Math 105AL – Lab 4

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1 Objective

In this project, I explored the concept of the growth factor p in Gaussian Elimination and examined how different pivoting strategies affect numerical stability. I implemented two functions, which are “gauss_elimination_standard.m” and “gauss_elimination_partial.m” to compare their behaviors and results on three test matrices A, B and C. The objective was to compute the growth factors and verify the correctness of the resulting solutions x in Mx=0.

2 Procedure

1. Implemented the two Gaussian Elimination functions according to the lab4 instructions.
2. The standard version performs forward elimination and back substitution without row swaps.
3. The partial one selects the largest pivot in magnitude for each column to improve the stability.
4. For the testing correctness, I ran the professor’s “test_ge.m” and “data.mat”.
5. For the growth-factor part, I used the matrices A, B and C provided in the Lab4 instruction file. Then I used my “main.m” script to compute the p factor.

$$\rho = \frac{\max_{i,j,k} |a_{ij}^{(k)}|}{\max_{i,j} |a_{ij}^{(0)}|}$$

   I recorded the results in the table and verified that the output x for Mx = 0 was the zero vector.

3 Results

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Table 1: Growth Factors for solving Mx = 0
------------------------------------------------------------
                A              B              C
------------------------------------------------------------
Standard Pivoting 1.142857e+00   9.999000e+03   5.368709e+08
Partial Pivoting 1.000000e+00   1.000000e+00   5.368709e+08
------------------------------------------------------------
```

Here is the table of results from matrices A, B and C above.

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Matrix A: ||x_std||=0.00e+00, ||x_pp||=0.00e+00, det≈-1.60e+01



  -> Output is (numerically) the zero vector: YES
Matrix B: ||x_std||=0.00e+00, ||x_pp||=0.00e+00, det≈-1.00e+00
  -> Output is (numerically) the zero vector: YES
Matrix C: ||x_std||=0.00e+00, ||x_pp||=0.00e+00, det≈5.37e+08
  -> Output is (numerically) the zero vector: YES
```

For all three matrices, A, B and C, solving Mx = 0.

4 Conclusion

Gaussian Elimination with partial pivoting drastically reduces growth and maintains numerical stability. Matrix B demonstrated how a tiny pivot can inflate entries without pivoting. And matrix C showed that it remains unstable even with pivoting. The results for matrix C are the same value. I think partial pivoting is really necessary for us in practice.

5 Challenges and Bugs

1. I used the “live script” to guess and try for the main function part in Q2 to display the better result and visual effect for the table. The algorithms to finding the solutions to Mx = 0 (right-hand side is the all-zero vector) is challenging.
2. I forgot to put the professor’s “test_ge.m” and “data.mat” files into my same

MATLAB folder. So, it cannot run at first and I have no ideas why.

3. The results table's format is not the same as the professor provided in the Lab4 instruction file.