Variables and assignments

Pronouns and quantifiers

Pronoun: She is smart.

 $\label{eq:Quantification: Every girl is smart.}$

Some boy is silly.

Multiple quantifiers: Every girl saw a boy.

Quantifier+pronoun: No kid likes their school.

What do a pronoun or a quantifier denote?

Variables

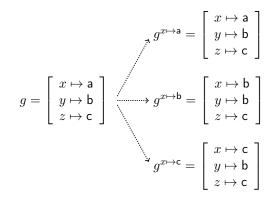
A pronoun does not have a deterministic meaning. Its value varies among different contexts.

- (1) Ada resolved the puzzle. She is smart.
- (2) Becky got an A. She is smart.

Formally, variables do not have deterministic values, either.

Universal quantification

 $[\![\text{everyone is smart}]\!]^{M,g} = \forall x.\mathsf{smart}(x)$



 $\forall x.\mathsf{smart}(x)$ means:

 $\operatorname{smart}(g^{x\mapsto \mathsf{a}}(x)) \wedge \operatorname{smart}(g^{x\mapsto \mathsf{b}}(x)) \wedge \operatorname{smart}(g^{x\mapsto \mathsf{c}}(x))$

Assignment (function)

$$g = \left[\begin{array}{c} x \mapsto \mathsf{a} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{array} \right] \qquad \qquad g = \left[\begin{array}{c} x \mapsto \mathsf{a} \\ y \mapsto \mathsf{a} \\ z \mapsto \mathsf{c} \end{array} \right]$$

$$[\![\operatorname{she}_x]\!]^{M,g} = g(x)$$

The concrete value of g is **contextually** determined.

• For (1),
$$g(x) = \mathbf{a}$$

$$[\![\mathbf{she}_x \text{ is smart}]\!]^{M,g} = [\![\mathbf{smart}]\!]^{M,g} ([\![\mathbf{she}_x]\!]^{M,g})$$

$$= \mathbf{smart}(g(x))$$

$$= \mathbf{smart}(\mathbf{a})$$

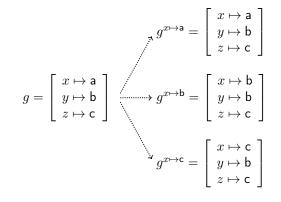
• For (2),
$$g(x) = b$$

$$[she_x \text{ is smart}]^{M,g} = [smart]^{M,g} ([she_x]^{M,g})$$

$$=$$

Existential quantification

 $[someone is silly]^{M,g} = \exists x.silly(x)$

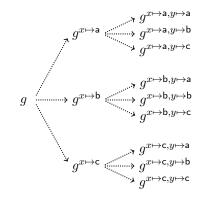


 $\exists x.\mathsf{silly}(x)$ means:

$$\mathsf{silly}(g^{x \mapsto \mathsf{a}}(x)) \vee \mathsf{silly}(g^{x \mapsto \mathsf{b}}(x)) \vee \mathsf{silly}(g^{x \mapsto \mathsf{c}}(x))$$

Multiple quantifiers

[everyone saw someone]] $^{M,g} = \forall x \exists y.see(y)(x)$



 $\forall x \exists y. see(y)(x)$ means:

$$\left\{ \begin{array}{l} \operatorname{see}(g^{x\mapsto \mathtt{a},y\mapsto \mathtt{a}}(y))(g^{x\mapsto \mathtt{a},y\mapsto \mathtt{a}}(x) \\ \operatorname{see}(g^{x\mapsto \mathtt{a},y\mapsto \mathtt{b}}(y))(g^{x\mapsto \mathtt{a},y\mapsto \mathtt{b}}(x)) \\ \operatorname{see}(g^{x\mapsto \mathtt{a},y\mapsto \mathtt{c}}(y))(g^{x\mapsto \mathtt{a},y\mapsto \mathtt{c}}(x)) \end{array} \right\} \\ \left\{ \begin{array}{l} \operatorname{see}(g^{x\mapsto \mathtt{a},y\mapsto \mathtt{c}}(y))(g^{x\mapsto \mathtt{b},y\mapsto \mathtt{a}}(x) \\ \operatorname{see}(g^{x\mapsto \mathtt{b},y\mapsto \mathtt{b}}(y))(g^{x\mapsto \mathtt{b},y\mapsto \mathtt{b}}(x)) \\ \operatorname{see}(g^{x\mapsto \mathtt{b},y\mapsto \mathtt{c}}(y))(g^{x\mapsto \mathtt{b},y\mapsto \mathtt{c}}(x)) \end{array} \right\} \\ \left\{ \begin{array}{l} \operatorname{see}(g^{x\mapsto \mathtt{c},y\mapsto \mathtt{a}}(y))(g^{x\mapsto \mathtt{c},y\mapsto \mathtt{a}}(x) \\ \operatorname{see}(g^{x\mapsto \mathtt{c},y\mapsto \mathtt{b}}(y))(g^{x\mapsto \mathtt{c},y\mapsto \mathtt{b}}(x)) \\ \operatorname{see}(g^{x\mapsto \mathtt{c},y\mapsto \mathtt{c}}(y))(g^{x\mapsto \mathtt{c},y\mapsto \mathtt{c}}(x)) \end{array} \right\} \\ \end{array} \right\}$$

How about the other meaning: $\exists y \forall x. see(y)(x)$

Bound pronouns

 $[\![\text{no one likes their}_x \text{ school}]\!]^{M,g} = \\ \neg \exists x. \mathsf{like}(\mathsf{school}\text{-of-}x)(x)$

The value of $their_x$ co-varies with $\exists x$