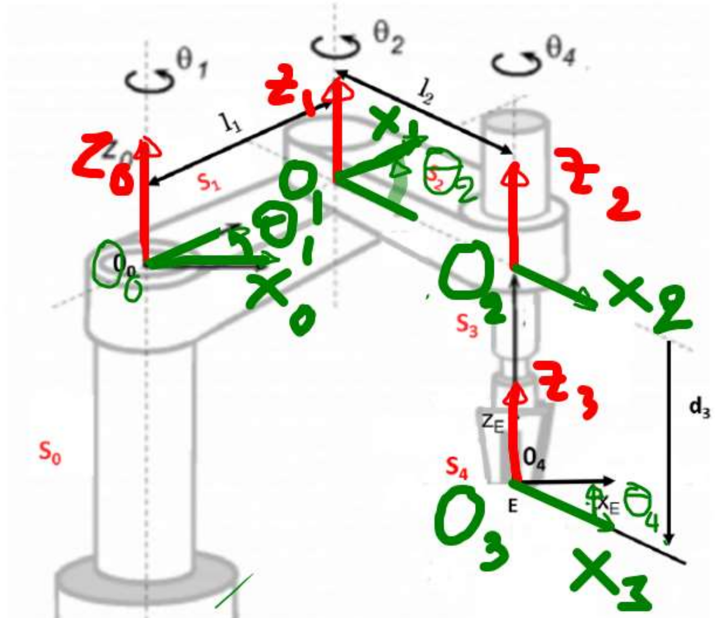
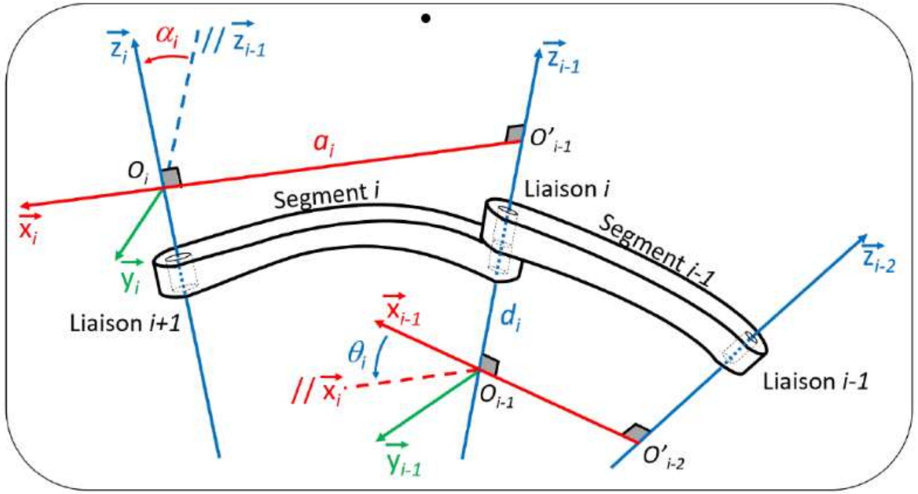
TD 3 – CORRECTION
Exerice 1

EXERCICE 1 : ROBOT SCARA

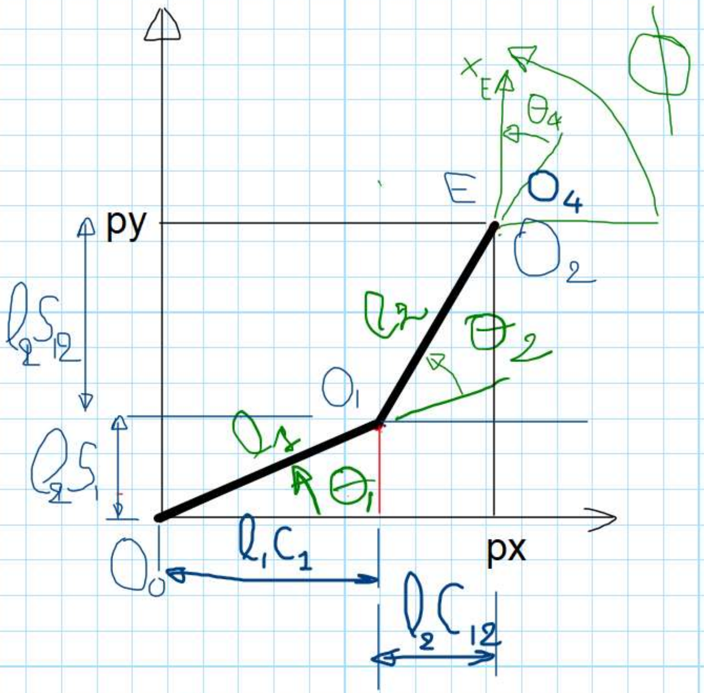




Liaison		θ_i	d_i	a_i	α_i
L_1	0→1	θ_1	0	l_1	0
L_2	1→2	θ_2	0	l_2	0
L_3	2→3	0	d_3	0	0
L_4	3→4	θ_4	0	0	0

Questions 1-2-3

Chasles $\overrightarrow{O_0O_4} = \overrightarrow{O_0O_1} + \overrightarrow{O_1O_2} + \overrightarrow{O_2O_3} + \overrightarrow{O_3O_4}$



$$\begin{cases} P_x = l_1 C_1 + l_2 C_{12} \\ P_y = l_1 S_1 + l_2 S_{12} \\ P_z = d_3 \\ \phi = \theta_1 + \theta_2 + \theta_4 \end{cases}$$

$C_1 = \cos\theta_1$ $\quad C_{12} = \cos\theta_1 + \theta_2$

$S_1 = \sin\theta_1$ $\quad S_{12} = \sin\theta_1 + \theta_2$

MGD $\quad X = f(q)$

4)

$$
{}^{i-1}\mathbf{T}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_{1(q_1)} = \begin{bmatrix} c_1 & s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^{1}T_{2(q_2)} = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_{3(q_3)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^{3}T_{4(q_4)} = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_{4(q)} = {}^{0}T_1\, {}^{1}T_2\, {}^{2}T_3\, {}^{3}T_4 = \begin{bmatrix} c_{124} & -s_{124} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{124} & c_{124} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 0 & d_3 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \underbrace{{}^{0}T_4(x)}_{\begin{bmatrix} c\phi & -s\phi & 0 & p_x \\ s\phi & c\phi & 0 & p_y \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}
$$

5) par identification dans l'eq matricielle ${}^0T_4(x) \stackrel{?}{=} {}^0T_4(q)$

$$\begin{cases} p_x = l_1 C_1 + l_2 C_{12} \\ p_y = l_1 S_1 + l_2 S_{12} \\ p_z = d_3 \\ \phi = \theta_1 + \theta_2 + \theta_4 \end{cases}$$

MGD $X = f(q)$

$$\xrightarrow{?} \underbrace{q = f^{-1}(x)}_{\text{MGI}}$$

$X = [p_x \ p_y \ p_z \ \phi]^T$

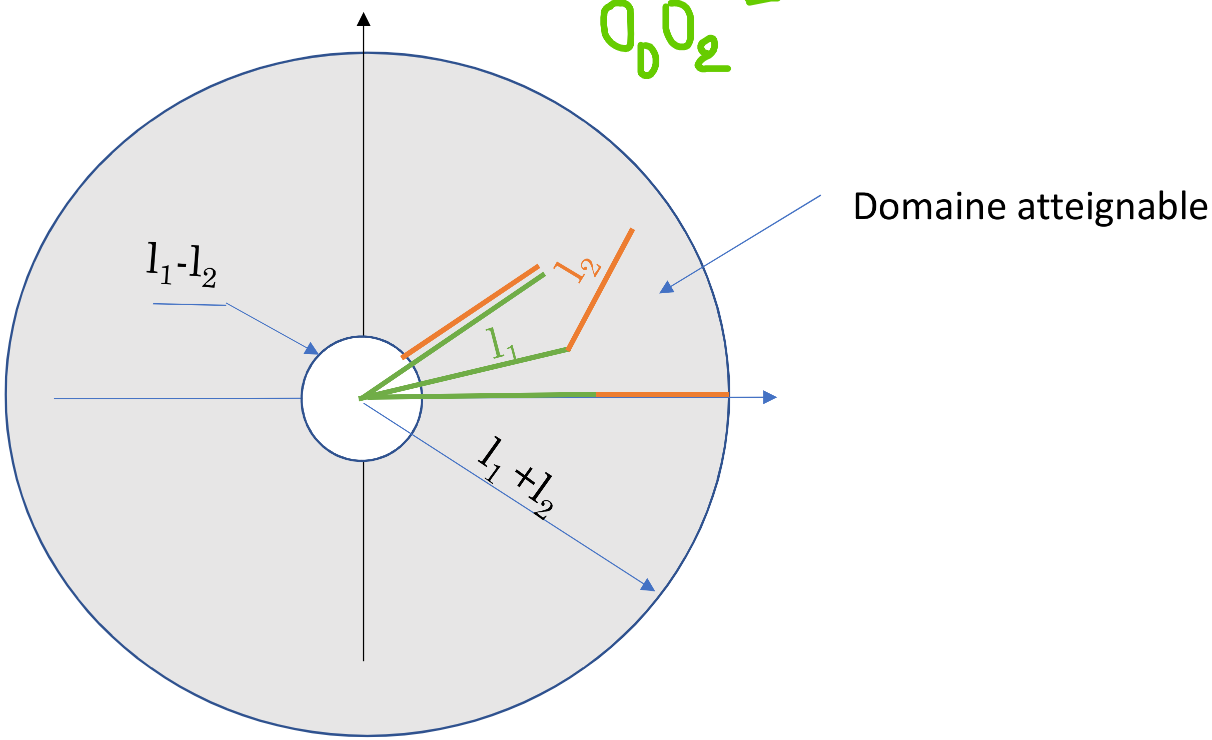
$q = [\theta_1 \ \theta_2 \ d_3 \ \theta_4]^T$

$$p_x^2 + p_y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta_2$$

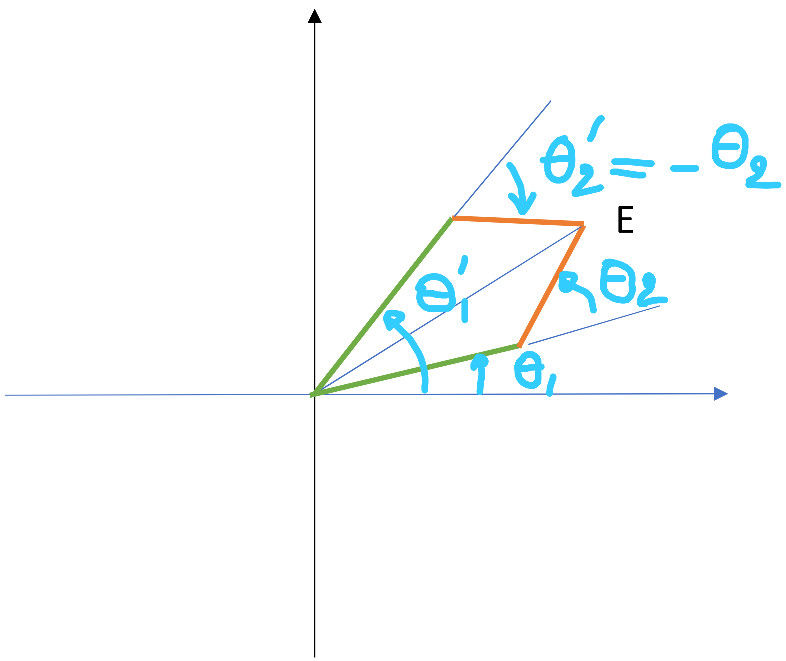
$$\rightarrow \theta_2 = \pm \arccos\left(\frac{p_x^2 + p_y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

θ_2 existe ssi $\quad -1 \leq \dfrac{p_x^2 + p_y^2 - l_1^2 - l_2^2}{2 l_1 l_2} \leq 1$

$$\longrightarrow (l_1 - l_2)^2 \leq \underbrace{p_x^2 + p_y^2}_{O_0 O_2^{\,2}} \leq (l_1 + l_2)^2$$



2 solutions pour $\theta_2 = \pm \arccos \ldots$



Résoudre θ_1 sachant θ_2 connu

$$\begin{cases} P_x = l_1 C_1 + l_2 C_{12} \\ P_y = l_1 S_1 + l_2 S_{12} \end{cases} \longrightarrow \begin{cases} P_x = (l_1 + l_2 C_2)\, C_1 - l_2 S_2 S_1 \\ P_y = \underbrace{(l_1 + l_2 C_2)}_{k_1} S_1 + \underbrace{l_2 S_2}_{k_2} C_1 \end{cases}$$

Soit $\Delta = \sqrt{k_1^2 + k_2^2}$

$$\begin{cases} \frac{k_1}{\Delta} C_1 - \frac{k_2}{\Delta} S_1 = \frac{P_x}{\Delta} = \cos(\theta_1 + \alpha) \\ \underbrace{\frac{k_1}{\Delta}}_{C\alpha} S_1 + \underbrace{\frac{k_2}{\Delta}}_{S\alpha} C_1 = \frac{P_y}{\Delta} = \sin(\theta_1 + \alpha) \end{cases}$$

On pose $\alpha = \operatorname{atan2}\left(\frac{k_2}{\Delta}, \frac{k_1}{\Delta}\right)$

$$\theta_1 + \alpha = \operatorname{atan2}\left(\frac{P_y}{\Delta}, \frac{P_x}{\Delta}\right)$$

$$\theta_1 = \operatorname{atan2}(P_y, P_x) - \operatorname{atan2}(k_2, k_1)$$

La fonction atan2 normalise qd les arguments $\notin [-1,1]$

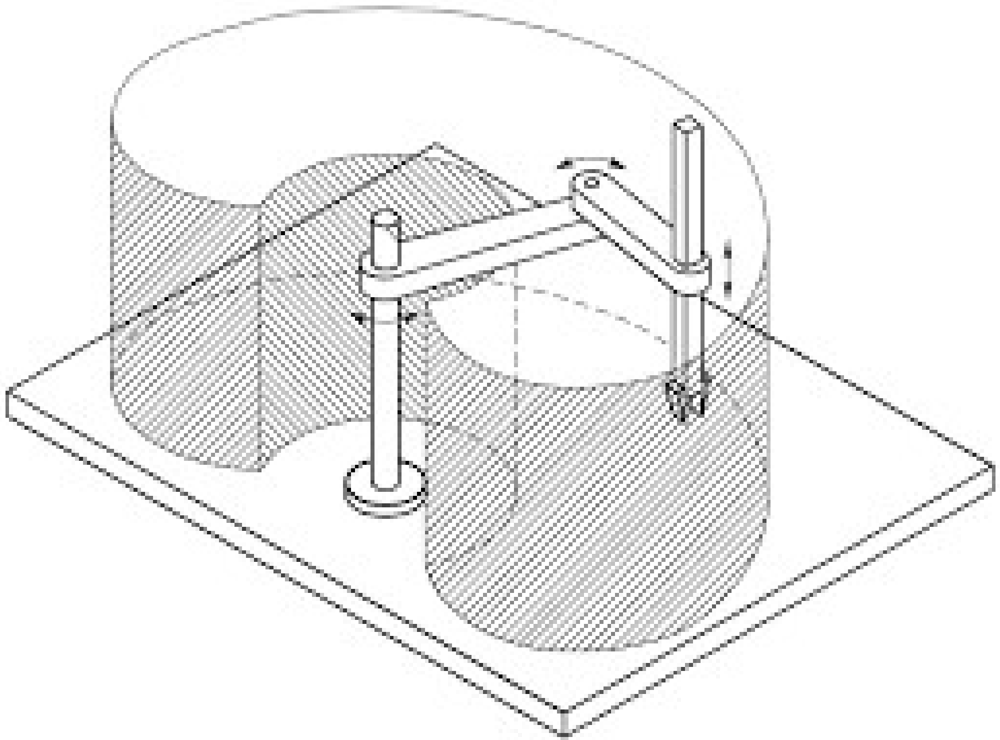
Attention . une seule valeur de θ_1 pour chaque valeur de θ_2

Enfin les 2 dernières éq. du MGD peuvent être inverses

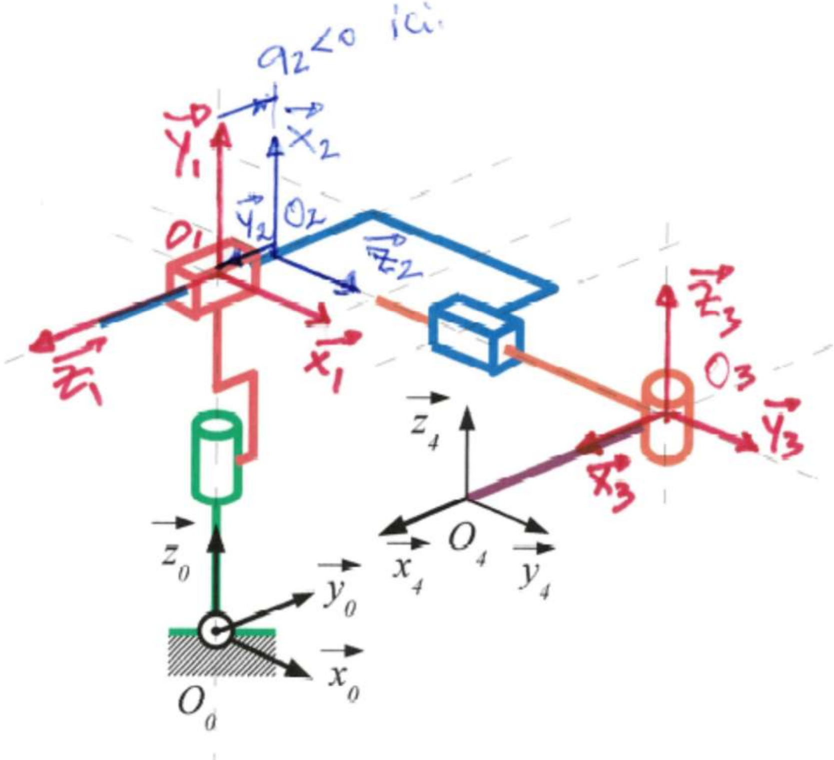
$$\begin{cases} d_3 = p_z \\ \theta_4 = \phi - \theta_1 - \theta_2 \end{cases}$$

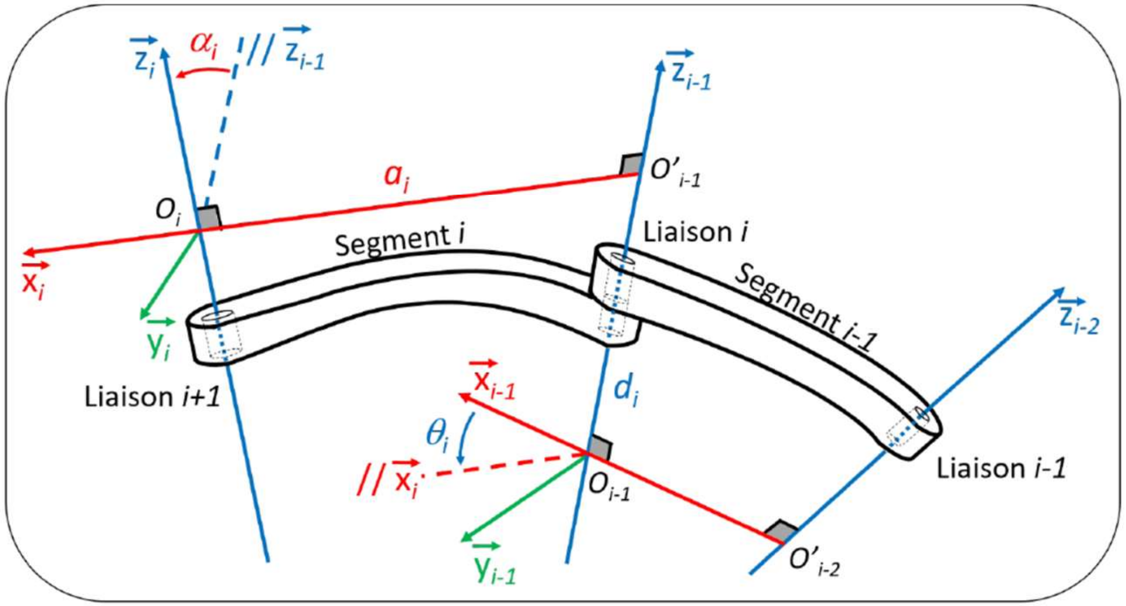
ce qui finit l'inversion du modèle géométrique deux solutions en tout du MGI

6) L'espace de travail du robot est l'espace atteignable (volumes entre deux cercles extrudées) qui sera restreint par la suite quand on considère en plus les butées articulaires dans les liaisons



TD 3 – CORRECTION
Exerice 2





Liaison	α_i	a_i	b_i	θ_i
1	$\pi/2$	0	l_1	q_1
2	$\pi/2$	0	q_2	$\pi/2$
3	$\pi/2$	0	q_3+l_3	$\pi/2$
4	0	l_4	0	q_4

Ex2.

$C_i = \cos(q_i)$
$S_i = \sin(q_i)$.

3) $${}^0T_4 = {}^0T_1(q_1)\,{}^1T_2(q_2)\,{}^2T_3(q_3)\,{}^3T_4(q_4)$$

$$= \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_3+l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & -S_4 & 0 & l_4C_4 \\ S_4 & C_4 & 0 & l_4S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) Seuls 3 paramètres opérationnels sont commandables, par exemple

* position suivant $\vec{x_0}$ de O_4
* —— —— $\vec{y_0}$ ——
* Orientation B_4/B_0 autour de $\vec{z_0}$

5).

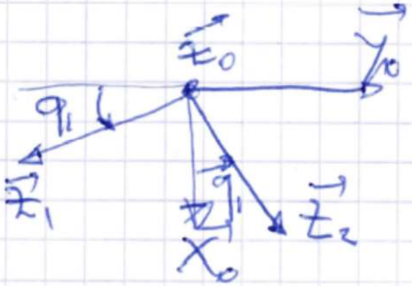
$$
{}^{0}T_3 = \begin{bmatrix} 0 & S_1 & C_1 & S_1 q_2 \\ 0 & -C_1 & S_1 & -C_1 q_2 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_3 + l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} S_1 & C_1 & 0 & C_1(q_3 + l_3) + S_1 q_2 \\ -C_1 & S_1 & 0 & S_1(q_3 + l_3) - C_1 q_2 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\overrightarrow{OD_3} = \begin{bmatrix} C_1(q_3 + l_3) + S_1 q_2 \\ S_1(q_3 + l_3) - C_1 q_2 \\ l_1 \end{bmatrix}_{R_0}
$$

d) $\overrightarrow{O_0D_3} = \overrightarrow{O_0A_1} + \overrightarrow{A_1B_2} + \overrightarrow{B_2D_3}$

$$= l_1 \vec{z_0} + q_2 \vec{z_1} + (l_3 + q_3)\vec{z_2}$$



$$= l_1 \vec{z_0} + q_2 (S_1 \vec{x_0} - C_1 \vec{y_0}) + (l_3 + q_3)(C_1 \vec{x_0} + S_1 \vec{y_0})$$

$$= \begin{pmatrix} S_1 q_2 + C_1 (l_3 + q_3) \\ -C_1 q_2 + S_1 (l_3 + q_3) \\ l_1 \end{pmatrix}_{R_0}$$

→ 2 positions x, y sont commandables à partir de q_1, q_2, q_3.

$${}^0R_3 = \begin{bmatrix} S_1 & C_1 & 0 \\ -C_1 & S_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$: l'orientation ne dépend que de q_1

On peut commander que l'orientation de R_3 suivant $\vec{z_0}$.

7) $q_1 = 0 \xrightarrow{MGD} \begin{cases} x = l_3 + q_3 \\ y = -q_2 \end{cases} \xrightarrow{MGI} \begin{cases} q_2 = -y \\ q_3 = x - l_3 \end{cases}$

position de D_3 % R_0.

8) $q_2 = 0 \xrightarrow{MGD} \begin{cases} x = C_1 (l_3 + q_3) \\ y = S_1 (l_3 + q_3) \end{cases} \xrightarrow{MGI} \begin{cases} q_1 = \operatorname{atan2}(y, x) \\ q_3 = \pm\sqrt{x^2 + y^2} - l_3 \end{cases}$