Chapter 1

Demo problem: Solution of a "free-boundary" Poisson problem in an "elastic" domain revisited -- this time with AlgebraicElements

Detailed documentation to be written. Here's a plot of the result and the already fairly well documented driver code...

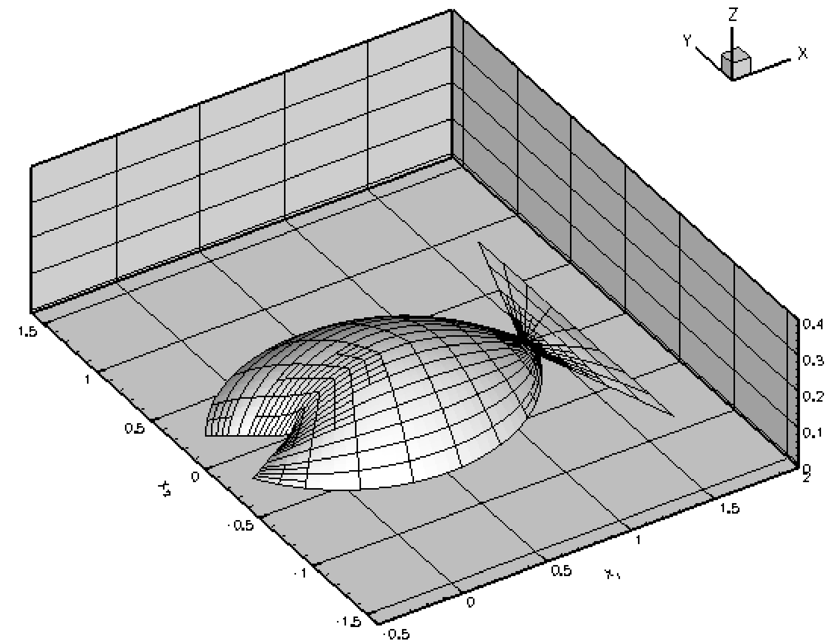

Figure 1.1 Adaptive solution of Poisson's equation in a fish-shaped domain for various ‘widths' of the domain.

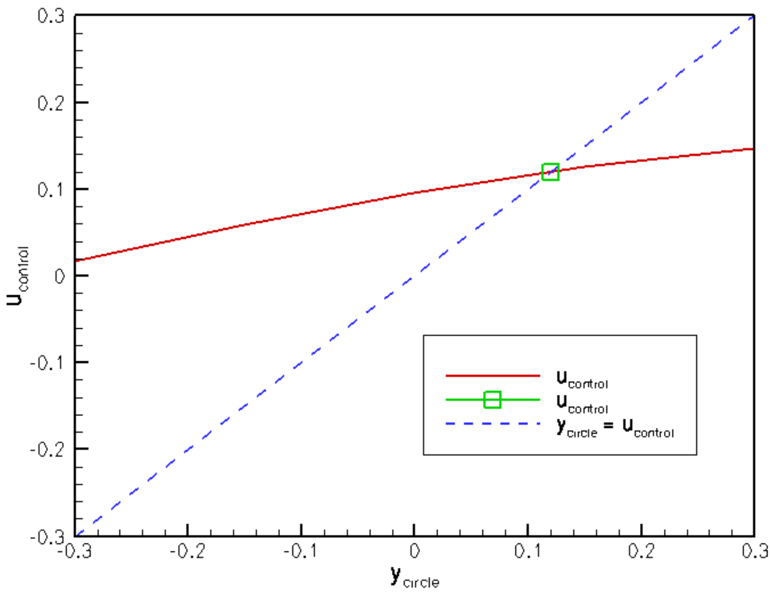

Figure 1.2 Solution of Poisson's equation at a control node as a function of the 'width' of the domain.

```
//LIC// ====================================================================
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
//LIC// at http://www.oomph-lib.org.
//LIC//
//LIC//    Version 1.0; svn revision $LastChangedRevision$
//LIC//
//LIC// $LastChangedDate$
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//LIC//
//LIC//====================================================================
// Driver for solution of "free boundary" 2D Poisson equation in
// fish-shaped domain with adaptivity


// Generic oomph-lib headers
#include "generic.h"

// The Poisson equations
#include "poisson.h"

// The fish mesh
#include "meshes/fish_mesh.h"

// Circle as generalised element:
#include "circle_as_generalised_element.h"

using namespace std;

using namespace oomph;

//////////////////////////////////////////////////////////////////////
```

```
//////////////////////////////////////////////////////////////////////////
//////////////////////////////////////////////////////////////////////////



//======================================================================
/// Namespace for const source term in Poisson equation
//======================================================================
namespace ConstSourceForPoisson
{
 /// Strength of source function: default value 1.0
 double Strength=1.0;

/// Const source function
 void get_source(const Vector<double>& x, double& source)
 {
  source = -Strength*(1.0+x[0]*x[1]);
 }

}


//////////////////////////////////////////////////////////////////////////
//////////////////////////////////////////////////////////////////////////
//////////////////////////////////////////////////////////////////////////



//======================================================================
/// Refineable Poisson problem in deformable fish-shaped domain.
/// Template parameter identify the elements.
//======================================================================
template<class ELEMENT>
class RefineableFishPoissonProblem : public Problem
{

public:

 /// \short  Constructor: Bool flag specifies if position of fish back is
 /// prescribed or computed from the coupled problem. String specifies
 /// output directory.
 RefineableFishPoissonProblem(
  const bool& fix_position, const string& directory_name,
  const unsigned& i_case);

 /// Destructor
 virtual ~RefineableFishPoissonProblem();

 /// \short Update after Newton step: Update mesh in response to
 /// possible changes in the wall shape
 void actions_before_newton_convergence_check()
  {
   fish_mesh_pt()->node_update();
  }

 /// Update the problem specs after solve (empty)
 void actions_after_newton_solve(){}

 /// Update the problem specs before solve: Update mesh
 void actions_before_newton_solve()
  {
   fish_mesh_pt()->node_update();
  }

 //Access function for the fish mesh
 AlgebraicRefineableFishMesh<ELEMENT>* fish_mesh_pt()
  {
   return Fish_mesh_pt;
  }

 /// Return value of the "load" on the elastically supported ring
 double& load()
  {
   return *Load_pt->value_pt(0);
  }


 /// \short Return value of the vertical displacement of the ring that
 /// represents the fish's back
 double& y_c()
  {
   return static_cast<ElasticallySupportedRingElement*>(fish_mesh_pt()->
                                                        fish_back_pt())->y_c();
  }

 /// Doc the solution
 void doc_solution();
```

```
 /// Access to DocInfo object
 DocInfo& doc_info() {return Doc_info;}

private:

 /// Helper fct to set method for evaluation of shape derivs
 void set_shape_deriv_method()
  {

   bool done=false;

   //Loop over elements and set pointers to source function
   unsigned n_element = fish_mesh_pt()->nelement();
   for(unsigned i=0;i<n_element;i++)
    {
     // Upcast from FiniteElement to the present element
     ELEMENT *el_pt = dynamic_cast<ELEMENT*>(fish_mesh_pt()->element_pt(i));

     // Direct FD
     if (Case_id==0)
      {
       el_pt->evaluate_shape_derivs_by_direct_fd();
       if (!done) std::cout << "\n\n [CR residuals] Direct FD" << std::endl;
      }
     // Chain rule with/without FD
     else if ( (Case_id==1) ||  (Case_id==2) )
      {
       // It's broken but let's call it anyway to keep self-test alive
       bool i_know_what_i_am_doing=true;
       el_pt->evaluate_shape_derivs_by_chain_rule(i_know_what_i_am_doing);
       if (Case_id==1)
        {
         el_pt->enable_always_evaluate_dresidual_dnodal_coordinates_by_fd();
         if (!done) std::cout << "\n\n [CR residuals] Chain rule and FD"
                              << std::endl;
        }
       else
        {
         el_pt->disable_always_evaluate_dresidual_dnodal_coordinates_by_fd();
         if (!done) std::cout << "\n\n [CR residuals] Chain rule and analytic"
                              << std::endl;
        }
      }
     // Fastest with/without FD
     else if ( (Case_id==3) ||  (Case_id==4) )
      {
       // It's broken but let's call it anyway to keep self-test alive
       bool i_know_what_i_am_doing=true;
       el_pt->evaluate_shape_derivs_by_fastest_method(i_know_what_i_am_doing);
       if (Case_id==3)
        {
         el_pt->enable_always_evaluate_dresidual_dnodal_coordinates_by_fd();
         if (!done) std::cout << "\n\n [CR residuals] Fastest and FD"
                              << std::endl;
        }
       else
        {
         el_pt->disable_always_evaluate_dresidual_dnodal_coordinates_by_fd();
         if (!done) std::cout << "\n\n [CR residuals] Fastest and analytic"
                              << std::endl;

        }
      }
     done=true;
    }

  }

 /// Node at which the solution of the Poisson equation is documented
 Node* Doc_node_pt;

 /// Trace file
 ofstream Trace_file;

 /// Pointer to fish mesh
 AlgebraicRefineableFishMesh<ELEMENT>* Fish_mesh_pt;

 /// Pointer to single-element mesh that stores the GeneralisedElement
 /// that represents the fish back
 Mesh* Fish_back_mesh_pt;

 /// \short Pointer to data item that stores the "load" on the fish back
 Data* Load_pt;

 /// \short Is the position of the fish back prescribed?
 bool Fix_position;
```

```
 /// Doc info object
 DocInfo Doc_info;

 /// Case id
 unsigned Case_id;

};




//=====================================================================
/// Constructor for adaptive Poisson problem in deformable fish-shaped
/// domain. Pass flag if position of fish back is fixed, and the output
/// directory.
//=====================================================================
template<class ELEMENT>
RefineableFishPoissonProblem<ELEMENT>::RefineableFishPoissonProblem
      (
 const bool& fix_position, const string& directory_name,
 const unsigned& i_case) : Fix_position(fix_position), Case_id(i_case)
{


 // Set output directory
 Doc_info.set_directory(directory_name);

 // Initialise step number
 Doc_info.number()=0;

 // Open trace file
 char filename[100];
 sprintf(filename,"%s/trace.dat",directory_name.c_str());
 Trace_file.open(filename);

 Trace_file
  << "VARIABLES=\"load\",\"y<sub>circle</sub>\",\"u<sub>control</sub>\""
  << std::endl;

 // Set coordinates and radius for the circle that will become the fish back
 double x_c=0.5;
 double y_c=0.0;
 double r_back=1.0;

 // Build geometric element that will become the fish back
 GeomObject* fish_back_pt=new ElasticallySupportedRingElement(x_c,y_c,r_back
      );

 // Build fish mesh with geometric object that specifies the fish back
 Fish_mesh_pt=new AlgebraicRefineableFishMesh<ELEMENT>(fish_back_pt);

 // Add the fish mesh to the problem's collection of submeshes:
 add_sub_mesh(Fish_mesh_pt);

 // Build mesh that will store only the geometric wall element
 Fish_back_mesh_pt=new Mesh;

 // So far, the mesh is completely empty. Let's add the
 // one (and only!) GeneralisedElement which represents the shape
 // of the fish's back to it:
 Fish_back_mesh_pt->add_element_pt(dynamic_cast<GeneralisedElement*>(
                                   Fish_mesh_pt->fish_back_pt()));

 // Add the fish back mesh to the problem's collection of submeshes:
 add_sub_mesh(Fish_back_mesh_pt);

 // Now build global mesh from the submeshes
 build_global_mesh();


 // Create/set error estimator
 fish_mesh_pt()->spatial_error_estimator_pt()=new Z2ErrorEstimator;

 // Choose a node at which the solution is documented: Choose
 // the central node that is shared by all four elements in
 // the base mesh because it exists at all refinement levels.

 // How many nodes does element 0 have?
 unsigned nnod=fish_mesh_pt()->finite_element_pt(0)->nnode();

 // The central node is the last node in element 0:
 Doc_node_pt=fish_mesh_pt()->finite_element_pt(0)->node_pt(nnod-1);

 // Doc
 cout << std::endl <<  "Control node is located at: "
```

```
      << Doc_node_pt->x(0) << " " << Doc_node_pt->x(1)
      << std::endl << std::endl;

 // Position of fish back is prescribed
 if (Fix_position)
  {
   // Create the load data object
   Load_pt=new Data(1);

   // Pin the prescribed load
   Load_pt->pin(0);

   // Pin the vertical displacement
   dynamic_cast<ElasticallySupportedRingElement*>(
    Fish_mesh_pt->fish_back_pt())->pin_yc();
  }
 // Coupled problem: The position of the fish back is determined
 // via the solution of the Poisson equation: The solution at
 // the control node acts as the load for the displacement of the
 // fish back
 else
  {
   // Use the solution (value 0) at the control node as the load
   // that acts on the ring. [Note: Node == Data by inheritance]
   Load_pt=Doc_node_pt;
  }


 // Set the pointer to the Data object that specifies the
 // load on the fish's back
 dynamic_cast<ElasticallySupportedRingElement*>(Fish_mesh_pt->fish_back_pt()
      )->
  set_load_pt(Load_pt);


 // Set the boundary conditions for this problem: All nodes are
 // free by default -- just pin the ones that have Dirichlet conditions
 // here.
 unsigned num_bound = fish_mesh_pt()->nboundary();
 for(unsigned ibound=0;ibound<num_bound;ibound++)
  {
   unsigned num_nod= fish_mesh_pt()->nboundary_node(ibound);
   for (unsigned inod=0;inod<num_nod;inod++)
    {
     fish_mesh_pt()->boundary_node_pt(ibound,inod)->pin(0);
    }
  }


 // Set homogeneous boundary conditions on all boundaries
 for(unsigned ibound=0;ibound<num_bound;ibound++)
  {
   // Loop over the nodes on boundary
   unsigned num_nod=fish_mesh_pt()->nboundary_node(ibound);
   for (unsigned inod=0;inod<num_nod;inod++)
    {
     fish_mesh_pt()->boundary_node_pt(ibound,inod)->set_value(0,0.0);
    }
  }

 /// Loop over elements and set pointers to source function
 unsigned n_element = fish_mesh_pt()->nelement();
 for(unsigned i=0;i<n_element;i++)
  {
   // Upcast from FiniteElement to the present element
   ELEMENT *el_pt = dynamic_cast<ELEMENT*>(fish_mesh_pt()->element_pt(i));

   //Set the source function pointer
   el_pt->source_fct_pt() = &ConstSourceForPoisson::get_source;
  }

 // Set shape derivative method
 set_shape_deriv_method();

 // Do equation numbering
 cout <<"Number of equations: " << assign_eqn_numbers() << std::endl;

}



//========================================================================
/// Destructor for Poisson problem in deformable fish-shaped domain.
//========================================================================
template<class ELEMENT>
RefineableFishPoissonProblem<ELEMENT>::~RefineableFishPoissonProblem
      ()
```

```
{
 // Close trace file
 Trace_file.close();

}



//==========================================================================
/// Doc the solution in tecplot format.
//==========================================================================
template<class ELEMENT>
void RefineableFishPoissonProblem<ELEMENT>::doc_solution
      ()
{

 ofstream some_file;
 char filename[100];

 // Number of plot points in each coordinate direction.
 unsigned npts;
 npts=5;


 // Output solution
 if (Case_id!=0)
  {
   sprintf(filename,"%s/soln_%i_%i.dat",Doc_info.directory().c_str(),
           Case_id,Doc_info.number());
  }
 else
  {
   sprintf(filename,"%s/soln%i.dat",Doc_info.directory().c_str(),
           Doc_info.number());
  }
 some_file.open(filename);
 fish_mesh_pt()->output(some_file,npts);
 some_file.close();

 // Write "load", vertical position of the fish back, and solution at
 // control node to trace file
 Trace_file
  << static_cast<ElasticallySupportedRingElement*>(fish_mesh_pt()->
                                             fish_back_pt())->load()
  << " "
  << static_cast<ElasticallySupportedRingElement*>(fish_mesh_pt()->
                                             fish_back_pt())->y_c()
  << " " << Doc_node_pt->value(0) << std::endl;
}






////////////////////////////////////////////////////////////////////////
////////////////////////////////////////////////////////////////////////
////////////////////////////////////////////////////////////////////////





//==========================================================================
/// Demonstrate how to solve 2D Poisson problem in deformable
/// fish-shaped domain with mesh adaptation.
//==========================================================================
template<class ELEMENT>
void demo_fish_poisson(const string& directory_name)
{

 // Set up the problem with prescribed displacement of fish back
 bool fix_position=true;
 RefineableFishPoissonProblem<ELEMENT> problem(fix_position,
      directory_name,0);

 // Doc refinement targets
 problem.fish_mesh_pt()->doc_adaptivity_targets(cout);

 // Do some uniform mesh refinement first
 //----------------------------------------
 problem.refine_uniformly();
 problem.refine_uniformly();

 // Initial value for the vertical displacement of the fish's back
```

```
 problem.y_c()=-0.3;

 // Loop for different fish shapes
 //------------------------------

 // Number of steps
 unsigned nstep=5;

 // Increment in displacement
 double dyc=0.6/double(nstep-1);

 // Valiation: Just do one step
 if (CommandLineArgs::Argc>1) nstep=1;

 for (unsigned istep=0;istep<nstep;istep++)
  {
   // Solve/doc
   unsigned max_solve=2;
   problem.newton_solve(max_solve);
   problem.doc_solution();

   //Increment counter for solutions
   problem.doc_info().number()++;

   // Change vertical displacement
   problem.y_c()+=dyc;
  }
}


//======================================================================
/// Demonstrate how to solve "elastic" 2D Poisson problem in deformable
/// fish-shaped domain with mesh adaptation.
//======================================================================
template<class ELEMENT>
void demo_elastic_fish_poisson(const string& directory_name)
{

 // Loop over all cases
 for (unsigned i_case=0;i_case<5;i_case++)
  //unsigned i_case=1;
  {
   std::cout << "[CR residuals] " << std::endl;
   std::cout << "[CR residuals]==================================================="
             << std::endl;
   std::cout << "[CR residuals] " << std::endl;
   //Set up the problem with "elastic" fish back
   bool fix_position=false;
   RefineableFishPoissonProblem<ELEMENT> problem(fix_position,
                                                  directory_name,
                                                  i_case);

   // Doc refinement targets
   problem.fish_mesh_pt()->doc_adaptivity_targets(cout);

   // Do some uniform mesh refinement first
   //--------------------------------------
   problem.refine_uniformly();
   problem.refine_uniformly();


   // Initial value for load on fish back
   problem.load()=0.0;

   // Solve/doc
   unsigned max_solve=2;
   problem.newton_solve(max_solve);
   problem.doc_solution();
  }


}





//======================================================================
/// Driver for "elastic" fish poisson solver with adaptation.
/// If there are any command line arguments, we regard this as a
/// validation run and perform only a single step.
//======================================================================
int main(int argc, char* argv[])
{
 // Store command line arguments
 CommandLineArgs::setup(argc,argv);
```

```
 // Shorthand for element type
  typedef AlgebraicElement<RefineableQPoissonElement<2,3> > ELEMENT;

  // Compute solution of Poisson equation in various domains
  demo_fish_poisson<ELEMENT>("RESLT");

  // Compute "elastic" coupled solution directly
  demo_elastic_fish_poisson<ELEMENT>("RESLT_coupled");

}
```

1.1 PDF file

A pdf version of this document is available.