Ex1

1)

$$ {}^{0}T_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & c+e \\ 0 & -1 & 0 & a-d \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_2 = \begin{bmatrix} \\ \\ \\ \end{bmatrix} $$

$$ {}^{2}T_1 = \begin{bmatrix} 0 & 0 & -1 & c+e \\ 0 & 1 & 0 & +d \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Vérifions que ${}^{0}T_2\,{}^{2}T_1 = {}^{0}T_1$

$$ \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & a \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & c+e \\ 0 & 1 & 0 & +d \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & c+e \\ 0 & -1 & 0 & a-d \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2)

$$ {}^{0}r_p = \begin{bmatrix} -b \\ c \\ a-d \\ 1 \end{bmatrix} \qquad {}^{1}r_p = \begin{bmatrix} b \\ 0 \\ e \\ 1 \end{bmatrix} $$

On vérifie que ${}^{0}r_p = {}^{0}T_1\,{}^{1}r_p$

$$ {}^{0}T_1\,{}^{1}r_p = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & c+e \\ 0 & -1 & 0 & a-d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \\ e \\ 1 \end{bmatrix} = \begin{bmatrix} -b \\ c \\ a-d \\ 1 \end{bmatrix} $$

3) Méthode Direct.
Lecture du Schéma.

$$ {}^{1}T_{0} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & a-d \\ 0 & -1 & 0 & c+e \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2nd méthode :

$$ T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \rightarrow T^{-1} = \begin{bmatrix} R^t & -R^t p \\ 0 & 1 \end{bmatrix} $$

$$ \left({}^{0}T_{1}\right)^{-1} = \begin{bmatrix} \; \end{bmatrix} $$

$$ -R^t t = -\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ c+e \\ a-d \end{bmatrix} = \begin{bmatrix} 0 \\ a-d \\ c+e \end{bmatrix} $$

4)

$$ {}^{0}R_{1} = Rot(\vec{y_0}, \pi) \times Rot(\vec{x_1}, \tfrac{\pi}{2}) $$ post-multiplication

Rotation autour d'axes non-fixes

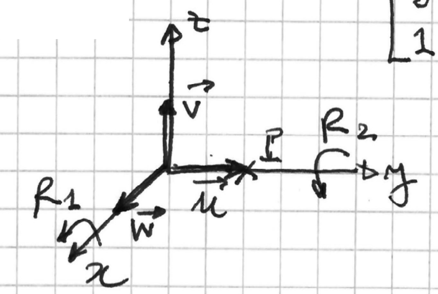
$$ = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} $$

$$ {}^{0}R_{1} = Rot(\vec{x_0}, \tfrac{-\pi}{2}) \times Rot(\vec{y_0}, \pi) $$ pré-multiplication

Rotation autour d'axes fixes.

$$ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} $$

EXERCICE 2



1)

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad {}^0\vec{v} = {}^0R\left(\vec{x}, \tfrac{\pi}{2}\right) {}^0\vec{u} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & C\frac{\pi}{2} & -S\pi/2 \\ 0 & S\frac{\pi}{2} & C\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad {}^0\vec{w} = {}^0R\left(\vec{y}, \tfrac{\pi}{2}\right) {}^0\vec{v} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} C\frac{\pi}{2} & 0 & S\pi/2 \\ 0 & 1 & 0 \\ -S\frac{\pi}{2} & 0 & C\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$ {}^0\vec{w} = {}^0R(\vec{y}, \pi/2)\, {}^0\vec{v} = \underbrace{{}^0R\left(\vec{y}, \tfrac{\pi}{2}\right) {}^0R\left(\vec{x}, \tfrac{\pi}{2}\right)}_{R_{Totale}} {}^0\vec{u}$$

R_{Totale} (prémultiplication) car Transf. exprimée dans la m̂ base.

$$R_{Totale} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

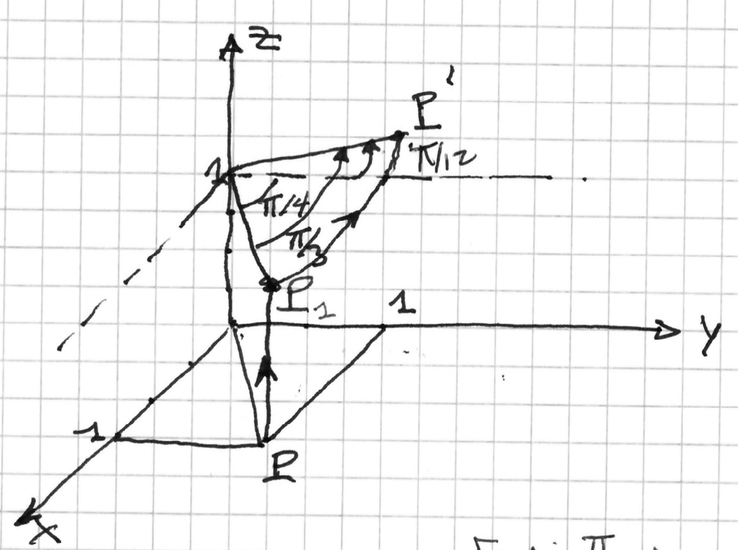
$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

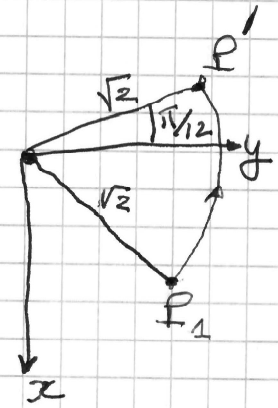
2) en coordonnée homogène

$$ {}^0r_{p'} = {}^0Rot\left(\vec{z}, \tfrac{\pi}{3}\right) \times {}^0Trans(\vec{z}, 1)\, {}^0r_p .$$

$$= \begin{bmatrix} \frac{1}{2} & -\sqrt{3}/2 & 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{3}}{2} + \frac{1}{2} \\ \frac{1}{2} + \frac{\sqrt{3}}{2} \\ 1 \\ 1 \end{bmatrix}$$





$$\vec{OP'} = \begin{pmatrix} -\sqrt{2}\sin\frac{\pi}{12} \\ \sqrt{2}\cos\frac{\pi}{2} \\ 1 \end{pmatrix} \cong \begin{pmatrix} -0{,}36 \\ 1{,}36 \\ 1 \end{pmatrix}$$

EXERCICE 3

1) $T = Tras(\vec{y}, 3) \times Rot(\vec{x}, \frac{\pi}{6})$

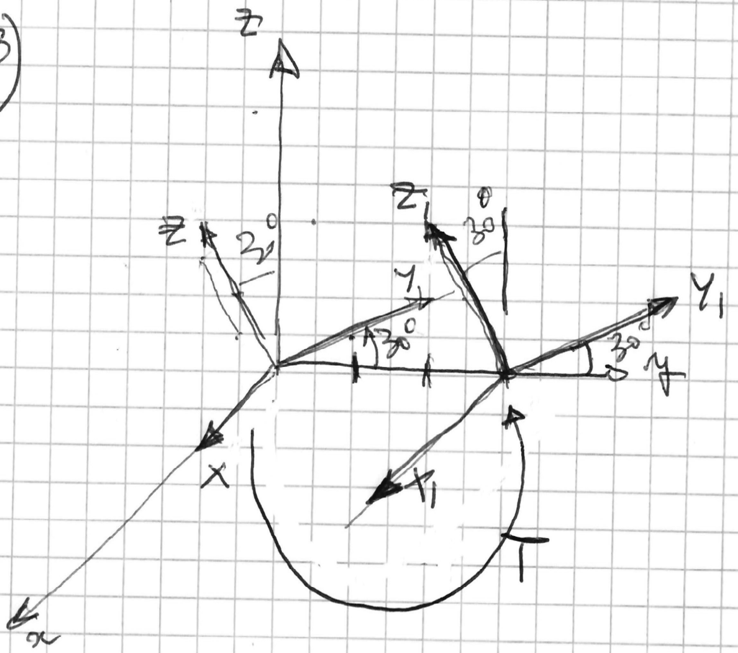
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & \frac{1}{2} & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

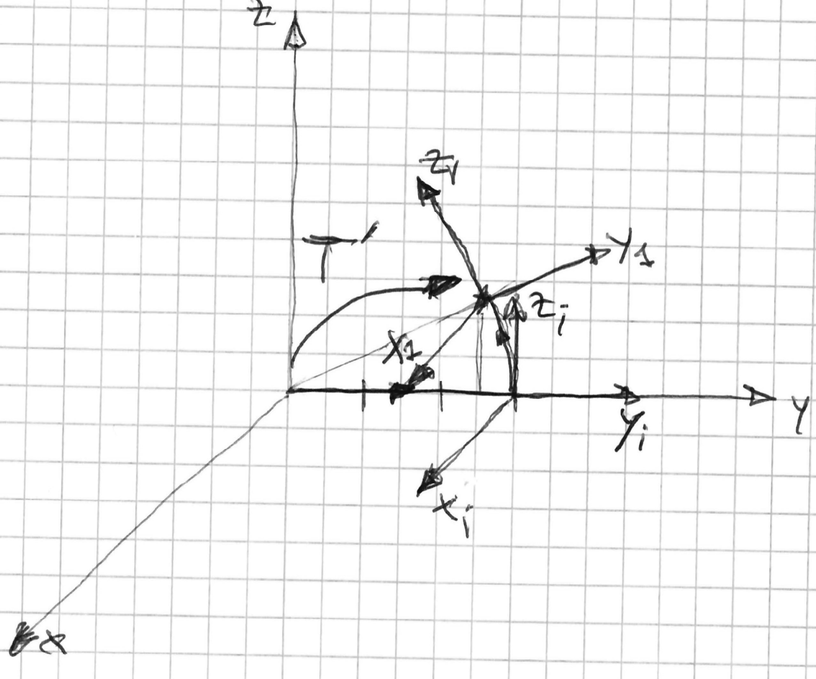
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 3 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) $$T' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 3\sqrt{3}/2 \\ 0 & 1/2 & \sqrt{3}/2 & 3/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3)





EXERCICE 4

— Formule d'Euler-Rodriguez

$$R(\vec{v},\theta) = R = I + \sin\theta \, S(\vec{v}) + 2\sin^2\left(\frac{\theta}{2}\right) S^2(\vec{v}).$$

$$\vec{v} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \longrightarrow S(\vec{v}) = \frac{1}{\sqrt{3}}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

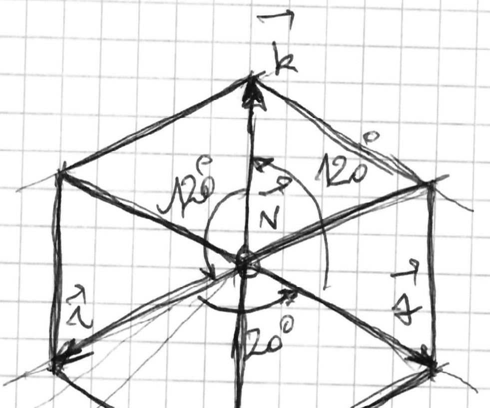
$$S^2(\vec{v}) = S(\vec{v}) \times S(\vec{v}) = \frac{1}{3}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$\theta = \frac{2\pi}{3}$

$$R = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{3}}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} + 2\,\frac{3}{4}\,\frac{1}{3}\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$R(\vec{i}) = \vec{j}$ $\quad R(\vec{j}) = \vec{k}$ $\quad R(\vec{k}) = \vec{i}$.



Vue suivant la grande diagonale du cube $\vec{v}$

EXERCICE 5

1)

$$T_{total} = Rot(\vec{z}, -\tfrac{\pi}{2}) \times Trans(\vec{x}, 2) \times Rot(\vec{y}, \tfrac{\pi}{2})$$

$$= \begin{bmatrix} \end{bmatrix} \times \begin{bmatrix} \end{bmatrix} \times \begin{bmatrix} \end{bmatrix}$$

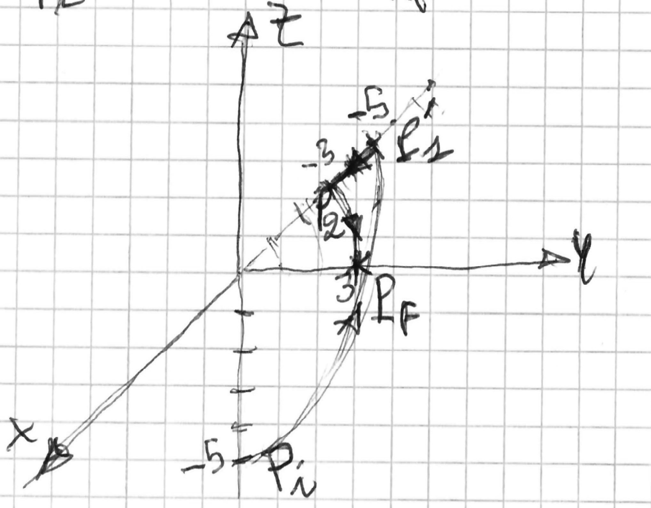
$$= \begin{bmatrix} \end{bmatrix} \times \begin{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \longrightarrow T^{-1} = \begin{bmatrix} R^T & -R^T t \\ 0 & 1 \end{bmatrix}$$

$$T^{-1}_{tot.} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{P_i} = T^{-1} r_{P_f} = T^{-1} \times \begin{bmatrix} 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -5 \\ 1 \end{bmatrix}$$



c)

$$r_Q = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$r_{Q'} = \begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 1 \end{bmatrix}$$