Assignment 4

CMPT307
Summer 2020
Assignment 4
Due Wed Aug 5 at 23:59
4 problems, 40 points.

1. Let $G = (V, E)$ be a directed graph with weighted edges, edge weights can be positive, negative, or zero. Suppose vertices of G are partitioned into k disjoint subsets $V_1, V_2, \ldots, V_k$; that is, every vertex of G belongs to exactly one subset V_i. For each i and j, let $\delta(i, j)$ denote the minimum shortest-path distance between vertices in V_i and vertices in V_j, that is

   $$\delta(i, j) = \min\{dist(u, v) \mid u \in V_i \text{ and } v \in V_j\}$$

   Describe an algorithm to compute $\delta(i, j)$ for all i and j. For full credit, your algorithm should run in $O(VE + kV \log V)$ time. (10 points)

2. Let $G = V, E$ be a flow network in which every edge has capacity 1 and the shortest-path distance from s to t is at least d. (10 points)

   (a) Prove that the value of the maximum (s, t)-flows is at most E/d.

   (b) Now suppose that G is simple, meaning that for all vertices u and v, there is at most one edge from u to v. Prove that the value of the maximum (s, t)-flow is at most $O(V^2/d^2)$.

3. A cycle cover of a given directed graph $G = (V, E)$ is a set of vertex-disjoint cycles that cover every vertex in G. Describe and analyze an efficient algorithm to find a cycle cover for a given graph, or correctly report that no cycle cover exists. (10 points)

   Hint: use bipartite matching. But G is not bipartite, so you'll have to use a graph derived from G.

4. Solve the equation by using an LUP decomposition. (For full credit, show your detail steps.) (10 points)

   $$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -9 \end{pmatrix}$$