# 8H element

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## 1 8H element

According to the method for advanced elements, we develop the 8H element. For details, please find the attachment of codes.
8H is a natural extension of 4Q. The displacement function is:

$$u = \alpha_1 + \alpha_2\psi + \alpha_3\eta + \alpha_4\zeta + \alpha_5\psi\eta + \alpha_6\eta\zeta + \alpha_7\psi\zeta + \alpha_8\psi\eta\zeta$$
$$v = \alpha_9 + \alpha_10\psi + \alpha_11\eta + \alpha_12\zeta + \alpha_13\psi\eta + \alpha_14\eta\zeta + \alpha_15\psi\zeta + \alpha_16\psi\eta\zeta$$
$$w = \alpha_17 + \alpha_18\psi + \alpha_19\eta + \alpha_20\zeta + \alpha_21\psi\eta + \alpha_22\eta\zeta + \alpha_23\psi\zeta + \alpha_24\psi\eta\zeta$$

Because 8H element has 24 degrees of freedom but the complete second order polynomial for 3 dimensions only has 10 terms, 2 terms should be eliminated. Concerning the symmetry, we eliminate 3 second-order terms and add one third-order term.

According to the characteristics of shape function, we can get:
$N_i = \frac{1}{8}(1 + \psi_i\psi)(1 + \eta_i\eta)(1 + \zeta_i\zeta)$
$i = 1, 2, ..., 8$

Then according to the geometry equations of 3D problem, we can get the strain matrix [B]:

$$B_i = \begin{pmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{pmatrix}$$

$$B = [B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8]$$

To transform the shape function into physical coordinate system, we use Jacobi matrix, which is similarly derived as 4Q. Then we have the local stiffness matrix:
$K^e = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} B^T DB|J|d\psi d\eta d\zeta$

## 2 Patch test-Single axis stretch

To test the convergence of 8H element, we first build a model of single axis stretch as Figure 1.The hexahedron is divided into 7 irregular elements. We set the load on each node(5,6,7,8) is 250, v=0.2, E=1000. Then we divide the model into 7 irregular part to process patch test.

It is easy to derive the exact displacement function should be :

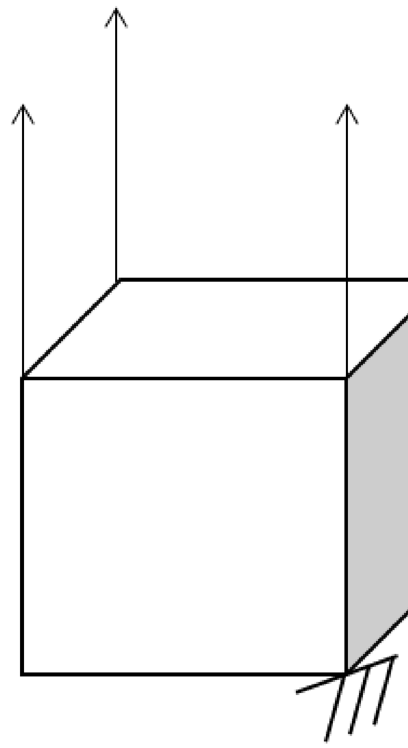

Figure 1: Single axis stretch.

$$u_x = -\nu \frac{4P}{El_z l_x} x$$
$$u_y = -\nu \frac{4P}{El_z l_y} y$$
$$u_z = \frac{4P}{El_z^2} z$$

Then we get the displacement perfectly match the exact displacement as Figure 2, which indicates it has the convergence of second order.

# 3 Example verification -Pure shearing

Secondly, we change the we of loading into pure shearing as Figure 3, as an example verification.

Then we get the displacement as Figure 4 which also matches the first order field correctly. So the 8H element also passes the pure shearing example analysis.
For details please find the 8Hpatch.out and 8Hshear.out in attachment.

D I S P L A C E M E N T S

| NODE | X-DISPLACEMENT | Y-DISPLACEMENT | Z-DISPLACEMENT |
|---|---|---|---|
| 1 | -4.00000e-002 | 0.00000e+000 | 0.00000e+000 |
| 2 | -4.00000e-002 | -4.00000e-002 | 0.00000e+000 |
| 3 | 2.08167e-017 | -4.00000e-002 | 0.00000e+000 |
| 4 | 0.00000e+000 | 0.00000e+000 | 0.00000e+000 |
| 5 | -4.00000e-002 | -9.02056e-017 | 2.00000e-001 |
| 6 | -4.00000e-002 | -4.00000e-002 | 2.00000e-001 |
| 7 | -1.45717e-016 | -4.00000e-002 | 2.00000e-001 |
| 8 | -2.05630e-016 | -4.85723e-017 | 2.00000e-001 |
| 9 | -2.40000e-002 | -8.00000e-003 | 4.00000e-002 |
| 10 | -2.40000e-002 | -2.40000e-002 | 8.00000e-002 |
| 11 | -8.00000e-003 | -2.40000e-002 | 4.00000e-002 |
| 12 | -8.00000e-003 | -8.00000e-003 | 4.00000e-002 |
| 13 | -2.40000e-002 | -8.00000e-003 | 1.20000e-001 |
| 14 | -2.40000e-002 | -2.40000e-002 | 1.20000e-001 |
| 15 | -8.00000e-003 | -2.40000e-002 | 1.20000e-001 |
| 16 | -8.00000e-003 | -8.00000e-003 | 1.20000e-001 |

Figure 2: Single axis displacement.

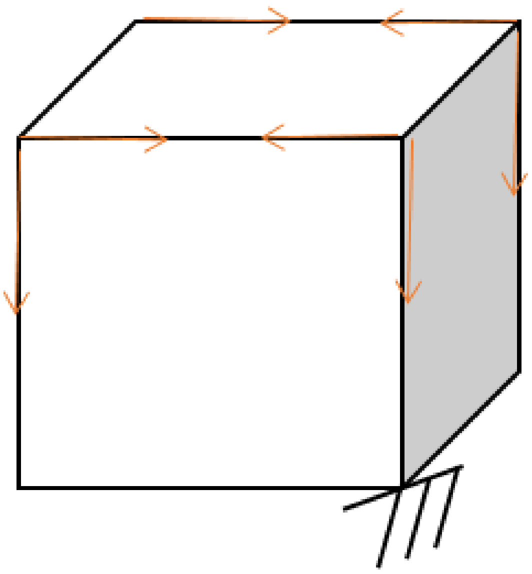

Figure 3: Pure shearing.

D I S P L A C E M E N T S

| NODE | X-DISPLACEMENT | Y-DISPLACEMENT | Z-DISPLACEMENT |
|---|---|---|---|
| 1 | 0.00000e+000 | 0.00000e+000 | 0.00000e+000 |
| 2 | 0.00000e+000 | 0.00000e+000 | 0.00000e+000 |
| 3 | 0.00000e+000 | 0.00000e+000 | 0.00000e+000 |
| 4 | 0.00000e+000 | 0.00000e+000 | 0.00000e+000 |
| 5 | 0.00000e+000 | 2.40000e+000 | 4.99600e-016 |
| 6 | 0.00000e+000 | 2.40000e+000 | -5.37672e-016 |
| 7 | -3.33067e-016 | 2.40000e+000 | -2.70884e-016 |
| 8 | -1.11022e-016 | 2.40000e+000 | -1.01581e-016 |

Figure 4: Pure shearing displacement.