8H element

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1 8H element

According to the method for advanced elements, we develop the 8H element. For details, please find the attachment of codes.
8H is a natural extension of 4Q. The displacement function is:

$$u = \alpha_1 + \alpha_2\psi + \alpha_3\eta + \alpha_4\zeta + \alpha_5\psi\eta + \alpha_6\eta\zeta + \alpha_7\psi\zeta + \alpha_8\psi\eta\zeta$$
$$v = \alpha_9 + \alpha_1 0\psi + \alpha_1 1\eta + \alpha_1 2\zeta + \alpha_1 3\psi\eta + \alpha_1 4\eta\zeta + \alpha_1 5\psi\zeta + \alpha_1 6\psi\eta\zeta$$
$$w = \alpha_1 7 + \alpha_1 8\psi + \alpha_1 9\eta + \alpha_2 0\zeta + \alpha_2 1\psi\eta + \alpha_2 2\eta\zeta + \alpha_2 3\psi\zeta + \alpha_2 4\psi\eta\zeta$$

Because 8H element has 24 degrees of freedom but the complete second order polynomial for 3 dimensions only has 10 terms, 2 terms should be eliminated. Concerning the symmetry, we eliminate 3 second-order terms and add one third-order term.

According to the characteristics of shape function, we can get:
$$N_i = \frac{1}{8}(1 + \psi_i\psi)(1 + \eta_i\eta)(1 + \zeta_i\zeta)$$
$$i = 1, 2, ..., 8$$

Then according to the geometry equations of 3D problem, we can get the strain matrix [B]:

$$B_i = \begin{pmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{pmatrix}$$

$$B = [B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8]$$

To transform the shape function into physical coordinate system, we use Jacobi matrix, which is similarly derived as 4Q. Then we have the local stiffness matrix:
$$K^e = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} B^T D B |J| d\psi d\eta d\zeta$$

2 Patch test-Single axis stretch

To test the convergence of 8H element, we first build a model of single axis stretch as Figure 1.The hexahedron is divided into 7 irregular elements. We set the load on each node(5,6,7,8) is 250, v=0.2, E=1000. Then we divide the model into 7 irregular part to process patch test.

It is easy to derive the exact displacement function should be :

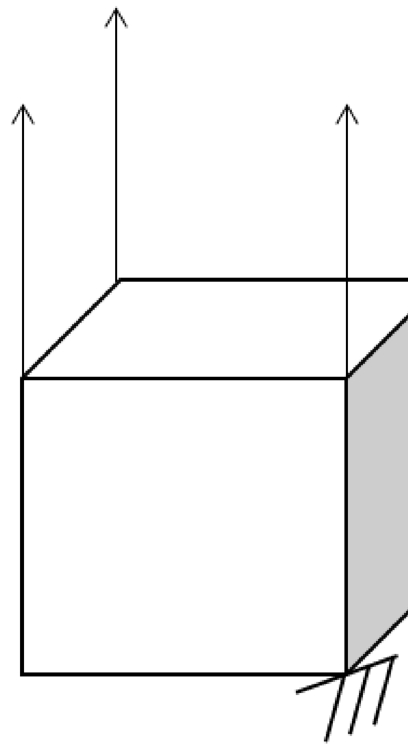

Figure 1: Single axis stretch.

$$u_x = -\nu \frac{4P}{El_z l_x} x$$
$$u_y = -\nu \frac{4P}{El_z l_y} y$$
$$u_z = \frac{4P}{El_z^2} z$$

Then we get the displacement perfectly match the exact displacement as Figure 2, which indicates it has the convergence of second order.

3 Example verification -Pure shearing

Secondly, we change the we of loading into pure shearing as Figure 3, as an example verification.

Then we get the displacement as Figure 4 which also matches the first order field correctly. So the 8H element also passes the pure shearing example analysis.
For details please find the 8Hpatch.out and 8Hshear.out in attachment.

D I S P L A C E M E N T S

NODE	X-DISPLACEMENT	Y-DISPLACEMENT	Z-DISPLACEMENT
1	-4.00000e-002	0.00000e+000	0.00000e+000
2	-4.00000e-002	-4.00000e-002	0.00000e+000
3	2.08167e-017	-4.00000e-002	0.00000e+000
4	0.00000e+000	0.00000e+000	0.00000e+000
5	-4.00000e-002	-9.02056e-017	2.00000e-001
6	-4.00000e-002	-4.00000e-002	2.00000e-001
7	-1.45717e-016	-4.00000e-002	2.00000e-001
8	-2.05630e-016	-4.85723e-017	2.00000e-001
9	-2.40000e-002	-8.00000e-003	4.00000e-002
10	-2.40000e-002	-2.40000e-002	8.00000e-002
11	-8.00000e-003	-2.40000e-002	4.00000e-002
12	-8.00000e-003	-8.00000e-003	4.00000e-002
13	-2.40000e-002	-8.00000e-003	1.20000e-001
14	-2.40000e-002	-2.40000e-002	1.20000e-001
15	-8.00000e-003	-2.40000e-002	1.20000e-001
16	-8.00000e-003	-8.00000e-003	1.20000e-001

Figure 2: Single axis displacement.

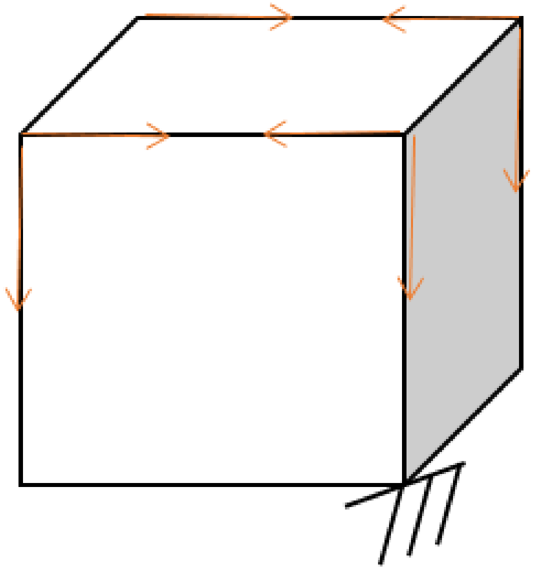

Figure 3: Pure shearing.

D I S P L A C E M E N T S

NODE	X-DISPLACEMENT	Y-DISPLACEMENT	Z-DISPLACEMENT
1	0.00000e+000	0.00000e+000	0.00000e+000
2	0.00000e+000	0.00000e+000	0.00000e+000
3	0.00000e+000	0.00000e+000	0.00000e+000
4	0.00000e+000	0.00000e+000	0.00000e+000
5	0.00000e+000	2.40000e+000	4.99600e-016
6	0.00000e+000	2.40000e+000	-5.37672e-016
7	-3.33067e-016	2.40000e+000	-2.70884e-016
8	-1.11022e-016	2.40000e+000	-1.01581e-016

Figure 4: Pure shearing displacement.