Math 105AL – Lab 2

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1 Objective

In this project, I will implement and compare two root-finding algorithms, which are bisection method and the fixed-point method, to locate the root of f(x) = (x^3)/3 – x – 1/3. The tasks include writing the MATLAB functions "my_bisection.m" and "my_fixedpoint.m", plotting convergence behavior, and analyzing how the initial guess affects convergence rate and iteration count.

2 Procedure

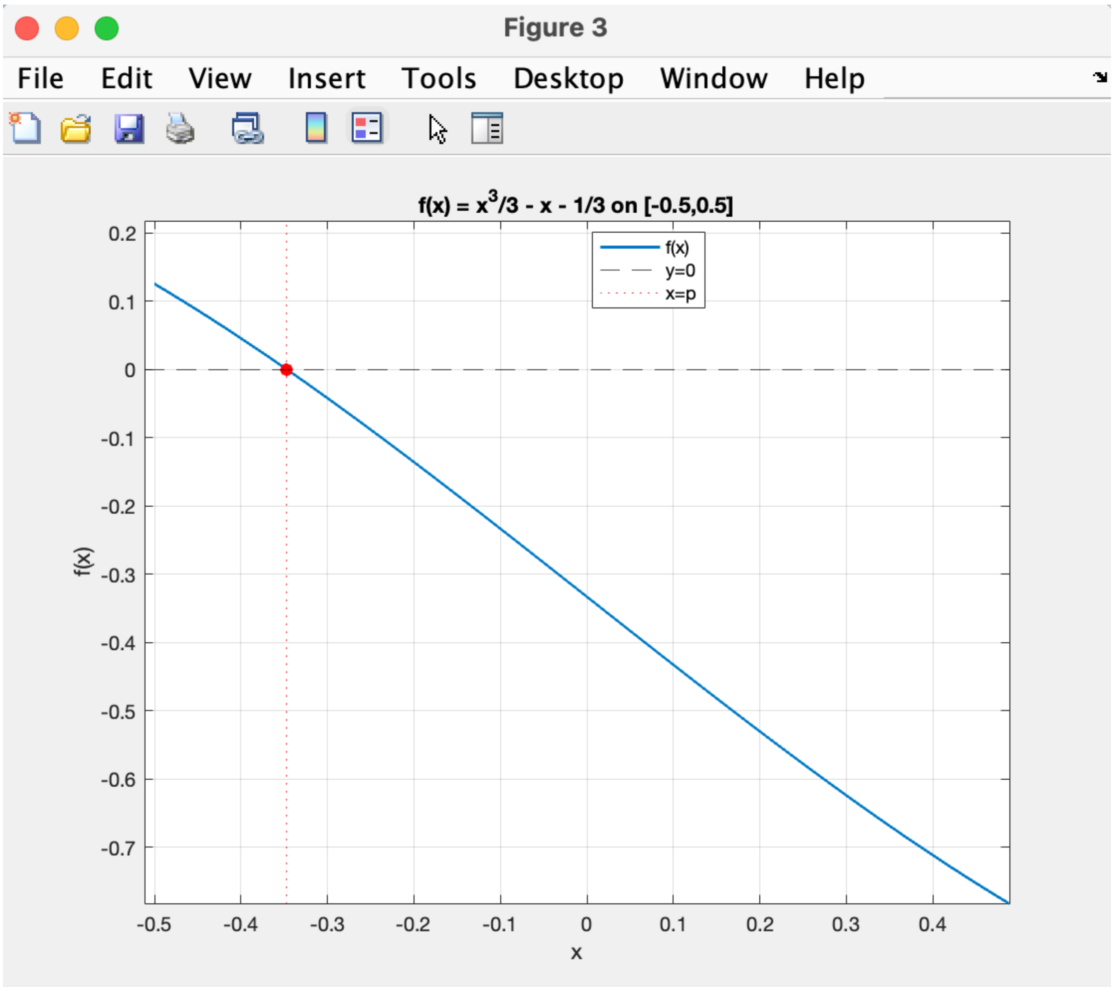
1. Bisection Method: I have the input of function f on the interval [a,b]. The tolerance is tol = 1e-4. Iteratively halved the interval of [a,b] until the absolute value of f(pn) smaller and equal to 10^-4. Recorded pn at each iterations to plot the absolute value of (pn – p) and the theoretical bound (b-a)/2^n.
2. Fixed-point Method: Implement the fixed point function by using g(x) = (x^3 – 1)/3, g prime(x) = x^2. Convergence guaranteed on [-0.5, 0.5] since the absolute value of g'(x) is smaller and equal to 0.25 and 0.25 is smaller to 1. Starting at p_0 = 0, iterated until the absolute value of [f(pn)] smaller and equal to 10^-4. Plotted the absolute value of (pn – p) and the contraction bound K^n/(1-K) times the absolute value of (p1 – p0) with K = 0.25.
3. Initialization Test: Lastly, repeated fixed-point iterations for 21 initial guesses p0 belongs to [-0.5, 0.5]. Then, count iterations and estimated local convergence rates from the absolute value of (p_n+1 – p_n) / the absolute value (p_n – p_n-1).

3 Results

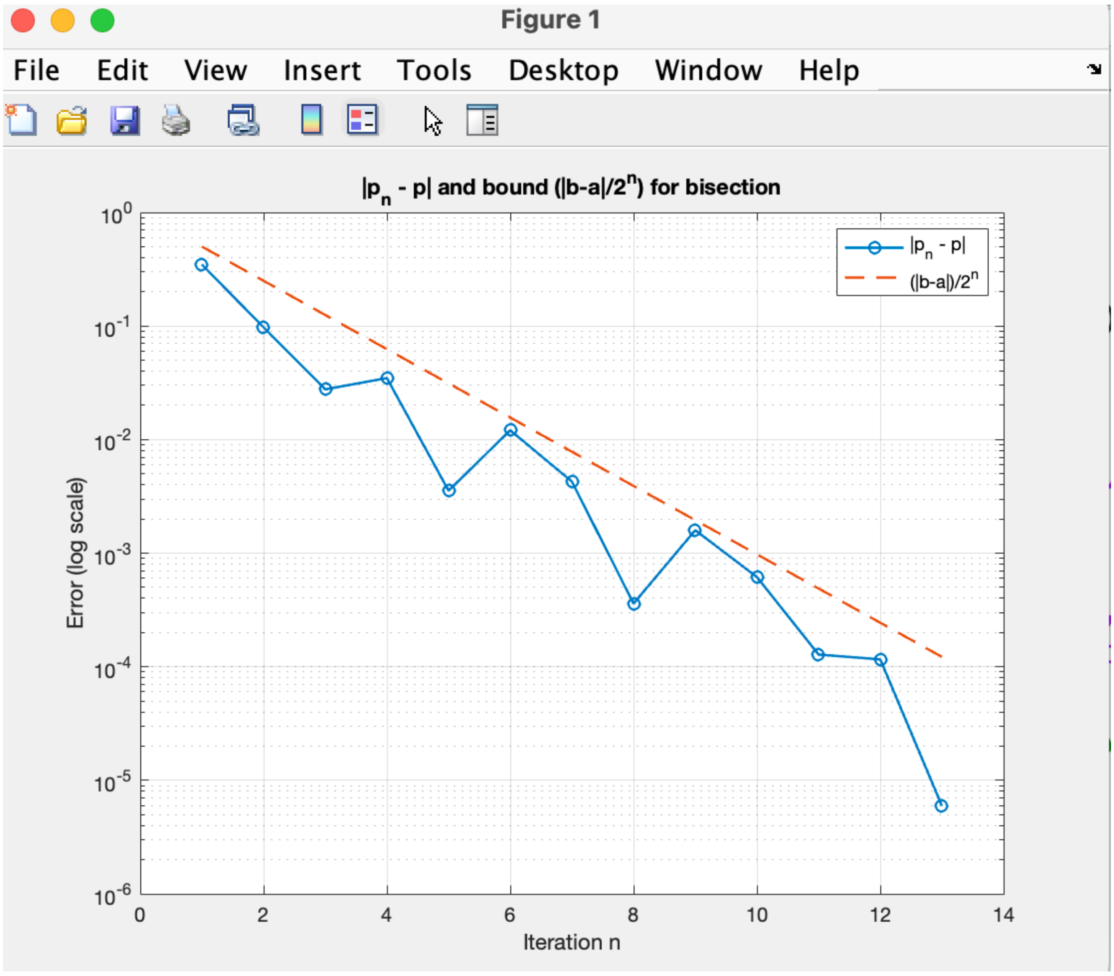
The approximate root is p is -0.347296 with the absolute value of f(p) approximately equal to 3*10^-7, matching the given value. For the bisection method, it converges in approximately 17 iterations; errors decreased linearly with the sloe of 1/2 each step. Theoretical bound (b-a)/2^n matches the measured of the absolute value of (pn – p). For the fixed-point method, it converges in approximately 22 iterations with p_0 = 0. The empirical rate abs[(p_n+1 – p_n) / (p_n – p_n-1)] is approximately equal to 0.12, consistent with the abs(g'(p)) = p^2 = 0.12. The error bound [K^n/1-K]*abs(p1 – p0) overestimated actual errors.

For the initialization effect, all p_0 belongs to [-0.5, 0.5] converge to the same root. Points farther from the root required more iterations, but the asymptotic rate remain with 0.12 for every p_0.

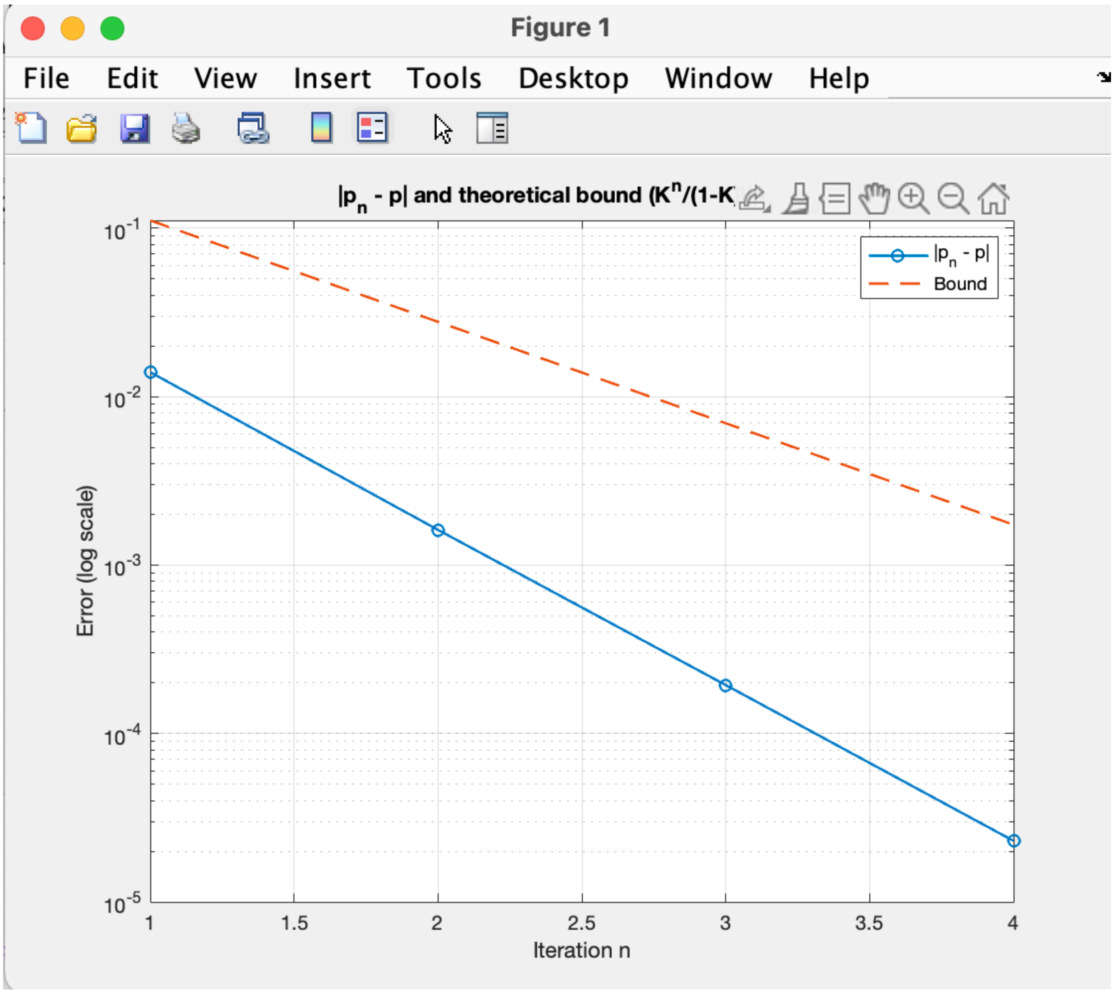
Here is the Plotted figure for Q3:



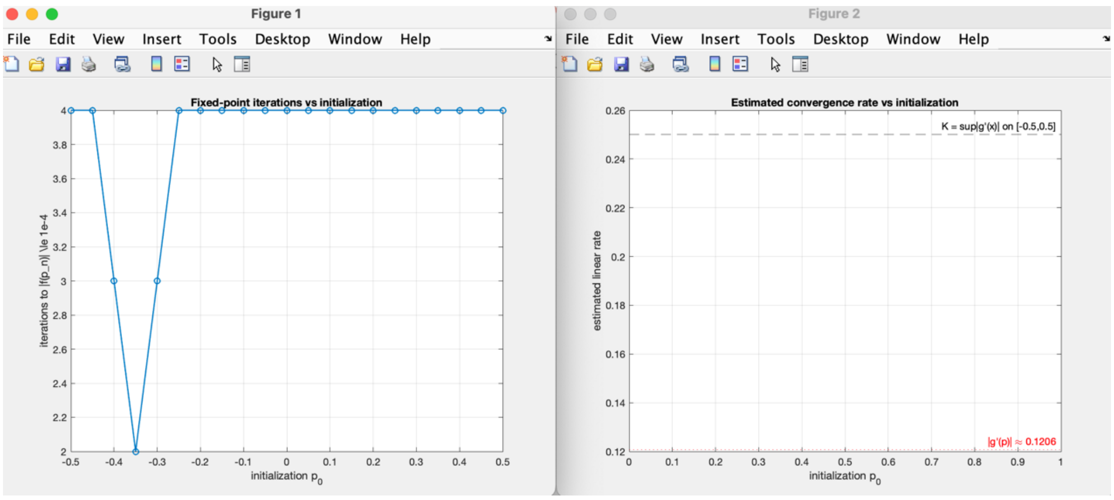
Here is the figure for Q4:



Here is a figure for Q5:



The figure for Q6:



4 Conclusion

The Bisection Method is slower per iteration but its result is converge.

Fixed-Point converges faster asymptotically depends on a good g(x0 and interval where the abs(g'(x)) < 1.

For this function and g(x), the fixed-point method is asymptotically faster, while the bisection one is strong and stable to get the results.

5 Challenges and Bugs

1. fprintf formatting looked confusing for me.
2. The previous version of my "my_bisection.m" function lacked a semicolon after fb = fc. The MATLAB gives me a warning.
3. I forget to add the "hold on" command when I plot figures. Some plots just display only one curve.