

CMPT 307

Summer 2020

Assignment 1

Due Wed June 3 at 23:59

Submit on CS Submission Server/CourSys.

4 problems; 10 points each.

1. Express $\sum_{i=0}^n (3i^3 - 6i + 2)$ as a polynomial $p(n)$. Then prove that the sum = $p(n)$ by induction.

2. *Aerosort* is a sorting algorithm.

Aerosort(A, i, j) // A is array to sort; i and j are start and end indices.

n = j - i + 1

If (n < 10) {

sort A[i...j] by insertion-sort

return

}

m₁ = i + 3 * n / 4

m₂ = i + n / 4

Aerosort(A, i, m₁)

Aerosort(A, m₂, j)

Aerosort(A, i, m₁)

- a. What is the asymptotic worst-case running time of Aerosort? Show your work.
 - b. Prove that Aerosort(A, 1, n) correctly sorts an array A of n elements.
3. Devise a comparison-based algorithm (no bucket or radix sort, for instance) to simultaneously find the minimum and the maximum element in a list of n numbers using at most $3n/2$ comparisons. Give pseudocode.
 4. Give an efficient algorithm to convert a given β -bit (binary) integer to a decimal representation. Argue that if multiplication or division of integers whose length is at most β takes time $M(\beta)$, then binary-to-decimal conversion can be performed in time $\Theta(M(\beta) \log \beta)$. (Hint: use a divide-and-conquer approach, obtaining the top and bottom halves of the result with separate recursions.)