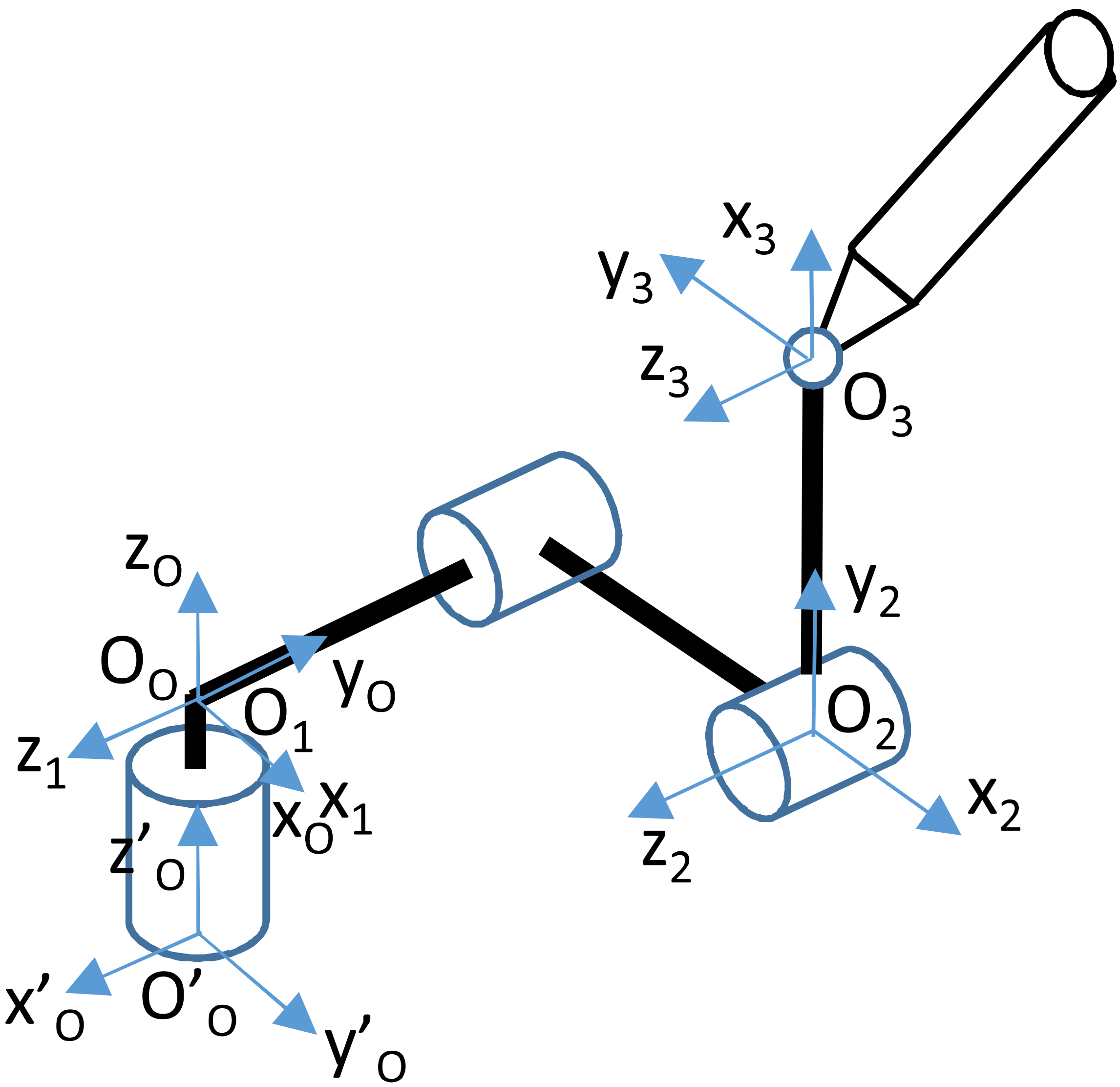


Configuration $\theta=[0,0,\pi/2]^T$

1) 1^{er} méthode Chasles

Interface Haptique PHANTOM.

$$\overrightarrow{O'_0 O_3} = \overrightarrow{O'_0 O_0} + \underbrace{\overrightarrow{O_0 O_1}}_{\vec{0}} + \overrightarrow{O_1 O_2} + \overrightarrow{O_2 O_3}$$

$$= L_0 \vec{z}'_0 - L_1 \vec{z}_1 + L_2 \vec{x}_2 + L_3 \vec{x}_3$$

$$= L_0 \vec{z}'_0 - L_1 (\cos\theta_1 \vec{x}'_0 + \sin\theta_1 \vec{y}'_0)$$

$$+ L_2 (\cos\theta_2 \vec{x}_1 + \sin\theta_2 \vec{z}_0)$$

$$+ L_3 \Big(\cos(\theta_2+\theta_3) \underbrace{\vec{x}_1}_{(\cos\theta_1 \vec{y}'_0 - \sin\theta_1 \vec{x}'_0)} + \sin(\theta_2+\theta_3) \vec{z}_0\Big).$$

$$O_3 : \begin{cases} x_{O_3} = -L_1 \cos\theta_1 - L_2 \cos\theta_2 \sin\theta_1 - L_3 \cos(\theta_2+\theta_3) \sin\theta_1 \\ y_{O_3} = -L_1 \sin\theta_1 + L_2 \cos\theta_2 \cos\theta_1 + L_3 \cos(\theta_2+\theta_3) \cos\theta_1 \\ z_{O_3} = L_0 + L_2 \sin\theta_2 + L_3 \sin(\theta_2+\theta_3). \end{cases}$$

2^e méthode paramètres DH.

	θ_i	d_i	a_i	α_i
1	θ_1	0	0	$\pi/2$
2	θ_2	$-L_1$	L_2	0
3	θ_3	0	L_3	0

+ Transformation fixe sur la base

$${}^{0'}T_0 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}T_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & L_2\cos\theta_2 \\ S\theta_2 & C\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & 1 & -L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_2 = \begin{bmatrix} C\theta_1 C\theta_2 & -C\theta_1 S\theta_2 & S\theta_1 & L_2 C\theta_2 C\theta_1 - L_1 S\theta_1 \\ S\theta_1 C\theta_2 & -S\theta_1 S\theta_2 & -C\theta_1 & L_2 C\theta_2 S\theta_1 + L_1 C\theta_1 \\ S\theta_2 & C\theta_2 & 0 & L_2 S\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{2}T_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & L_3 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & L_3 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{0}T_3 = \left[\begin{array}{c|c} {}^{0}R_3 & \begin{matrix} C\theta_1(L_3C\theta_2C\theta_3 - L_3S\theta_3S\theta_2) + L_2C\theta_2C\theta_1 - L_1S\theta_1 \\ S\theta_1(L_3C\theta_3C\theta_2 - L_3S\theta_3S\theta_2) + L_2C\theta_2S\theta_1 + L_1C\theta_1 \\ L_3C\theta_3S\theta_2 + L_3S\theta_3C\theta_2 + L_2S\theta_2 \end{matrix} \\ \hline 0 & 1 \end{array}\right] $$

Modèle Géométrique Direct restreint à la position

$$ {}^{0'}T_3 = {}^{0'}T_0 \, {}^{0}T_3 = \left[\begin{array}{c|c} {}^{0'}R_3 & \left.\begin{matrix} -L_1C\theta_1 - L_2C\theta_2S\theta_1 - L_3C(\theta_2+\theta_3)S\theta_1 \\ -L_1S\theta_1 + L_2C\theta_2C\theta_1 + L_3C(\theta_2+\theta_3)C\theta_1 \\ L_2S\theta_2 + L_3S(\theta_2+\theta_3) + L_0 \end{matrix}\right\} \to \overrightarrow{O'_0O_3} \\ \hline 0 & 1 \end{array}\right] $$

c.q.f.d.

Première méthode : Dérivation % temps du MGD

2). M.G.D. $X = F(\theta)$. $\longrightarrow$ M.C.D $\dot{X} = \left[\frac{\partial F}{\partial \theta}\right]\dot{\theta} = J\dot{\theta}$

$$ \dot{X} = \begin{bmatrix} +L_1S\theta_1 - L_2C\theta_2C\theta_1 - L_3C\theta_{2+\theta_3}C\theta_1 & +L_2S\theta_1S\theta_2 + L_3S\theta_1S\theta_{2+\theta_3} & L_3S\theta_1S\theta_{2+\theta_3} \\ -L_1C\theta_1 - L_2C\theta_2S\theta_1 - L_3C\theta_{2+\theta_3}S\theta_1 & -L_2C\theta_1S\theta_2 - L_3C\theta_1S\theta_{2+\theta_3} & -L_3C\theta_1S\theta_{2+\theta_3} \\ 0 & L_2C\theta_2 + L_3C\theta_{2+\theta_3} & L_3C\theta_{2+\theta_3} \end{bmatrix} \times \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} $$

2ème méthode : Coordonnées plückeriennes des axes.

$$ \begin{pmatrix} \omega_{3/0} \\ V_{3/0} \end{pmatrix} = \begin{bmatrix} \vec{e}_1 & \vec{e}_i & \vec{e}_3 \\ \vec{e}_1 \times \vec{U}_1 & \vec{e}_i \times \vec{U}_i & \vec{e}_3 \times \vec{U}_3 \end{bmatrix} \begin{bmatrix} \dot{\theta} \end{bmatrix} $$

Axe i

$$ = \begin{bmatrix} J_\omega \\ \hline J_v \end{bmatrix} \dot{\theta} $$

$$ \to J_v = \left[\, \vec{Z}_0 \times \overrightarrow{O_0O_3} \;\middle|\; \vec{Z}_1 \times \overrightarrow{O_1O_3} \;\middle|\; \vec{Z}_2 \times \overrightarrow{O_2O_3} \,\right] $$

exprimés dans R_0' lié à la base

car $\dot{X} = \frac{d}{dt}\overrightarrow{O'_0O_3} \leftarrow$ exprimé dans R_0'

$$\vec{Z}_0 \times \overrightarrow{O_0O_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -L_1 C\theta_1 - L_2 S\theta_1 C\theta_2 - L_3 S\theta_1 C\theta_{2+\theta_3} \\ -L_1 S\theta_1 + L_2 C\theta_1 C\theta_2 + L_3 C\theta_1 C\theta_{2+\theta_3} \\ L_2 S\theta_2 + L_3 S\theta_{2+\theta_3} + L_0 \end{bmatrix}$$

$$= \begin{bmatrix} L_1 S\theta_1 - L_2 C\theta_1 C\theta_2 - L_3 C\theta_1 C\theta_{2+\theta_3} \\ -L_1 C\theta_1 - L_2 S\theta_1 C\theta_2 - L_3 S\theta_1 C\theta_{2+\theta_3} \\ 0 \end{bmatrix}_{R_0}$$

$$\vec{Z}_1 \times \overrightarrow{O_1O_3} = \begin{bmatrix} C\theta_1 \\ S\theta_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} -L_1 C\theta_1 - L_2 S\theta_1 C\theta_2 - L_3 S\theta_1 C\theta_{2+\theta_3} \\ -L_1 S\theta_1 + L_2 C\theta_1 C\theta_2 + L_3 C\theta_1 C\theta_{2+\theta_3} \\ L_2 S\theta_2 + L_3 S\theta_{2+\theta_3} \end{bmatrix}$$

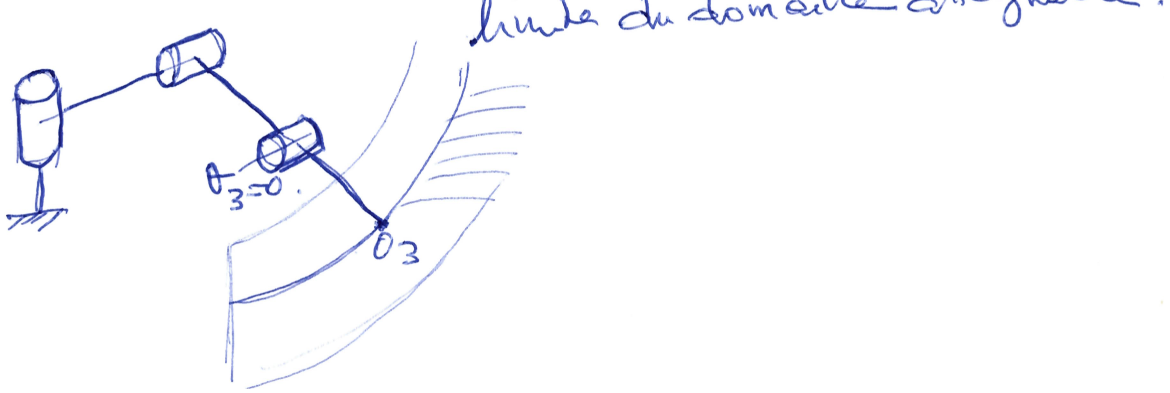
$$= \begin{bmatrix} L_2 S\theta_1 S\theta_2 + L_3 S\theta_1 S\theta_{2+\theta_3} \\ -L_2 C\theta_1 S\theta_2 - L_3 C\theta_1 S\theta_{2+\theta_3} \\ L_2 C\theta_2 + L_3 C\theta_{2+\theta_3} \end{bmatrix}_{R_0}$$

$$\vec{Z}_2 \times \overrightarrow{O_2O_3} = \begin{bmatrix} C\theta_1 \\ S\theta_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} -L_3 S\theta_1 C\theta_{2+\theta_3} \\ L_3 C\theta_1 C\theta_{2+\theta_3} \\ L_3 S\theta_{2+\theta_3} \end{bmatrix}$$

$$= \begin{bmatrix} L_3 S\theta_1 S\theta_{2+\theta_3} \\ -L_3 C\theta_1 S\theta_{2+\theta_3} \\ L_3 C\theta_{2+\theta_3} \end{bmatrix}_{R_0}$$ c.q.f.d

3) Comme $\vec{Z}_1 \equiv \vec{Z}_2$, les colonnes 2 et 3 de J sont colinéaires si $\overrightarrow{O_1O_3}$ colinéaire à $\overrightarrow{O_2O_3}$ c'est à dire O_1, O_2, O_3 alignés ou $\theta_3 = 0$ ou π.

limite du domaine atteignable.



4) $\theta_1 = 0 \quad \theta_2 = 0 \quad \underline{\theta_3 = 0}$ Conf. singulière.

$$J = \begin{bmatrix} -L_2 - L_3 & 0 & 0 \\ -L_1 & 0 & 0 \\ 0 & L_2 + L_3 & L_3 \end{bmatrix}$$

$\tau = J^T F$ ← force créer en O_3.

Couples actionneur

$$\begin{cases} \tau_1 = -(L_2 + L_3) F_x - L_1 F_y \\ \tau_2 = (L_2 + L_3) F_z \\ \tau_3 = L_3 F_z \end{cases}$$

← F_x, F_y ne peuvent être contrôlés séparément

5) $\tau = J^T F + \dfrac{\partial V}{\partial \theta}$ ← Energie potentielle.

Si compensation de gravité seule

$$\tau = \frac{\partial V}{\partial \theta} = \begin{pmatrix} \frac{\partial V}{\partial \theta_1} \\ \frac{\partial V}{\partial \theta_2} \\ \frac{\partial V}{\partial \theta_3} \end{pmatrix}$$

$$V = m_1 g \underbrace{z_1}_{\text{Cste}} + m_2 g \underbrace{d_2 \sin\theta_2}_{z_2} + m_3 g \underbrace{(d_3 \sin\theta_2 + \theta_3 + L_2 \sin\theta_2)}_{z_3}.$$

z_i = hauteur des centre de gravité du corps i.

$$\tau = \begin{pmatrix} 0 \\ m_2 g d_2 \cos\theta_2 + m_3 g L_2 \cos\theta_2 + m_3 g d_3 \cos(\theta_2 + \theta_3) \\ m_3 g d_3 \cos\theta_2 + \theta_3 \end{pmatrix}$$

6) $\tau^d = J^T F^d$ ← F désiré.

tension de commande du moteur u $= k(\tau^d - \tau^{\text{courant}})$

mesuré grâce au courant

Ex 2

RRPR

1) $${}^{b}T_{0} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & L_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) $${}^{0}T_{1} = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & -L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}T_{2} = \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -1 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{2}T_{3} = \begin{bmatrix} 1 & & & -L_3 \\ & 1 & & 0 \\ & & 1 & d_3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad {}^{3}T_{4} = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) $${}^{0}T_{2} = \begin{bmatrix} C_1C_2 & -S_1 & -C_1S_2 & L_2S_1 \\ S_1C_2 & C_1 & -S_1S_2 & -L_2C_1 \\ S_2 & 0 & C_2 & -L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_{3} = \left[\begin{array}{c|c} {}^{0}R_{2} & \begin{matrix} -L_3C_1C_2 - d_3C_1S_2 + L_2S_1 \\ -L_3S_1C_2 - d_3S_1S_2 - L_2C_1 \\ -L_3S_2 + d_3C_2 - L_1 \end{matrix} \\ \hline 0 & 1 \end{array}\right]$$

$${}^{b}T_{3} = \begin{bmatrix} S_2 & 0 & C_2 & -L_3S_2 + d_3C_2 - L_1 \\ -S_1C_2 & -C_1 & S_1S_2 & L_3S_1C_2 + d_3S_1S_2 + L_2C_1 \\ C_1C_2 & -S_1 & -C_1S_2 & -L_3C_1C_2 - d_3C_1S_2 + L_2S_1 + L_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

dans R_b $$\begin{cases} p_x = -L_3S_2 + d_3C_2 - L_1 \\ p_y = L_3S_1C_2 + d_3S_1S_2 + L_2C_1 \\ p_z = -L_3C_1C_2 - d_3C_1S_2 + L_2S_1 + L_0 \end{cases}$$

4) Configuration de la figure

$\theta_1 = 0 \qquad \theta_2 = \frac{\pi}{2} \qquad \theta_4 = 0$

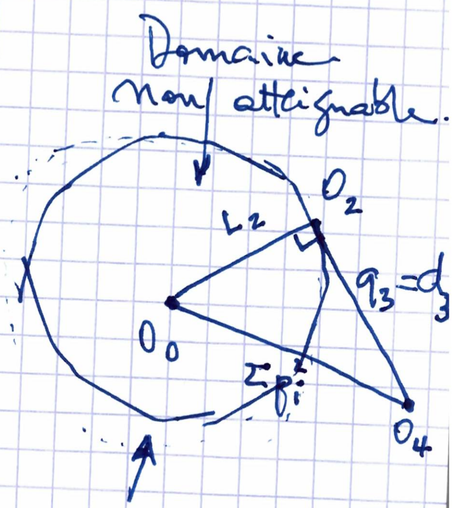
$$^bT_3 = \begin{bmatrix} 1 & 0 & 0 & -L_3 - L_1 \\ 0 & -1 & 0 & L_2 \\ 0 & 0 & -1 & L_0 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

cohérent avec la figure

5) $L_3 = 0. \quad L_1 = 0$

dans $\mathcal{R}_b$

$$\begin{cases} p_x = d_3 C_2 \\ p_y = d_3 S_1 S_2 + L_2 C_1 \\ p_z = -d_3 C_1 S_2 + L_2 S_1 + L_0 \end{cases}$$



6). $p_y^2 + (p_z - L_0)^2 = d_3^2 S_2^2 + L_2^2$

$p_x^2 + p_y^2 + (p_z - L_0)^2 = d_3^2 + L_2^2 \longrightarrow d_3 = \pm\sqrt{\underbrace{p_x^2 + p_y^2 + (p_z - L_0)^2 - L_2^2}_{>0}}$

$\theta_2 = \operatorname{acos}(p_x / d_3) \qquad$ si $d_3 \neq 0$.

$p_y C_1 + p_z S_1 = L_2 \rightarrow \theta_1 = $ —— voir cours.

7)

$$\begin{pmatrix} \dot p_x \\ \dot p_y \\ \dot p_z \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -d_3 S_2 & C_2 \\ \left(\begin{smallmatrix} d_3 C_1 S_2 \\ - L_2 S_1 \end{smallmatrix}\right) & d_3 S_1 C_2 & S_1 S_2 \\ \left(\begin{smallmatrix} d_3 S_1 S_2 \\ + L_2 C_1 \end{smallmatrix}\right) & -d_3 C_1 C_2 & -C_1 S_2 \end{pmatrix}}_{J} \begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot d_3 \end{pmatrix}$$

$d_3 = 0 \qquad \det(J) = 0 \qquad$ pt O_4 sur l'axe liaison (2).

le robot perd une mobilité opérationnelle

8) $$J = \left(\vec{z}_0 \times \overrightarrow{O_0O_4} \;\middle|\; \vec{z}_1 \times \overrightarrow{O_1O_4} \;\middle|\; \vec{z}_2 \right)$$

dans R_b. $${}^b\vec{z}_0 \times {}^b\overrightarrow{O_0O_4} = \begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix} d_3 C_2 \\ d_3 S_1 S_2 + L_2 C_1 \\ -d_3 C_1 S_2 + L_2 S_1\end{pmatrix}$$

$$\underset{{}^1\vec{z}_1}{=} \begin{pmatrix}0\\ d_3 C_1 S_2 - L_2 S_1 \\ d_3 S_1 S_2 + L_2 C_1\end{pmatrix}_{R_b}$$

$${}^b\vec{z}_1 \times {}^b\overrightarrow{O_1O_4} = {}^bR_0\,{}^0R_1 \begin{pmatrix}0\\0\\1\end{pmatrix} \times \begin{pmatrix} - \\ - \\ - \end{pmatrix}$$

$$= \begin{pmatrix}0\\C_1\\S_1\end{pmatrix} \times \begin{pmatrix} - \\ - \\ - \end{pmatrix}$$

$$= \begin{pmatrix} -d_3 S_2 \\ d_3 S_1 C_2 \\ -d_3 C_1 C_2\end{pmatrix}_{R_b}$$

$${}^b\vec{z}_2 = \underbrace{{}^bR_2}_{= {}^bR_3} \underbrace{\begin{pmatrix}0\\0\\1\end{pmatrix}}_{{}^2\vec{z}_2} = \begin{pmatrix} C_2 \\ S_1 S_2 \\ -C_1 S_2\end{pmatrix}$$

voir ${}^bL_3$

9) $h_{G_3} = L_0 + L_2 S_1 - (d_3 - \lambda_3) C_1 S_2$

$h_{G_2} = L_0 + \lambda_2 S_1$

$h_{G_1} = L_0$

10) $$E_p = m_1 g L_0 + m_2 g (L_0 + \lambda_2 S_1) + m_3 g \left(L_0 + L_2 S_1 - (d_3 - \lambda_3) C_1 S_2 \right).$$

$$\tau_{\text{gravité}} = \frac{\partial E_p}{\partial q} = \begin{pmatrix} (m_2 \lambda_2 + m_3 L_2) g C_1 + (d - \lambda_3) S_1 S_2 \times m_3 g \\ - m_3 g (d - \lambda_3) C_1 C_2 \\ - m_3 g C_1 S_2 \end{pmatrix}$$

11) $$\tau = \tau_{\text{gravité}} - J^T F_{(\text{Env.} \to \text{Robot})}.$$