板壳单元综合调研

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Outline

- 板壳位移元简述
- 板壳单元基本构造思路
- 两个问题及其解决方案
  - 有限元试函数的构造
  - θ_z 向抗扭刚度缺失
- 讨论

板壳位移元简述

- 基本理论
  - 建立途径
    - 3D弹性力学方程→构造内力分量
    - 构造能量泛函→泛函极值问题
  - 板壳理论基本假设：平截面假设

$$\begin{cases} 薄板\ h \ll l(h/l < 1/5) \begin{cases} 小挠度理论\ w \ll h \rightarrow 中面无伸缩剪切 \\ 大挠度理论\ w \sim h \ll l \rightarrow 中面有伸缩剪切 \end{cases} \\ 厚板\ h < l(1/5 < h/l < 1) \rightarrow 直法线假设\times，无挤压、等挠度假设\surd \\ 薄壳\ h \ll l(h/l < 1/20) \left. \begin{cases} 二维壳:扁壳/深壳 \\ 一维壳:轴对称壳 \end{cases} \right\} \rightarrow 中面有伸缩剪切\ [弯曲应变？] \end{cases}$$

板壳位移元简述

- 板的控制方程
  - 混合形式：$\begin{cases} L^T DL\boldsymbol{\theta}+\boldsymbol{S}=0 \\ \nabla\boldsymbol{S}+\overline{q}=0 \\ \boldsymbol{S}=\alpha(\nabla w-\boldsymbol{\theta}) \end{cases}$ $L=\begin{pmatrix} \partial_x & & \partial_y \\ & \partial_y & \partial_x \end{pmatrix}^T$ $\alpha=\kappa Gt$

    Mindliln板单元及其各种形式

    $\begin{cases}\text{进化}\\ \text{超进化}\\ \text{究极进化}\end{cases}$

    (略*3)

  - 不可约形式：引入薄板假设 $\nabla w-\boldsymbol{\theta}=0$，有

    Kirchhoff 板

    (略*3)

    $$(L\nabla)^T DL\nabla w-\overline{q}=0$$

板壳位移元简述

- 壳的控制方程 (略*3)
  - 平板壳单元
    - 一般壳：三角形板
    - 圆柱壳：矩形板
  - 轴对称壳 → 截锥单元
    - 二维降为一维
  - 扁壳 → 扁壳单元
    - 流动坐标系近似正交坐标系
    - 深壳：单元尺寸非常小时也适用
  - “厚壳” → 退化型壳单元
    - 各种闭锁：剪切闭锁、薄膜闭锁

板壳单元基本构造思路

- 有限元格式构造方法Ⅰ-能量原理
  - 协调位移元——最小势能原理
  - 非协调位移元——分区势能原理
  - 广义协调位移元 (eneralized conforming element)

    →分区势能原理的退化形式
  - 应力杂交元（采用应力试函数，满足平衡微分方程）

    →最小余能原理
  - 混合元（采用混合试函数，含位移、应力和应变）

    →广义变分原理
  - 分区混合元（试函数：部分单元采用位移，其余采用应力）

    →分区混合能量原理
- 有限元格式构造方法Ⅱ-控制方程的弱形式

龙驭球，龙志飞，岑松. 新型有限元论. 北京：清华大学出版社，2004.

两个问题

- 试函数选择：如何保证**收敛性**？
  - 可约形式各变量直接独立插值：完备性
    - → 各种闭锁：选择性缩减积分——零能模态？
    - → 一致插值、**假设剪应变√**
  - 通过人工近似缩减为不可约形式：协调性
    - → 广义协调元！
- θ_z 方向的刚度缺失：板/平板壳单元疑难
    - → 含旋转自由度的膜单元！

龙驭球，龙志飞，岑松. 新型有限元论. 北京：清华大学出版社，2004.
Ibrahimberovic A., et al. A robust quadrilateral membrane finite element with drilling degrees of freedom. IJNME, VOL. 30,445-457 (1990)
Kugler S., et al. A highly efficient membrane finite element with drilling degrees of freedom. Acta Mech 213, 323–348 (2010)

广义协调元

$$\Pi_{mp} = \sum_{e} (\Pi_p^e + H_p^e)$$

$$H_p^e = \int_{\Gamma^e} \boldsymbol{p}^T (\boldsymbol{u} - \boldsymbol{u}_a) \mathrm{d}\Gamma =: \int_{\Gamma^e} \boldsymbol{p}^T \boldsymbol{u}_\lambda \mathrm{d}\Gamma$$

$$\lim_{h \to 0} H_p^e = 0$$

龙驭球，龙志飞，岑松. 新型有限元论. 北京：清华大学出版社，2004.

含旋转自由度的膜单元

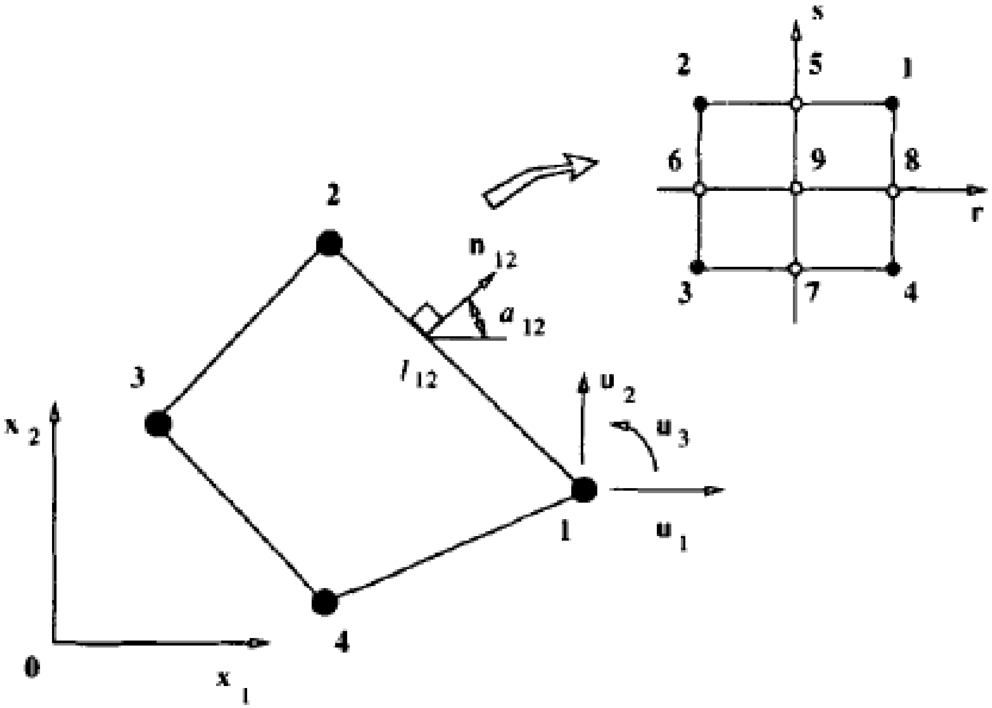

Figure 1. A quadrilateral element with drilling degrees of freedom

$$u_3 \equiv \psi^h = \sum \sum^{4} N_I^e(r,s)\,\psi_I$$

$$\text{symm}\,\nabla\mathbf{u}^e = \mathbf{B}_I^e\mathbf{u}_I + \mathbf{G}_I^e\psi_I$$

$$\text{skew}\,\nabla\mathbf{u}^e - \psi^e = \mathbf{b}_I^e\mathbf{u}_I + g_I^e\psi_I$$

$$\mathbf{h}^e = \int_{\Omega^e} \langle \mathbf{b}^e; \mathbf{g}^e \rangle^{\mathrm{T}}\, \mathrm{d}\Omega$$

$$\begin{bmatrix} \mathbf{K}^e & \mathbf{h}^e \\ \mathbf{h}^{e\,\mathrm{T}} & -\gamma^{-1}\Omega^e \end{bmatrix} \begin{Bmatrix} \mathbf{a} \\ \tau_0^e \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ 0 \end{Bmatrix}; \quad \mathbf{a} = \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\psi} \end{Bmatrix}$$

$$\hat{\mathbf{K}}^e\mathbf{a} = \mathbf{f}; \quad \hat{\mathbf{K}}^e = \mathbf{K}^e + \frac{\gamma}{\Omega^e}\mathbf{h}^e\,\mathbf{h}^{e\,\mathrm{T}}$$

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~~讨论：近期工作计划~~

感谢大家的聆听！

讨论：近期工作计划

- 1. 前处理：材料性质输入
- 2. 后处理：六面体
  - 哪些应力分量？
  - “最大”应力？

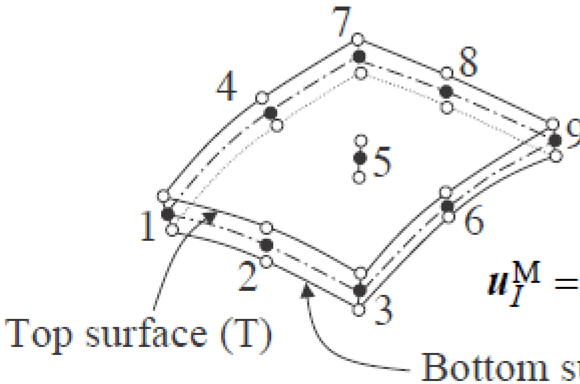
- Kirchoff plate
  - C_1 trial functions — difficult to construct
  - Only valid for thin plates
- Mindlin plate
  - C_0 trial functions
  - The interpolation of rotation and deflection are done independently
  - Shear locking
    - Consistent interpolation
    - Selective reduced integration
    - Assumed shear strain

5.5 板单元

- Triangular elements ?
  - Nine d.o.f. are required: three d.o.f. per node
  - A complete cubic polynomial has 10 terms !
    - Omit one term from x^3, x^2y, xy^2 and y^3 ? Lack geometric isotropy !
    - Combine two cubic terms: $x^2y + xy^2$? Produce singular matrix !
    - Supplemented by lateral displacement at the centroid ? Fail to converge !
  - Employ a complete quartic (15 terms) ?
    - Nine d.o.f. by vertex d.o.f. Unfavorable arrangement of nodal dof !
    - $w_{,xy}$ and normal rotations at midsides
- Discrete Kirchhoff (DK) elements
  - Independent fields for lateral displacement and for rotational of a midsuface-normal line
  - Enforce zero transverse shear strain at selected locations
  - DKT — Discrete Kirchhoff Triangle
  - DKQ — Discrete Kirchhoff Quadrilateral

5.5 壳单元 Shell element

Nine-node quadrilateral shell element



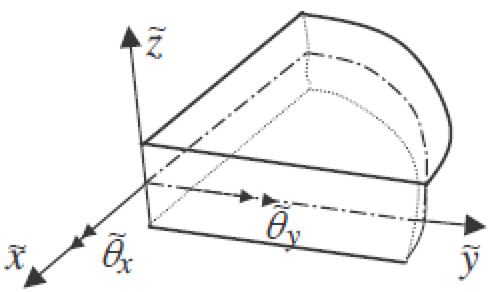
- ● Primary nodes (midplane)
- ○ Secondary nodes

$$\boldsymbol{u}_I^{\mathrm{M}} = (\boldsymbol{u}_I^{\mathrm{B}} + \boldsymbol{u}_I^{\mathrm{T}})/2$$

$$N_I^{\mathrm{top}} = N_2^{2\mathrm{L}}(\varsigma) N_I^{9\mathrm{Q}}(\xi,\eta)$$

$$N_I^{\mathrm{bot}} = N_1^{2\mathrm{L}}(\varsigma) N_I^{9\mathrm{Q}}(\xi,\eta)$$

Boundary conditions



Clamped

$$\tilde{u}_z = \tilde{u}_y = 0$$

$$\tilde{\theta}_x = \tilde{\theta}_y = 0$$

$$\tilde{\theta}_z = 0$$

Simple supported

$$\tilde{u}_z = \tilde{u}_y = 0$$

$$\tilde{\theta}_y = 0$$

$$\tilde{\theta}_z = 0$$