MANIPULATOR MODELING

ACCOUNTING FOR FLEXIBILITIES

M2R SAR

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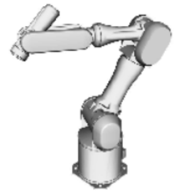
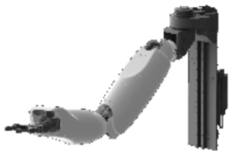
Flexibility in robots

Intrinsic sources of flexibility

- Joints
  - Gears, Harmonic drives
  - Belt/cable transmission
  - Fluid/ducts deformability of hydraulic actuator
- Links
  - New materials
  - slender designs
- Effectors/grips

General trends

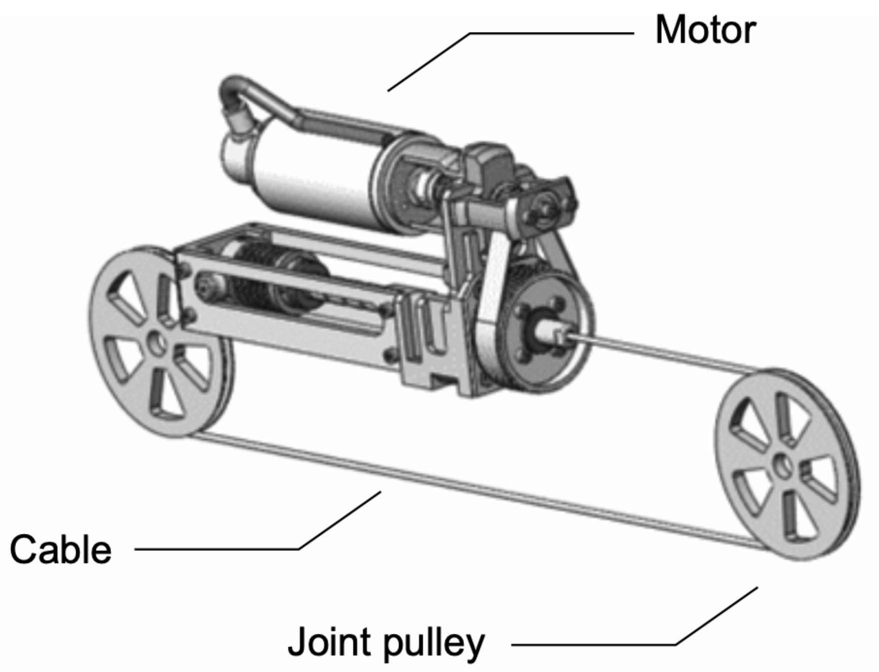
- Increase payload / reach
- Reduce mass
- Light-weight robotics

Robot	Specifications*	Actuation technology and instrumentation
 KUKA-DLR LWR	$m_r = 16$ kg $m_c = 7$ kg $l^p = 0.936$ m	– Harmonic Drive® gears – integrated strain gauges, motor and joint position sensors
 Barrett WAM	$m_r^{\text{arm}} = 5.8$ kg $m_r^{\text{total}} = 27$ kg $m_c = 3$ kg $l^p = 1$ m	– gearless cable transmission – motor (and joint) position sensors
 Mitsubishi PA10-7CE	$m_r = 38$ kg $m_c = 10$ kg $l^a = 0.93$ m	– AC servo motors, Harmonic Drive® gears – motor position sensors
 ASSIST arm (CEA-LIST)	$m_r = 9.3$ kg $m_c = 3$ kg $l^a = 0.8$ m	– cable transmissions, gear-motors – motor position sensors

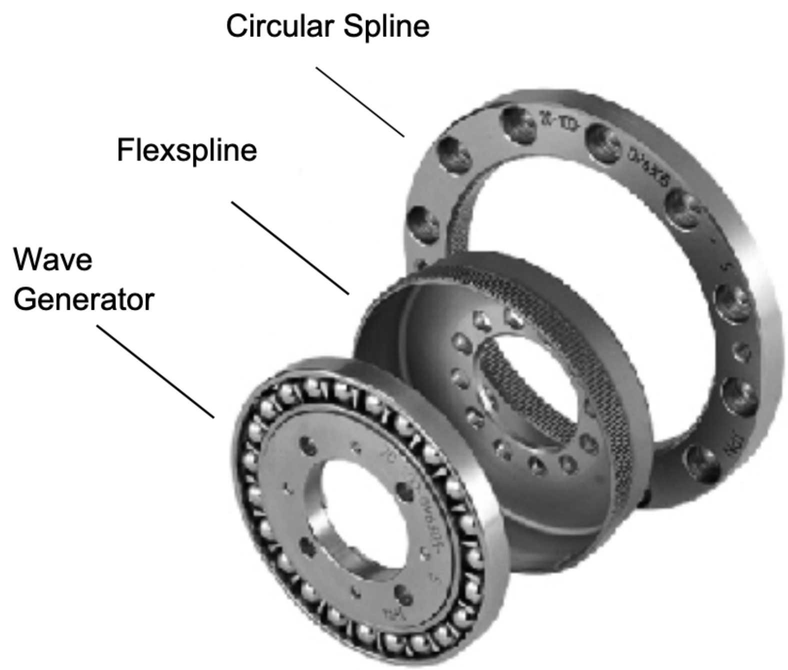
*m_r, robot mass; m_c, payload; l, characteristic dimension (maximum distance between joints l^a or reach l^p).

ref [Makarov & Grossard 13]

Flexibility in robots



a) Actuator with cable transmission (CEA)



b) Harmonic Drive® gear

ref [Makarov & Grossard 13]

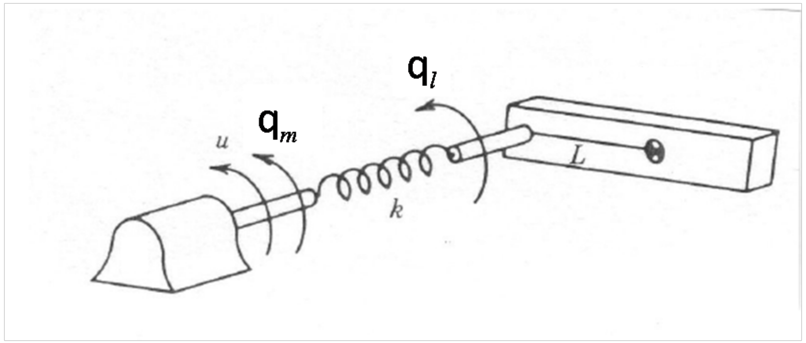
Modeling approaches for flexible in robots

- level 1 : Flexible joints
- level 2 : Flexible links/grips

Joint flexibility

Equation of motion for 1 link system :

$$
\begin{aligned}
J_l \ddot{q}_l + B_l \dot{q}_l + k(q_l - q_m) &= 0 \\
J_m \ddot{q}_m + B_m \dot{q}_m - k(q_l - q_m) &= u
\end{aligned}
$$



Joint flexibility-related assumptions

- Elastic deformations are concentrated at the joint level and the robot is composed of rigid links connected to each other by linear torsion springs with constant stiffness (a realistic hypothesis for deformations of low amplitudes).
- The motor rotors do not present any movement other than that around their rotation axis and do not present any eccentricity, i.e. they can be modelled as uniform bodies for which the center of mass is located on their rotation axis. This implies that the inertia matrix and the gravity term of the system are independent of the angular positions of the motor.
- Kinetic energy of the rotors is due solely to their own rotation, which amounts to neglecting the inertial coupling between the motors and the segments of the robot.

[Makarov et Grossard 2008]

Serial system dynamics with flexible joints

Rigid system

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{f}} + \boldsymbol{\tau}_{\mathrm{ext}}$$

Flexible joints : $\tau_i \ \leftarrow \ \tau_{ki} = k_i(p_i - q_i)$

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{S}(\mathbf{q})\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{c}_1(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{p}}) + \mathbf{g}(\mathbf{q}) + \mathbf{K}(\mathbf{q}-\mathbf{p}) = \boldsymbol{\tau}_{\mathrm{fl}} + \boldsymbol{\tau}_{\mathrm{ext}}$$

$$\mathbf{S}^{\mathsf{T}}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{J}_m(\mathbf{q})\,\ddot{\mathbf{p}} + \mathbf{c}_2(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{p}}) - \mathbf{K}(\mathbf{q}-\mathbf{p}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{fm}}$$

with $\mathbf{S}$, $\mathbf{c}_1$, $\mathbf{c}_2$ inertial coupling terms

$\mathbf{J}_{\mathrm{m}}$ motorization inertia matrix $\mathbf{K}$ stiffness matrix (often assumed diagonal)

$\Rightarrow$ extended system

$$\hat{\mathbf{M}}(\hat{\mathbf{q}})\ddot{\hat{\mathbf{q}}} + \hat{\mathbf{C}}(\hat{\mathbf{q}},\dot{\hat{\mathbf{q}}})\dot{\hat{\mathbf{q}}} + \hat{\mathbf{K}}\hat{\mathbf{q}} + \hat{\mathbf{g}}(\hat{\mathbf{q}}) = \hat{\boldsymbol{\tau}} + \hat{\boldsymbol{\tau}}_{\mathrm{f}} + \hat{\boldsymbol{\tau}}_{\mathrm{ext}}$$

Small perturbations in joints

Simplified equations ($\mathbf{S}, \mathbf{c}_1, \mathbf{c}_2 \approx \mathbf{0}$)

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{K}(\mathbf{q} - \mathbf{p}) = \boldsymbol{\tau}_{\mathrm{fj}} + \boldsymbol{\tau}_{\mathrm{ext}}$$

$$\mathbf{J}_{\mathrm{m}}\,\ddot{\mathbf{p}} - \mathbf{K}(\mathbf{q} - \mathbf{p}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{fm}}$$

NB $\mathbf{J}_{\mathrm{m}}$, $\mathbf{K}$, $\boldsymbol{\tau}_{\mathrm{fm}}$ to be identified in addition to rigid model

Alternative writing with $\mathbf{q} = \mathbf{p} + \mathbf{w}$

$$\mathbf{M}(\mathbf{p})(\ddot{\mathbf{p}} + \ddot{\mathbf{w}}) + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}} + \dot{\mathbf{w}})\,(\dot{\mathbf{p}} + \dot{\mathbf{w}}) + \mathbf{g}(\mathbf{p}) + \mathbf{K}\mathbf{w} = \boldsymbol{\tau}_{\mathrm{fj}} + \boldsymbol{\tau}_{\mathrm{ext}}$$

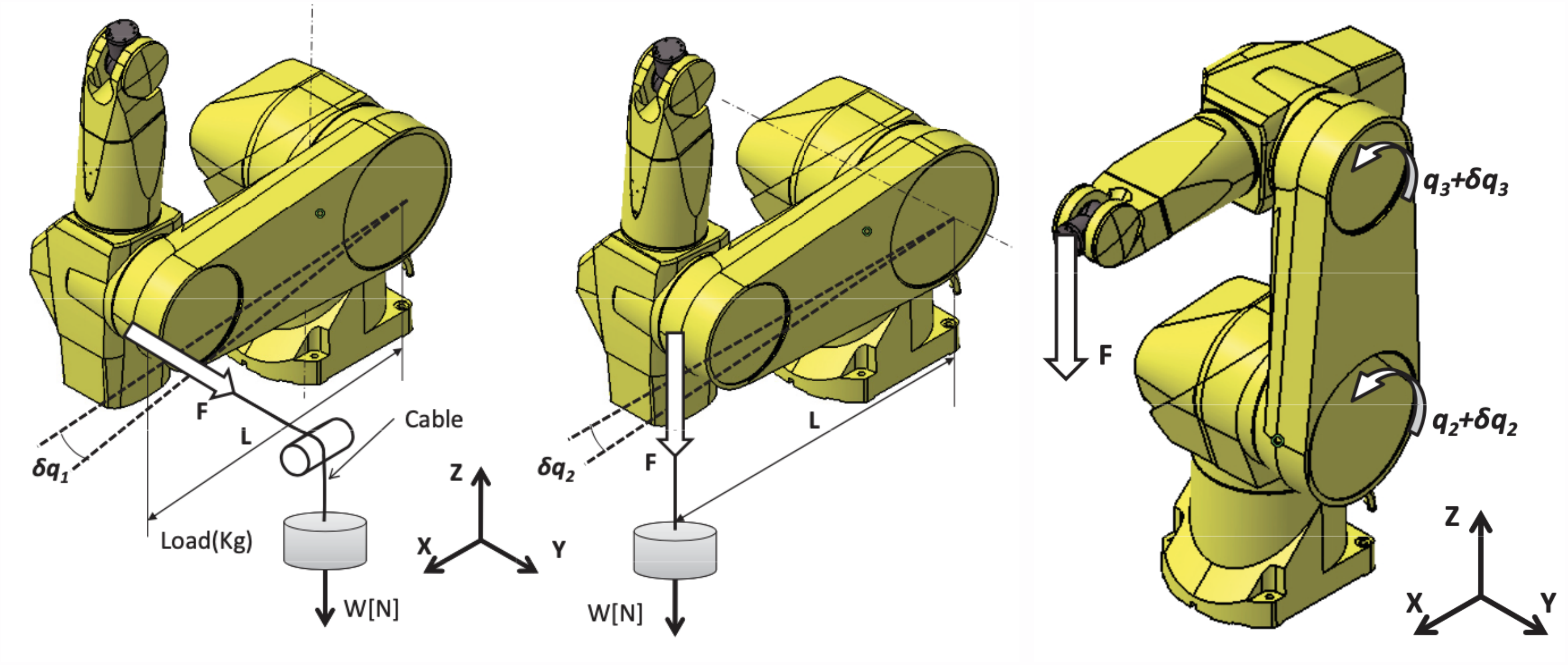
$$\mathbf{J}_{\mathrm{m}}\,\ddot{\mathbf{p}} - \mathbf{K}\mathbf{w} = \boldsymbol{\tau} + \boldsymbol{\tau}_{\mathrm{fm}}$$

Modal analysis in configuration $\mathbf{p}$, with $\dot{\mathbf{p}}, \ddot{\mathbf{p}} = 0$

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{w}} + \mathbf{C}(\mathbf{p}, \mathbf{0})\,\dot{\mathbf{w}} + \mathbf{K}\mathbf{w} = \mathbf{0} \quad \Rightarrow \quad \left(\lambda \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{K} & \mathbf{C} \end{bmatrix}\right) \mathbf{V} = \mathbf{0}$$

Local identification

Static tests per joint



ref [Olabi et al 12]

Global identification

Exciting trajectories

$$\hat{\mathbf{Y}}(\hat{\mathbf{q}}, \dot{\hat{\mathbf{q}}}, \ddot{\hat{\mathbf{q}}})\,\hat{\boldsymbol{\pi}} = \hat{\boldsymbol{\tau}}$$

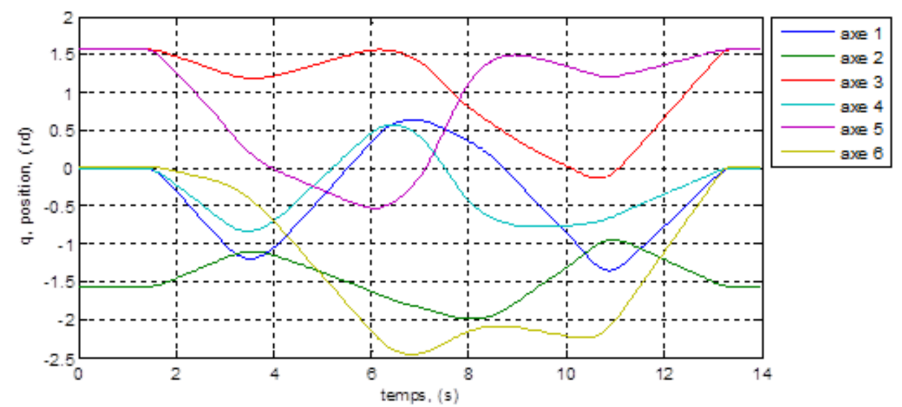


FIGURE 4.5 – Positions articulaires mesurées pour l'essai du Kuka KR500-2MT

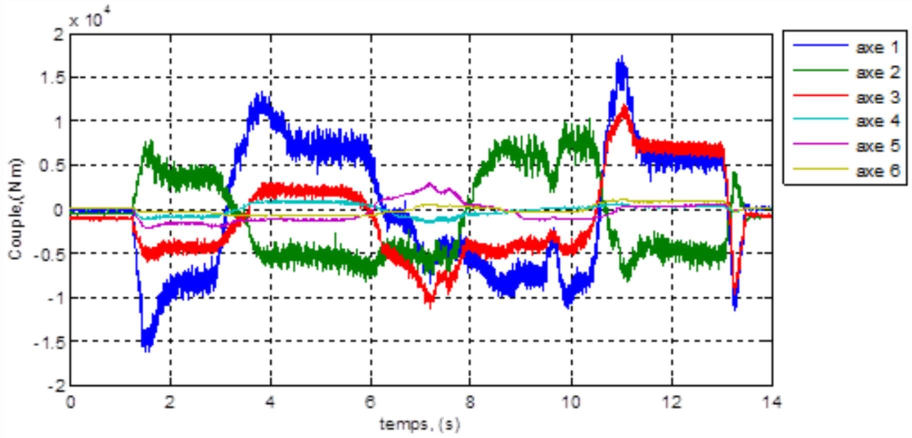
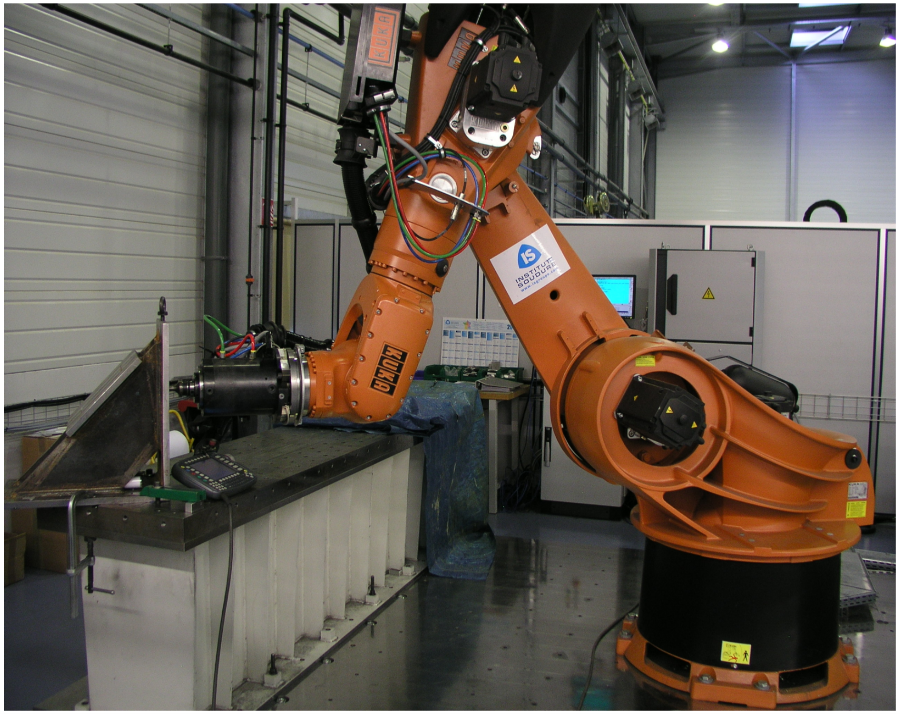


FIGURE 4.6 – Couples mesurés pour l'essai du Kuka KR500-2MT

Static tests

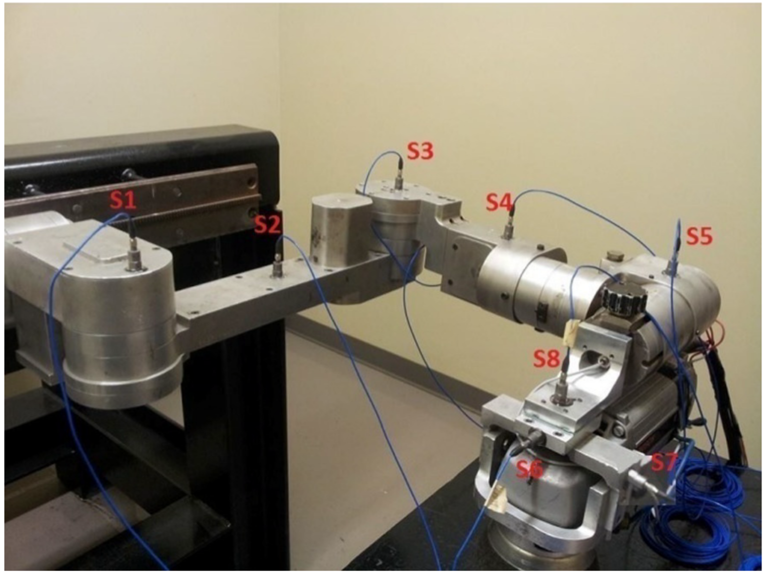
$$\bar{\mathbf{K}}\bar{\mathbf{w}} = \bar{\mathbf{f}}_{\mathrm{ext}}$$

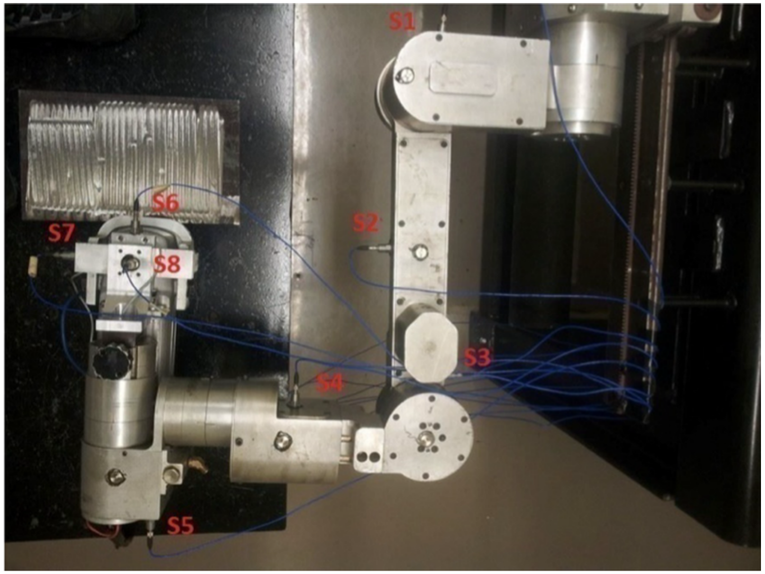
$$\bar{\mathbf{K}} = \mathbf{J}^{-\mathsf{T}}\mathbf{K}\mathbf{J}^{-1}, \quad \bar{\mathbf{w}} = \mathbf{J}\mathbf{w}$$



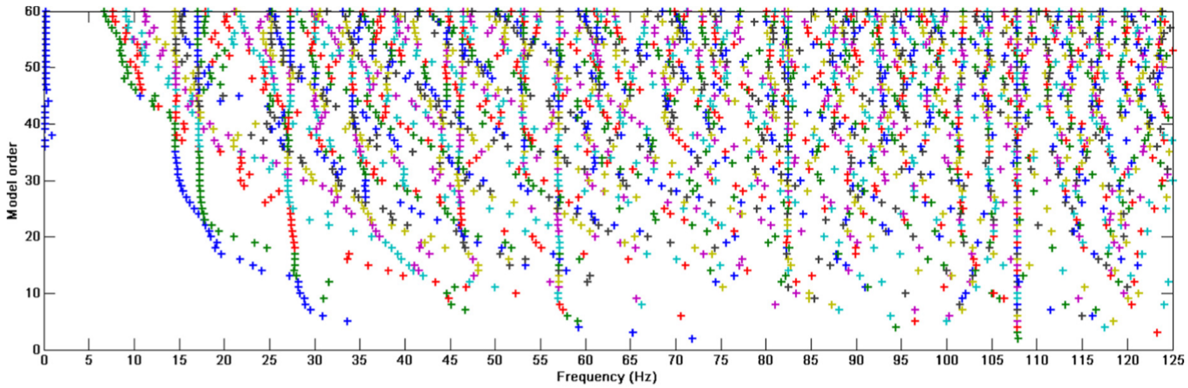
Analyse d'un robot avec flexible ref [Qin 13]

Global identification

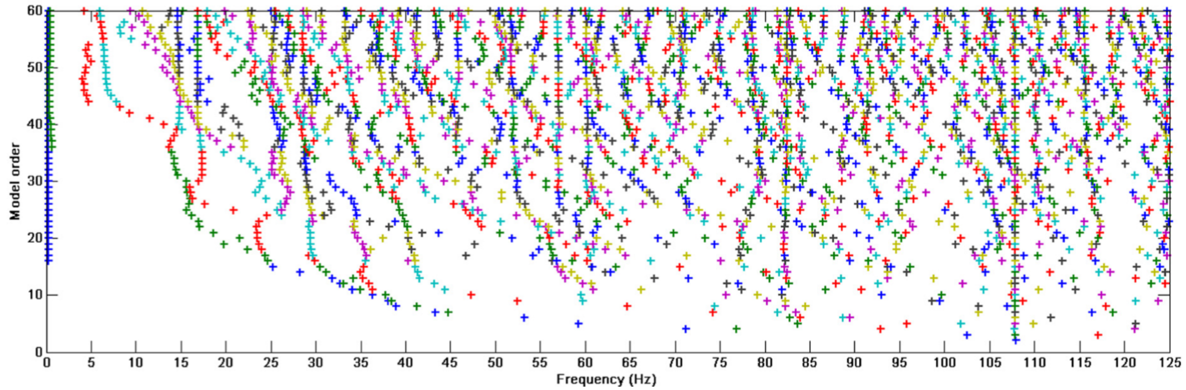




Modal analysis



a) Direction Z, vertical measurement



b) Direction Y, horizontal measurement

Analyse modale expérimentale [Vu 16]

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