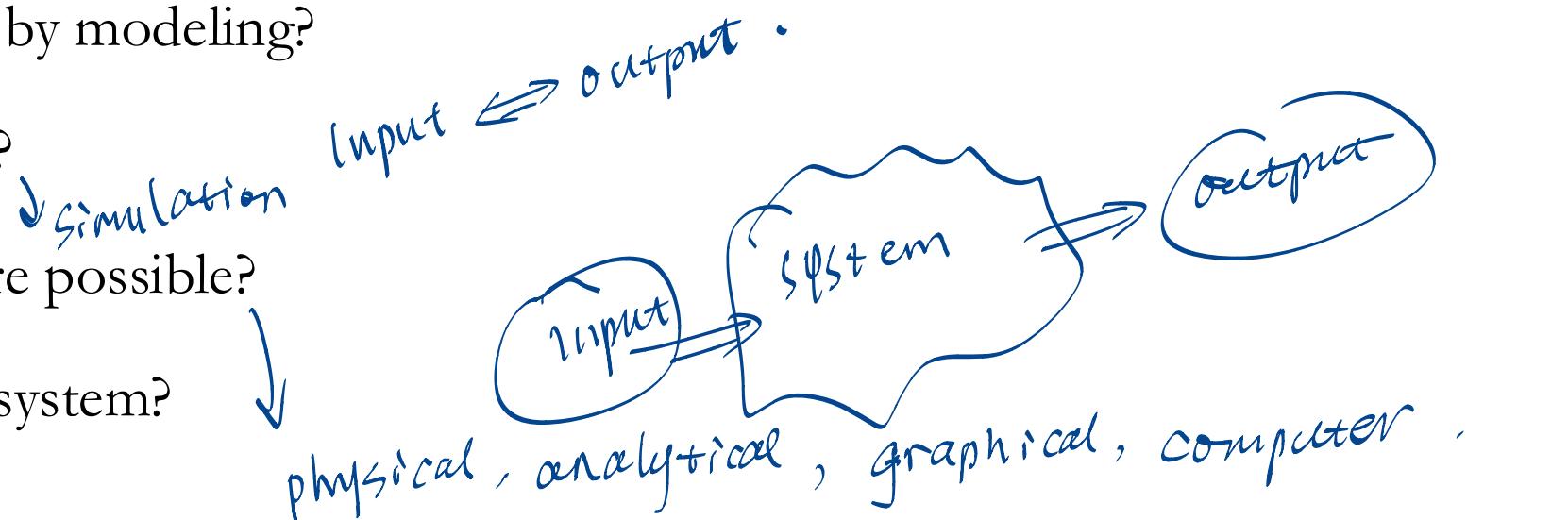




Mechatronic Modeling and Design with Applications in Robotics

Introduction to Modeling

1. What is specifically meant by modeling?
2. What is the use of models?
3. What types of modeling are possible?
4. How to model a dynamic system?



A way to develop an analytical model that has the four characteristics: Integrated, Unified, Unique and Systematic.

↓ State space.
(not unique)

Integrated, Unified, *Unique and Systematic*.
↓ multi-domain.
(Mechanical, electrical
thermal fluid)

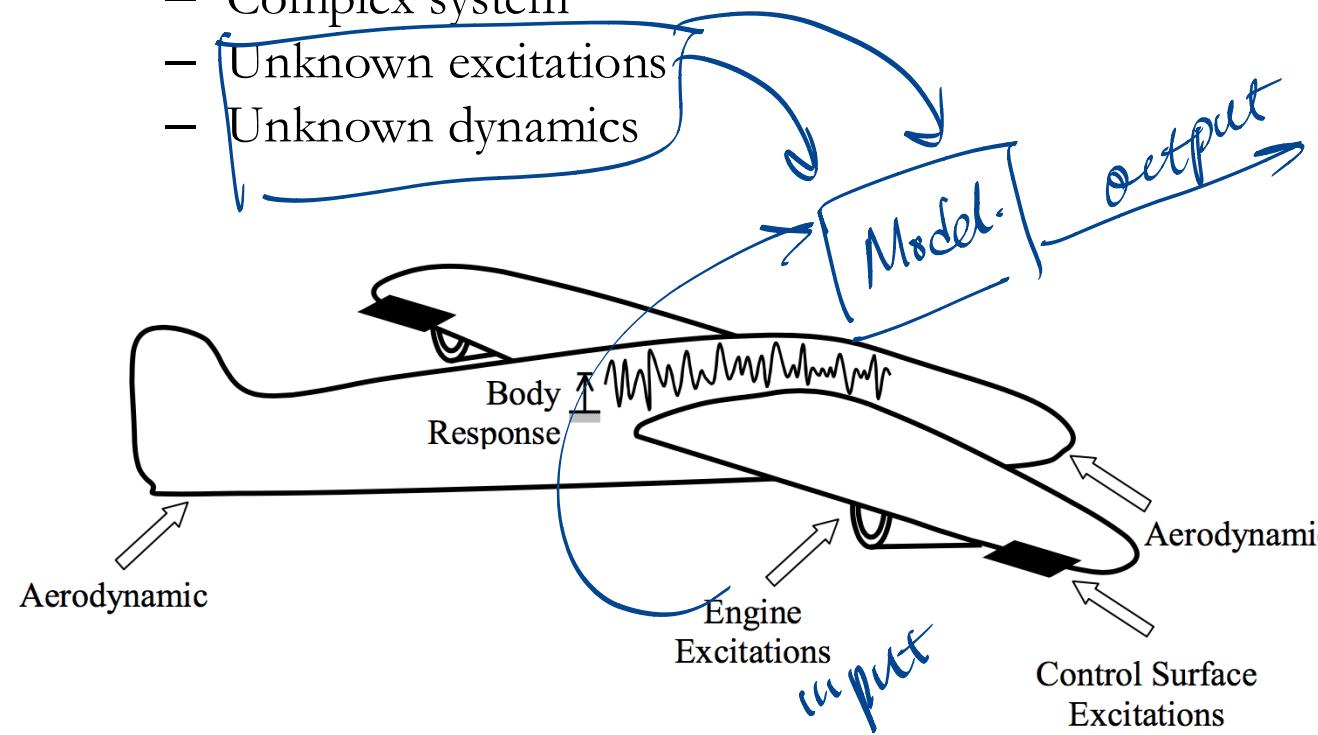
at same time
similarity

Make a **Dynamic System** behave in a desired manner, according to some **Performance Specifications**.

Note:

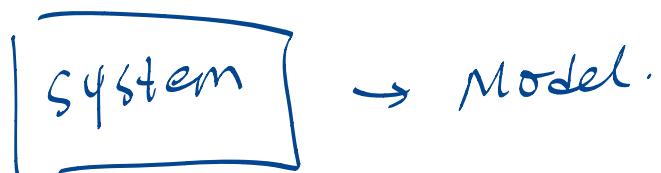
- Complex system
- Unknown excitations
- Unknown dynamics

Modeling is an optimistic approach where we attempt to accurately represent the system or required system behavior

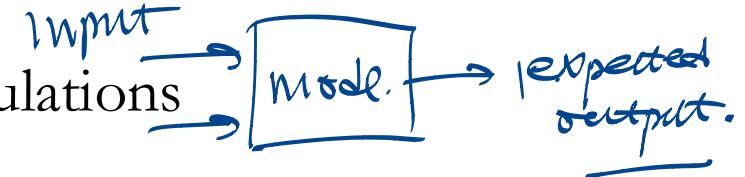
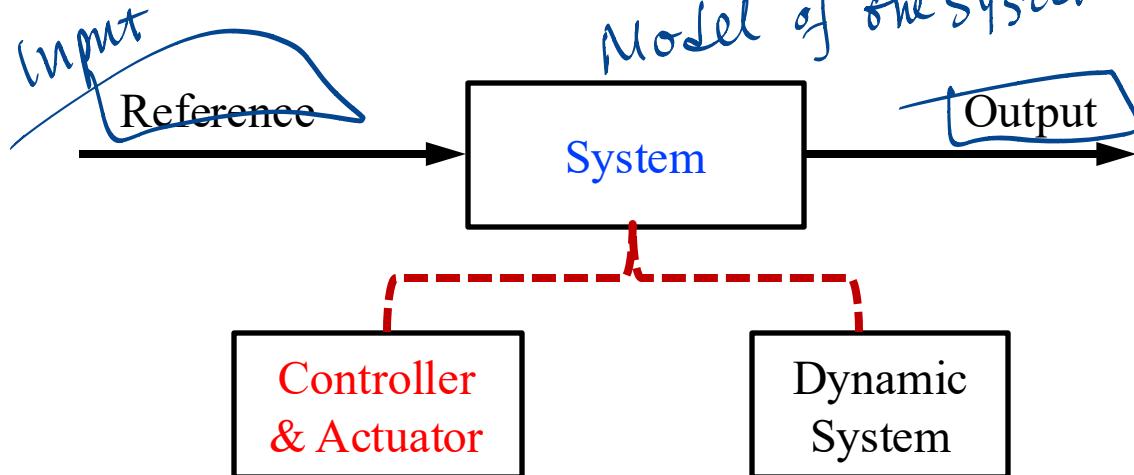


- Room temperature control
- Car/bicycle driving
- Balanced of bank account
- Laundry machine
- Airplane, rocket, satellite.
- DC motor

- **Initial Design:** The beginning of a design process (desired system does not exist)
- **Design Optimization:** design iteration, particularly prototyping can become very costly and time consuming

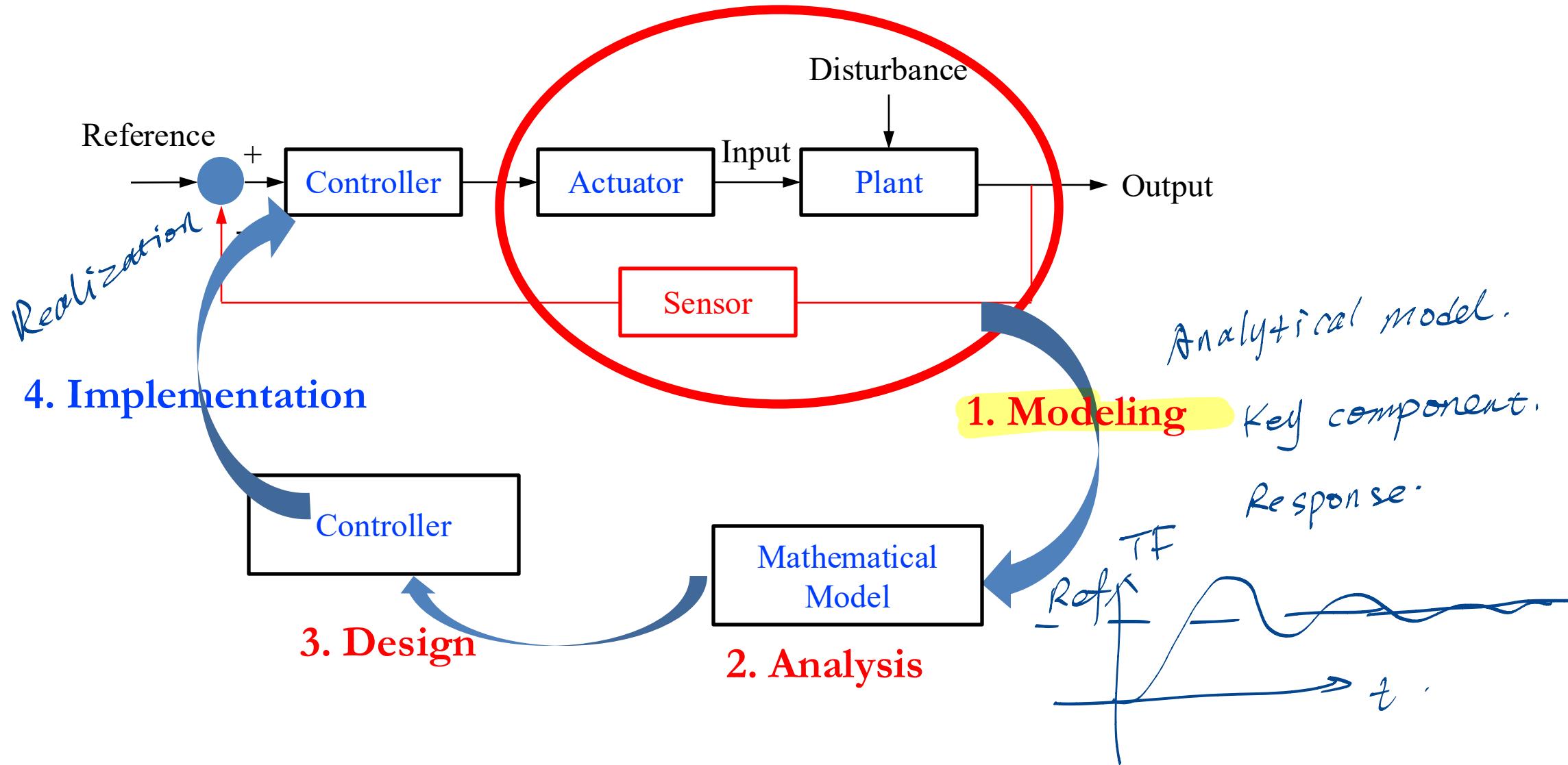


- **Monitoring and Fault Diagnosis:** identify faulty/degraded parts, provide suggestions
- **Design Evolution:** guide the design process by computer simulations



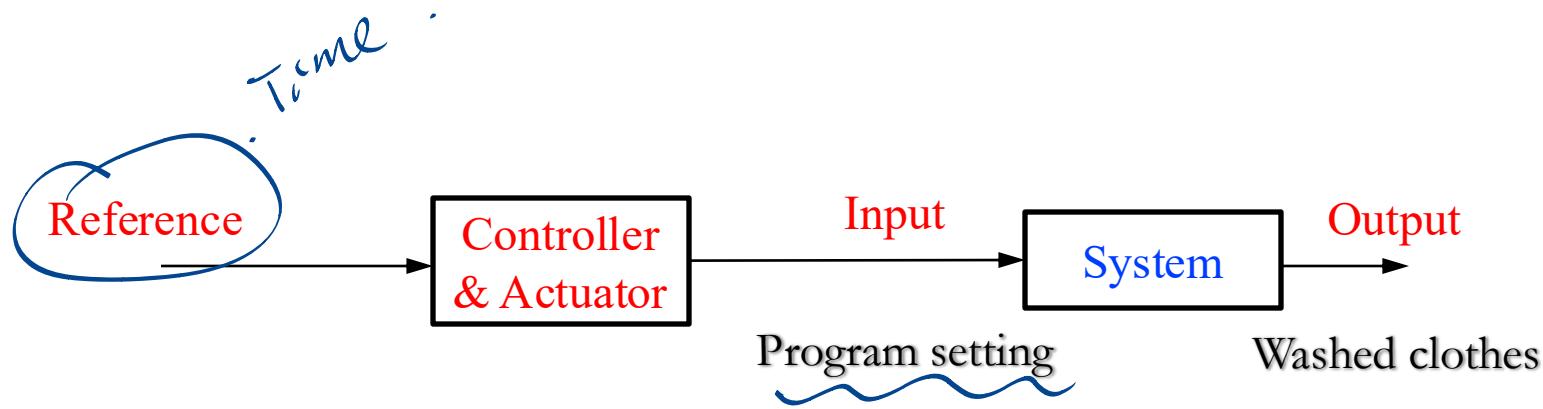
Engineering Examples:

- ✓ Room temperature control
- ✓ Liquid-level control
- ✓ Steam pressure control
- ✓ Voice volume control



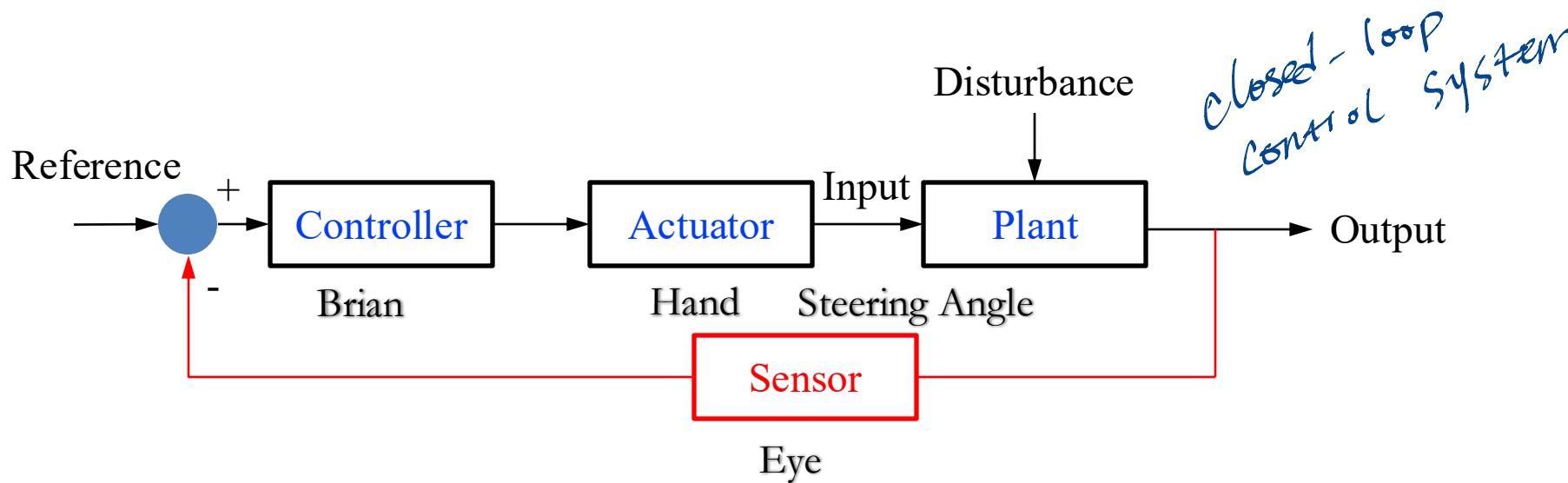
An Example: Laundry Machine

- A laundry machine washes clothes, by setting a program.



- A laundry machine does **not measure** how clean the clothes become.
- Control without measuring devices (sensors) are called **open-loop control**.

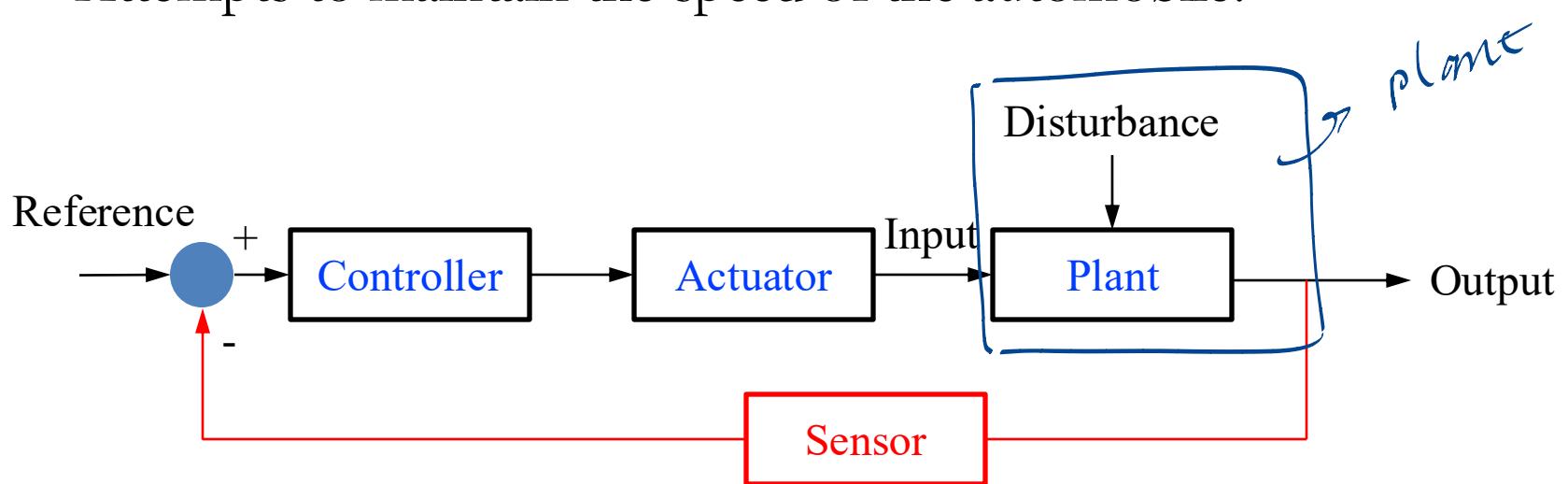
- Attempts to change the direction of the automobile.



- Manual closed-loop (**feedback**) control.
- Although the controlled system is “Automobile”, the **input** and the **output** of the system can be different, depending on **control objectives!**

An Example: Automobile Cruise Control

- Attempts to maintain the speed of the automobile.

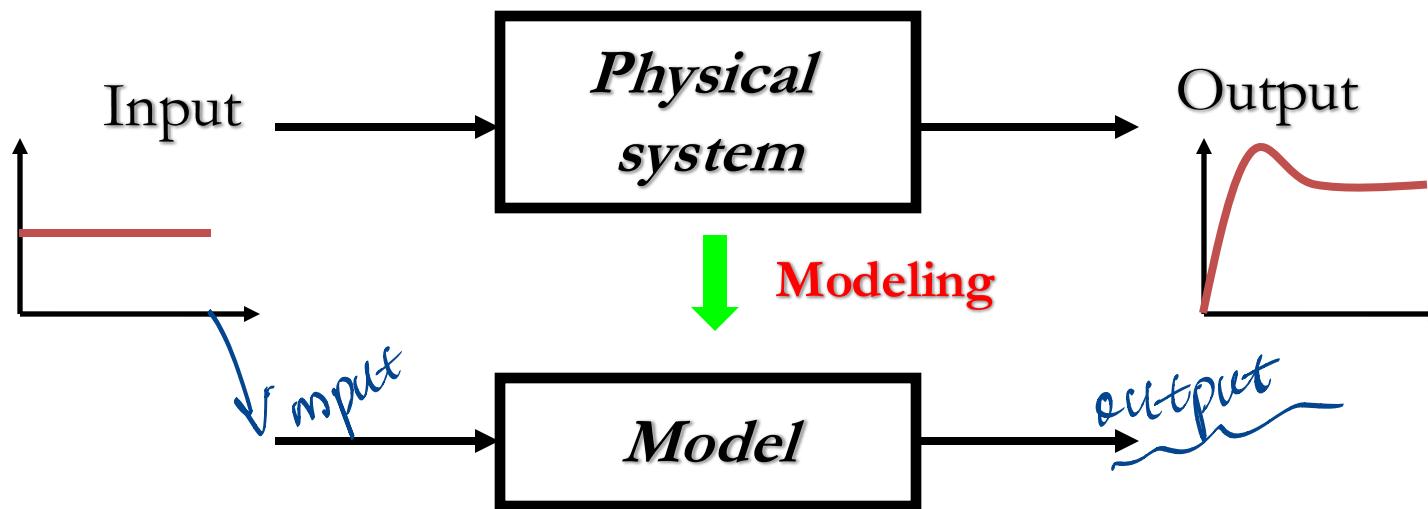


- Cruise control can be both manual and automatic.
~~~~~
- Note the similarity of the diagram above to the diagram in the previous slide!

| System      | Typical Input           | Typical Output                            |
|-------------|-------------------------|-------------------------------------------|
| Human Body  | Neuroelectric pulses    | Muscle contraction, body movements        |
| Company     | Information             | Decisions, finished products              |
| Power plant | Fuel rate               | Electric power,<br>Pollution rate         |
| Automobile  | Steering wheel movement | Front wheel turn, direction of<br>heading |
| Robot       | Voltage to Joint        | Joint motions, effector motion            |

Dynamical  
systems

- Representation of the input-output relation of a physical system



- A model is used for the **analysis** and **design** of control systems.

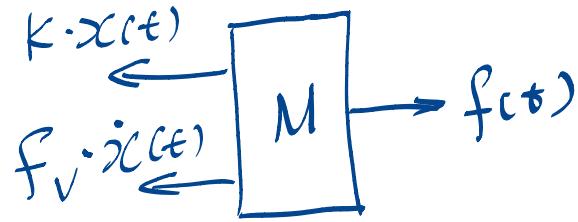
- Modeling is the **most important and difficult task** in control system design.
- No mathematical model exactly represents a physical system.

Math model  $\neq$  Physical system  
Math model  $\approx$  Physical system

- Do not confuse **models** with **physical systems**!
- Selecting a model **close enough** to a physical system and yet **simple enough** to be studied analytically is the **most important and difficult task**.
- In this course, we may use the term "**system**" to mean a mathematical/analytical model.

# An Example

Draw Free Body Diagram (FBD)



Apply Newton's 2nd law of motion:

$$\sum F = m \cdot a = M \ddot{x}(t)$$

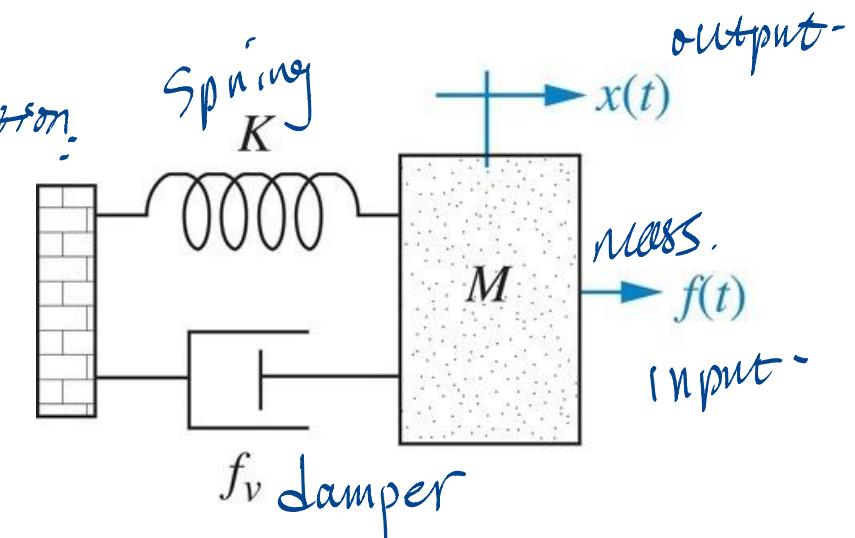
$$f(t) - Kx(t) - f_v \dot{x}(t) = M \ddot{x}(t)$$

$$M \ddot{x}(t) + f_v \dot{x}(t) + Kx(t) = f(t)$$

Laplace Transform: Assume zero initial conditions:

$$M \cdot X(s) s^2 + f_v X(s) s + K X(s) = F(s)$$

$$TF: G(s) = \frac{\text{Output}}{\text{Input}} = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$



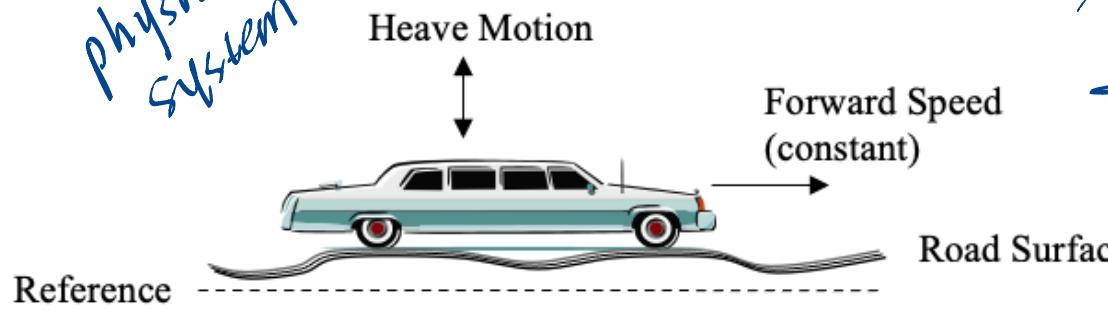
$$\begin{aligned} x(0) &= 0 \\ \dot{x}(0) &= 0 \\ \ddot{x}(0) &= 0. \end{aligned}$$

s: s domain

# Another Example

Modeling of automobile suspension system

*physical system*



*simplification*

automobile

suspension

- 1: Analytical  $\approx$  physical wheel
- 2: Simple & close enough.

FBD of  $M_1$  and  $M_2$ .  $\sum F = ma$ .

$$\left\{ -K_1(x_1 - x_2) - B(\dot{x}_1 - \dot{x}_2) = M_1 \ddot{x}_1 \right. \quad (1)$$

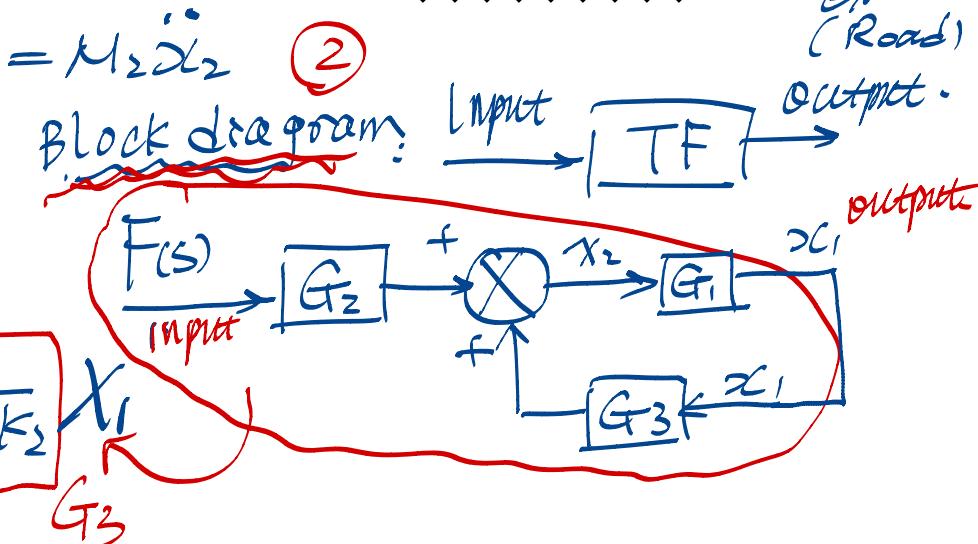
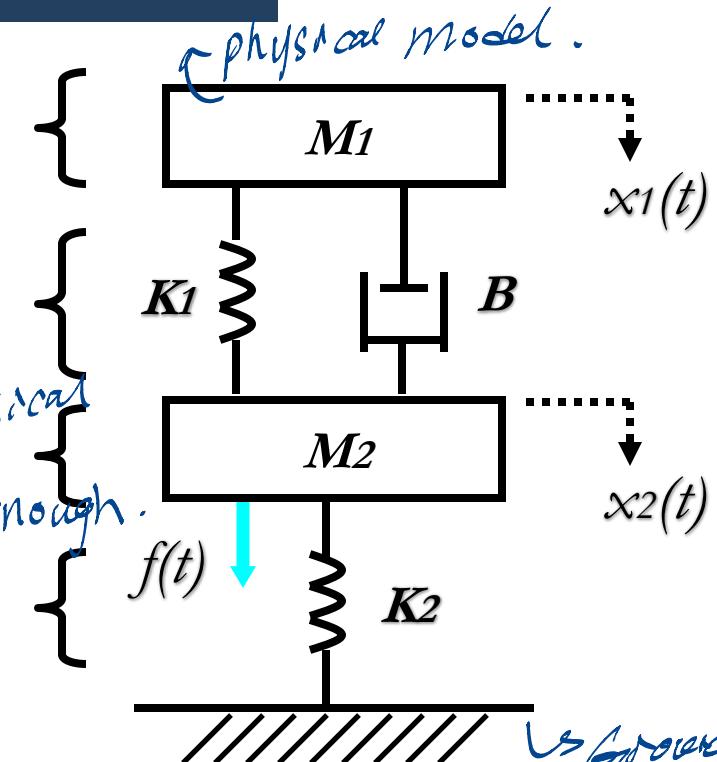
$$\left\{ -K_2x_2 + f(t) + K_1(x_1 - x_2) + B(\dot{x}_1 - \dot{x}_2) = M_2 \ddot{x}_2 \right. \quad (2)$$

TF (Transfer Function)

$$X_1 = \frac{Bs + K_1}{M_1 s^2 + Bs + K_1} X_2 \quad G_1$$

$$X_2 = \frac{1}{M_2 s^2 + Bs + K_1 + K_2} F(s) \quad G_2$$

$$F(s) + \frac{Bs + K_1}{M_2 s^2 + Bs + K_1 + K_2} X_1 \quad G_3$$



**Model:** A “representation” of a system

## Types of Models:

- Physical Models (Prototypes)
- Analytical Models
- Computer (Numerical) Models (Data Tables, Curves, Programs, Files, etc.)
- Experimental Models (use input/output experimental data for model “identification”)

**Dynamic System:** Response variables are functions of time, with non-negligible rates of changes about an operating condition.

Note the implication of “approximation” in modeling

**Universal Model** (which considers all aspects of the system) is **unrealistic**

E.g.: An automobile model that represents ride quality, energy consumption, traction characteristics, handling, structural strength, capacity, load characteristics, cost, safety, control, etc. can be complex and impractical

A model may address a few specific aspects of interest/application

Model should be as simple as possible (Approximate modeling, model reduction, etc. are applicable here)

- Modern, high-capacity, high-speed computers can accommodate complex analytical models
- Models can be modified quickly, and conveniently, at low cost
- High flexibility of making structural and parametric changes
- Naturally amenable to computer simulation
- Can be integrated with computer/numerical/experimental/ hardware models
- Can be done well before a prototype is built (and can be instrumental in deciding whether to prototype)

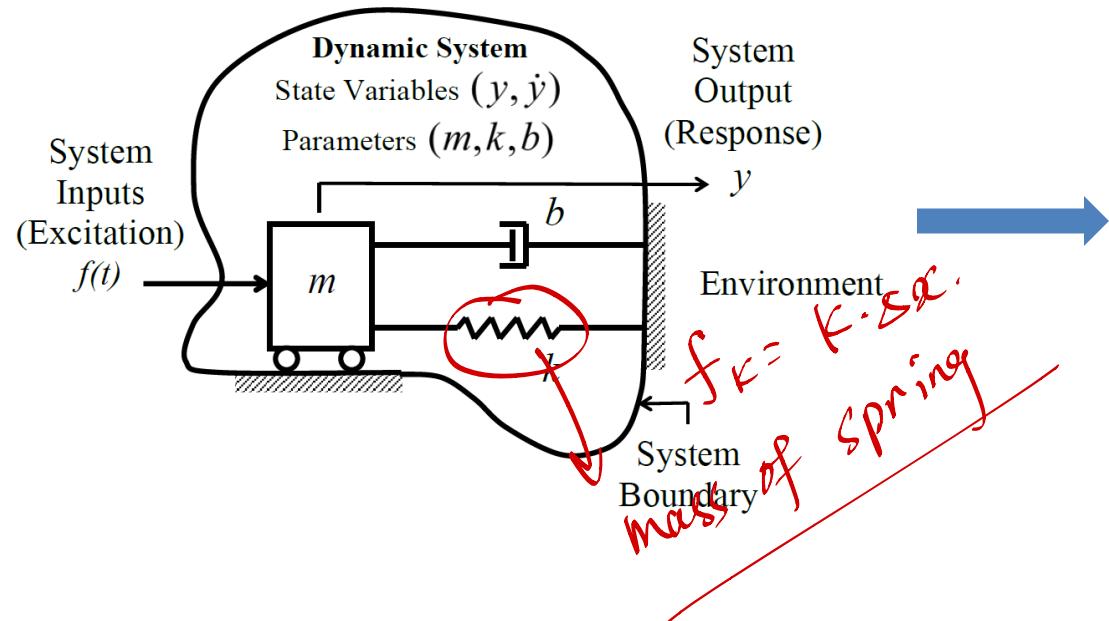
**Note:** Can't be easily understand and modified (an abstraction of the physical system)

TF      State-Space model      → Graphical Model.

- **Nonlinear:** Nonlinear differential equations (**principle of superposition** does not hold)
- **Linear:** Linear differential equations (principle of superposition holds)
- **Distributed (Continuous)-parameter:** Partial differential equations (Dependent variables are functions of time and space)
- **Lumped-parameter:** Ordinary differential equations (Dependent variables are functions of time, not space)
- **Time-varying (Non-stationary, Non-autonomous):** Differential equations with time-varying coefficients (Model parameters vary with time)
- **Time-invariant (Stationary, Autonomous):** Differential equations with constant coefficients (Model parameters are constant)
- **Random (Stochastic):** Stochastic differential equations (Variables and/or parameters are governed by probability distributions)
- **Deterministic:** Non-stochastic differential equations (not governed by probabilities—repeat test under same conditions → same results)
- **Continuous-time:** Differential equations (Time variable is **continuously** defined)
- **Discrete-time:** Difference equations (Time variable is defined as **discrete** values at a sequence of time points)

# An Example

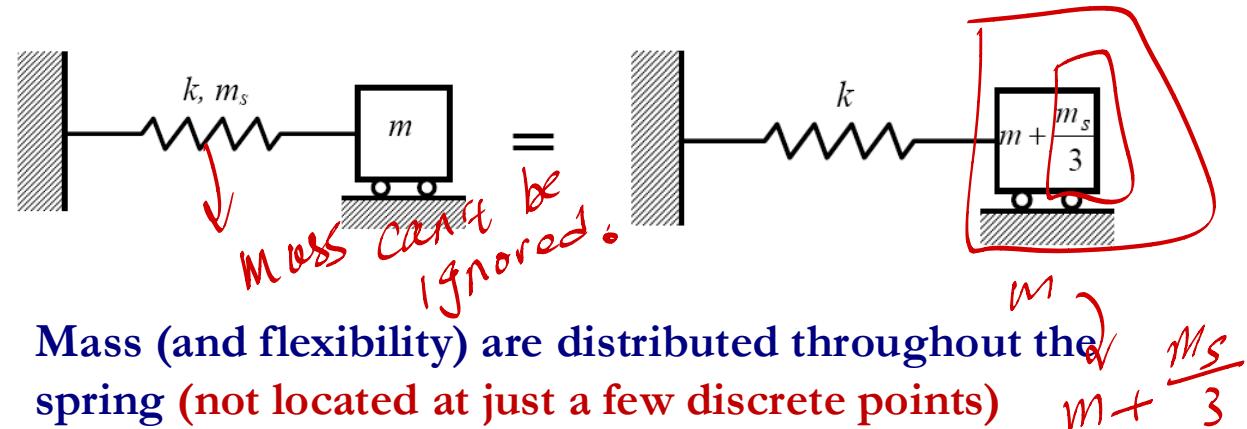
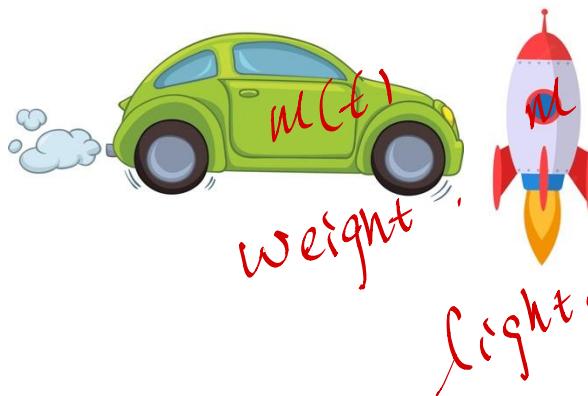
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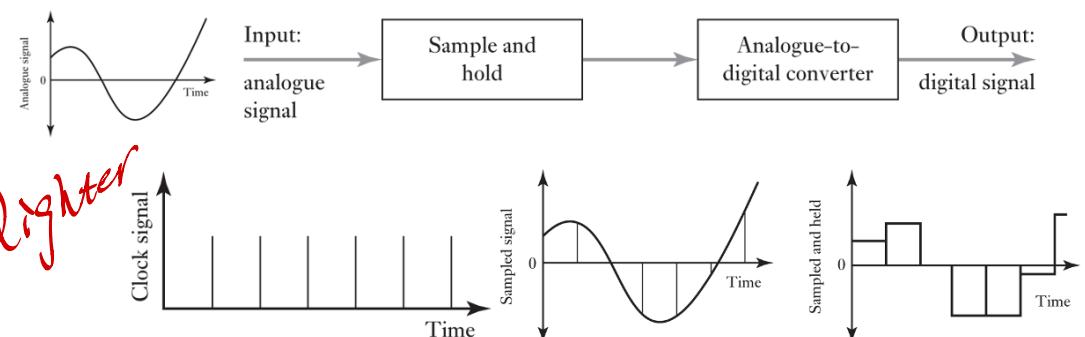
$$m\ddot{y} + b\dot{y} + ky = f(t)$$

Nonlinear Damping

$$m\ddot{y} + c(\dot{y})\dot{y}^2 + ky = f(t)$$



## Analog to Digital Conversion



## Differential Equations to Difference Equations

- Modeling: It is a representation of a dynamic system
  - Useful in analysis, simulation, design, modification, control/operation, and evaluation/testing (e.g., qualification)
  - An engineering (e.g., Mechatronic) physical system consists of a mixture of different types of components
  - An engineering (e.g., Mechatronic) system is typically a multi-domain (or multi-physics or mixed) system
- DC Motor. { mechanical  
electrical . }*
- systematic modeling tools*

Next:

- Integrated: All domains are modeled together (concurrent)
- Unified: Use analogous procedures to model all components (in analytical modeling)
- Analogies exist in mechanical, electrical, fluid, and thermal systems

