MANIPULATOR MODELING

ACCOUNTING FOR FLEXIBILITIES

M2R SAR

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Vibrations around a stable equilibrium configuration

consider a mechanical system $\mathcal{S}$, described by a set of n kinematical parameters $\mathbf{q} = [q_1(t), ..., q_n(t)]^\mathsf{T}$, as well as by energetic functions $\mathcal{T}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathcal{V}(\mathbf{q})$ (kinetic and potential energy resp.), featuring a stable equilibrium at $\mathbf{q} = 0$: $\left[\frac{\partial \mathcal{V}}{\partial q_i}\right]_{\mathbf{q}=0} = 0$

Lagrange equations

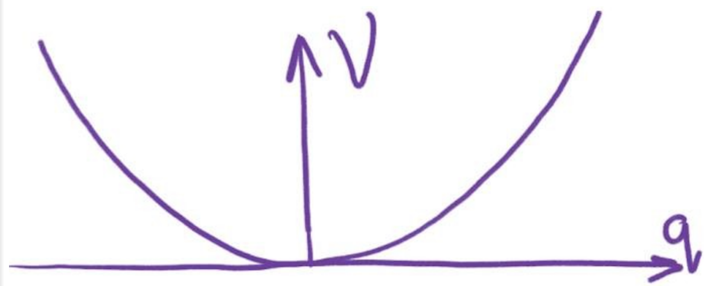
Lagrangian $\mathcal{L} = \mathcal{T} - \mathcal{V}$

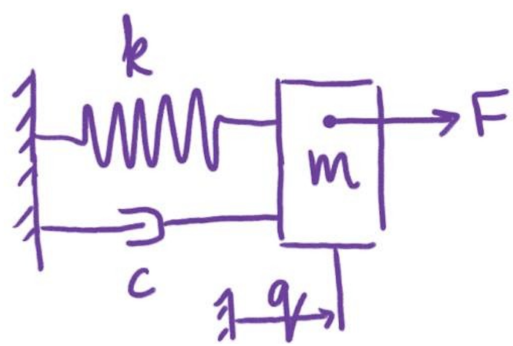
$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = Q_{\mathrm{nc}i}$$

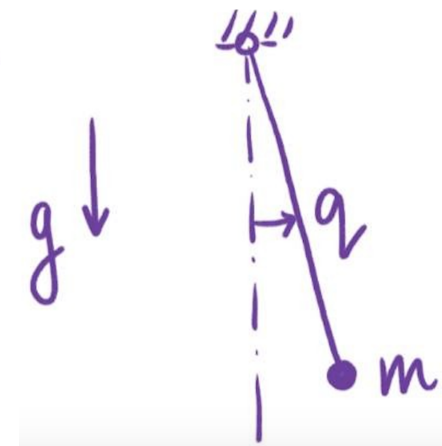
$$\frac{d}{dt}\frac{\partial \mathcal{T}}{\partial \dot{q}_i} + \frac{\partial \mathcal{V}}{\partial q_i} = Q_{\mathrm{nc}i}$$

Force balance

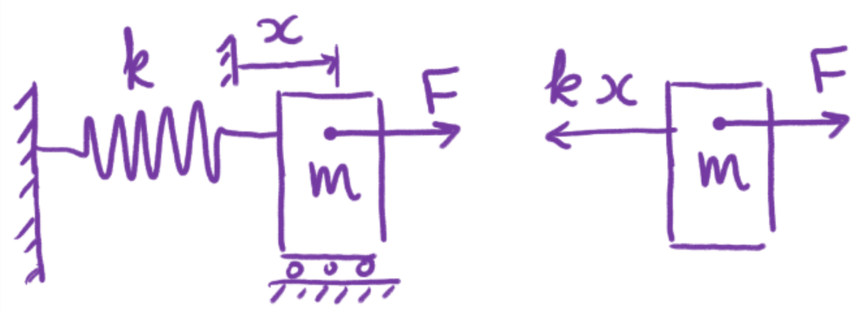
$$m_i \mathbf{v}_i = \mathbf{f}_i$$







Conservative system



Equation of motion

$$m\ddot{x} + kx = 0$$

Caracteristic equation

$$m\lambda^2 + k = 0$$

Eigensolution

$$\begin{aligned} \lambda_{1,2} &= \pm \boldsymbol{i}\sqrt{\frac{k}{m}} \\ &= \pm \boldsymbol{i}\omega_0 \end{aligned}$$

with $\omega_0 = \sqrt{\frac{k}{m}}$ natural frequency

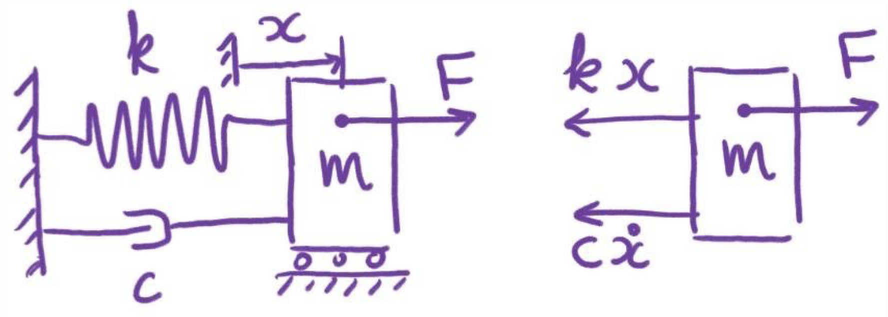
Solution

$$\begin{aligned} x(t) &= A_1 \cos \omega_0 t + A_2 \sin \omega_0 t \\ &= A_0 \cos(\omega_0 t + \phi_0) \\ &= B_1 e^{\boldsymbol{i}\omega_0 t} + B_2 e^{-\boldsymbol{i}\omega_0 t} \\ &= \mathrm{Re}\, B_0 e^{\boldsymbol{i}(\omega_0 t + \phi_0)} \end{aligned}$$

initial conditions :

$$\begin{aligned} A_1 &= x(0); \\ A_2 &= \frac{\dot{x}(0)}{\omega_0}; \end{aligned}$$

Dissipative system



Equation of motion

$$m\ddot{x} + c\dot{x} + kx = 0$$

Caracteristic equation

$$m\lambda^2 + c\lambda + k = 0$$

Eigensolution

$$\lambda_{1,2} = -\frac{c \pm \sqrt{c^2 - 4km}}{2m}$$

- if $c < 2\sqrt{km}$ (underdamped)

$$\begin{aligned}\lambda_{1,2} &= -\omega_0\left(\zeta \pm \boldsymbol{i}\sqrt{1-\zeta^2}\right)\\ &= -\omega_0\zeta \pm \boldsymbol{i}\omega_d\end{aligned}$$

- if $c = 2\sqrt{km}$ (critical)

$$\lambda_{1,2} = -\omega_0\zeta$$

- if $c > 2\sqrt{km}$ (overdamped)

$$\lambda_{1,2} = -\omega_0\left(\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

with $\zeta = \frac{c}{2\sqrt{km}}$, $\omega_d = \omega_0\sqrt{1-\zeta^2}$.

Dissipative system – 2

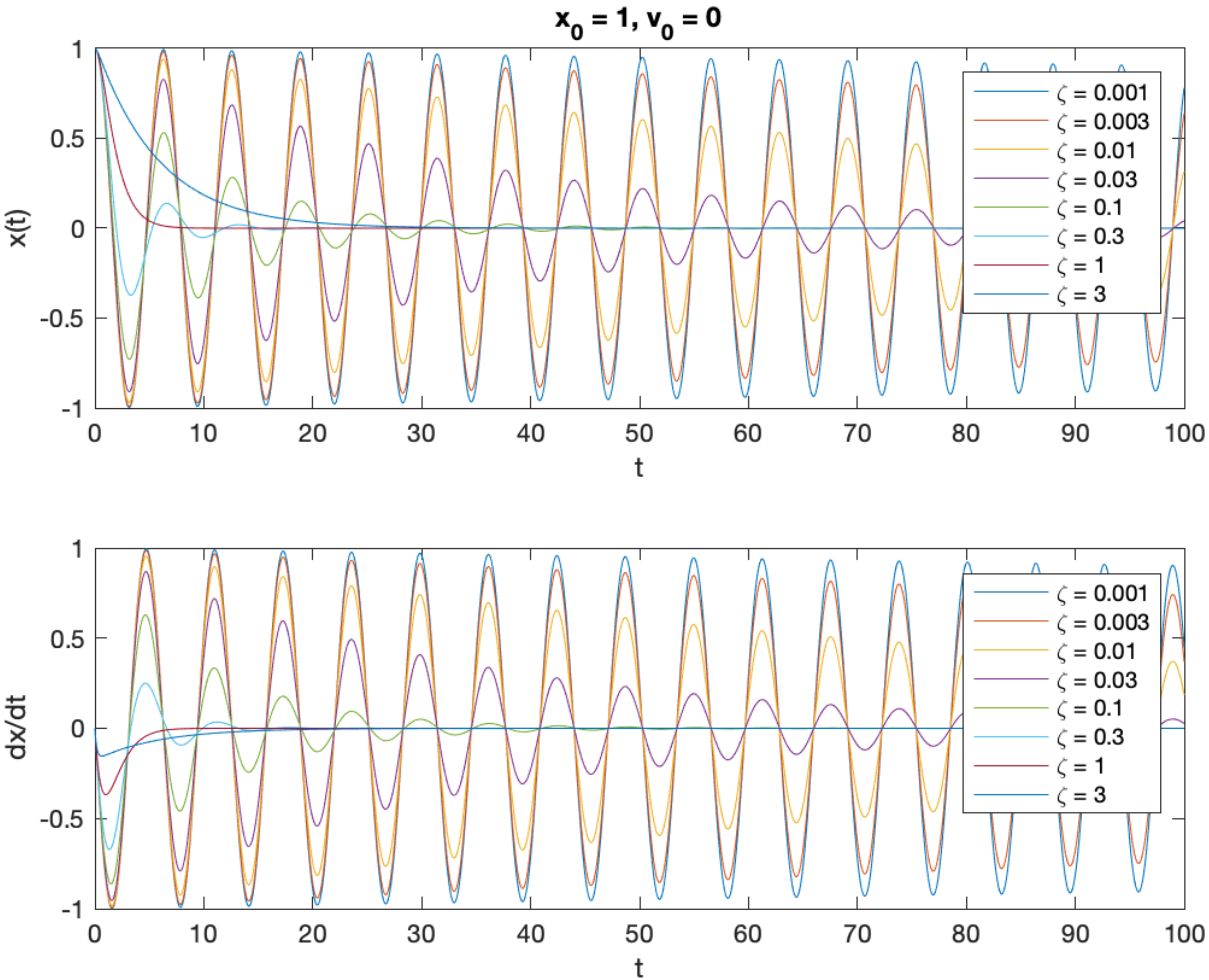
Solution

- if $c < 2\sqrt{km}$ $\quad x(t) = e^{-\zeta\omega_0 t} A_0 \cos(\omega_d t + \phi_0)$
- if $c = 2\sqrt{km}$ $\quad x(t) = e^{-\omega_0 t} (A_1 + A_2 \omega_0 t)$
- if $c > 2\sqrt{km}$ $\quad x(t) = e^{-\zeta\omega_0 t} \left(B_1 e^{\omega_0 \sqrt{\zeta^2 - 1} t} + B_2 e^{-\omega_0 \sqrt{\zeta^2 - 1} t} \right)$

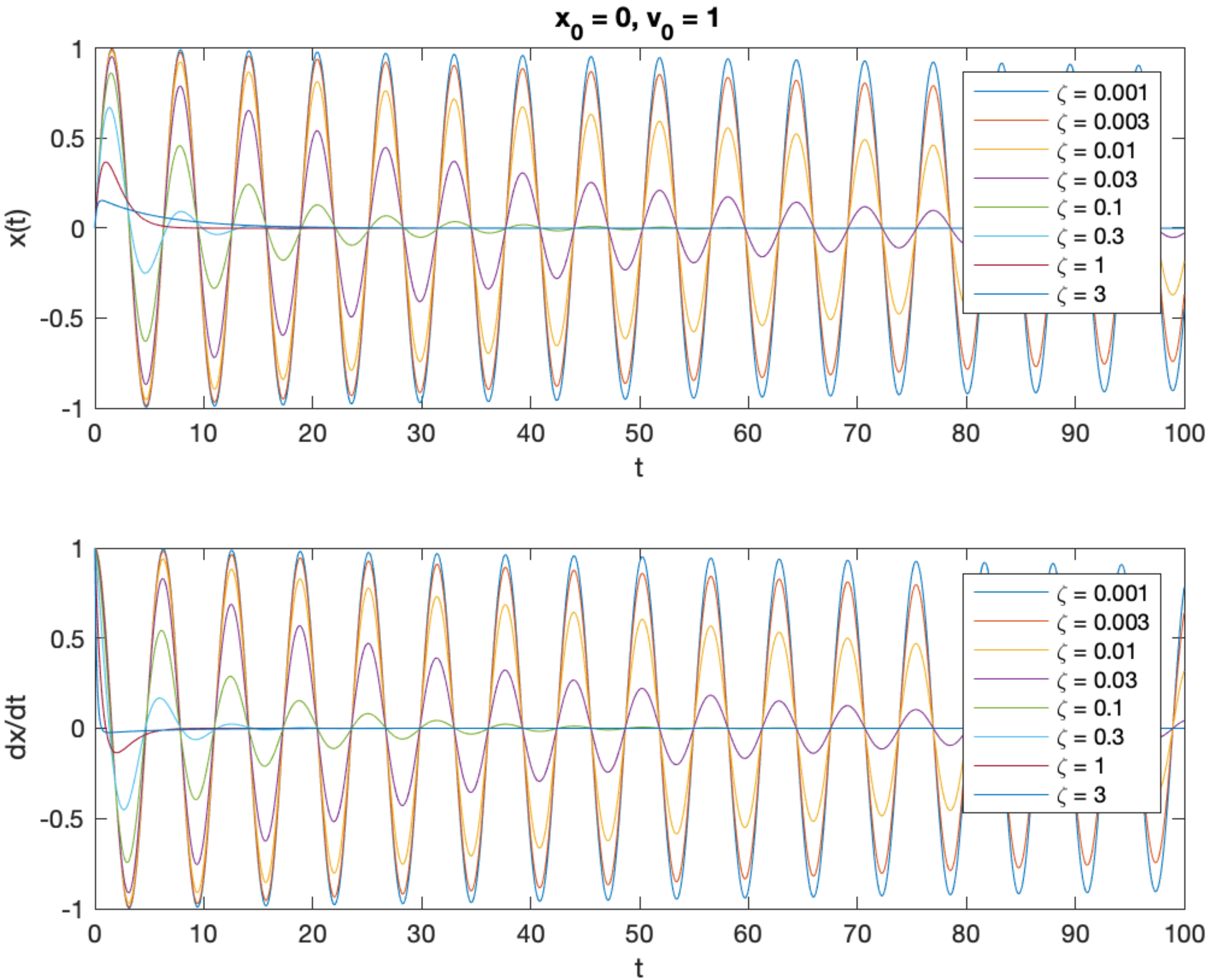
Damping characteristics

- damping coefficient c
- damping ratio ζ
- log decrement $\delta = \ln \frac{x(t)}{x(t+T_d)} = \omega_0 \zeta T_d$
- $Q = 1/2\zeta$ quality factor

Dissipative system – 3



Dissipative system – 4



Harmonic response : undamped case

Harmonic forcing

$$F = F_0 \cos(\omega t)$$

Undamped system

$$m\ddot{x} + kx = F$$

Undamped response

- if $\omega \neq \omega_0$

$$x(t) = A_0 \cos(\omega_0 t + \phi_0) + \frac{F_0 \cos(\omega t + \phi_F)}{m(\omega_0^2 - \omega^2)}$$

- if $\omega = \omega_0$

$$x(t) = A_0 \cos(\omega_0 t + \phi_0) + \frac{F_0 t \cos(\omega_0 t + \phi_F)}{m\omega_0}$$

Harmonic response : damped case

Damped system

$$m\ddot{x} + c\dot{x} + kx = F$$

Damped response

$$\begin{aligned} x(t) &= e^{-\zeta\omega_0 t} A_0 \cos(\omega_d t + \phi_0) + A_F \cos(\omega t + \phi_F) \\ &= x_0(t) + x_F(t) \\ &= \text{general} + \text{particular} \\ &= \text{transient} + \text{steady} \\ &= \text{free} + \text{forced} \end{aligned}$$

with $A_F = F_0/m\sqrt{(\omega^2 - \omega_0^2)^2 + (2\zeta\omega\omega_0)^2}$, $\tan\phi = \frac{2\zeta\omega\omega_0}{\omega_0^2 - \omega^2}$

Transfer function

Complex notations

$$F(t) = \text{Re } F_0 e^{\boldsymbol{i}(\omega t)}$$
$$x_F(t) = \text{Re } B_F e^{\boldsymbol{i}(\omega t)}$$

$$\dot{x}_F(t) = \text{Re } \boldsymbol{i}\omega B_F e^{\boldsymbol{i}(\omega t)}$$
$$\ddot{x}_F(t) = -\text{Re } \omega^2 B_F e^{\boldsymbol{i}(\omega t)}$$

Transfer function

$$B_F = H(\boldsymbol{i}\omega) F_0$$

$$\begin{aligned} H(\boldsymbol{i}\omega) &= \frac{1}{-m\omega^2 + \boldsymbol{i}c\omega + k} \\ &= \frac{1}{m(-\omega^2 + 2\boldsymbol{i}\zeta\omega\omega_0 + \omega_0^2)} \\ &= \frac{1}{k(-\Omega^2 + 2\boldsymbol{i}\zeta\Omega + 1)} \end{aligned}$$

Resonance : max $||H||$

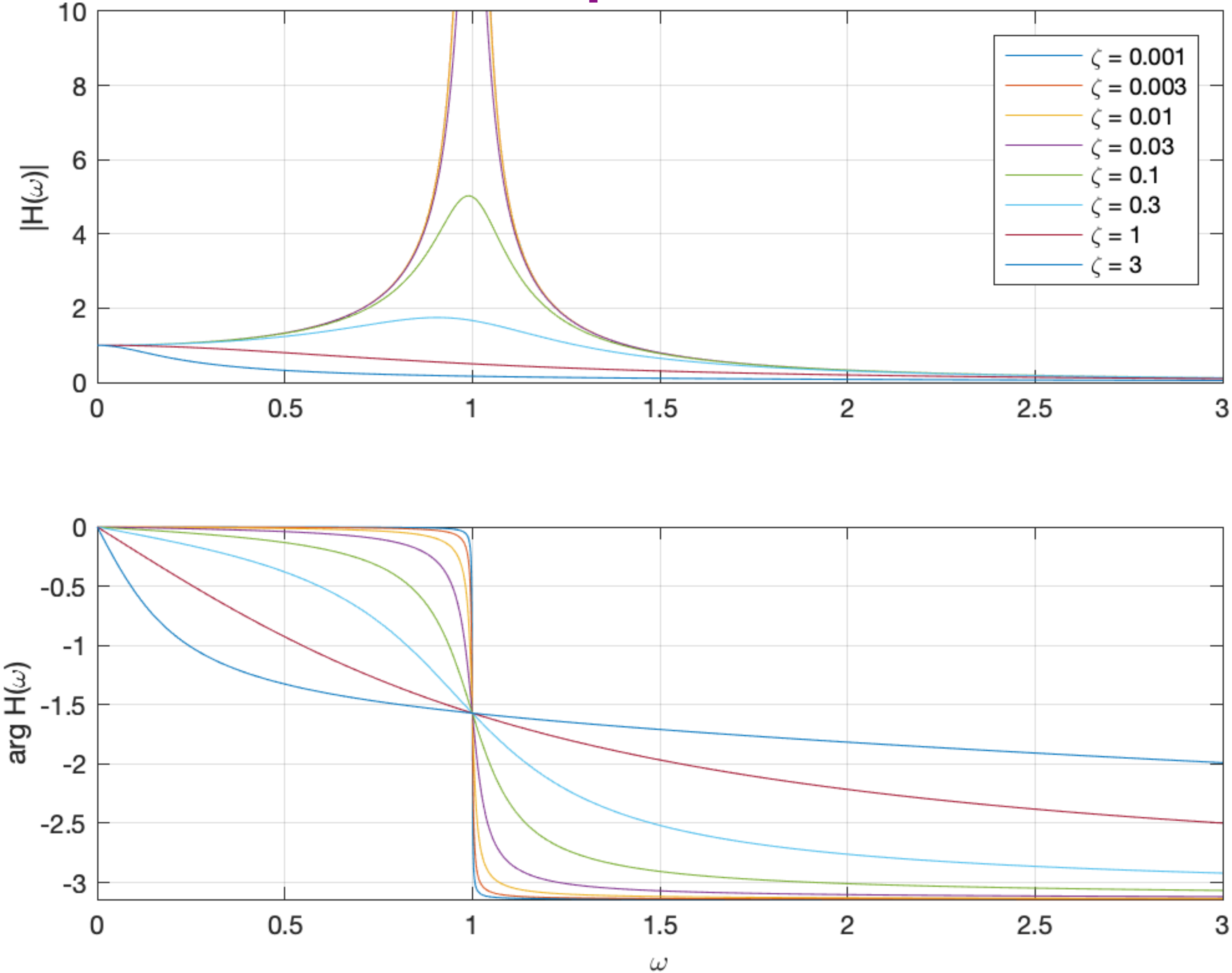
$$\omega_r = \omega_0 \sqrt{1 - 2\zeta^2}$$

$$H_r = H(\boldsymbol{i}\omega_r) = \frac{1}{2k\zeta\left(\zeta + \boldsymbol{i}\sqrt{1 - 2\zeta^2}\right)}$$

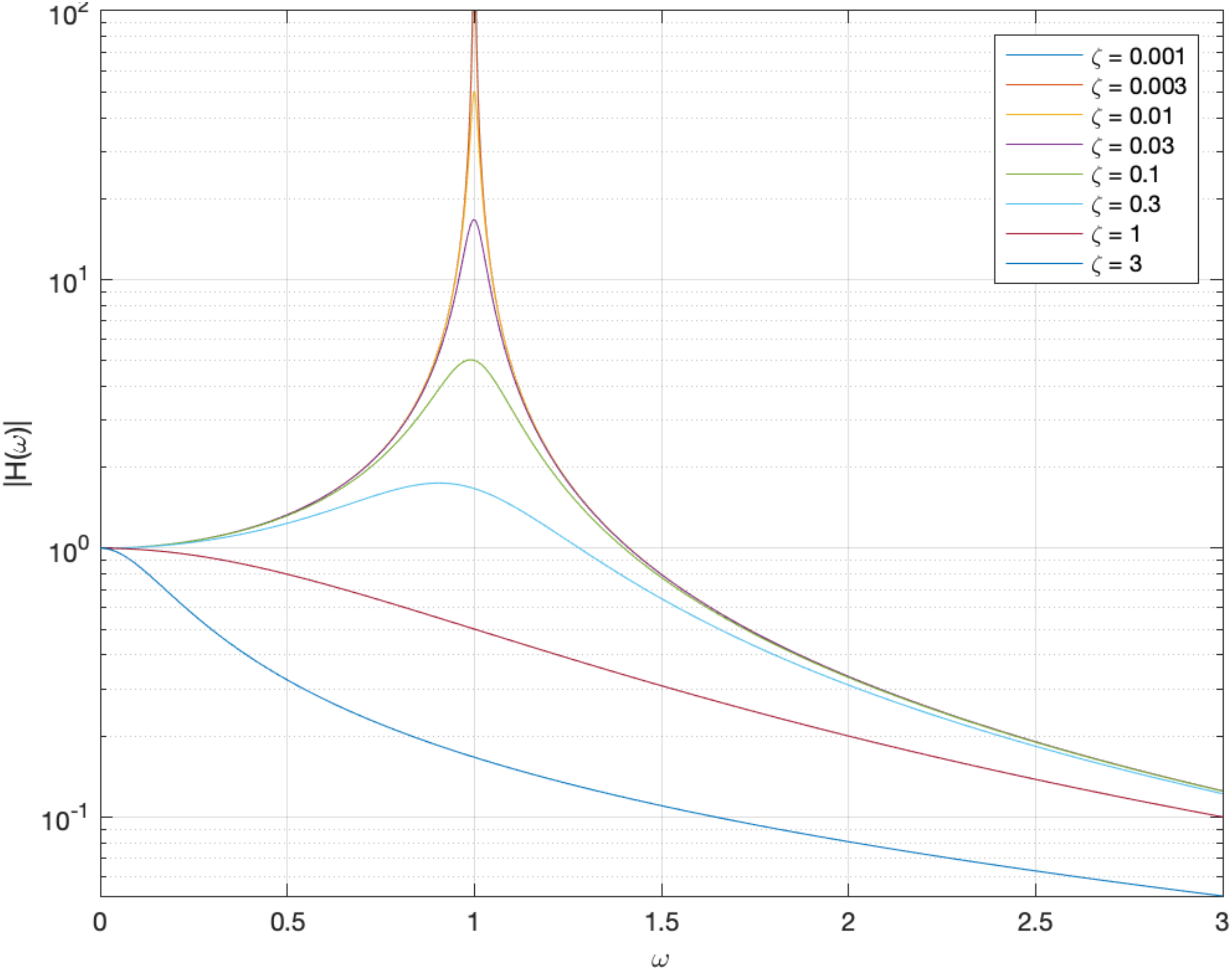
Other force transfer functions

Output	Transfer function	Inverse transfer function
displacement	receptance / admittance / compliance	dyn stiffness
velocity	mobility	impedance
acceleration	accelerance	apparent mass

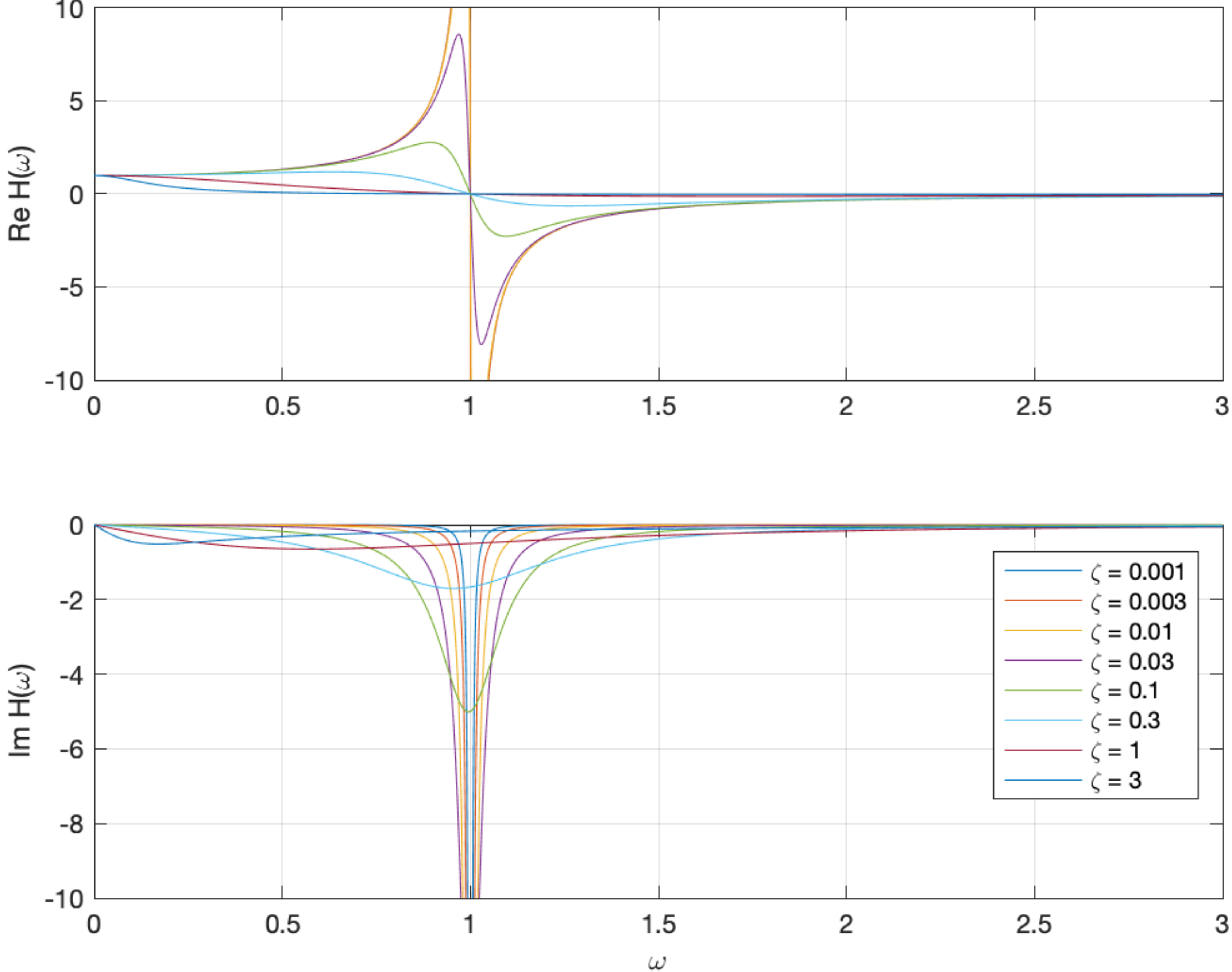
Transfer function. Bode plot



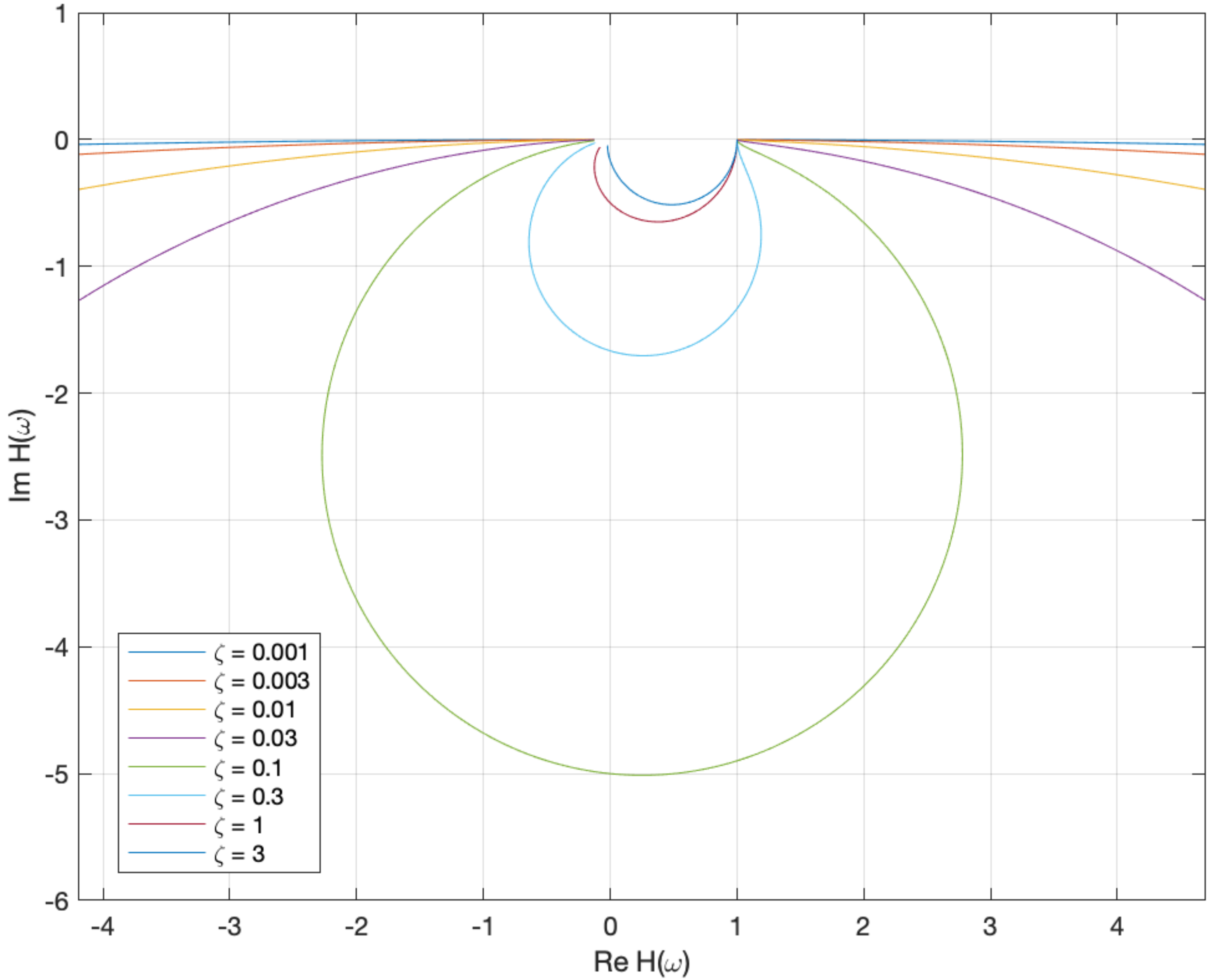
Transfer function



Transfer function. Real/Imaginary part



Transfer function. Nyquist plot



Damping considerations

Orders of magnitudes

$\zeta \approx 10^{-4}$ –	monomaterial metallic parts
$\zeta \approx 10^{-3} \sim$	plastic parts
$\zeta \approx 10^{-2} +$	mechanical assemblies
$\zeta \approx 10^{-1} +$	machines

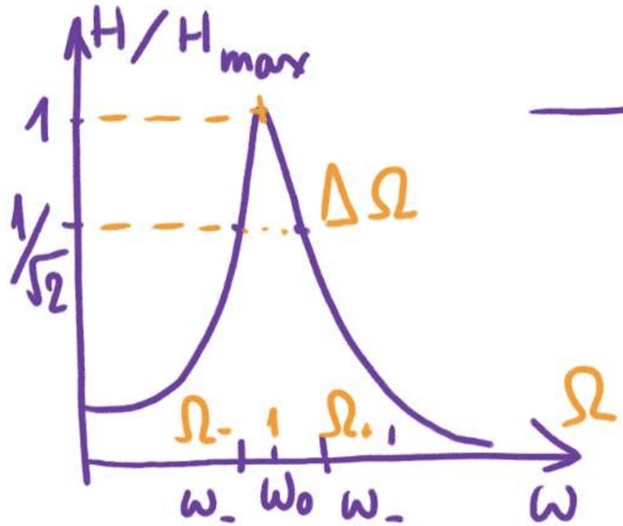
Half-power bandwidth

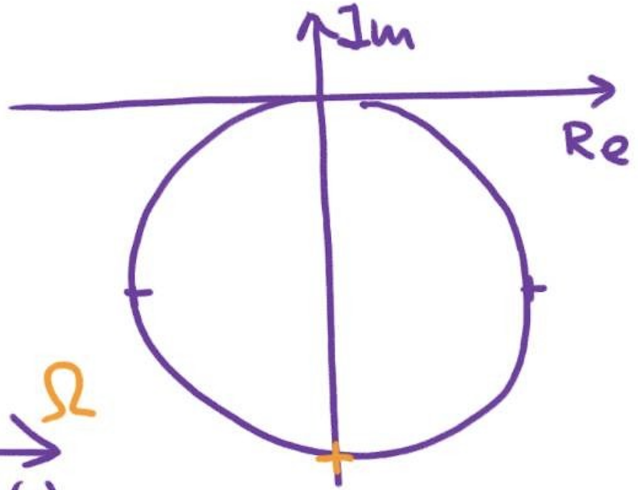
$$\zeta \approx \frac{1}{2}\Delta\Omega \approx \frac{2\Delta\omega}{\omega_{\max}}$$

$$\Delta\omega = \omega_+ - \omega_-$$

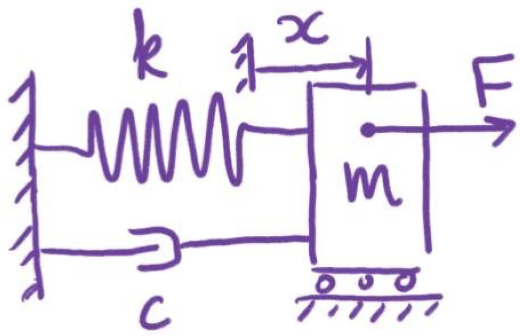
$$|H(\omega_\pm)| = \frac{|H_{\max}|}{\sqrt{2}}$$

practical but limited approximation





Time domain methods



Analytical : Duhamel integral

seeing $F(t)$ as a "sequence of pulses"

$$x(t) = x_0\dot{h} + v_0 h t + \int\limits_0^t F(\tau)h(t-\tau)d\tau$$

with $h(t)$ impulse response function

$$h(t) = \frac{1}{m\,\omega_d} e^{\zeta\omega_0 t}\sin\omega_d t$$

Numerical : Time marching

Direct integration (Newmark method)
or
Cauchy form $\dot{\mathbf{x}} = \phi(\mathbf{x}, t)$ with $\mathbf{x} = [x, v]^{\mathrm{T}}$
(Runge-Kutta, Adams methods etc)

$$\begin{cases} \dot{x} = v \\ \dot{v} = \frac{1}{m}(-k\ x - c\ v + F(t)) \\ x(0) = x_0,\ \ v(0) = v_0 \end{cases}$$

(cf also state space)

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{x}(0) = [x_0, v_0]^{\mathrm{T}} \end{cases}$$

Frequency domain methods. Fourier and Laplace

Fourier transform

$$\left(-m\,\omega^2 + \boldsymbol{i}\,c\,\omega + k\right) X(\boldsymbol{i}\omega) = F(\boldsymbol{i}\omega)$$

$$X(\boldsymbol{i}\omega) = H(\boldsymbol{i}\omega)F(\boldsymbol{i}\omega)$$

$$\begin{aligned} H(\boldsymbol{i}\omega) &= \frac{1}{-m\,\omega^2 + \boldsymbol{i}c\,\omega + k} \\ &= \frac{1}{m\left(-\omega^2 + 2\boldsymbol{i}\zeta\omega\omega_0 + \omega_0^2\right)} \\ &= \frac{1}{k(-\Omega^2 + 2\boldsymbol{i}\zeta\Omega + 1)} \end{aligned}$$

Laplace transform

$$\left(m\,s^2 + c\,s + k\right) X(s) + (\mathrm{IC}) = F(s)$$

$$X(s) = H(s)\left(F(s) - (\mathrm{IC})\right)$$

$$\begin{aligned} H(s) &= \frac{1}{m\,s^2 + c\,s + k} \\ &= \frac{1}{m}\left(\frac{1}{s+\lambda_1} + \frac{1}{s+\lambda_2}\right) \end{aligned}$$

$$\lambda_{1,2} = \omega_0\,\left(-\zeta \pm \boldsymbol{i}\sqrt{1-\zeta^2}\right)$$

$$(\mathrm{IC}) = k\,x(0) + s\,c\,\dot{x}(0)$$

Free vibrations of basic conservative systems

Equations of motion

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0$$

Eigenproblem

Fundamental solutions $\mathbf{q} = \phi e^{\lambda t}$

$$(\mathbf{M}\lambda + \mathbf{K})\phi = 0$$

Characteristic equation

$$\det(\mathbf{M}\lambda + \mathbf{K}) = 0$$

Eigenmodes

- eigenvalues $\lambda_i = -\omega_i^2$
  $\rightarrow \omega_i$ eigenfrequencies
  $\rightarrow \mathbf{\Omega} = \mathrm{diag}\omega_i$
- eigenvectors ϕ_i
  $\rightarrow$ modal shapes
  $\rightarrow \mathbf{\Phi} = [\phi_1 \ldots \phi_n]$
- Rayleigh quotient $\omega_i^2 = \frac{\phi_i^{\mathsf{T}} \mathbf{K} \phi_i}{\phi_i^{\mathsf{T}} \mathbf{M} \phi_i}$

Eigenmodes. Orthogonality, normalization, modal expansion

Orthogonality

$$\begin{cases} \phi_i^T \mathbf{M} \phi_i = \mu_i & \text{modal mass} \\ \phi_i^T \mathbf{M} \phi_j = 0 \text{ if } i \neq j \end{cases}$$

$$\begin{cases} \phi_i^T \mathbf{K} \phi_i = \kappa_i \text{ modal stiffness} \\ \phi_i^T \mathbf{K} \phi_j = 0 \text{ if } i \neq j \end{cases}$$

Normalization

different conventions

$$\begin{array}{ll} \bar{\mu}_i = 1 \; \forall i & (\text{norm wrt mass}, \Rightarrow \hat{\kappa}_j = \omega_j^2) \\ \hat{\phi}_{ji} = 1 & (\text{norm wrt } j^{th} \text{ DOF}) \\ & \hat{\kappa}_i = k_{jj},\; \hat{\mu}_i = \frac{k_{jj}}{\omega_i^2} \text{ apparent mass} \end{array}$$

Modal expansion

$$\mathbf{q} = \sum \phi_i \tilde{q}_i = \mathbf{\Phi} \tilde{\mathbf{q}}$$

$$\tilde{\mathbf{M}} = \mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi}$$

$$\tilde{\mathbf{K}} = \mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi}$$

$$\ddot{\tilde{\mathbf{q}}} + \mathbf{\Omega}^2 \tilde{\mathbf{q}} = 0$$

Uncoupled : $\ddot{\tilde{q}}_i + \omega_i^2 \tilde{q}_i = 0$

$$\mathbf{\Omega} = \text{diag}\omega_i$$

$$\mathbf{\Omega}^2 = \begin{bmatrix} \omega_1^2 & 0 & \dots & 0 \\ 0 & \omega_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \omega_n^2 \end{bmatrix} = \tilde{\mathbf{M}}^{-1} \tilde{\mathbf{K}}$$

Free vibrations of basic dissipative systems

Linear dissipative forces

(viscous damping) : $\mathbf{f}_{\text{nc}} = -\mathbf{C}\dot{\mathbf{q}}$

Equations of motion

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0$$

Cons. Eigenproblem

$$(\mathbf{M}\lambda + \mathbf{K})\phi = 0 \;\; \Rightarrow \omega_i, \phi_i$$

Modal expansion

$$\ddot{\tilde{\mathbf{q}}} + \mathbf{Z}\dot{\tilde{\mathbf{q}}} + \mathbf{\Omega}^2\tilde{\mathbf{q}} = 0$$

with $\mathbf{Z} = \tilde{\mathbf{C}} = \mathbf{\Phi}^{\mathsf{T}}\mathbf{C}\mathbf{\Phi}$

Modal damping

if $\mathbf{Z}$ is diagonal, damping is modal :

$$\mathbf{Z} = \mathrm{diag} z_i$$

$$z_i = 2\zeta_i\omega_i = \phi_i^{\mathsf{T}}\mathbf{C}\phi_i$$

- Rayleigh damping : $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$

Forced response computation

Matrix equations of motion

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}$$

Time domain

$$\dot{\mathbf{y}} = \mathbf{g}(t, \mathbf{y}); \qquad \mathbf{y} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$$

$$\mathbf{y} = \mathbf{y}_0 + \int_0^t \mathbf{g}(\tau, \mathbf{y}) d\tau$$

$$\mathbf{g} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \mathbf{y} + \begin{bmatrix} 0 \\ \mathbf{f} \end{bmatrix}$$

Frequency domain

$$\left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)\mathbf{Q} = \mathbf{F}$$

$$\mathbf{Q}(\omega) = \mathbf{H}(\omega)\mathbf{F}(\omega)$$

$$\mathbf{H} = \left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)^{-1}$$

Forced response modal expansion

Modal equations of motion : modal approach

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}$$

$$\ddot{\tilde{\mathbf{q}}} + \mathbf{Z}\dot{\tilde{\mathbf{q}}} + \mathbf{\Omega}^2\tilde{\mathbf{q}} = \tilde{\mathbf{f}}$$

$$\tilde{\mathbf{Q}}(\omega) = \tilde{\mathbf{H}}(\omega)\tilde{\mathbf{F}}(\omega)$$

$$\tilde{\mathbf{H}}(\omega) = \mathrm{diag}\,\frac{1}{\mu_k\,\left(\omega_k^2 + 2\boldsymbol{i}\zeta_k\omega_k\omega - \omega^2\right)}$$

Modal expansion of physical transfer function

$$\begin{aligned}\mathbf{H} &= \left(-\omega^2\mathbf{M} + \boldsymbol{i}\omega\mathbf{C} + \mathbf{K}\right)^{-1} \\ &= \mathbf{\Phi}\tilde{\mathbf{H}}\mathbf{\Phi}^{\mathsf{T}}\end{aligned}$$

$$h_{ij}(\omega) = \sum_k \frac{\phi_{ik}\phi_{jk}}{\mu_k\,\left(\omega_k^2 + 2\boldsymbol{i}\zeta_k\omega_k\omega - \omega^2\right)}$$

modal estimation* : $h_{ii}(\omega_k) \approx \frac{\bar{\phi}_{ik}^2}{2\boldsymbol{i}\zeta_k\omega_k^2}$

accelerance : $h_{ii}^{(\mathrm{acc})}(\omega_k) \approx \frac{\bar{\phi}_{ik}^2}{2\boldsymbol{i}\zeta_k}$

*isolated modes, low damping

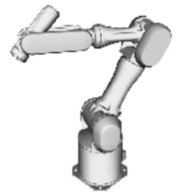
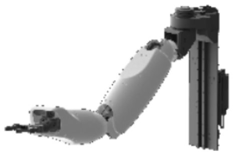
Flexibility in robots

Intrinsic sources of flexibility

- Joints
  - Gears, Harmonic drives
  - Belt/cable transmission
  - Fluid/ducts deformability of hydraulic actuator
- Links
  - New materials
  - slender designs
- Effectors/grips

General trends

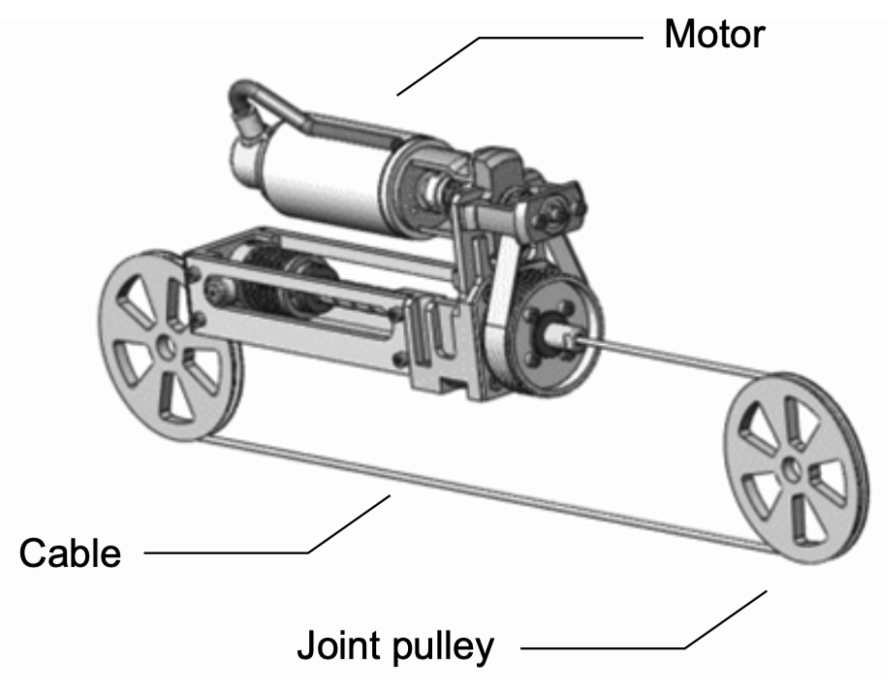
- Increase payload / reach
- Reduce mass
- Light-weight robotics

Robot	Specifications*	Actuation technology and instrumentation
 KUKA-DLR LWR	$m_r = 16$ kg $m_c = 7$ kg $l^p = 0.936$ m	– Harmonic Drive® gears – integrated strain gauges, motor and joint position sensors
 Barrett WAM	$m_r^{\text{arm}} = 5.8$ kg $m_r^{\text{total}} = 27$ kg $m_c = 3$ kg $l^p = 1$ m	– gearless cable transmission – motor (and joint) position sensors
 Mitsubishi PA10-7CE	$m_r = 38$ kg $m_c = 10$ kg $l^a = 0.93$ m	– AC servo motors, Harmonic Drive® gears – motor position sensors
 ASSIST arm (CEA-LIST)	$m_r = 9.3$ kg $m_c = 3$ kg $l^a = 0.8$ m	– cable transmissions, gear-motors – motor position sensors

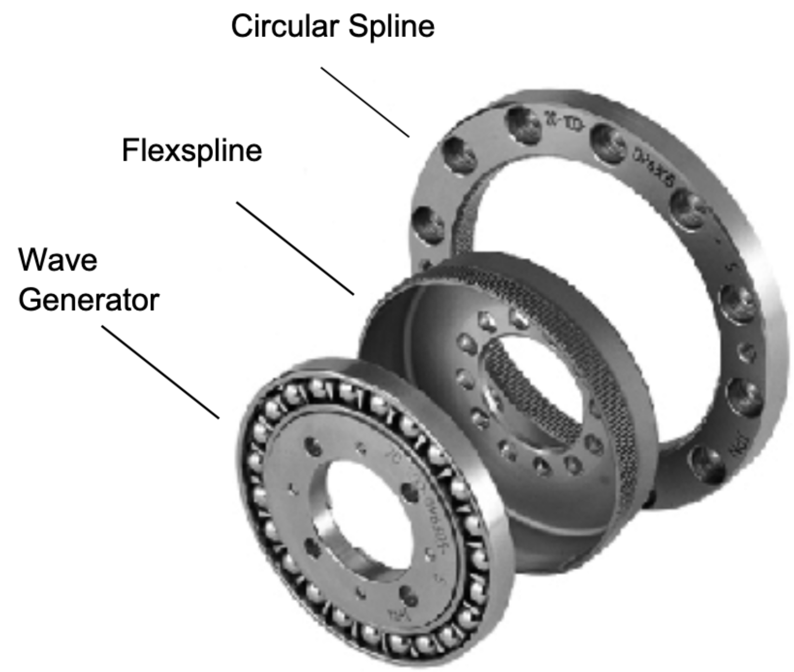
*m_r, robot mass; m_c, payload; l, characteristic dimension (maximum distance between joints l^a or reach l^p).

ref [Makarov & Grossard 13]

Flexibility in robots



a) Actuator with cable transmission (CEA)



b) Harmonic Drive® gear

ref [Makarov & Grossard 13]

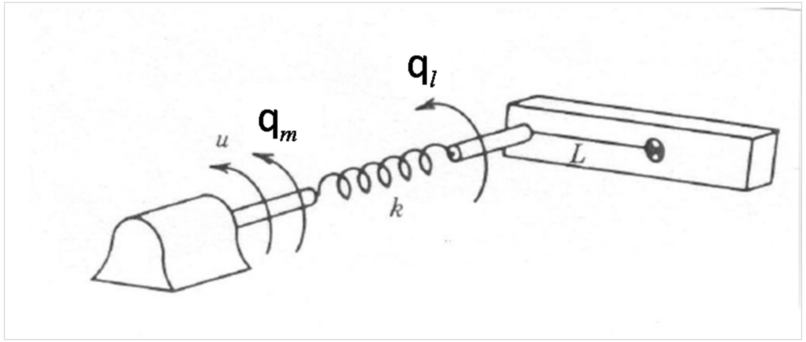
Modeling approaches for flexible in robots

- level 1 : Flexible joints
- level 2 : Flexible links/grips

Joint flexibility

Equation of motion for 1 link system :

$$
\begin{aligned}
J_l \ddot{q}_l + B_l \dot{q}_l + k(q_l - q_m) &= 0 \\
J_m \ddot{q}_m + B_m \dot{q}_m - k(q_l - q_m) &= u
\end{aligned}
$$



Joint flexibility-related assumptions

- Elastic deformations are concentrated at the joint level and the robot is composed of rigid links connected to each other by linear torsion springs with constant stiffness (a realistic hypothesis for deformations of low amplitudes).
- The motor rotors do not present any movement other than that around their rotation axis and do not present any eccentricity, i.e. they can be modelled as uniform bodies for which the center of mass is located on their rotation axis. This implies that the inertia matrix and the gravity term of the system are independent of the angular positions of the motor.
- Kinetic energy of the rotors is due solely to their own rotation, which amounts to neglecting the inertial coupling between the motors and the segments of the robot.

[Makarov et Grossard 2008]

Serial system dynamics with flexible joints

Rigid system

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{f}} + \boldsymbol{\tau}_{\mathrm{ext}}$$

Flexible joints : $\tau_i \leftarrow \tau_{ki} = k_i(p_i - q_i)$

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{S}(\mathbf{q})\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{c}_1(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{p}}) + \mathbf{g}(\mathbf{q}) + \mathbf{K}(\mathbf{q}-\mathbf{p}) = \boldsymbol{\tau}_{\mathrm{fl}} + \boldsymbol{\tau}_{\mathrm{ext}}$$

$$\mathbf{S}^{\mathsf{T}}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{J}_m(\mathbf{q})\,\ddot{\mathbf{p}} + \mathbf{c}_2(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{p}}) - \mathbf{K}(\mathbf{q}-\mathbf{p}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{fm}}$$

with $\mathbf{S}$, $\mathbf{c}_1$, $\mathbf{c}_2$ inertial coupling terms

$\mathbf{J}_{\mathrm{m}}$ motorization inertia matrix $\mathbf{K}$ stiffness matrix (often assumed diagonal)

$\Rightarrow$ extended system

$$\hat{\mathbf{M}}(\hat{\mathbf{q}})\ddot{\hat{\mathbf{q}}} + \hat{\mathbf{C}}(\hat{\mathbf{q}},\dot{\hat{\mathbf{q}}})\dot{\hat{\mathbf{q}}} + \hat{\mathbf{K}}\hat{\mathbf{q}} + \hat{\mathbf{g}}(\hat{\mathbf{q}}) = \hat{\boldsymbol{\tau}} + \hat{\boldsymbol{\tau}}_{\mathrm{f}} + \hat{\boldsymbol{\tau}}_{\mathrm{ext}}$$

Small perturbations in joints

Simplified equations $(\mathbf{S}, \mathbf{c}_1, \mathbf{c}_2 \approx \mathbf{0})$

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{K}(\mathbf{q} - \mathbf{p}) = \boldsymbol{\tau}_{\mathrm{fj}} + \boldsymbol{\tau}_{\mathrm{ext}}$$

$$\mathbf{J}_{\mathrm{m}}\,\ddot{\mathbf{p}} - \mathbf{K}(\mathbf{q} - \mathbf{p}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{fm}}$$

NB $\mathbf{J}_{\mathrm{m}}$, $\mathbf{K}$, $\boldsymbol{\tau}_{\mathrm{fm}}$ to be identified in addition to rigid model

Alternative writing with $\mathbf{q} = \mathbf{p} + \mathbf{w}$

$$\mathbf{M}(\mathbf{p})(\ddot{\mathbf{p}} + \ddot{\mathbf{w}}) + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}} + \dot{\mathbf{w}})\,(\dot{\mathbf{p}} + \dot{\mathbf{w}}) + \mathbf{g}(\mathbf{p}) + \mathbf{K}\mathbf{w} = \boldsymbol{\tau}_{\mathrm{fj}} + \boldsymbol{\tau}_{\mathrm{ext}}$$

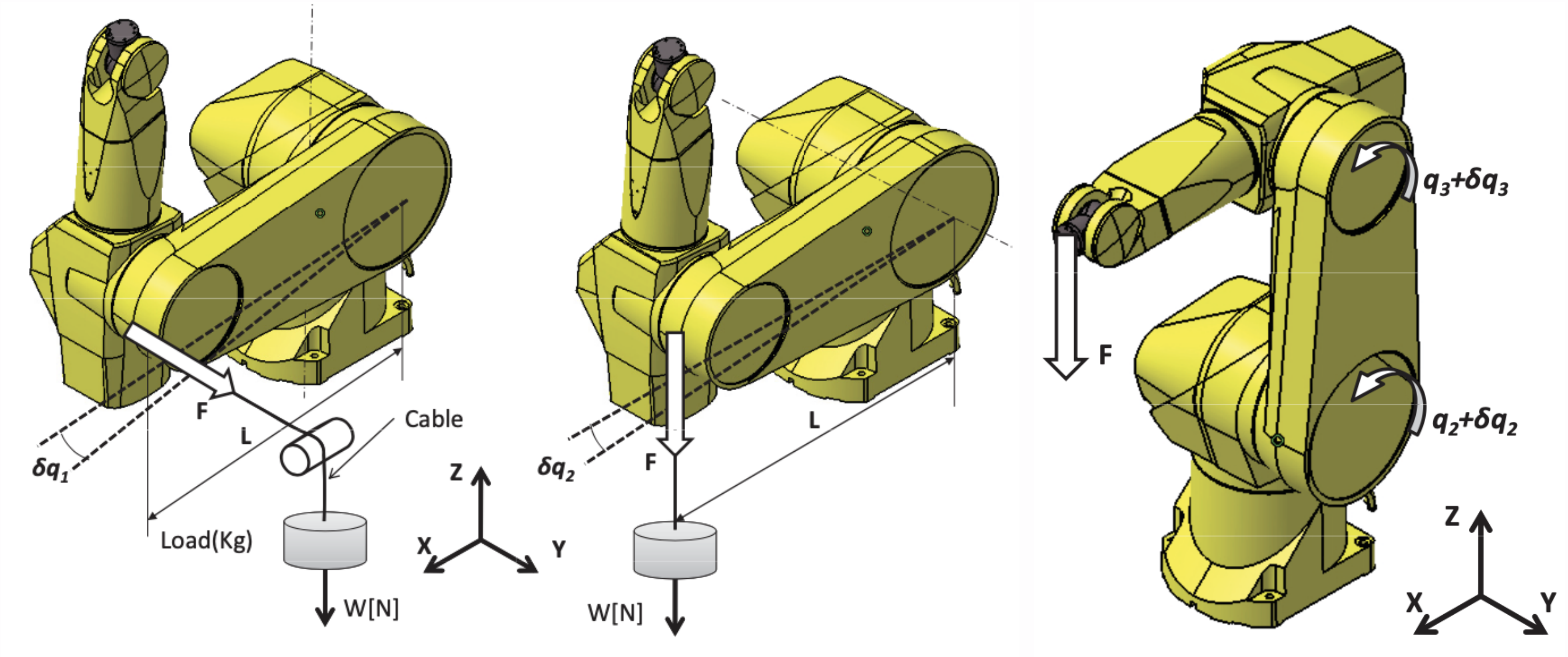
$$\mathbf{J}_{\mathrm{m}}\,\ddot{\mathbf{p}} - \mathbf{K}\mathbf{w} = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{fm}}$$

Modal analysis in configuration $\mathbf{p}$, with $\dot{\mathbf{p}}, \ddot{\mathbf{p}} = 0$

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{w}} + \mathbf{C}(\mathbf{p}, \mathbf{0})\,\dot{\mathbf{w}} + \mathbf{K}\mathbf{w} = \mathbf{0} \quad \Rightarrow \quad \left(\lambda \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{K} & \mathbf{C} \end{bmatrix}\right)\mathbf{V} = \mathbf{0}$$

Local identification

Static tests per joint



ref [Olabi et al 12]

Global identification

Exciting trajectories

$$\hat{\mathbf{Y}}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})\,\hat{\pi} = \hat{\tau}$$

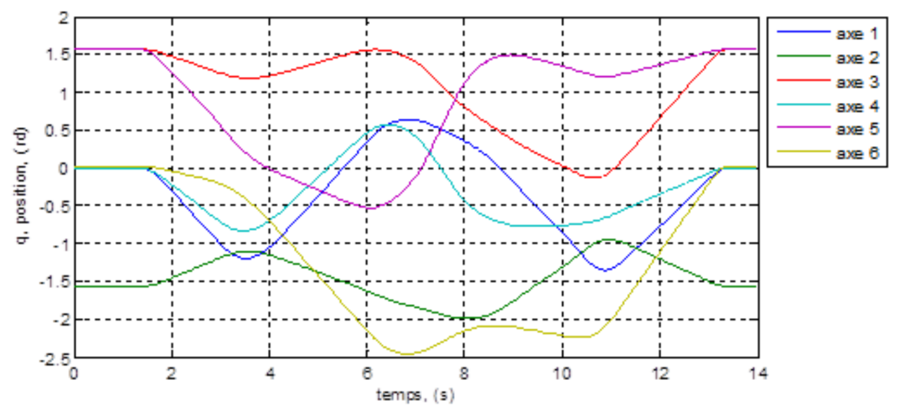


FIGURE 4.5 – Positions articulaires mesurées pour l'essai du Kuka KR500-2MT

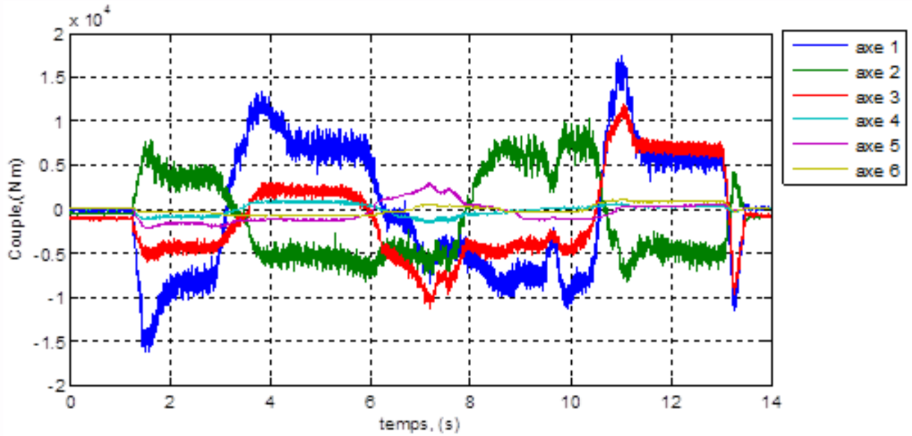
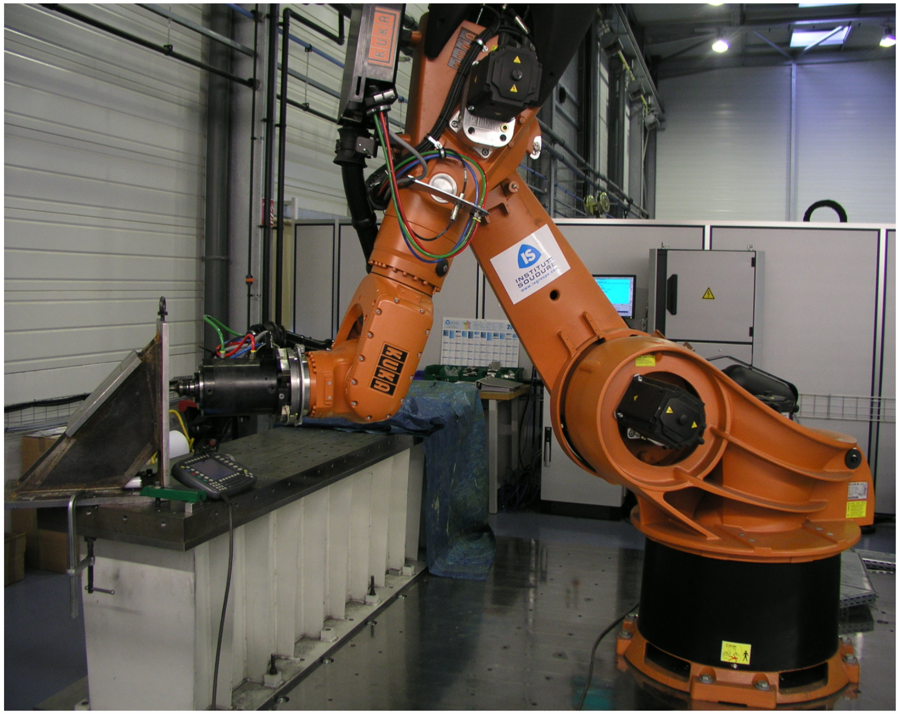


FIGURE 4.6 – Couples mesurés pour l'essai du Kuka KR500-2MT

Static tests

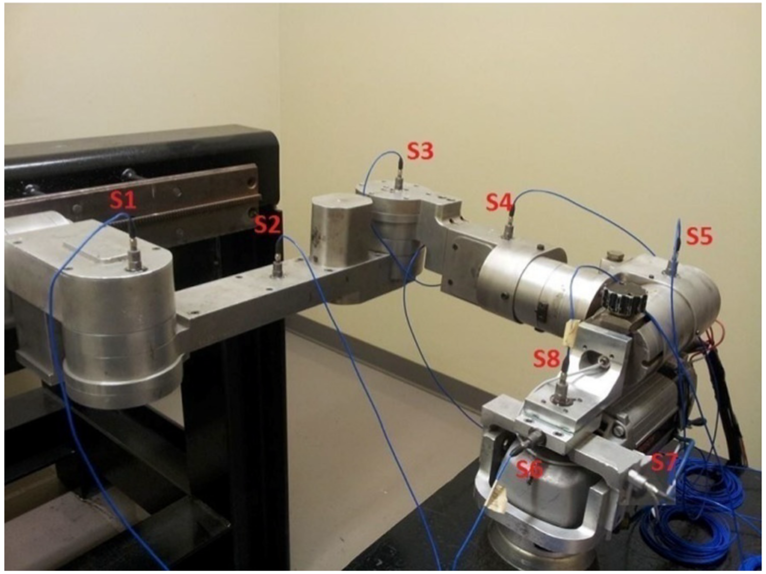
$$\bar{\mathbf{K}}\bar{\mathbf{w}} = \bar{\mathbf{f}}_{\text{ext}}$$

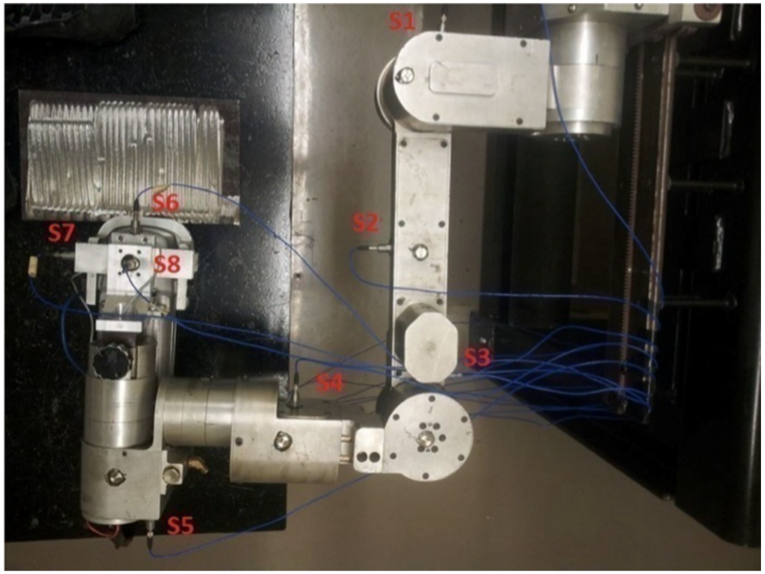
$$\bar{\mathbf{K}} = \mathbf{J}^{-\mathsf{T}}\mathbf{K}\mathbf{J}^{-1}, \quad \bar{\mathbf{w}} = \mathbf{J}\mathbf{w}$$



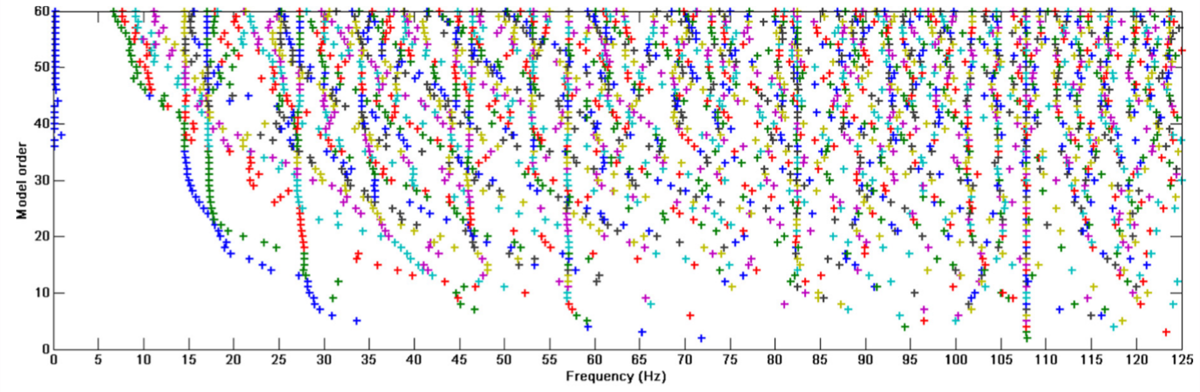
Analyse d'un robot avec flexible ref [Qin 13]

Global identification

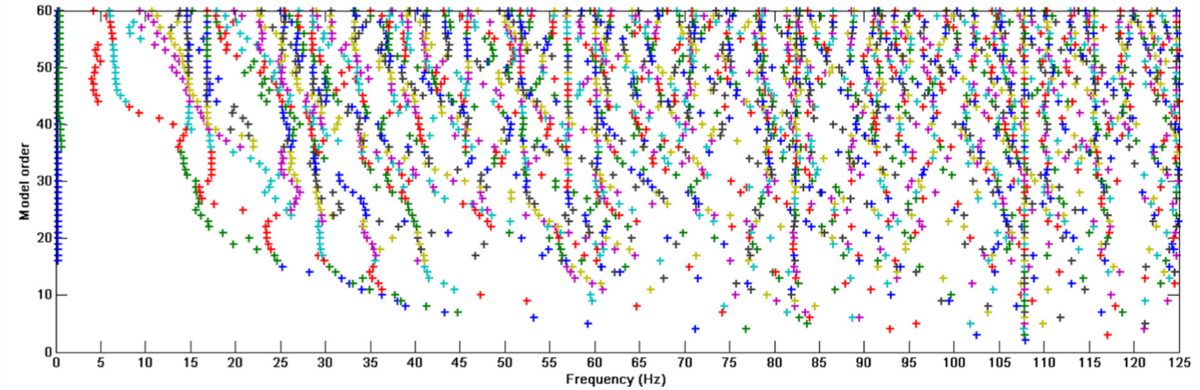




Modal analysis



a) Direction Z, vertical measurement



b) Direction Y, horizontal measurement

Analyse modale expérimentale [Vu 16]

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