Linear control I

2021-2021

Tutorial 1: Analysis of linear time-invariant systems in time and frequency domains

Exercise 1: modeling of a mechanical system

Lets consider the dynamic equation of a mass-spring-damper system as follows:

$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = e(t)$

$x(t)$ and $e(t)$ are respectively the output and the input of the system.

m, c and k are respectively the mass, the damping and the stiffness parameters.

1. Find the transfer function, $F(s)$, of this system.
2. When the input, $e(t)$, is a pulse of unit amplitude, the time response of the output is (it is assumed in this question that $c^2 > 4km$):

   $h(t) = \frac{1}{\sqrt{c^2-4km}}(e^{p_1 t} - e^{p_2 t})$

   with: $p_1 = -\frac{c}{2m} + \frac{1}{2m}\sqrt{c^2 - 4km}$ and $p_2 = -\frac{c}{2m} - \frac{1}{2m}\sqrt{c^2 - 4km}$
   Find the expression of the transfer function, $F(s)$, using the impulse response.
3. What are the poles, zeros and static gain of the transfer function $F(s)$?
4. Plot the asymptotic Bode diagram of the frequency response of $F(s)$ given $k > 1$ N/m.

Exercise 2: Bode plot

Plot the asymptotic Bode diagram of the following transfer functions:

1. $G_1(s) = \frac{1}{s^n}$, where $n > 0$
2. $G_2(s) = \frac{K}{s^2(1+\tau s)}$, where $0 < K < 1$ and $0 < \tau < 1$
3. $G_3(s) = \frac{Ks(1+\tau_1 s)}{(1+\tau_2 s)(1+\tau_3 s)}$, where $0 < K < 1$ and $0 < \tau_3 < \tau_2 < \tau_1 < 1$