Linear control for LTI systems

Dynamic control of a MEMS active force sensor

Although the "Fantastic Voyage"imagined by Isaac Asimov is still not possible, numerous scientific advances point to the possibility of piloting micro-robots that can transport and characterize objects as small as biological cells while sensing the interaction forces.

The development of sensors capable of measuring forces at the micro Newton and nano Newton scales has been a strategic research focus over the last 20 years to analyze interaction phenomena at very small scales. The need for extreme miniaturization of these measuring instruments has led to the abandonment of traditional engineering techniques for the fabrication of sensors in favor of clean room manufacturing processes. Cleanrooms have made the production of micro-electro-mechanical systems (MEMS) possible. MEMS engineering allows the production of systems with much smaller elements and better resolution than conventional techniques.

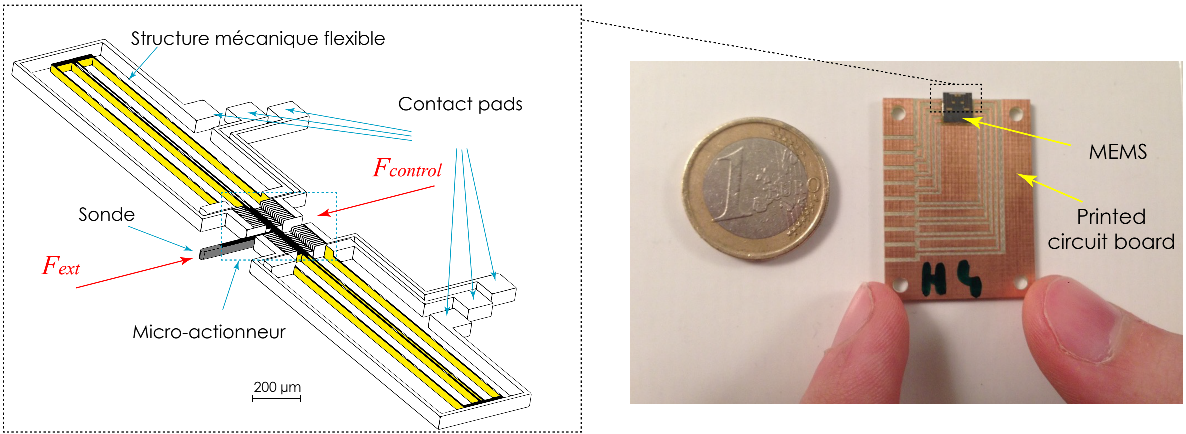

Рис. 1: Active force sensor developed at ISIR.

In this document, we will focus on the control of an active MEMS force sensor recently developed at ISIR (voir Fig 1). This sensor consists of a probe on which the external force F_{ext} to be measured is applied, a micro-actuator, and a flexible micro-mechanical structure. When the force F_{ext} is applied to the probe, the actuator produces an opposing force F_{act} to counteract F_{ext} and maintain the probe position at a fixed value. The measurement of F_{ext} is thus deduced from the voltage u applied to the actuator to within a conversion factor k_e.

The sensor can be assimilated to a monovariable system with input $u(t)$ (electrical voltage applied to the micro-actuator) and output $y(t)$ (deviation of the probe position in response to a force F_{ext}). An identification has shown that the behavior of this system can be approximated by the following second order differential equation:

$$m\frac{d^2y\left(t\right)}{dt^2}+\nu\frac{dy\left(t\right)}{dt}+ky\left(t\right)=F_{act}-F_{ext} \tag{1}$$

Where: $F_{act}=k_eu$.

$k_e=0.3\ \mu\text{N/V}$.

$m=7.85\times10^{-9}$ kg: mass of the moving part of the MEMS.

$\nu=5\times10^{-6}$ N/ms: damping factor.

$k=1.5$ N/m: stiffness of the flexible structure.

The principle of the control is illustrated on Fig.2. When a force F_{ext} (e.g. $F_{ext} = 15\mu N$) is applied to the probe, the controller supplies a voltage u so as to generate a steady-state actuation force F_{act} equal in magnitude and opposite in direction to F_{ext}. This ensures a reference position of the probe at y =0. In steady state $|F_{act}|$ being equal to $|F_{ext}|$, the force measurement is deduced by the measurement of u multiplied by k_e.

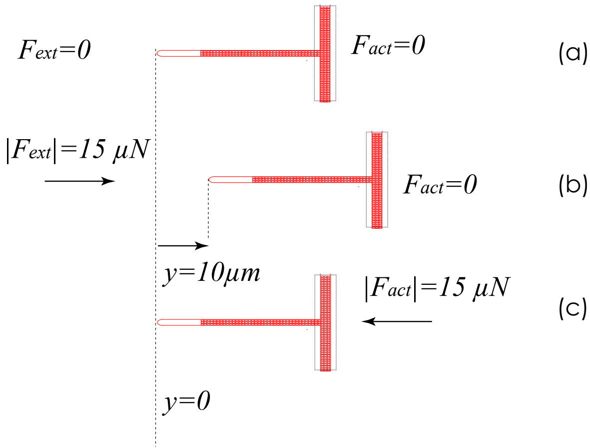

Рис. 2: (a) $F_{ext} = 0$ et $F_{act} = 0\mu N$, the probe position is at $y = 0$. (b) $|F_{ext}| = 15\mu N$ et $|F_{act}| = 0\mu N$, the probe position is at $y = 10\mu m$. (c) $|F_{ext}| = 15\mu N$ et $|F_{act}| = 15\mu N$, the probe position is at $y = 0$.

The electrical voltage $u_0(t)$ generated by the control law is amplified by a linear amplifier of gain $K_{amp} = 10$, such that $u(t) = K_{amp} \times u_0(t)$. The block diagram of the force sensor is shown in Fig.3.

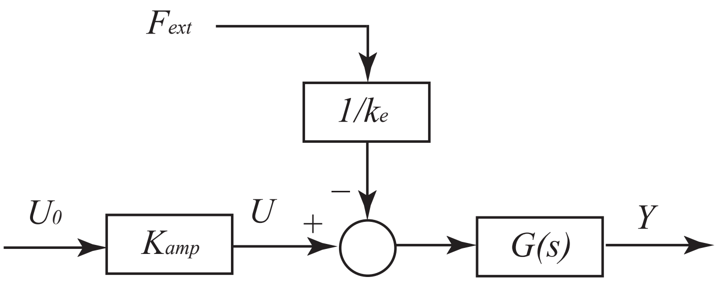

Рис. 3: Block diagram of the force sensor.

$$G(s) = \frac{k_e}{ms^2 + \nu s + k} \tag{2}$$

We consider the controlled position and the measured force in μm and μN respectively. Thus:

$$G(s) = \frac{0.3}{7.85 \times 10^{-9}s^2 + 5 \times 10^{-6}s + 1.5} \tag{3}$$

0.1 Study of the non-forced sensor: -study of the system with an input U_0 and an output Y for $F_{ext} = 0$ -

0.1.1 System definition in Matlab

In this section, we consider $F_{ext} = 0$. The objective is to analyze the static and dynamic performances of the sensor's movement in the unforced state. This study will allow to establish the first lines of the specifications for a dynamic force measurement in closed loop.

The program must be written on a script (file.m)

1. Define the transfer function $G_c(s) = K_{amp}G(s)$.
2. Run this program and check the value of $G_c(s)$.

0.1.2 Calculation of poles, zeros and static gain

The commands *pole*, *zero* and *dcgain* allow to calculate respectively the poles, the zeros and the static gain of a transfer function. For example, if the transfer function H has been defined, the instructions *pole(H)*, *zero(H)* and *dcgain(H)* allow to calculate its poles, its zeros and its static gain. The command *damp* allows to calculate the damping factor and the natural frequency.

Calculate, in Matlab, the poles, zeros and static gain of the transfer function $G_c(s)$. According to the results obtained,

1. Is this system stable?
2. Will the step response exhibit oscillations? If so, deduce from the value of the poles, the pulsation of these oscillations.
3. What is the value, in steady state, of the response to a step of unit amplitude?

0.1.3 Calculation and plotting of temporal responses

Impulse response

In order to make a plot in Matlab, you must first define a window that will contain your plot. To do this, you must type the command *figure*. The *impulse* command allows you to plot the impulse response of an LTI system. For example, if the transfer function H has already been created in Matlab, you can plot the impulse response by typing *impulse(H)*. You can draw a grid on this figure to facilitate the reading. To do this, use the command *grid on*. Finally, you can add a title to your figure using the command *title*.

1. Plot the impulse response of the system $G_c(s)$. From the response, is this system stable?

Step response

The command *step* allows to plot the response to a step of unit amplitude of an LTI system.

1. Plot the step response of the system $G_c(s)$. Measure the overshoot, the settling time at 5%, the steady state value and the pulsation of the oscillations. Are these measurements consistent with the values of the poles and the static gain calculated previously?

0.1.4 Bode diagram

The *bode* command allows you to plot the Bode diagram of a system. For example, if the transfer function F has been defined, we can plot the bode diagram of the system by typing *bode(F)*. By clicking with the left mouse button on a point of the gain (resp. phase) curve, one obtains the corresponding value of the gain (resp. phase) and of the pulsation. By clicking with the right mouse button on the white part of the figure, then selecting Characteristics, then Peak response in the menu, we can determine the resonance pulsation and the value of the gain at resonance. The command *freqresp* allows to calculate the value of the transfer function for a given pulsation. We can then calculate the value of the modulus and the value of the phase corresponding to this pulsation by using respectively the commands *abs* and *angle*. The angle command provides the value of the phase in radians. The command *margin(F)* allows to draw the bode diagram of a function F by showing the phase margin, the gain margin at the pulsation ω_{odb} and ω_{-180}. To visualize this information, read the title at the top of the Bode diagram drawn from the function *margin(F)*.

Example:

F130=freqresp(F,130); % Calculation of F(j 130)

module=abs(F130) % Calculation of the modulus

gaindB=20*log10(module) % gain in dB

phaseRad=angle(F130) % phase in rad

phaseDeg=180/pi*phaseRad % phase in deg.

margin(F)% plot of the Bode diagram of F showing the phase margin and the gain margin at the corresponding pulsations. Example G_m=Inf db (at Inf rad/s), P_m=14.6 deg (at 1.51e+04 rad/s). In this case, the gain margin is infinite, the phase margin is 14.6 deg, ω_{odb}=1.51e+04 rad/s and $\omega_{-180}=\infty$.

1. Draw the Bode diagram of the system $G_c(s)$. Determine the resonance pulsation and the corresponding gain value. Compare the value of the gain at low frequencies with the value of the static gain calculated previously.

2. Determine the value of the gain (in dB) and the value of the phase (in rad/s) of $G_c(s)$ at the pulse 1.45e+04 rad/s.

3. Determine the phase margin, the gain margin and the pulses ω_{odb} and ω_{-180}.

0.2 Study of the forced sensor: -study of the system with the input U_0 and the output Y for $F_{ext} \neq 0$ -

The objective here is to evaluate in steady state, the maximum amplitude of the force that can be measured by the MEMS sensor. As a reminder, the force measurement is performed by compensating the position of the probe to a fixed value, $y = 0$ in our case.

1. The maximum electrical voltage applicable on the actuator is $u = 200V$, what is the maximum amplitude of the force F_{ext} that can be measured? Justify the answer.

0.3 Dynamic force measurement: -Dynamic control-

The objective is to be able to measure a dynamic force in closed loop at a frequency of 1 kHz. The input of the closed loop system is the reference position of the probe $Y_{ref} = 0$. The output is the position Y. The static error in steady state must be zero. The force to be measured is considered as a disturbance. In order to be able to measure this force, we wish to obtain a low oscillation behavior in closed loop and a response time sufficient to follow the dynamics of the force to be measured (1 kHz).

The control scheme is illustrated in Fig.4. The controller $C(s)$ provides to the probe a desired dynamic (speed of movement, vibration reduction ,stability, etc.) so that it can remain at the $Y_{ref} = 0$ position when an external force F_{ext} is applied to its tip. The movement of the probe must be able to follow the dynamics of F_{ext} for an efficient measurement in dynamic mode. The measured force $F_{measuree}$ is deduced from the control voltage U_0. If the control $C(s)$ is well adjusted, then $F_{measuremente} = F_{ext}$ on the whole frequency range considered in the specifications.

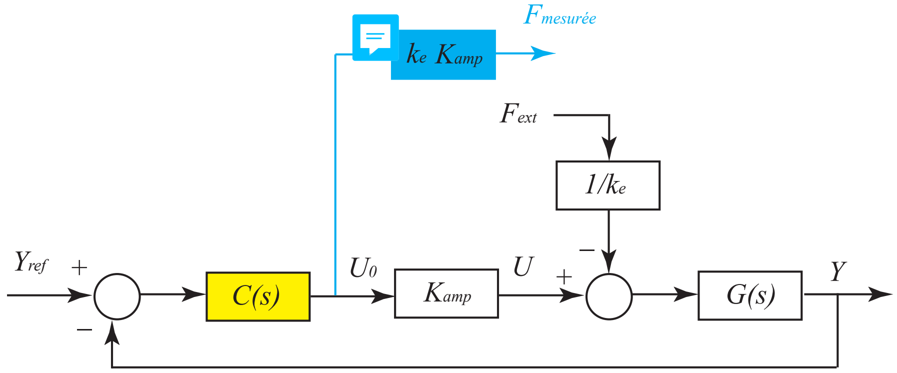

Рис. 4: Block diagram of the closed loop force sensor.

The specifications are as follows:

- static error in steady state equal to zero, i.e. $y(t) = y_{ref} = 0$ in steady state.
- Phase margin of the corrected open loop $F_{bo}(s) = C(s)G_c(s)$ equal to $frac\pi 4$ in order to reduce the vibrations of the probe.
- Pulse ω_{odb} of the corrected open loop $F_{bo}(s)$ sufficient for a force measurement at 1 kHz.

1. Propose a structure of $C(s)$ to satisfy the specifications and define the numerical values of its parameters.
2. Program $C(s)$ on the .m file
3. On Simulink, simulate the Fig.4.
   (a) F_{ext} must be a sinusoidal signal of frequency 1 kHz and amplitude 15 μ N.
   (b) Draw on a single *scope* the curves $F_{measure}$ and F_{ext}. It is possible to use a multiplexer to superimpose the two curves. Verify that the controller has met the specifications by making sure that $F_{measure}$ is the image of F_{ext}.
   (c) Does the control voltage $u(t)$ match the system limit (u(t)<200 V?).

Reference

J. Cailliez, M. Boudaoud, A. Mohand-Ousaid, A. Weill–Duflos, S. Haliyo and S. Régnier. Modeling and Experimental Characterization of an Active MEMS Based Force Sensor, Journal of Micro-Bio Robotics, 2019 https://doi.org/10.1007/s12213-019-00115-1