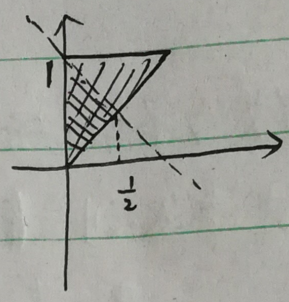
第六周作业.

12. (1) $\int_0^1 dy\int_0^x f(x,y)\,dx = \frac{1}{6}C = 1 \Rightarrow C=6$

(2) $P(X+Y\le 1)=\int_0^{\frac12}dx\int_x^{1-x} f(x,y)\,dy=\frac12$

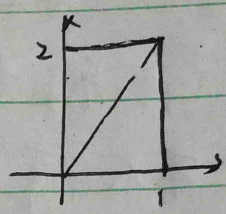
(3) $P\{X\le 0.5\}=\int_0^{\frac12}dx\int_x^{1} f(x,y)\,dy=\frac78$



13. (1) $f_X(x)=\int_0^2 f(x,y)\,dy=2x \quad x\in(0,1)$

$f_Y(y)=\int_0^1 f(x,y)\,dx=\frac12 \quad y\in(0,2)$

(2) $P\{Y\le 2X\}=\int_0^1 dx\int_0^{2x} f(x,y)\,dy=\frac23$

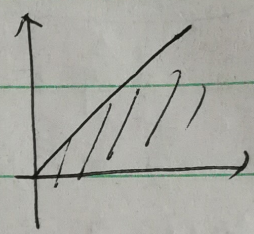


15. (1) $f_X(x)=\int_0^x f(x,y)\,dy = xe^{-x} \quad x\in(0,\infty)$

$f_Y(y)=\int_y^{\infty} f(x,y)\,dx = e^{-y} \quad y\in(0,\infty)$

(2) $f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\frac{1}{x} \quad y\in$

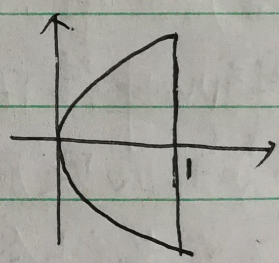
(3) 是均匀分布. $f_{Y|X}(y|x)$为常数.



17. (1) $f_Y(y)=\int_{y^2}^1 f(x,y)\,dx=\frac58(1-y^4) \quad y\in(-1,1)$

(2) $f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\frac{2x}{1-y^4}$

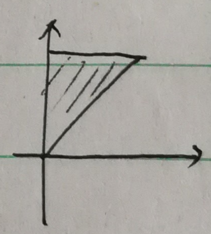
(3) $P\{X>\frac12 | Y=\frac12\} = \int_{\frac12}^1 f_{X|Y}(x|y)\big|_{y=\frac12}\,dx=\frac45$



18. (1) 由题: $f_X(x)=1$, $f_{Y|X}(y|x)=\frac{1}{1-x}$

$\therefore f(x,y)=f_X(x)\cdot f_{Y|X}(y|x)=\frac{1}{1-x}$, $(0<x<y<1)$

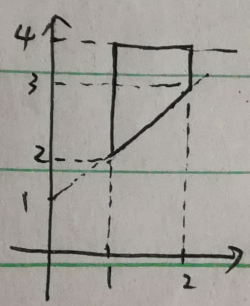
(2) $f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\frac{f(x,y)}{\int_0^y f(x,y)\,dx}=\frac{1}{(x-1)\ln(1-y)}$



19. (1) $f_X(x)=1$

$\therefore f(x,y)=f_X(x)\cdot f_{Y|X}(y|x)=\frac{2(4-y)}{(3-x)^2} \quad \begin{pmatrix} x\in(1,2) \\ y\in(x+1,4)\end{pmatrix}$

$P(Y<3)=\int_1^2 dx\int_{x+1}^3 f(x,y)\,dy=\frac12$



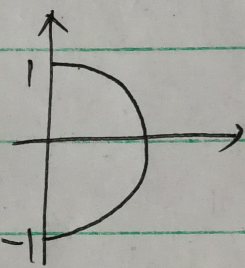
(2) $f_Y(y)=\begin{cases}\int_1^{y-1} f(x,y)\,dx = y-2 & y\in(2,3)\\ \int_1^{2} f(x,y)\,dx = 4-y & y\in(3,4)\end{cases}$

(3) $\cancel{y}$ $Y=3$ 有

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\frac{2}{(3-x)^2}\quad x\in(1,2)$$

$$\therefore P(X<1.5\,|\,Y=3)=\int_1^{1.5} f_{X|Y}(x|y=3)\,dx=\frac{1}{3}$$

21. (1) $f(x,y)=\dfrac{1}{\frac{1}{2}\cdot\pi\cdot 1^2}=\dfrac{2}{\pi}$



$$\therefore f_X(x)=\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)\,dy=\frac{4}{\pi}\sqrt{1-x^2}$$

(2) $P\{X<\frac{1}{2}\}=\int_0^{\frac{1}{2}}dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)\,dy$

$$=\int_0^{\frac{1}{2}}\frac{4}{\pi}\sqrt{1-x^2}\,dx \xlongequal{x=\sin\theta}\int_0^{\frac{\pi}{6}}\frac{4}{\pi}\cos^2\theta\,d\theta$$

$$=\int_0^{\frac{\pi}{6}}\frac{4}{\pi}\cdot\frac{1+\cos 2\theta}{2}\,d\theta=\frac{1}{3}+\frac{\sqrt{3}}{2\pi}$$

(3) $f_Y(y)=\int_0^{\sqrt{1-y^2}} f(x,y)\,dx=\frac{2}{\pi}\sqrt{1-y^2}$

$\therefore f(x,y)\neq f_X(x)\cdot f_Y(y)$.

∴不独立.