Generalized quantifiers: Basics

Quantification in FOL

(1) Every **lady danced**. (2) A **cat danced**.

Suppose that we have two ladies, one boy and one cat

- Ada and Becky danced
- Carl also danced.
- The cat Kitty didn't dance.

$\forall x \Rightarrow g$

$g^{x\mapsto \mathsf{a}}$ $\mathsf{lady}(g^{x\mapsto \mathsf{a}}(x))$ $\mathsf{dance}(g^{x\mapsto \mathsf{a}}(x))$

$g^{x\mapsto \mathsf{b}}$ $\mathsf{lady}(g^{x\mapsto \mathsf{b}}(x))$ $\mathsf{dance}(g^{x\mapsto \mathsf{b}}(x))$

$g^{x\mapsto \mathsf{c}}$ $\mathsf{lady}(g^{x\mapsto \mathsf{c}}(x))$ $\mathsf{dance}(g^{x\mapsto \mathsf{c}}(x))$

$g^{x\mapsto \mathsf{k}}$ $\mathsf{lady}(g^{x\mapsto \mathsf{k}}(x))$ $\mathsf{dance}(g^{x\mapsto \mathsf{k}}(x))$

Given the fact,
(1) should be translated as: $\forall x.\mathsf{lady}(x) \to \mathsf{dance}(x)$

How about (2)?

Problem

(3) Most ladies danced.

Suppose there're three ladies, one boy and one cat:

Scenario I: (3) is true

- Ada danced and Becky, but Cindy didn't
- Donald didn't danced.
- The cat Kitty didn't dance.

Scenario II: (3) is false

- Ada danced, but Becky and Cindy didn't
- Donald danced.
- The cat Kitty didn't dance.

Consider the following formulas:

1. $\mathsf{most}\ x.\mathsf{lady}(x) \to \mathit{dance}(x)$ — 1 in I; $\boxed{1}$ in II
2. $\mathsf{most}\ x.\mathsf{lady}(x) \wedge \mathit{dance}(x)$ — $\boxed{0}$ in I; 0 in II
3. $\mathsf{most}\ x.\mathsf{lady}(x) \vee \mathit{dance}(x)$ — 1 in I; $\boxed{1}$ in II

Operations on sets

$[\![\text{every lady danced}]\!] = \{x \mid \mathsf{lady}(x)\} \subseteq \{x \mid \mathsf{dance}(x)\}$

$[\![\text{a cat danced}]\!] = \{x \mid \mathsf{cat}(x)\} \cap \{x \mid \mathsf{dance}(x)\} \neq \emptyset$

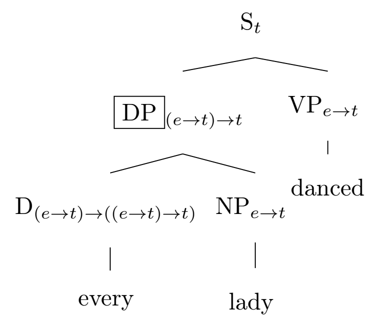
$[\![\text{most ladies danced}]\!] = \dfrac{|\mathsf{lady} \cap \mathsf{dance}|}{|\mathsf{lady}|} > \dfrac{1}{2}$

$[\![\text{more than two ladies danced}]\!] = |\mathsf{lady} \cap \mathsf{dance}| > 2$

$[\![\text{at most two ladies danced}]\!] = |\mathsf{lady} \cap \mathsf{dance}| \leqslant 2$

$[\![\text{no lady danced}]\!] = |\mathsf{lady} \cap \mathsf{dance}| = \emptyset$

What are quantifiers



Suppose there're three ladies and two boys:

- Ada, Becky, Cindy and Donald danced
- Ada, Becky, Cindy and Eric sang

$$\mathsf{lady} = \{\mathsf{a},\mathsf{b},\mathsf{c}\} \subseteq \begin{array}{l} \mathsf{dance} = \{\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{d}\} \\ \mathsf{lady} = \{\mathsf{a},\mathsf{b},\mathsf{c}\} \\ \mathsf{sing} = \{\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{e}\} \end{array}$$

$[\![\text{every lady}]\!] = \{\{\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{d}\}, \{\mathsf{a},\mathsf{b},\mathsf{c}\}, \{\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{e}\}\}$

1. $[\![\text{every lady is a lady}]\!] = [\![\text{lady}]\!] \in [\![\text{every lady}]\!]$
2. $[\![\text{every lady danced}]\!] = [\![\text{dance}]\!] \in [\![\text{every lady}]\!]$
3. $[\![\text{every lady sang}]\!] = [\![\text{sang}]\!] \in [\![\text{every lady}]\!]$

$[\![\text{every lady}]\!] = \lambda P.\mathsf{lady} \subseteq P$ Type: $(e \to t) \to t$

Boolean compounds

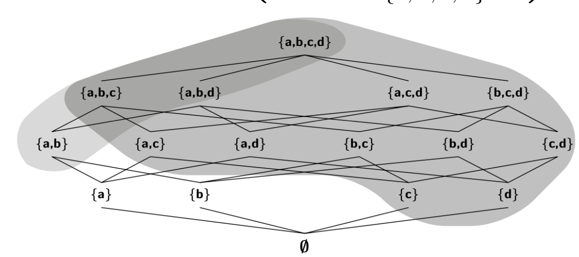
(4) Every lady **and** at least one boy danced.
(5) Every lady **or** at least one boy danced.
(6) Every lady **but** not every boy danced.

Suppose that we have two ladies and two boys:

- Ada and Becky danced and sang.
- Carl danced but not sing.
- Donald sang but not dance.
- Ada and Carl wore a hat.
- Becky and Carl are tall.
- Ada and Donald likes swimming.
- Becky and Donald wore a scarf.

$$[\![\text{every lady}]\!] = \left\{ \begin{array}{l} \mathsf{lady} = \{\mathsf{a},\mathsf{b}\} \\ \mathsf{dance} = \{\mathsf{a},\mathsf{b},\mathsf{c}\} \\ \mathsf{sing} = \{\mathsf{a},\mathsf{b},\mathsf{d}\} \\ \mathsf{human} = \{\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{d}\} \end{array} \right\}$$

$$[\![\text{at least one boy}]\!] = \left\{ \begin{array}{l} \mathsf{boy\ who\ danced} = \{\mathsf{c}\} \\ \mathsf{boy\ who\ sang} = \{\mathsf{d}\} \\ \mathsf{boy} = \{\mathsf{c},\mathsf{d}\} \\ \mathsf{wear.hat} = \{\mathsf{a},\mathsf{c}\} \\ \mathsf{wear.scarf} = \{\mathsf{b},\mathsf{d}\} \\ \mathsf{tall} = \{\mathsf{b},\mathsf{c}\} \\ \mathsf{like.swim} = \{\mathsf{a},\mathsf{d}\} \\ \mathsf{dance} = \{\mathsf{a},\mathsf{b},\mathsf{c}\} \\ \mathsf{sing} = \{\mathsf{a},\mathsf{b},\mathsf{d}\} \\ \mathsf{human} = \{\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{d}\} \end{array} \right\}$$



$[\![\text{every lady } \textbf{and} \text{ at least one boy}]\!]$