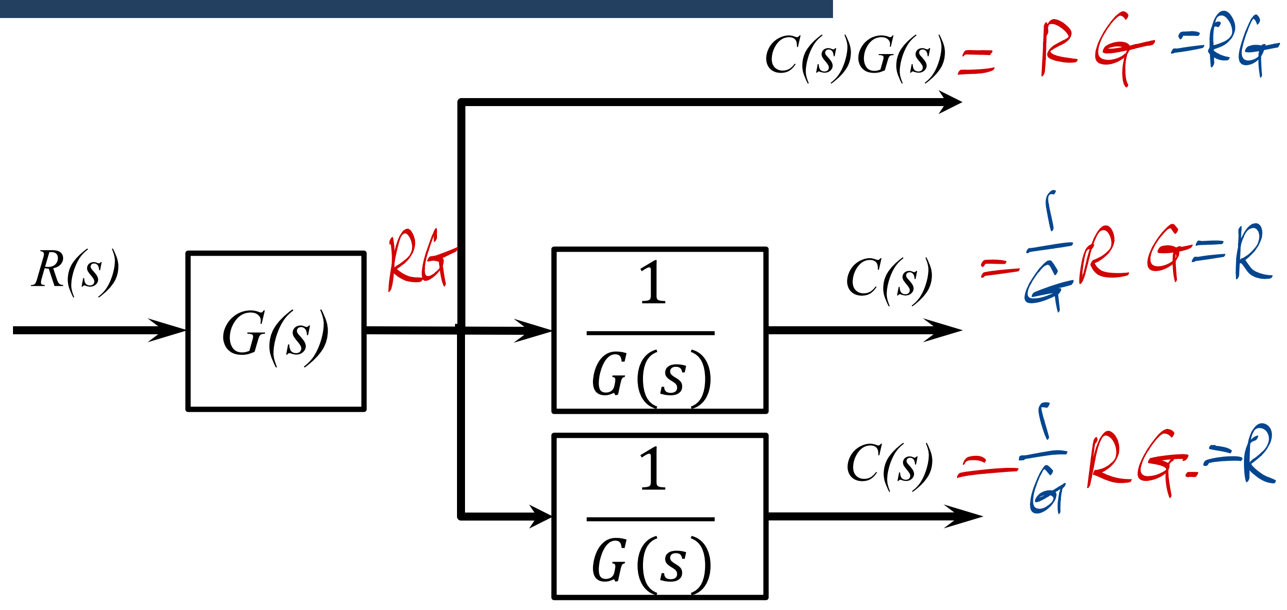

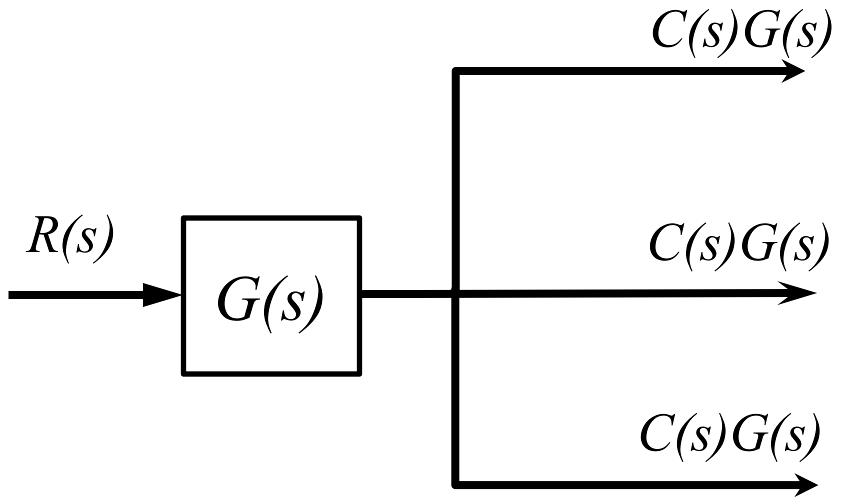

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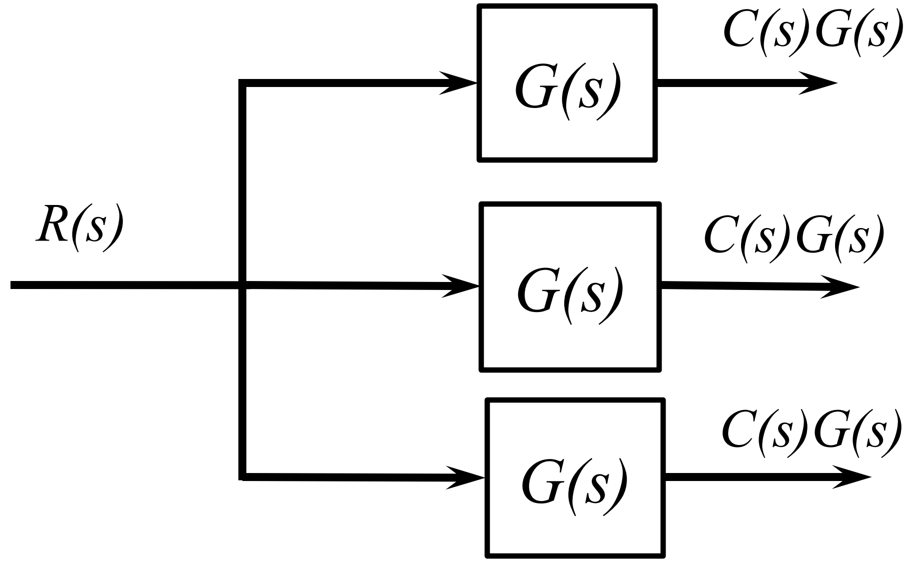
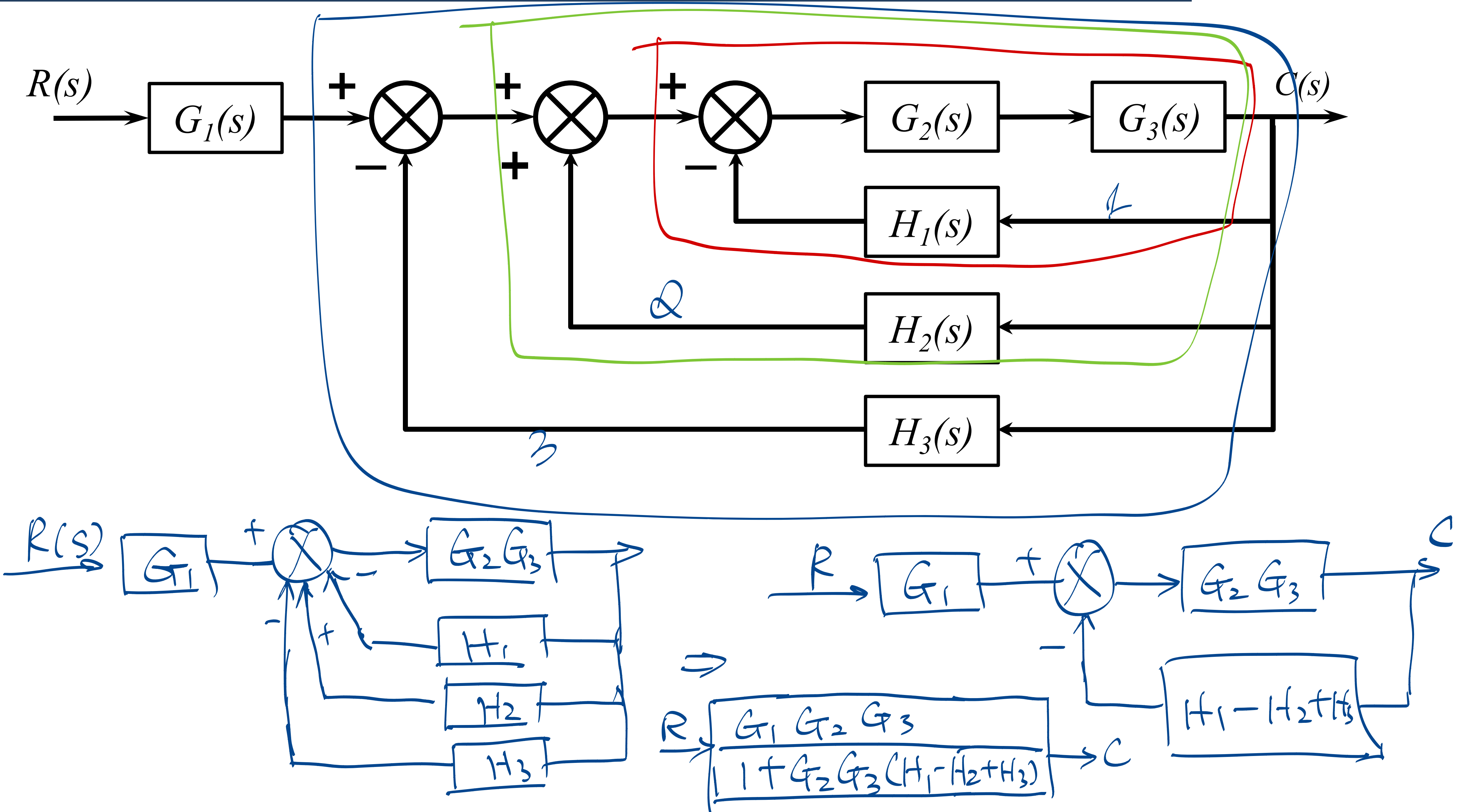
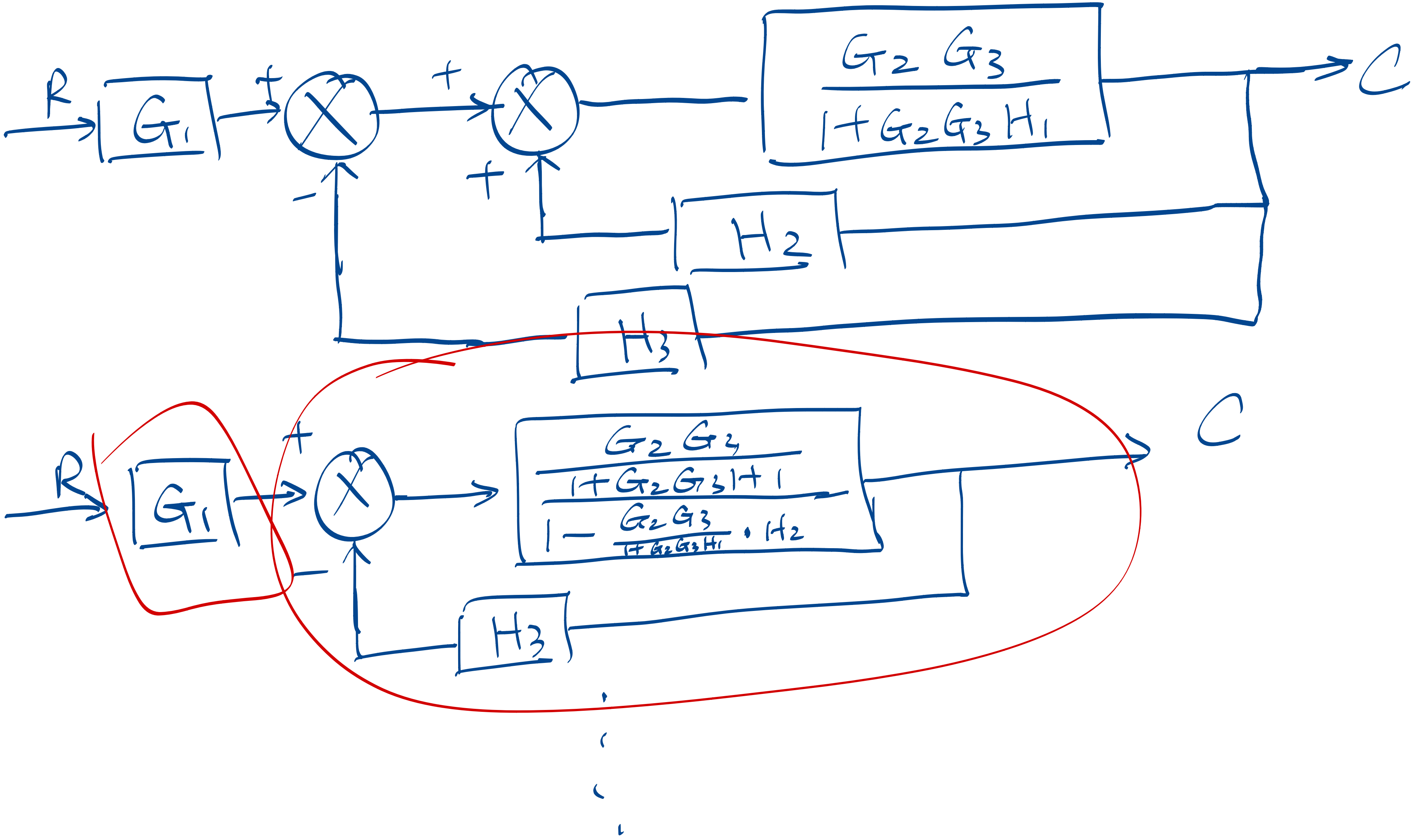


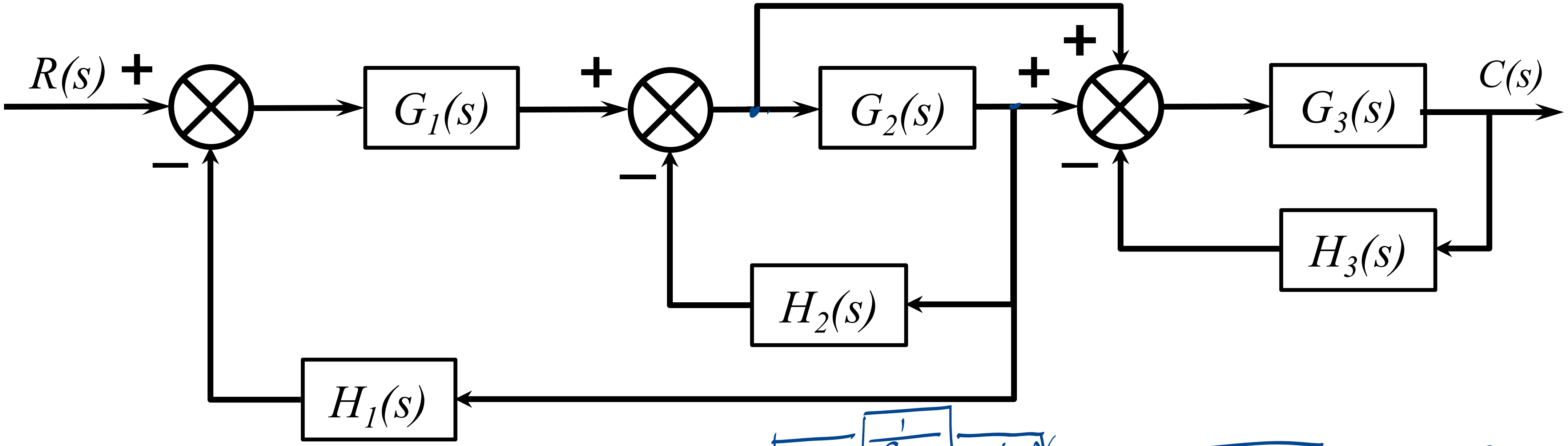
Example 1

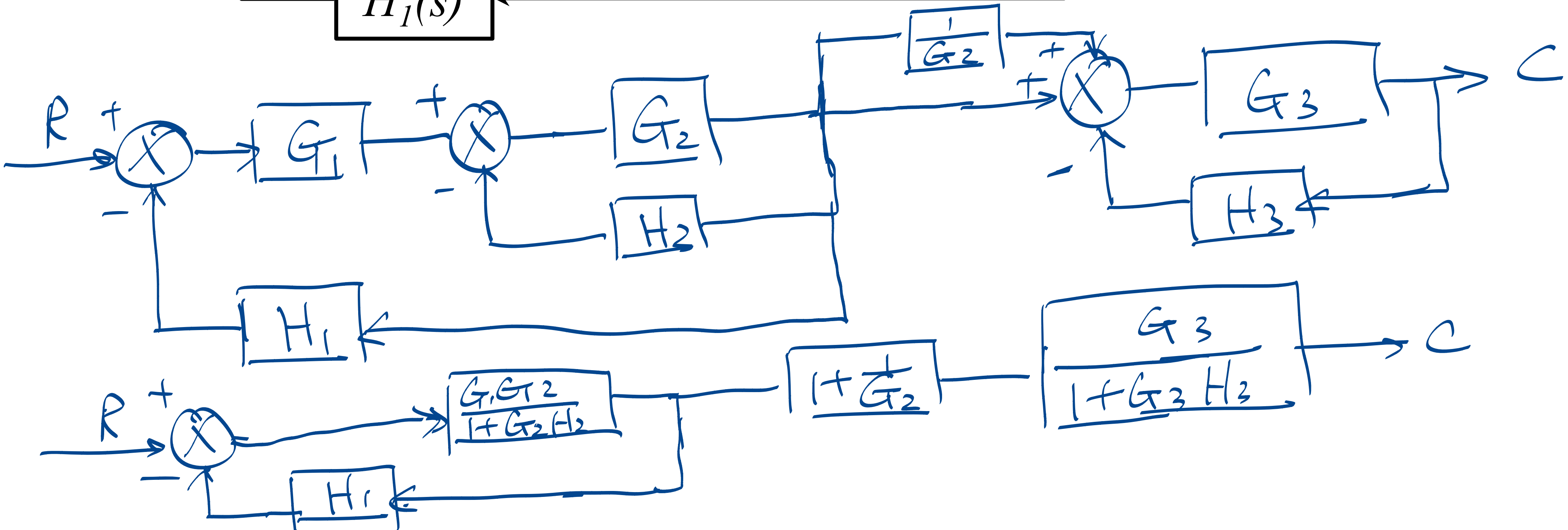

R(s)
G1(s)
+
−
+
+
+
−
G2(s)
G3(s)
C(s)
H1(s)
H2(s)
H3(s)
1
2
3
R(s)
G1
G2G3
H1
H2
H3
R
G1
G2 G3
H1 − H2 + H3
C
R
G1 G2 G3
1 + G2G3(H1 − H2 + H3)
C

Example 1 (Cont'd)

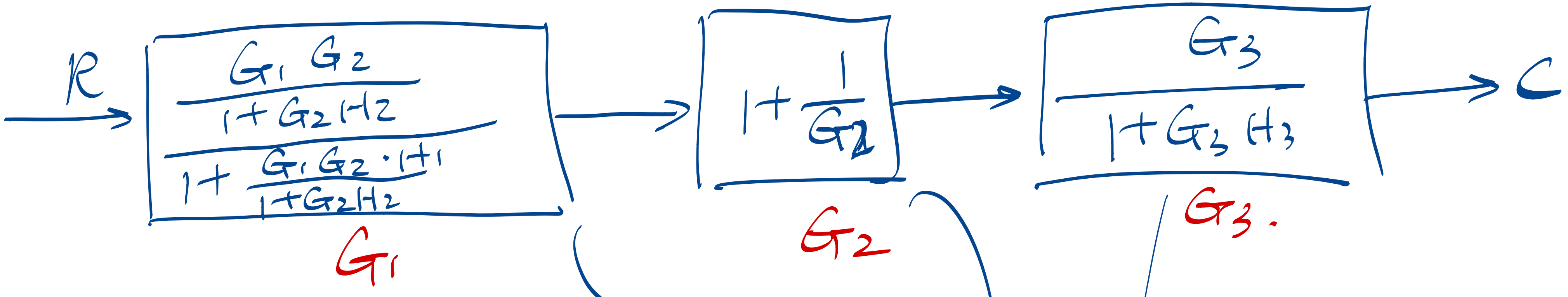


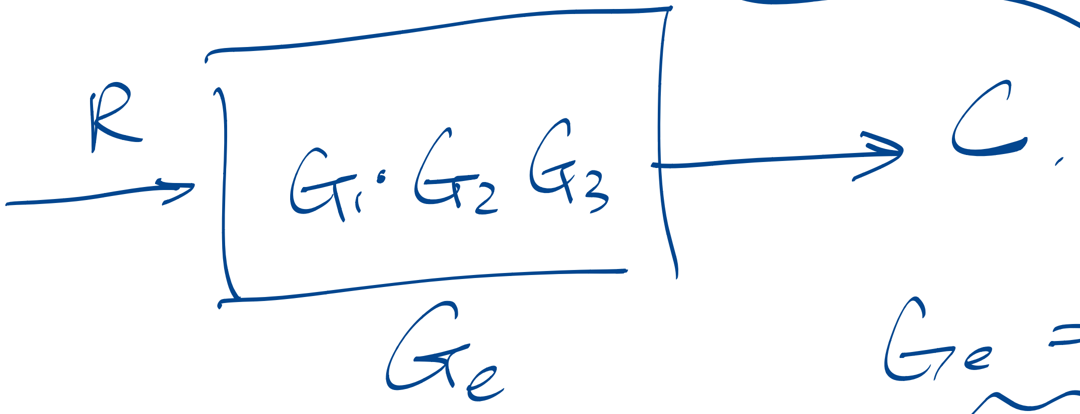
Example 2


R(s)
+
−
G1(s)
+
−
G2(s)
+
+
−
G3(s)
C(s)
H3(s)
H2(s)
H1(s)


R
+
−
G1
+
−
G2
H2
H1
1/G2
+
+
−
G3
H3
C
G1G2 / 1+G2H2
1+G2
G3 / 1+G3H3
C

Example 2 (Cont'd)





$$G_e = G_1 \cdot G_2 \cdot G_3$$

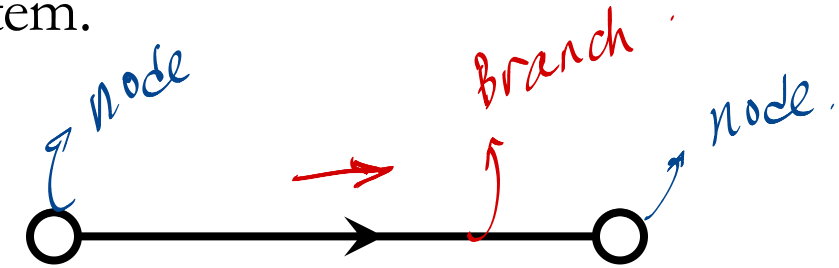
$$TF = G(s) = \frac{C(s)}{R(s)}$$

Graphical Model $\Longleftrightarrow$ Analytical Model.

Signal Flow Graph

Definition

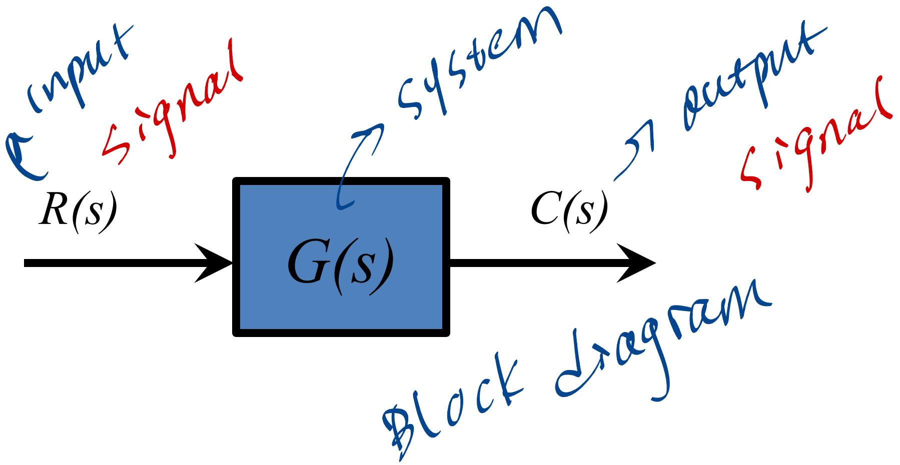
A system is represented by a line with an arrow showing the direction of signal flow through the system.

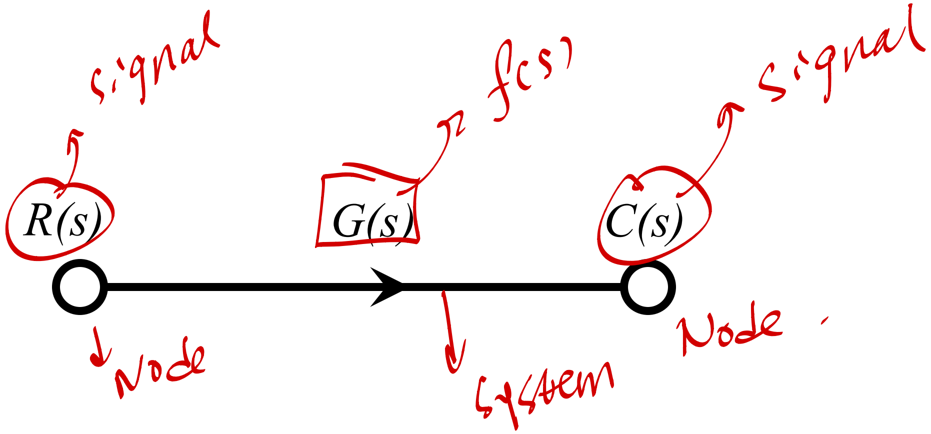


A signal-glow graph consists only **branches** and **nodes**:

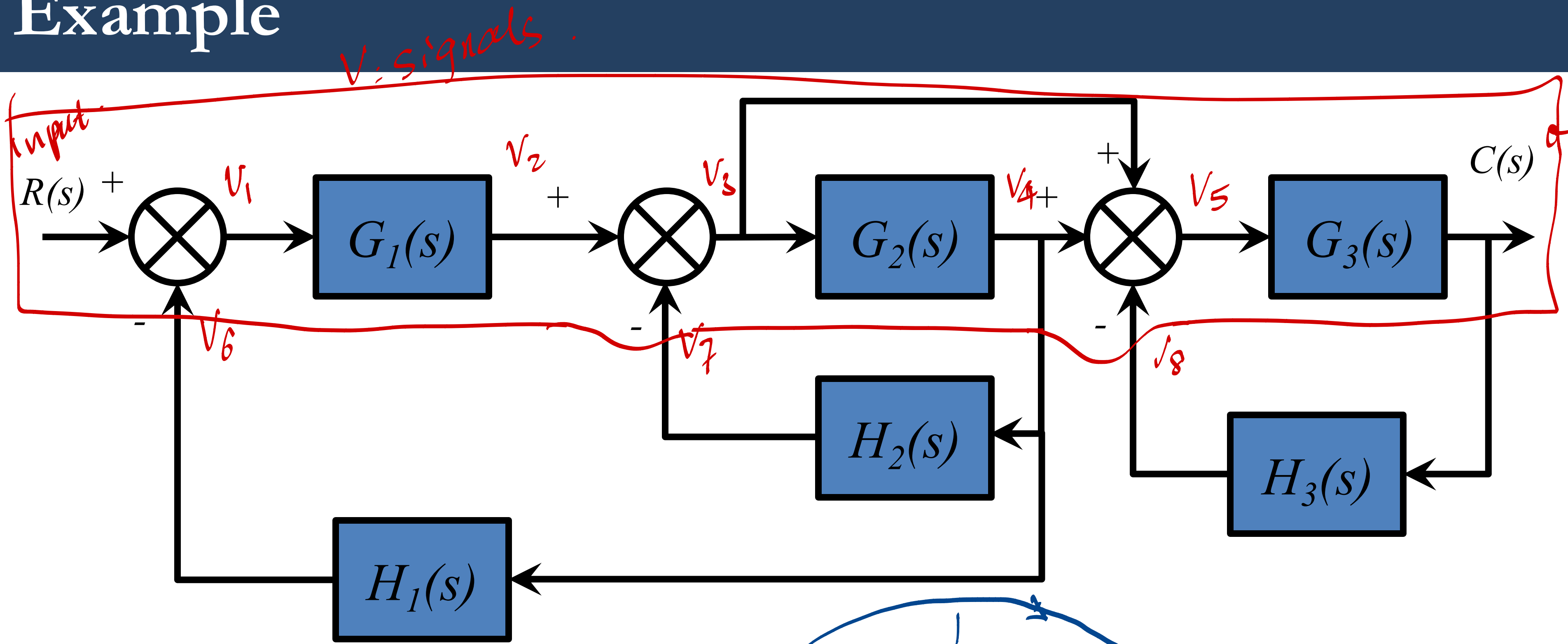
Branches: represent **systems**

Nodes: represent **signals**



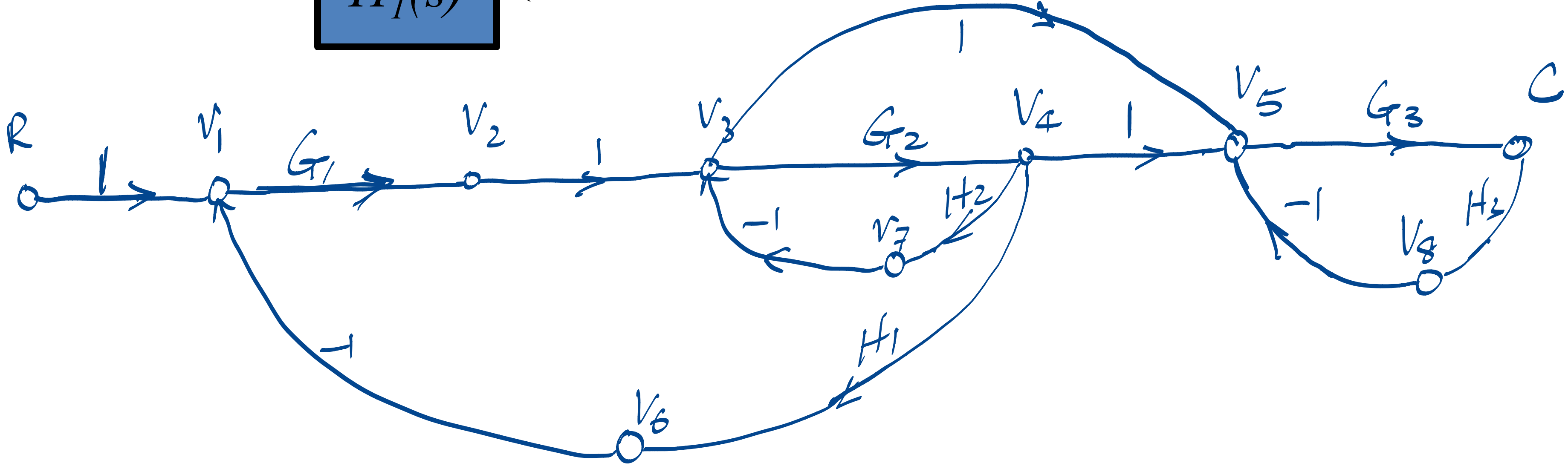


Example



* systems.
*signals.

Block Diagram



Signal flow Graph

Analytical model

TF. input/output.

?

Mason's Rule: Definition

Loop Gain:

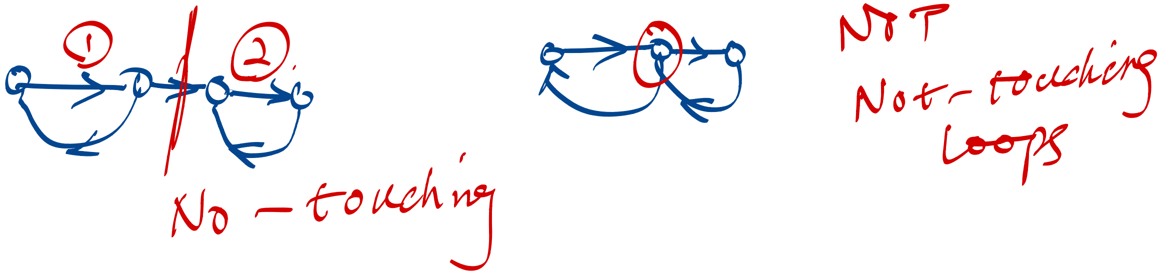
The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once.

Forward-path Gain:

The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

Non-touching Loops:

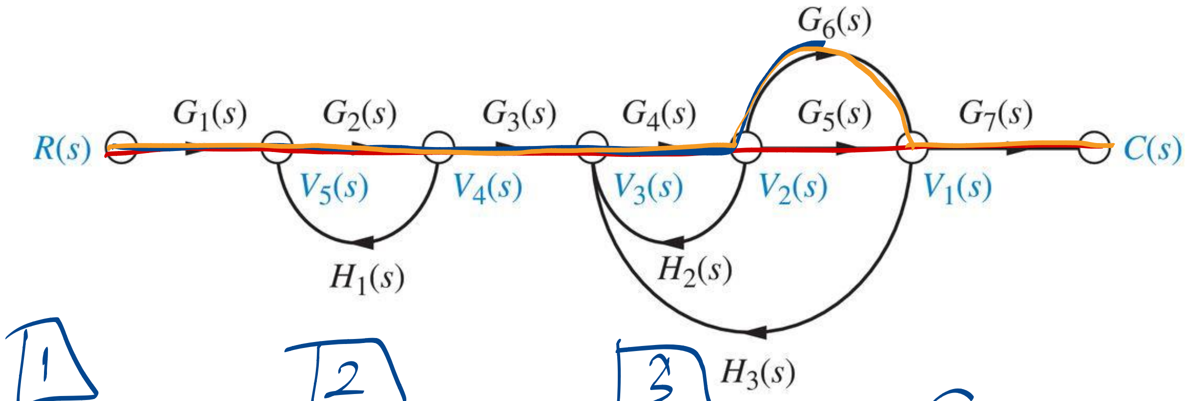
Loops that do not have any nodes in common.

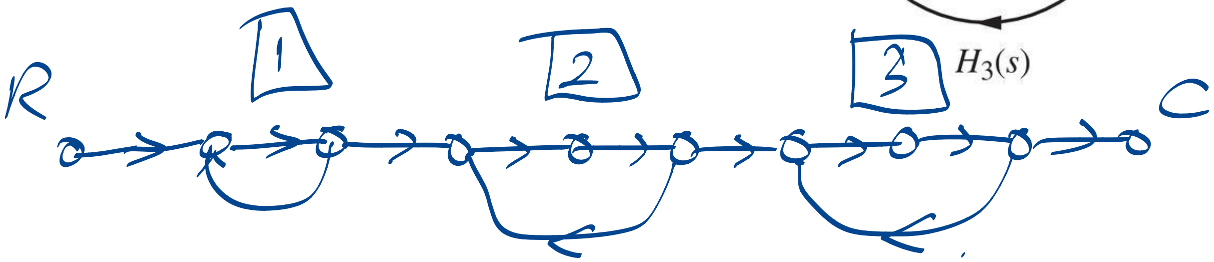


Non-Touching-Loop Gain:

The product of loop gains from non-touching loops taken two, three four, or more at a time

Example





Loop Gain:

$G_2 H_1$

$G_4 H_2$

$G_4 G_5 H_3$

$G_4 G_6 H_3$

Forward-path Gain:

Red: $G_1 G_2 G_3 G_4 G_5 G_7$.

Orange: $G_1 G_2 G_3 G_4 G_6 G_7$.

Non-touching Loops: 1 $G_2 H_2$ 2 $G_4 H_2$ 3 $G_4 G_5 H_3$ 4. $G_4 G_6 H_3$.

Non-Touching-Loop Gain: $[G_2 H_2][G_4 H_2]$ (1, 2) $[G_2 H_2][G_4 G_5 H_3]$ (1, 3)

$[G_2 H_2][G_4 G_6 H_3]$ (1, 4)

Mason's Rule

system TF

output

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

input

$K=2$

k = number of forward paths

T_k = the kth forward-path gain

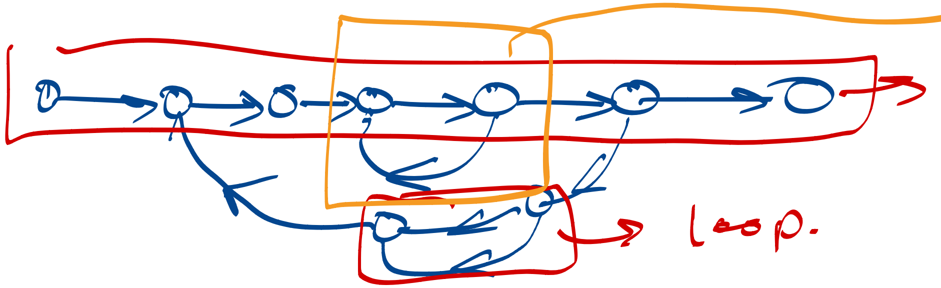
T_k

Δ = 1- Σ loop gains + Σ non-touching loop gains taken two at a time – Σ non-touching loop grains taken three at a time + Σ non-touching loop gains taken four at a time …

$= 1 - G_7 H_4$

$G_7 H_4$

Δ_k = Δ – Σ loop gain terms in Δ that touch the kth forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the kth forward path.



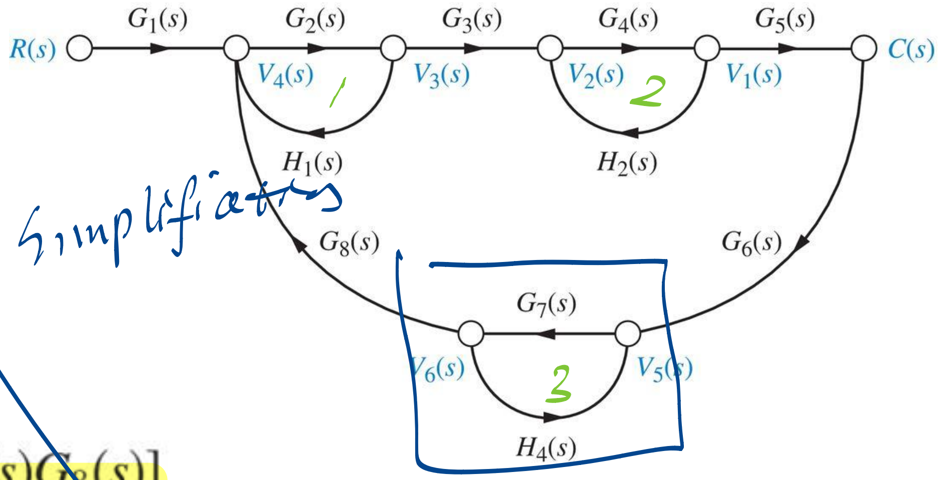
forward path.

loop.

one forward path

Find the transfer function, $C(s)/R(s)$ for the signal-flow-graph:

$\Delta K = \Delta -$



simplifications

$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{\boxed{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)]}[1 - G_7(s)H_4(s)]}{\Delta}$$

ΔK

G7 H4

$$\begin{aligned}\Delta = 1 &- [G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) \\ &+ G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ &+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ &+ G_4(s)H_2(s)G_7(s)H_4(s)] \\ &- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned}$$

forward path gain: $G_1 G_2 G_3 G_4 G_5$

loop Gain: G_2H_1 $\quad G H_2$ $\quad G_7H_4,$ $\quad G_2 G_3 G_4 G_5 G_6 G_7 G_8$

non-touching loop gain (taken two): $G_2H_1G_4H_2$ $\quad G_2H_1G_7H_4.$ $\quad G_4H_2G_7H_2$

Loop 1,2 $\quad$ Loop 1,3 $\quad$ Loop 2,3.

- - - - - - - - - - - - - - (taken three): $G_2H_1G_4H_2G_7H_4$

Signal-Flow Graphs of State Equations

Consider the following state and output equations:

state space model.

$$\begin{cases} \dot{x}_1 = \boxed{2x_1} - \boxed{5x_2} + \boxed{3x_3} + \boxed{2r} \\ \dot{x}_2 = -6x_1 - 2x_2 + 2x_3 + 5r \\ \dot{x}_3 = x_1 - 3x_2 - 4x_3 + 7r \\ y = -4x_1 + 6x_2 + 9x_3 \end{cases}$$

input

output

x: states

$x_1 \; x_2 \; x_3$.

$\dot{x}_1 \; \dot{x}_2 \; \dot{x}_3$

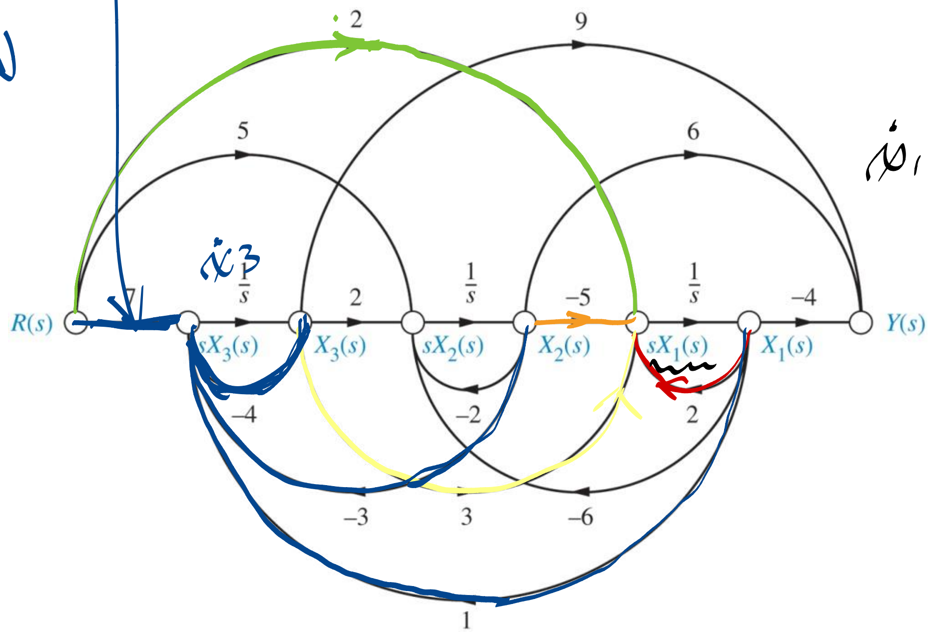
$\dot{x} \xrightarrow{\frac{1}{s}} x$

$\dot{x}_1 \xrightarrow{\frac{1}{s}} x_1$, 2 $\Rightarrow \dot{x}_1 = 2x_1$

where r is the input, y is the output, x_1, x_2 and x_3 are the state variables, please draw its signal-flow graph.

$\dot{x} = Ax + Bu$

$y = Cx + Du$



→ TF

signal flow graph.

The End!!