8H element

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1 8H element

According to the method for advanced elements, we develop the 8H element. For details, please find the attachment of codes.
8H is a natural extension of 4Q. The displacement function is:

$$u = \alpha_1 + \alpha_2\psi + \alpha_3\eta + \alpha_4\zeta + \alpha_5\psi\eta + \alpha_6\eta\zeta + \alpha_7\psi\zeta + \alpha_8\psi\eta\zeta$$
$$v = \alpha_9 + \alpha_10\psi + \alpha_11\eta + \alpha_12\zeta + \alpha_13\psi\eta + \alpha_14\eta\zeta + \alpha_15\psi\zeta + \alpha_16\psi\eta\zeta$$
$$w = \alpha_17 + \alpha_18\psi + \alpha_19\eta + \alpha_20\zeta + \alpha_21\psi\eta + \alpha_22\eta\zeta + \alpha_23\psi\zeta + \alpha_24\psi\eta\zeta$$

Because 8H element has 24 degrees of freedom but the complete second order polynomial for 3 dimensions only has 10 terms, 2 terms should be eliminated. Concerning the symmetry, we eliminate 3 second-order terms and add one third-order term.

According to the characteristics of shape function, we can get:
$N_i = \frac{1}{8}(1 + \psi_i\psi)(1 + \eta_i\eta)(1 + \zeta_i\zeta)$
$i = 1, 2, ..., 8$

Then according to the geometry equations of 3D problem, we can get the strain matrix [B]:

$$B_i = \begin{pmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{pmatrix}$$

$$B = [B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8]$$

To transform the shape function into physical coordinate system, we use Jacobi matrix, which is similarly derived as 4Q. Then we have the local stiffness matrix:
$K^e = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} B^T DB|J|d\psi d\eta d\zeta$

2 Patch test-Single axis stretch

To test the convergence of 8H element, we first build a model of single axis stretch as Figure 1.The hexahedron is divided into 7 irregular elements. We set the load on each node(5,6,7,8) is 250, v=0.2, E=1000. Then we divide the model into 7 irregular part to process patch test.

It is easy to derive the exact displacement function should be :

$$u_x = -\nu\frac{4P}{El_z l_x}x$$
$$u_y = -\nu\frac{4P}{El_z l_y}y$$

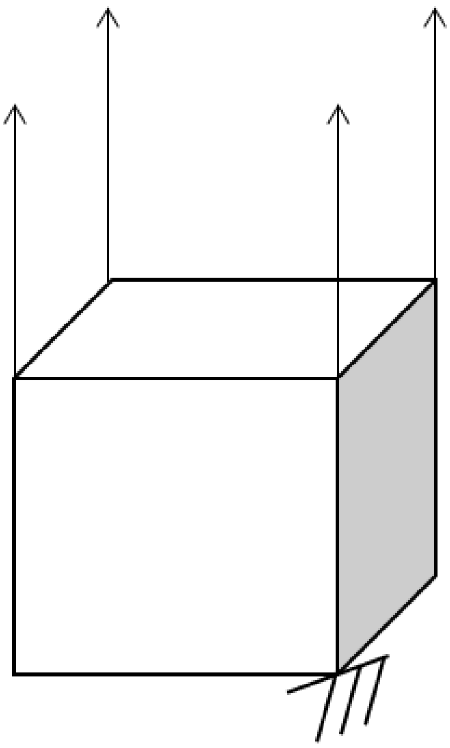

Figure 1: Single axis stretch.

$$u_z = \frac{4P}{El_z^2} z$$

Then we get the displacement perfectly match the exact displacement as Figure 2, which indicates it has the convergence of second order.

3 Patch test-Pure shearing

Secondly, we change the we of loading into pure shearing as Figure 3.(But the patch test of single axis stretch should be already enough.)

Then we get the displacement as Figure 4 which also matches the first order field correctly. For details please find the 8Hpatch.out and 8Hshear.out in attachment.

D I S P L A C E M E N T S

NODE	X-DISPLACEMENT	Y-DISPLACEMENT	Z-DISPLACEMENT
1	-4.00000e-002	0.00000e+000	0.00000e+000
2	-4.00000e-002	-4.00000e-002	0.00000e+000
3	2.08167e-017	-4.00000e-002	0.00000e+000
4	0.00000e+000	0.00000e+000	0.00000e+000
5	-4.00000e-002	-9.02056e-017	2.00000e-001
6	-4.00000e-002	-4.00000e-002	2.00000e-001
7	-1.45717e-016	-4.00000e-002	2.00000e-001
8	-2.05630e-016	-4.85723e-017	2.00000e-001
9	-2.40000e-002	-8.00000e-003	4.00000e-002
10	-2.40000e-002	-2.40000e-002	8.00000e-002
11	-8.00000e-003	-2.40000e-002	4.00000e-002
12	-8.00000e-003	-8.00000e-003	4.00000e-002
13	-2.40000e-002	-8.00000e-003	1.20000e-001
14	-2.40000e-002	-2.40000e-002	1.20000e-001
15	-8.00000e-003	-2.40000e-002	1.20000e-001
16	-8.00000e-003	-8.00000e-003	1.20000e-001

Figure 2: Single axis displacement.

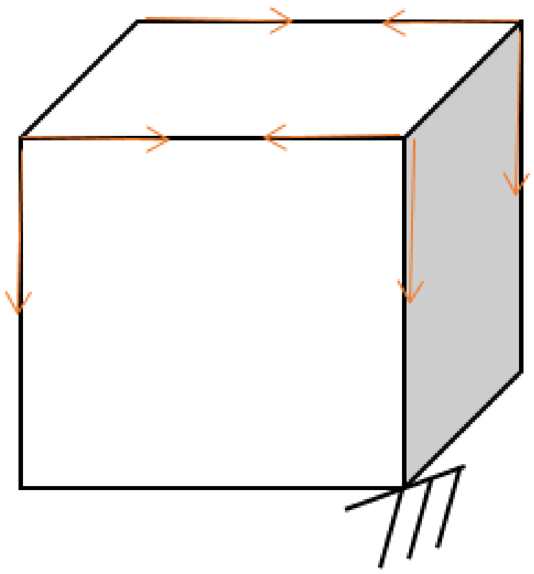

Figure 3: Pure shearing.

D I S P L A C E M E N T S

NODE	X-DISPLACEMENT	Y-DISPLACEMENT	Z-DISPLACEMENT
1	0.00000e+000	0.00000e+000	0.00000e+000
2	0.00000e+000	0.00000e+000	0.00000e+000
3	0.00000e+000	0.00000e+000	0.00000e+000
4	0.00000e+000	0.00000e+000	0.00000e+000
5	0.00000e+000	2.40000e+000	4.99600e-016
6	0.00000e+000	2.40000e+000	-5.37672e-016
7	-3.33067e-016	2.40000e+000	-2.70884e-016
8	-1.11022e-016	2.40000e+000	-1.01581e-016

Figure 4: Pure shearing displacement.