Robust H_∞ and $\mu-$Synthesis to Active Suspension Control

The text and the example treated her are from the robust control toolbox user guide of Mathworks.

The aim is to design a robust controller for an active suspension system. The example describes the quarter-car suspension model. An H_∞ controller is first computed for a nominal system using the *hinfsyn* command. Then, a μ-synthesis is computed to design a robust controller for the full uncertain system

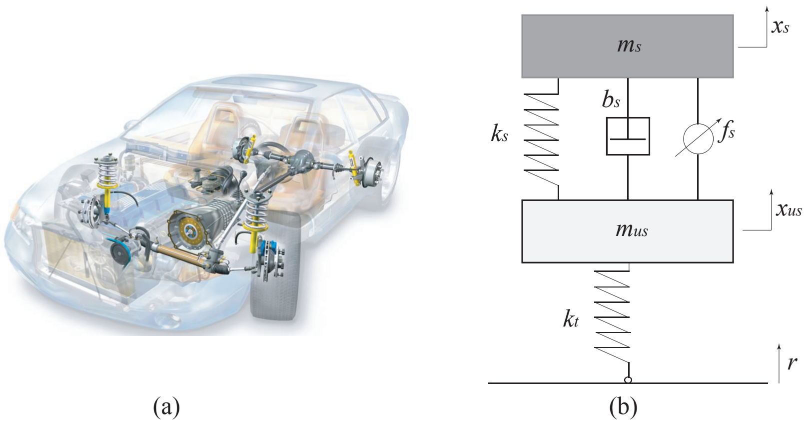

Рис. 1: CAD scheme of a car (a) and its equivalent quarter car model (b).

Conventional passive suspensions use a spring and damper between the car body and wheel assembly. The spring-damper characteristics are selected to emphasize one of several conflicting objectives such as passenger comfort, road handling, and suspension deflection. Active suspensions allow the designer to balance these objectives using a feedback-controller hydraulic actuator between the chassis and wheel assembly. This example uses a quarter-car model of the active suspension system (Fig.1). The mass m_s represents the car chassis (body) and the mass m_{us} represents the wheel assembly. The spring k_s and damper b_s represent the passive spring and shock absorber placed between the car body and the wheel assembly. The spring k_t models the compressibility of the pneumatic tire. The variables x_s, x_{us}, and r are the body travel, wheel travel, and road disturbance, respectively. The force f_s applied between the body and wheel assembly is controlled by feedback and represents the active component of the suspension system.

I- Quarter Car Suspension Model

Defining $x_1 = x_s$, $x_2 = \dot{x}_s$, $x_3 = x_{us}$ and $x_4 = \dot{x}_{us}$, the following is the state-space description of the quarter car dynamics.

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -\frac{1}{m_s}[k_s(x_1 - x_3) + b_s(x_2 - x_4) - 10^3 f_s],$$

$$\dot{x}_3 = x_4,$$

$$\dot{x}_4 = \frac{1}{m_{us}}[k_s(x_1 - x_3) + b_s(x_2 - x_4) - k_t(x_3 - r) - 10^3 f_s].$$

The following component values are taken from reference [2]:

$m_s = 300$ kg, $m_{us} = 60$ kg; $b_s = 1000$ N/m/s, $k_s = 16000$ N/m, $k_t = 190000$ N/m.

A linear, time-invariant model of the quarter car model is defined in the state space, called *qcar*, from the equations of motion and parameter values. The inputs to the model are the road disturbance r and actuator force f_s, respectively, and the outputs are the car body deflection x_s, suspension deflection $x_s - x_{us}$, and acceleration $\ddot{x}_s$. The state space vector is $[x_1 \;\; x_2 \;\; x_3 \;\; x_4]^T$.

1- Compute the state space model *qcar* in Matlab using the command *ss*.

The transfer function from actuator force f_s to body travel x_s and acceleration $\ddot{x}_s$ has an imaginary-axis zero with natural frequency $\omega_{tire-hop}$. This is called the *tire-hop frequency*. Similarly, the transfer function from actuator force f_s to suspension deflection $x_s - x_{us}$ has an imaginary-axis zero with natural frequency $\omega_{rattlespace}$. This is called the *rattlespace* frequency.

Road disturbances influence the motion of the car and suspension. Passenger comfort is associated with small body acceleration. The allowable suspension travel is constrained by limits on the actuator displacement.

2- Plot, using the commande *bodemag*, the open-loop gain from road disturbance r and actuator force f_s to body acceleration $\ddot{x}_s$ and suspension displacement $s_d = x_s - x_{us}$. Deduce $\omega_{tire-hop}$ and $\omega_{rattlespace}$.

Because of the imaginary-axis zeros, feedback control cannot improve the response from road disturbance r to body acceleration $\ddot{x}_s$ at the tire-hop frequency, and from r to suspension deflection s_d at the rattlespace frequency. Moreover, because of the relationship $x_{us} = x_s - s_d$ and the fact that the wheel position x_{us} roughly follows r at low frequency (less than 5 rad/s), there is an inherent trade-off between passenger comfort and suspension deflection: any reduction of body travel at low frequency will result in an increase of suspension deflection.

II- Linear H$_\infty$ Controller Design

The main control objectives are formulated in terms of passenger comfort and road handling, which relate to body acceleration $\ddot{x}_s$ and suspension travel s_d. Other factors that influence the control design include the characteristics of the road disturbance, the quality of the sensor measurements for feedback, and the limits on the available control force. To use H$_\infty$ synthesis algorithms, it is necessary to express these objectives as a single cost function to be minimized. This can be done as indicated in Fig.2.

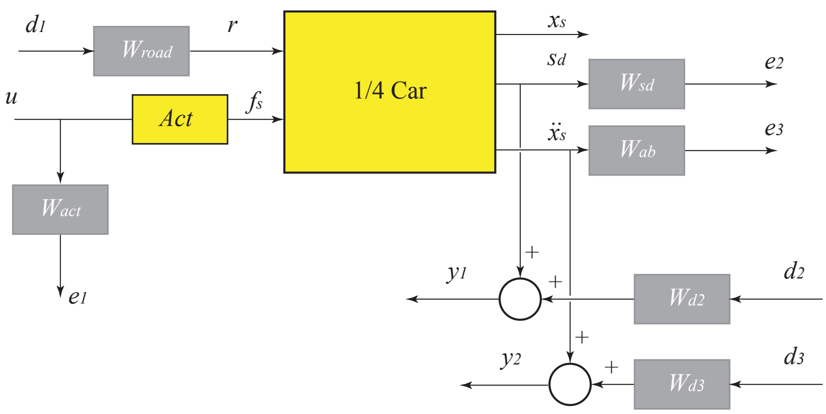

Рис. 2: Augmented control scheme.

The feedback controller uses measurements y_1, y_2 of the suspension travel s_d and body acceleration $\ddot{x}_s$ to compute the control signal u driving the hydraulic actuator. The transfer function between f_s and u (actuator model) is:

$Act = \frac{1}{(1/60)s+1}$

s is Laplace variable.

There are three external sources of disturbance:

- The road disturbance r, modeled as a normalized signal d_1 shaped by a weighting function W_{road}. To model broadband road deflections of magnitude seven centimeters, we use the constant weight W_{road}=0.07.
- Sensor noise on both measurements, modeled as normalized signals d_2 and d_3 shaped by weighting functions W_{d2} and W_{d3}. These functions are selected as W_{d2}=0.01 and W_{d3}=0.5 to model broadband sensor noise of intensity 0.01 and 0.5, respectively. In a more realistic design, these weights would be frequency dependent to model the noise spectrum of the displacement and acceleration sensors.

The control objectives can be reinterpreted as a disturbance rejection goal: Minimize the impact of the disturbances d_1, d_2 and d_3 on a weighted combination of control effort u, suspension travel s_d, and body acceleration $\ddot{x}_s$.

When using the H$_\infty$ norm (peak gain) to measure "impact", this amounts to designing a controller that minimizes the H$_\infty$ norm from disturbance inputs d_1, d_2 and d_3 to error signals e_1, e_2 and e_3.

3- Compute the functions W_{road}, W_{d2} and W_{d3}. Define W_{act} as a high-pass filter to penalize high-frequency content of the control signal and thus to limit the control bandwidth.

4- Specify closed-loop targets for the gain from road disturbance r to suspension deflection s_d (handling) and body acceleration $\ddot{x}_s$ (comfort). Because of the actuator uncertainty and imaginary-axis zeros, only seek to attenuate disturbances below 10 rad/s.

5- Plot, using the commande *bodemag*, the open-loop gain from road disturbance r to body acceleration $\ddot{x}_s$ and suspension displacement $s_d = x_s - x_{us}$ and compare the curve with closed loop target.

The corresponding performance weights W_{sd}, W_{ab} are the reciprocals of these comfort and handling targets.

6- To investigate the trade-off between passenger comfort and road handling, construct three sets of weights (βW_{sd},$(1-\beta)W_{ab}$) corresponding to three different trade-offs: comfort (β=0.01), balanced (β=0.5), and handling (β=0.99).

7- Define the standard from of the augmented system corresponding to the H$_\infty$ control design.

For each value of β, the controller K_i has two inputs s_d and $\ddot{x_s}$ and one output u. The following notations will be used: K_1 for comfort objective (β=0.01), K_2 for balanced objective (β=0.5) and K_3 for handling objective (β=0.99).

8- Compute the controller K_i using the Matlab function *hinfsyn*. What is the value of the norm γ_i obtained for each synthesis?

III- Closed-loop analysis with the linear H$_\infty$ Controller Design

The three controllers K_1, K_2 and K_3 achieve closed-loop H$_\infty$ norms of γ_1, γ_2 and γ_3 respectively.

9- Construct the corresponding closed-loop models $Gcl_{i_{11}} = \frac{x_s}{r}$, $Gcl_{i_{21}} = \frac{s_d}{r}$ and $Gcl_{i_{31}} = \frac{\ddot{x_s}}{r}$ where $i = 1, 2, 3$ corresponds to a controller K_i.

10- Compare the gains from road disturbance to x_s, s_d and $\ddot{x_s}$ for the passive and active suspensions. Observe that all three controllers reduce suspension deflection and body acceleration below the rattlespace frequency (23 rad/s).