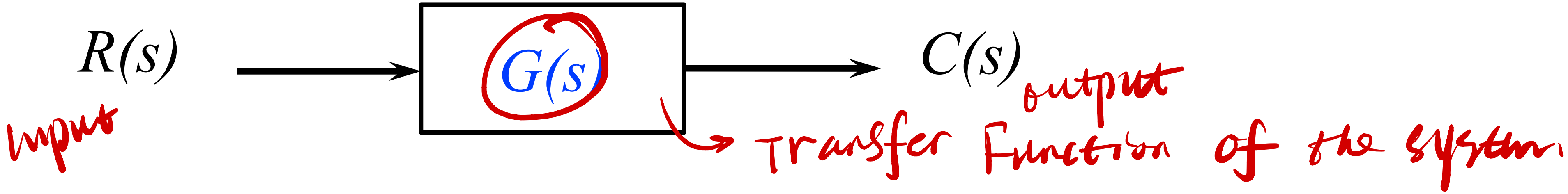
GRASP Laboratory
General Robotics & Autonomous Systems and Processes

Mechatronic Modeling and Design with Applications in Robotics

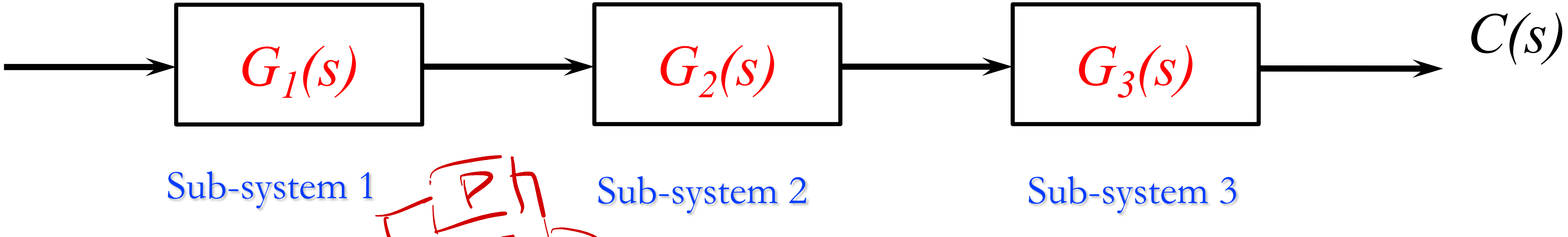
Graphical Models

Block Diagram

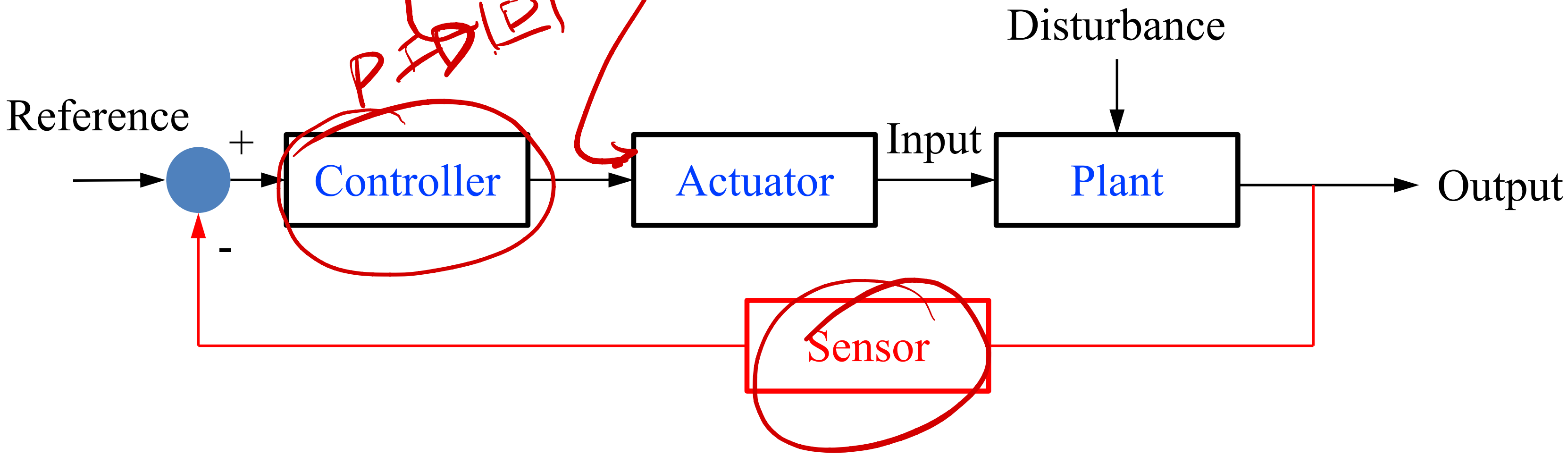
Block Diagram



Systems usually are composed of multiple subsystems:

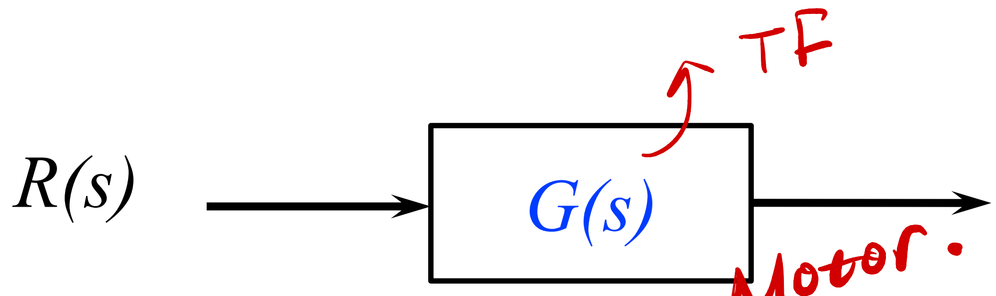


More complex control block diagram (e.g., Feedback)



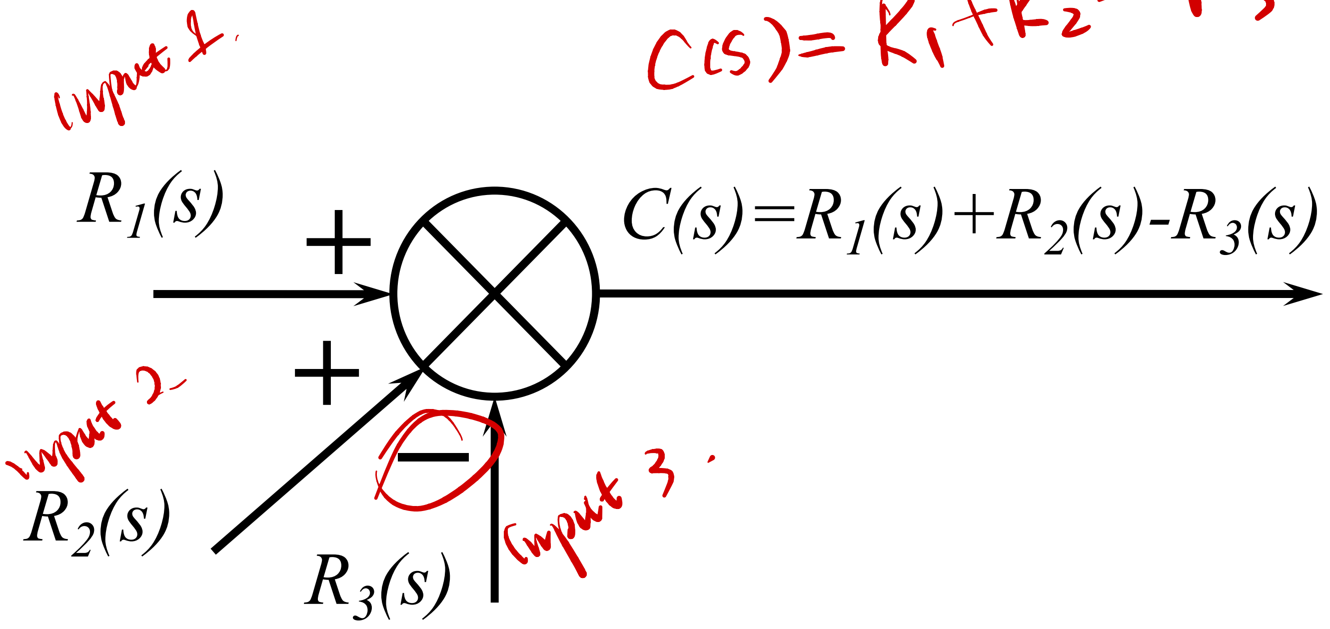
Basic Components of Block Diagrams

Signals

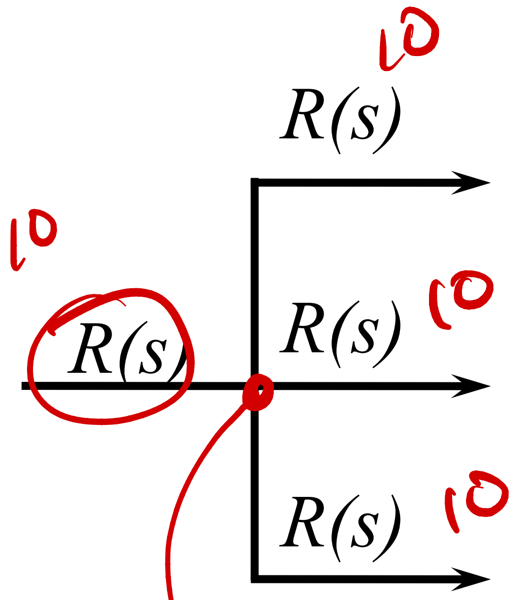


System

$C(s) = R_1 + R_2 - R_3$

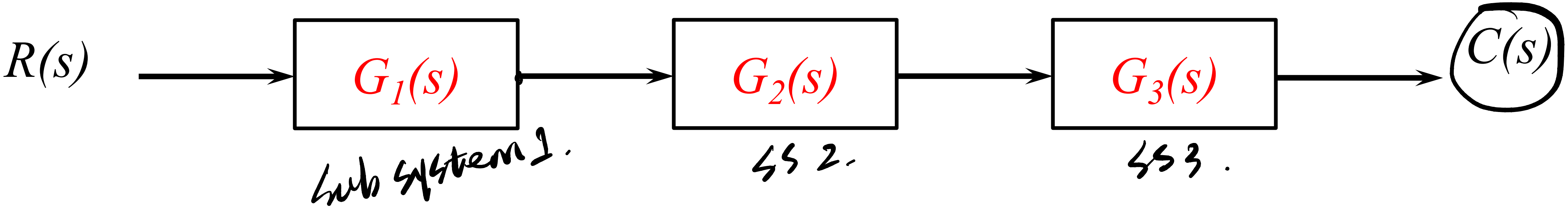


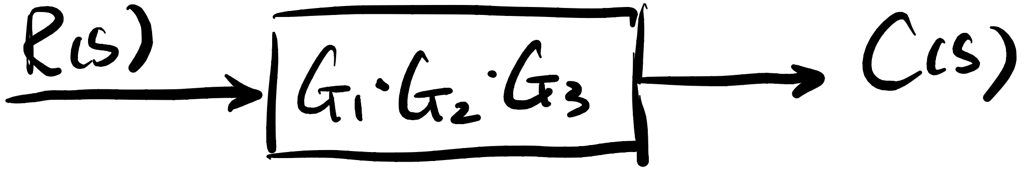
Summing Junction

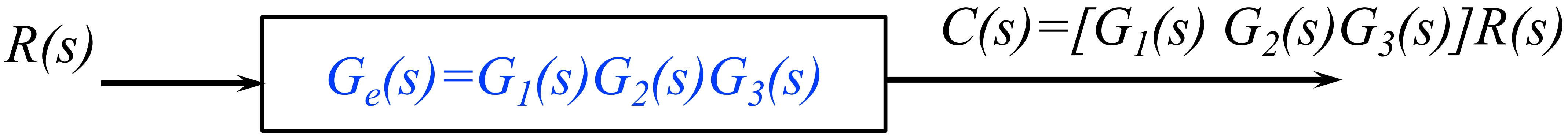


Pickoff Point

Cascade Form

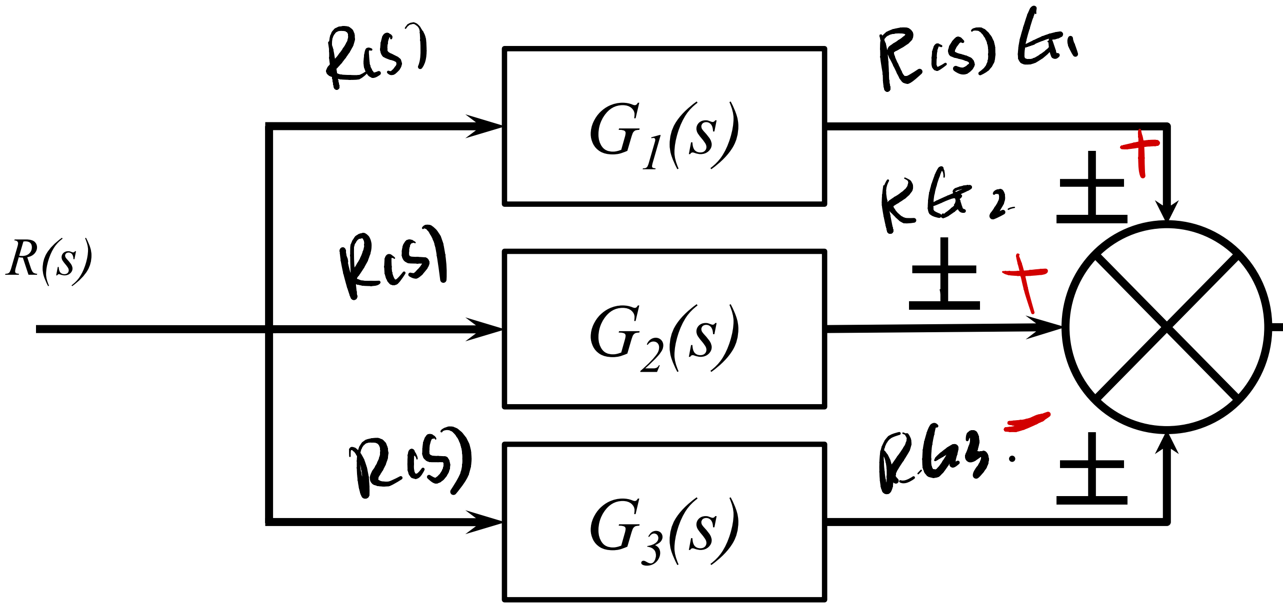






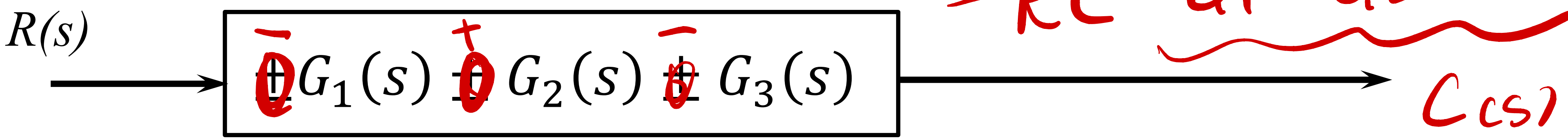
Parallel Form

$R(s)(G_1+G_2-G_3).$

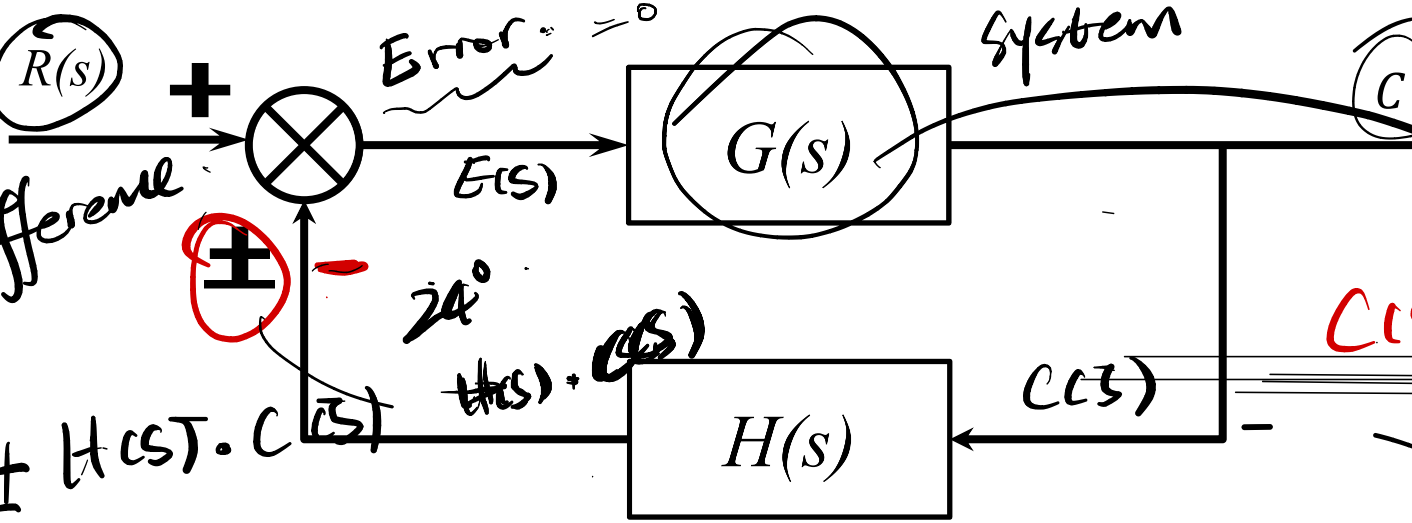


$C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]R(s)$

$C(s) = R(-G_1+G_2-G_3).$



Feedback Form: Eliminating a Feedback Loop



20° Error $=0$ system 200°

4° difference 24°

$H(s)\cdot C(s)$

$$C(s) = \frac{G(s)}{1 \mp G(s)H(s)}$$

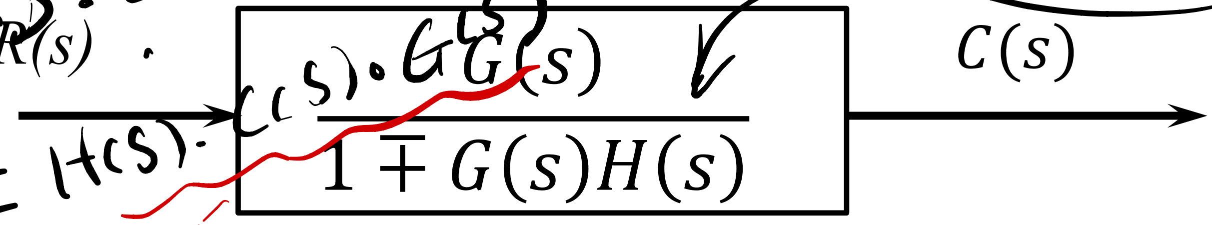
$$E(s) = R(s) \pm H(s)\cdot C(s)$$

$$C(s) = E(s)\,G(s) \quad ②$$

$$E(s) = \frac{C(s)}{G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp H(s)\cdot G(s)}$$

open loop Transfer function



$$\frac{C(s)}{G(s)} = R(s) \pm H(s)\cdot C(s)$$

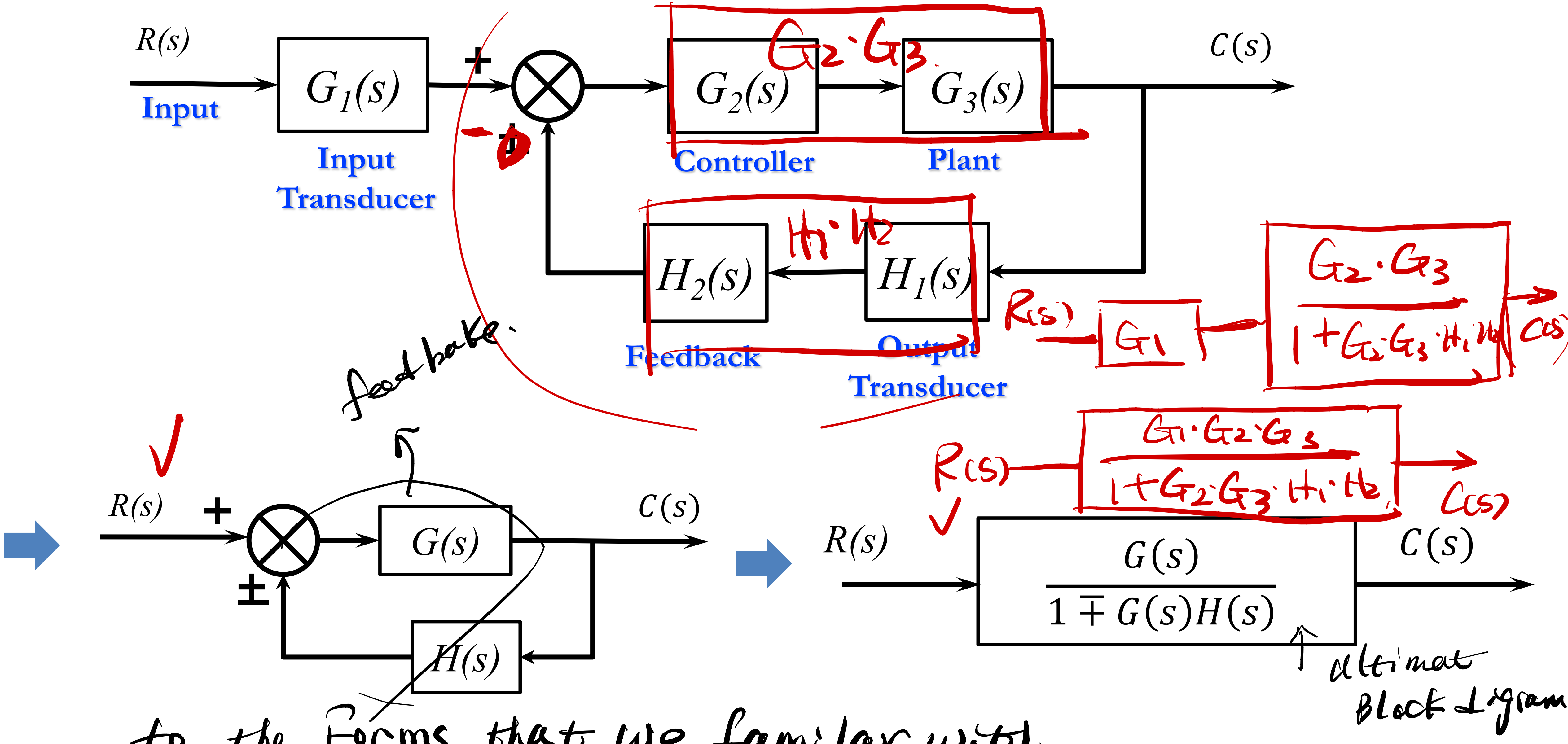
$$C(s) = G(s)\,R(s) \pm H(s)\cdot C(s)\cdot G(s)$$

$$T(s) = \frac{C(s)}{R(s)}$$

TF

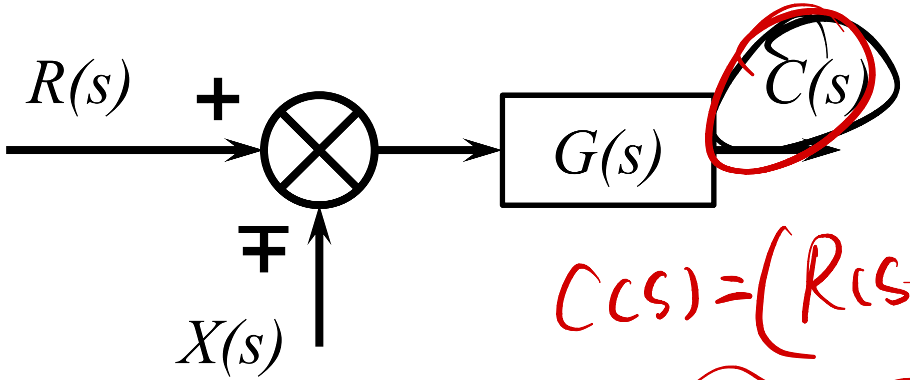
$$C(s)\left(1 \mp H(s)\cdot G(s)\right) = G(s)\,R(s)$$

Moving Blocks to Create Familiar Forms

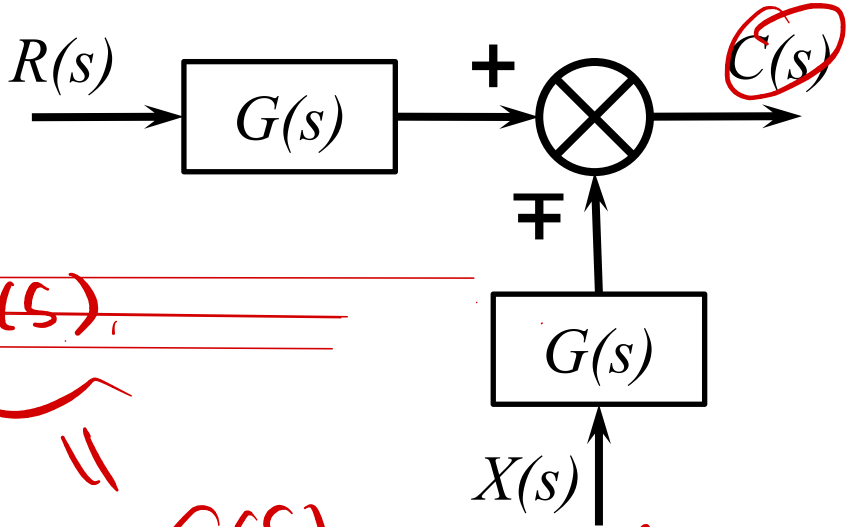


to the forms that we familar with.

Moving a Summing Junction



≡

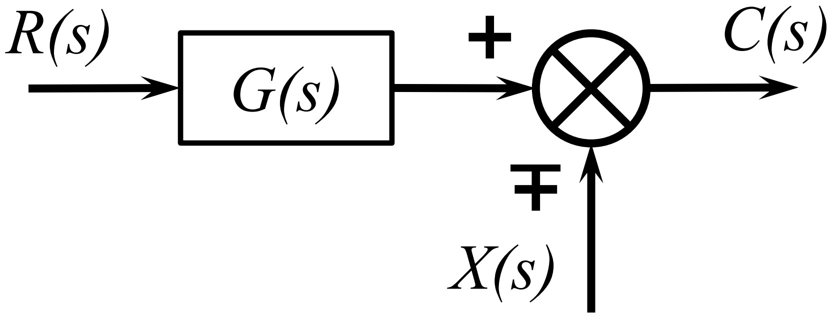


$C(s) = (R(s) \mp X(s))\, G(s)$

$\parallel$

$C(s) = R(s)\, G(s) \mp G(s)\, X(s)$

$C(s) = R(s) \cdot G(s) \mp X(s)$



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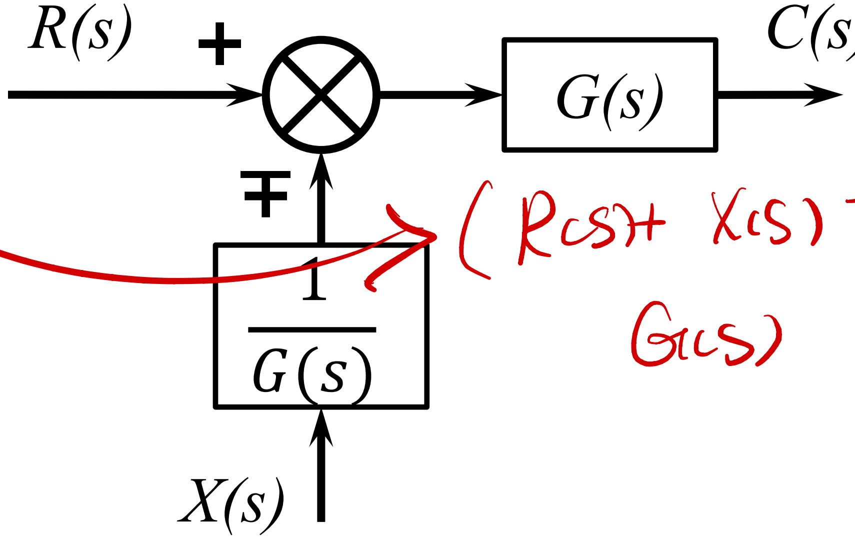
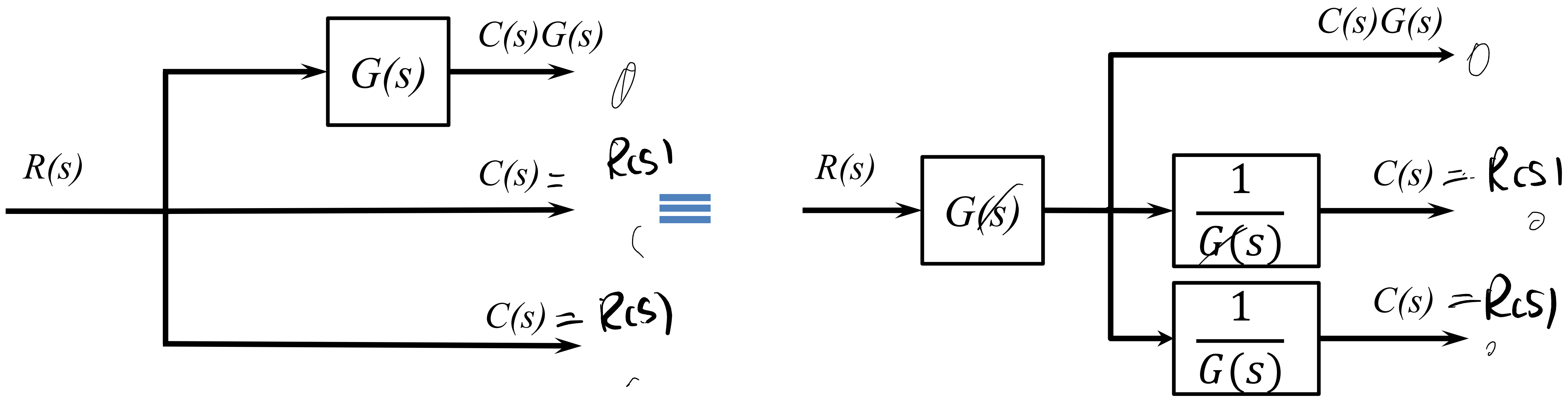
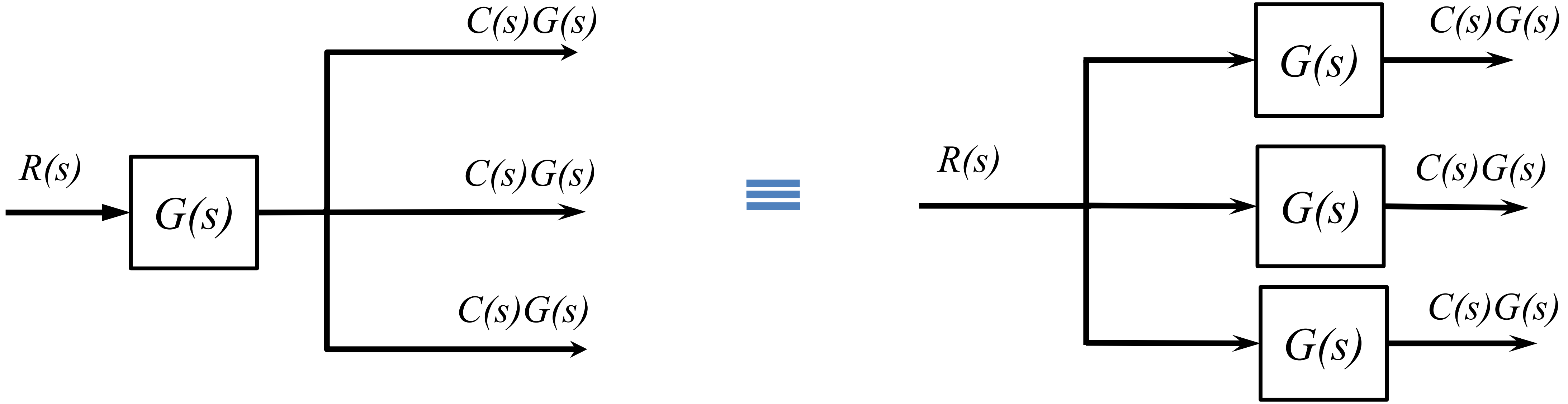


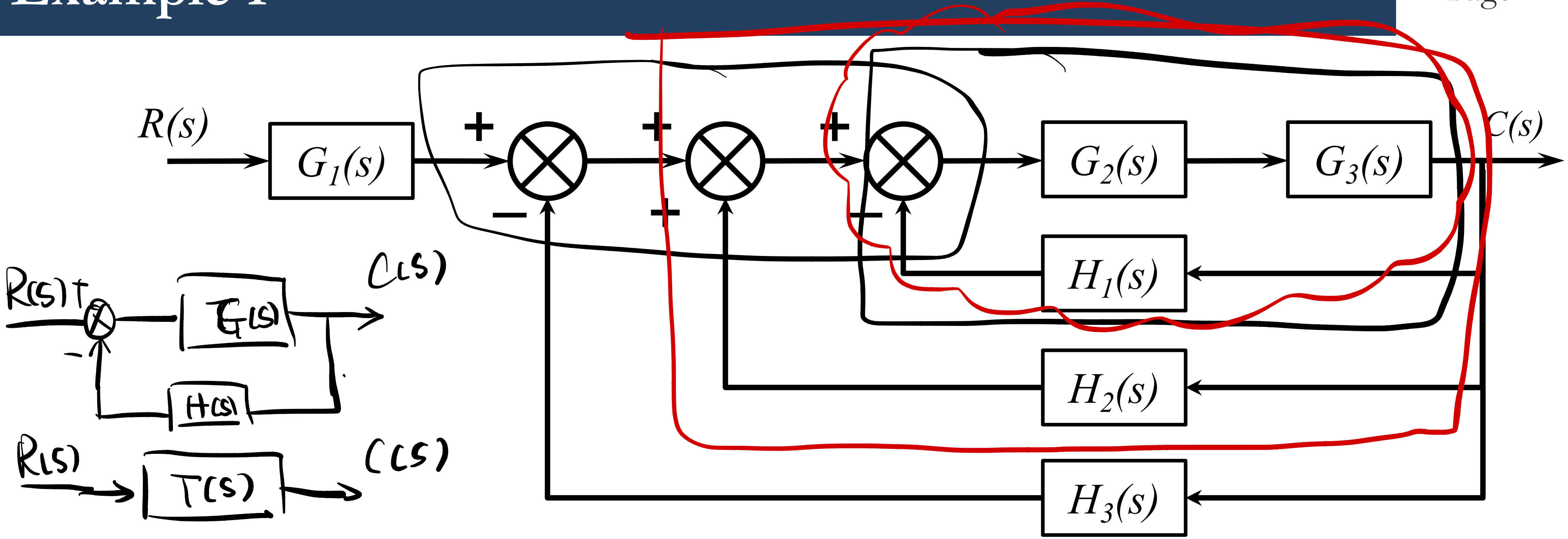
$\left(R(s) + X(s)\,\frac{1}{G(s)}\right) \cdot G(s)$

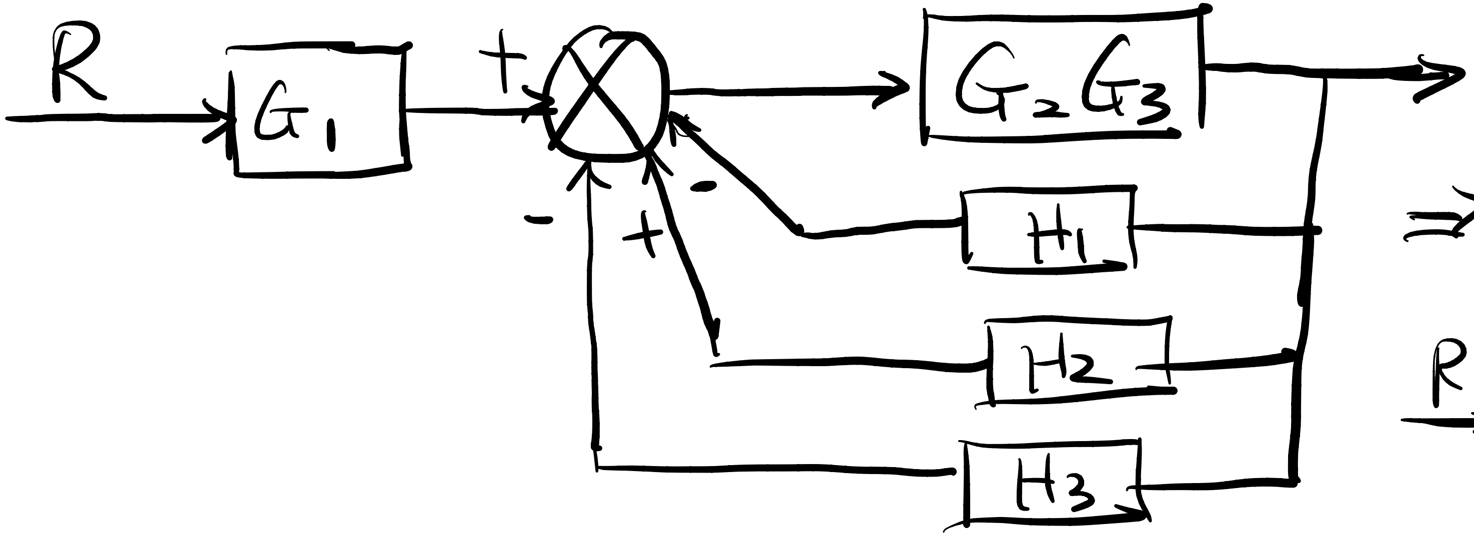
Moving a Pickoff Point

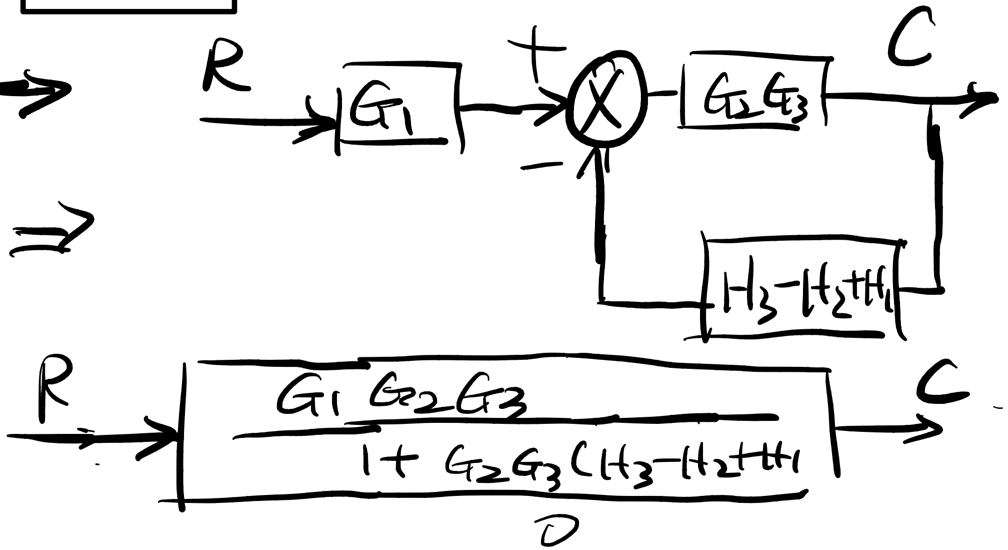




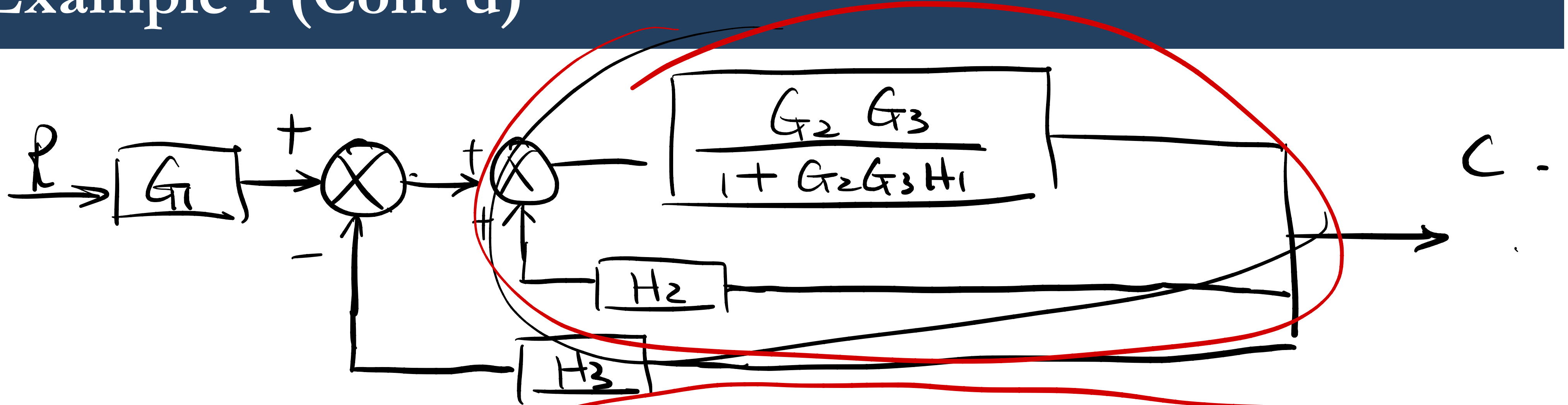
Example 1


R(s)
G1(s)
G2(s)
G3(s)
C(s)
H1(s)
H2(s)
H3(s)
R(s)
C(s)
G(s)
H(s)
R(s)
T(s)
C(s)


R
G1
G2G3
H1
H2
H3

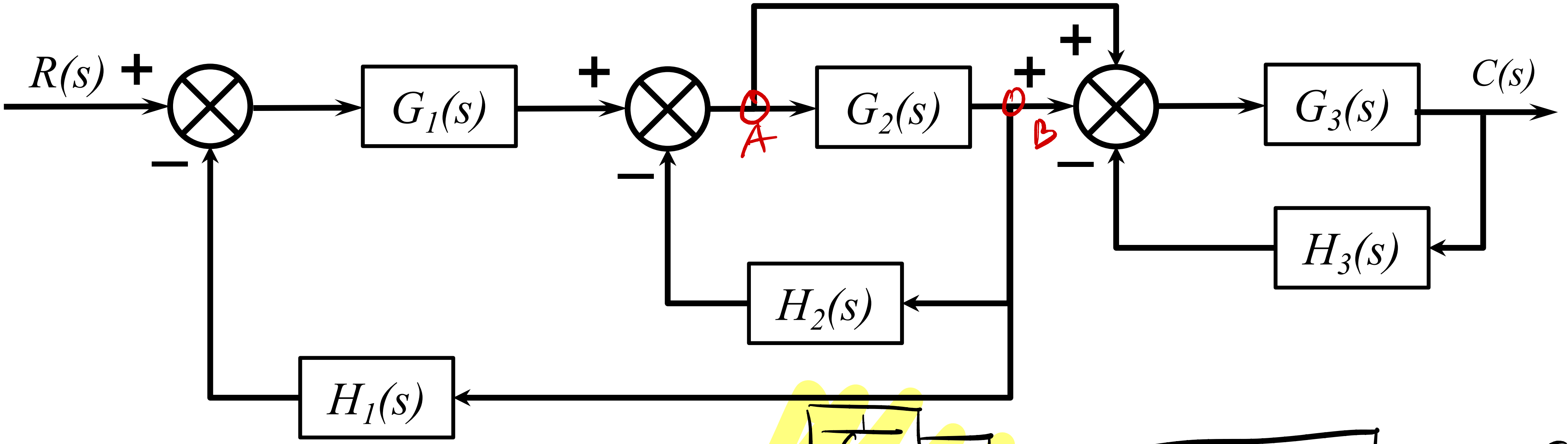

R
G1
G2G3
C
H3-H2+H1
R
G1G2G3
1+G2G3(H3-H2+H1)
C

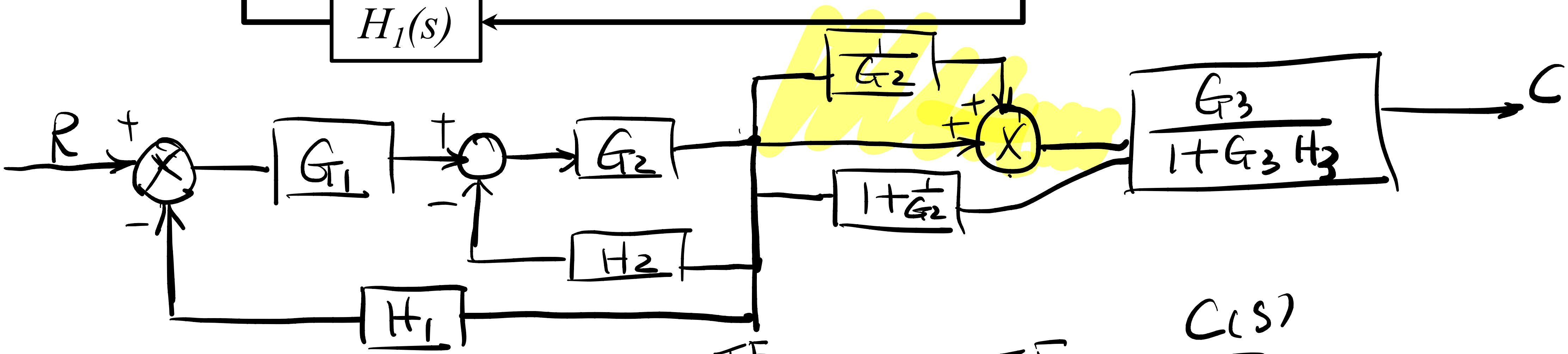
Example 1 (Cont'd)

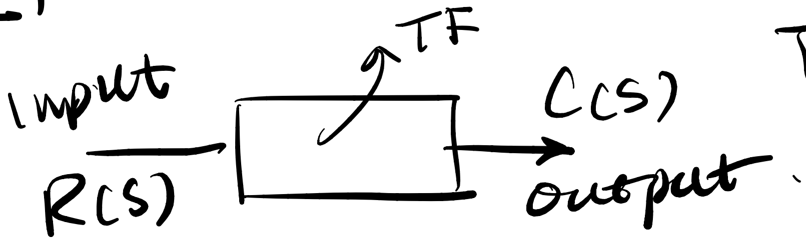


$$T(s) = \boxed{\dfrac{\dfrac{G_2 G_3}{1+G_2G_3H_1}}{1 - \dfrac{G_2G_3}{1+G_2G_3H_1} \cdot H_2}}$$

Example 2







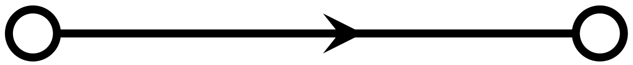
$$TF = \frac{C(s)}{R(s)}.$$

Example 2 (Cont’d)

Signal Flow Graph

Definition

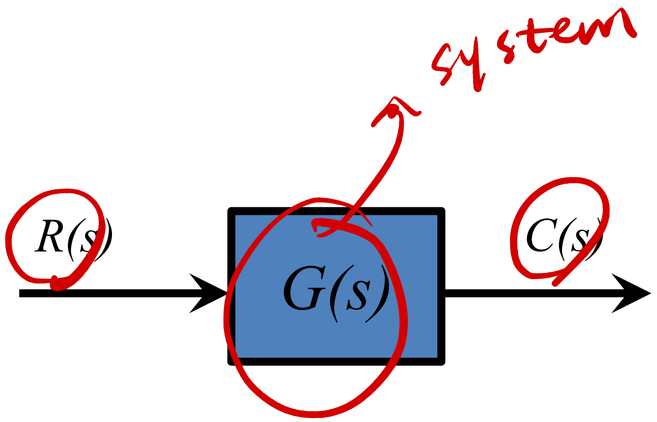
A system is represented by a line with an arrow showing the direction of signal flow through the system.

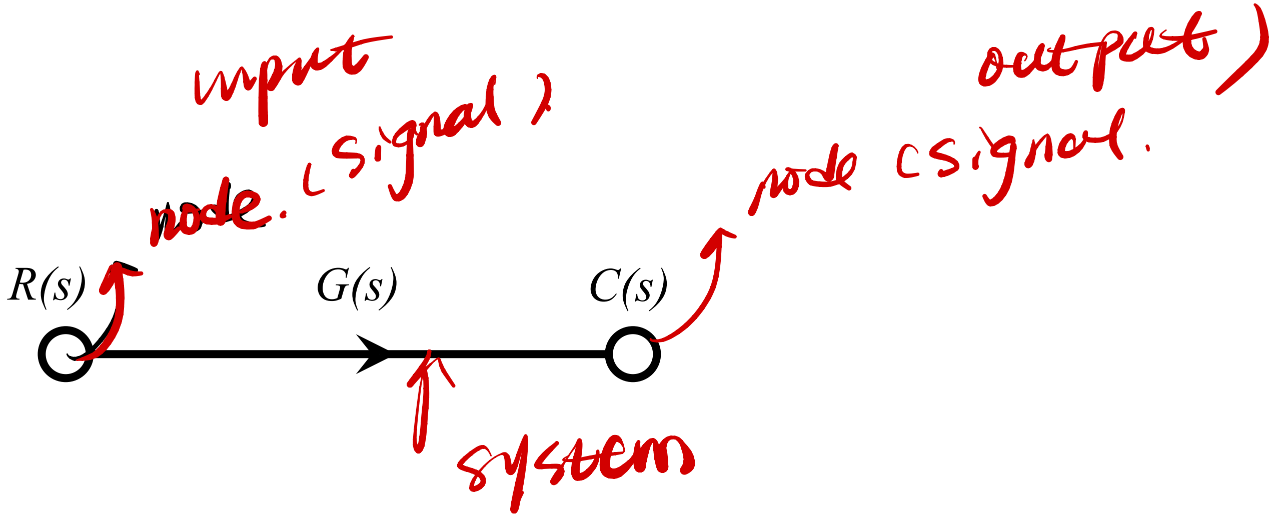


A signal-glow graph consists only **branches** and **nodes**:

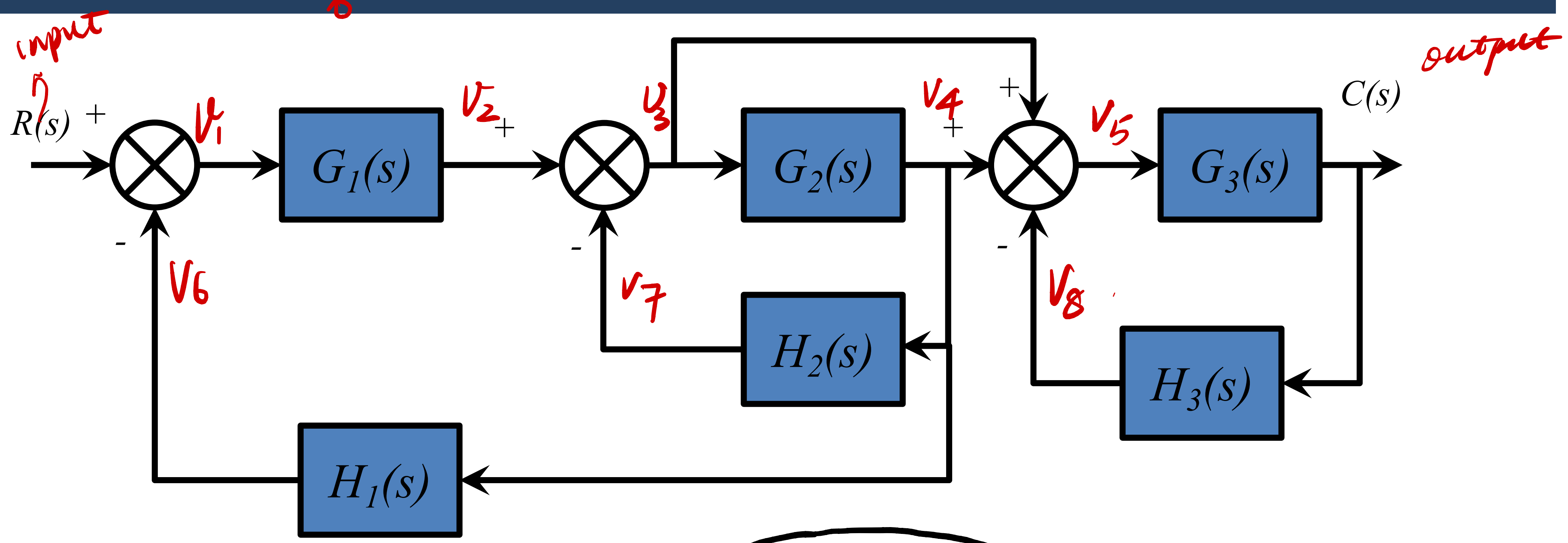
Branches: represent **systems**

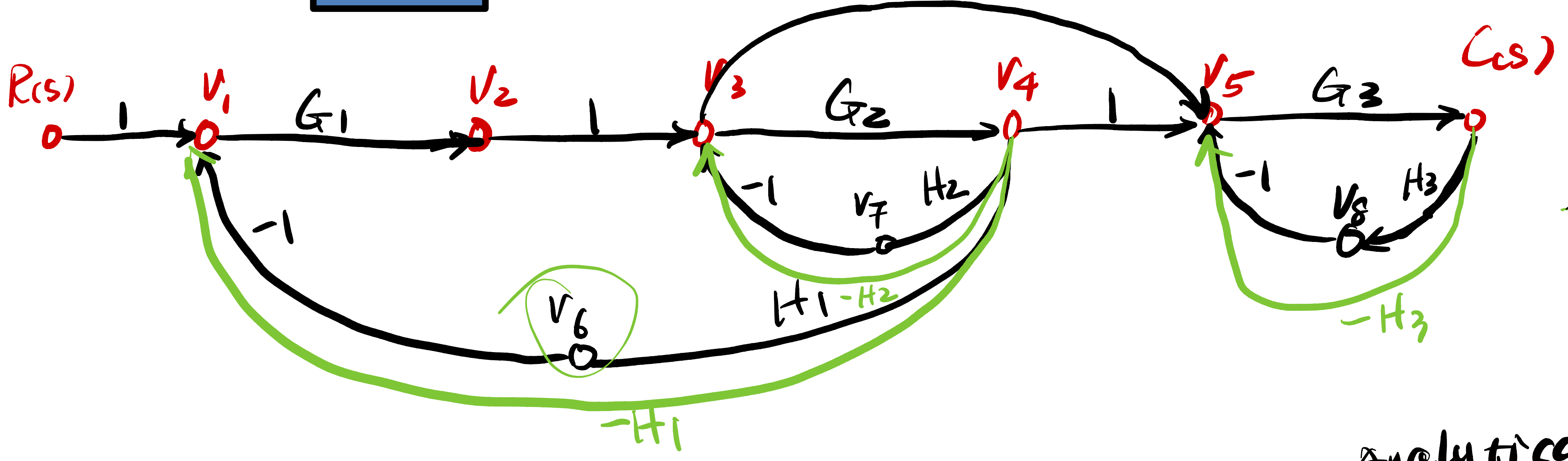
Nodes: represent **signals**





Example





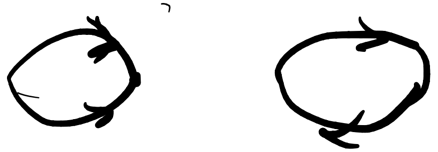
$$TF = \frac{C(s)}{R(s)}$$

Transfer function.

analytical model.

Mason's Rule: Definition

Loop Gain:



The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once.

starts: input

ends: output

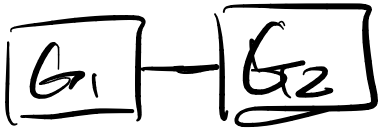
Forward-path Gain:

The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

Non-touching Loops:

Loops that do not have any nodes in common.

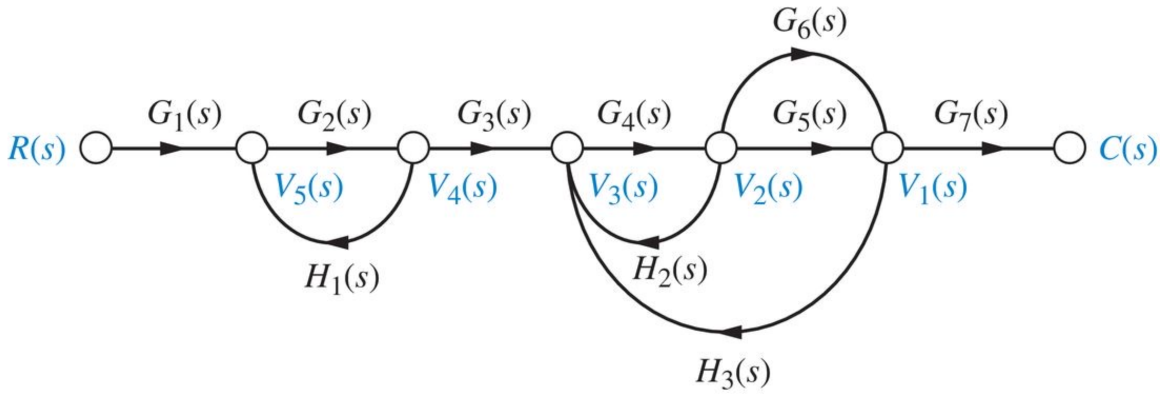
Non-Touching-Loop Gain:



$G_1 G_2$

The product of loop gains from non-touching loops taken two, three four, or more at a time

Example



Loop Gain:

$G_2 \cdot H_1$

$G_4 \cdot H_2$

$G_4 \cdot G_5 \cdot H_3$

$G_4 \cdot G_6 \cdot H_3$

Forward-path Gain:

$G_1 G_2 G_3 G_4 G_5 G_7$

$G_1 G_2 G_3 G_4 G_6 G_7$

Non-touching Loops:

1 $G_2 H_1$ 2 $G_4 H_2$ 3 $G_4 G_5 H_3$ 4 $G_4 G_6 H_3$.

Non-Touching-Loop Gain: $[G_2 H_1][G_4 H_2]$ $[G_2 H_1][G_4 G_6 H_3]$

$[G_2 H_1][G_4 \cdot G_5 \cdot H_3]$

Mason’s Rule

TF.

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

k = number of forward paths

T_k = the kth forward-path gain

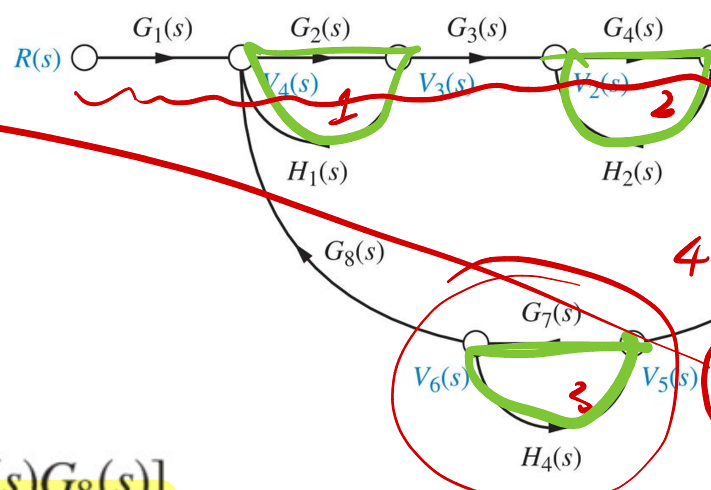
Δ = (1- Σ loop gains + Σ non-touching loop gains taken two at a time – Σ non-touching loop grains taken three at a time + Σ non-touching loop gains taken four at a time …)

Δ_k = Δ – Σ loop gain terms in Δ that touch the kth forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the kth forward path.

Example

1. forward path

Find the transfer function, $C(s)/R(s)$ for the signal-flow-graph:



$G_7 H_4$

Forward path gain

$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta}$$

$$\begin{aligned}\Delta = 1 -&[G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) \\ &+ G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ &+[G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ &+ G_4(s)H_2(s)G_7(s)H_4(s)] \\ &-[G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned}$$

non touching gain (taken three)
$G_2H_1\, G_4H_2\, G_7H_4$ (loop 1, 2, 3)

Forward path gain: $G_1G_2G_3G_4G_5$

loop gain: $G_2 \cdot H_1$, $G_4 \cdot H_2$, $G_7 G_4$, $G_2G_3G_4G_5G_6G_7G_8$

non touching loop gain (taken two) $G_2H_1\, G_4H_2$ (Loop 1, 2) $G_2H_1\, G_7H_4$ (loop 1, 3)

$G_4H_2\, G_7H_4$ (loop 2, 3)

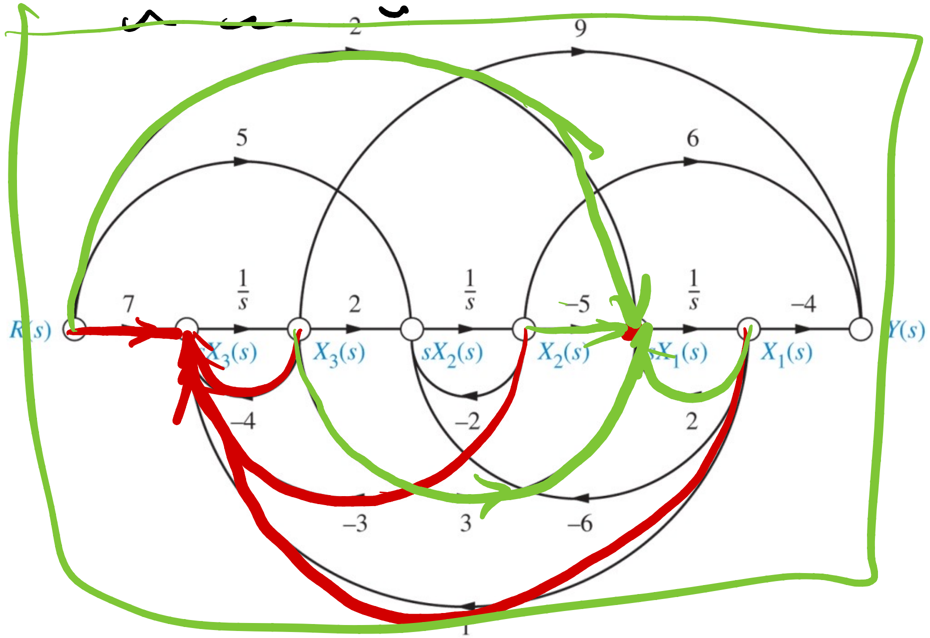
Consider the following state and output equations:

$$
\begin{cases}
\dot{x}_1 = 2x_1 - 5x_2 + 3x_3 + 2r \\
\dot{x}_2 = -6x_1 - 2x_2 + 2x_3 + 5r \\
\dot{x}_3 = x_1 - 3x_2 - 4x_3 + 7r \\
\quad y = -4x_1 + 6x_2 + 9x_3
\end{cases}
$$

$\dot{x}_3 \xrightarrow{\frac{1}{s}} x_3 \qquad \dot{x}_2 \xrightarrow{\frac{1}{s}} x_2 \qquad \dot{x}_1 \xrightarrow{\frac{1}{s}} x_1 \quad 2$

where r is the input, y is the output, x_1, x_2 and x_3 are the state variables, please draw its signal-flow graph.

$$
\begin{cases}
\dot{x} = Ax + Bu \\
y = Cx + Du
\end{cases}
$$



The End!!