Linear control I

Sorbonne University

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Tutorial 2: Output feedback control strategies for LTI systems

Problem:

Let's consider the output feedback control scheme of Fig.1 where $G(s)$ and $C(s)$ are respectively the transfer functions of the controlled system and that of the controller.

$G(s) = \frac{100}{(10^{-3}s+1)(10^{-4}s+1)}$.

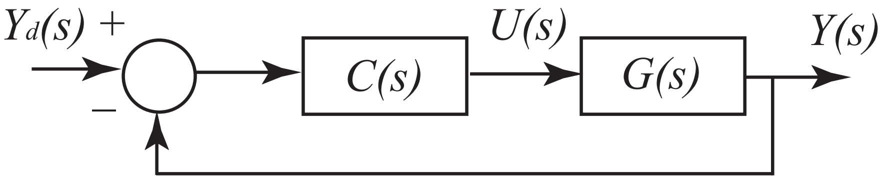

Fig. 1: Output feedback control scheme.

The objective is to design a dynamic controller $C(s)$ satisfying specifications in terms of stability, bandwidth and precision for the closed loop system. Three different kinds of control strategies will be studied.

Proportional control:

Let's consider a proportional controller with the transfer function $C(s) = C_1(s) = K_p$.

1. Calculate the gain K_p so that the phase margin is equal to $\frac{\pi}{4}$. Let's note $\omega_{0db} = \omega_0$ the gain crossover frequency for the calculated K_p .
2. What is the value of the steady state tracking error in closed loop?
3. Is it possible to find a value of K_p so that $\omega_{0db} = \omega_1 = 2 \times \omega_0$ while keeping the phase margin at $\frac{\pi}{4}$?
4. Conclude on the drawbacks of the proportional controller.

Phase-lead control:

Let's consider a phase-lead controller with the transfer function $C(s) = C_2(s) = K\frac{1+a\tau s}{1+\tau s}$ with $a > 1$. The maximum phase φ_M produced by this controller is obtained at $\omega_M = \frac{1}{\tau\sqrt{a}}$, where:
$Sin(\varphi_M) = \frac{a-1}{a+1}$

1. Compute the parameters of $C_2(s)$ satisfying the following specifications:
   (a) Phase margin equal to $\frac{\pi}{4}$.
   (b) Gain crossover frequency of the open loop system $\omega_1 = 2 \times \omega_0$.
   (c) Optimal control design, i.e. $\omega_M = \omega_1$.
2. What is the value of the steady state tracking error in closed loop?
3. Conclude on the advantages and drawbacks of the phase-lead controller.

Dynamic controller with integral action:

The aim is to design a dynamic controller $C_3(s)$ so that the specifications in terms of bandwidth, stability margin and precision are all satisfied. More importantly, the objective is that the structure of $C_3(s)$ allows setting these three performances independently. The control specifications are as follows:

- Phase margin equal to $\frac{\pi}{4}$.
- Gain crossover frequency of the open loop system $\omega_1 = 2 \times \omega_0$.
- Steady state tracking error in closed loop equal to zero.

The controller $C_3(s)$ must be also able to compensate the poles of $G(c)$.

1. Propose a structure for $C_3(s)$.
2. Compute the parameters of $C_3(s)$ satisfying the control specifications.
3. Conclude on the control structures studied in this tutorial.

The closed loop step responses obtained with the controllers $C_1(p)$, $C_2(p)$ and $C_3(p)$ are shown in Fig.2.

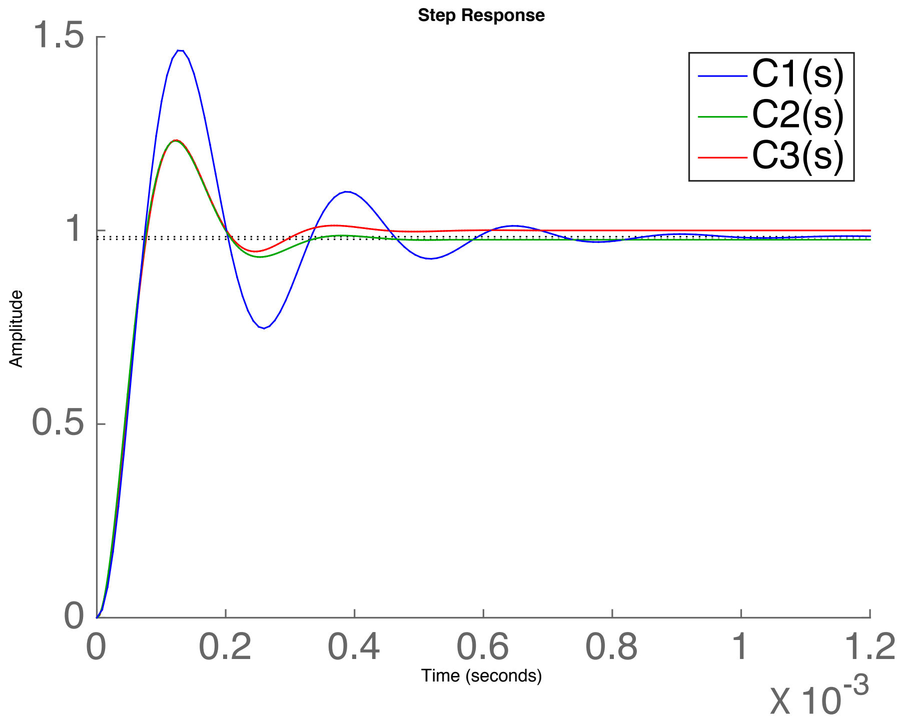

Fig. 2: Closed loop step responses obtained with the controllers $C_1(p)$, $C_2(p)$ and $C_3(p)$. The three controllers used to aobtain these curves have been designed so that the gain crossover frequency of the open loop system is $\omega_{0db} = \omega_1 = 2 \times \omega_0$.