Empirical properties of quantifiers

Any

1. *Ann ate **any** fish last night.
2. *A lady ate **any** fish last night.
3. Ann didn't eat **any** fish last night.
4. No lady ate **any** fish last night.
5. *Every lady who read a book talked to **any** professor.
6. Every lady who read **any** book talked to a professor.

Monotonic increasing

Suppose we have three ladies, Ann, Becky, and Cindy, and one body, Donald,

- Ann, Cindy, and Donald walked.
- Ann, Becky, and Cindy talked.

$$(\mathsf{walk} \cap \mathsf{talk}) \subseteq \mathsf{walk} \subseteq (\mathsf{walk} \cup \mathsf{talk})$$

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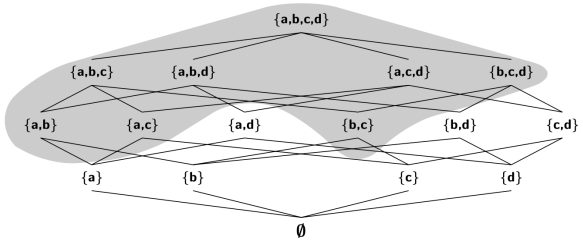
1. More than one lady walked and talked.
2. More than one lady walked.
3. More than one lady walked or talked.

1 entails 2, 2 entails 3, and 1 entails 3

⟦more than one lady⟧ =



The generalized quantifier GQ is **monotonically increasing** iff whenever X is an element of GQ, all supersets of X are elements of GQ.

Monotonic decreasing

Suppose we have two ladies, Ann and Becky, and two boys, Carl and Donald

- Ann and Becky jogged.
- Carl walked.
- Carl and Donald talked.

$$(\mathsf{walk} \cap \mathsf{talk}) \subseteq \mathsf{walk} \subseteq (\mathsf{walk} \cup \mathsf{talk})$$

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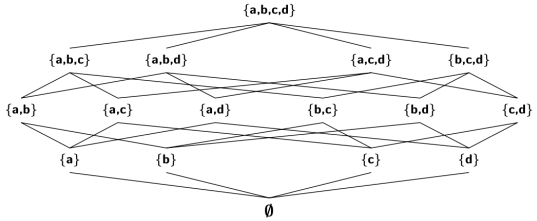
1. No lady walked and talked.
2. No lady walked.
3. No lady walked or talked.

3 entails 2, 2 entails 1, and 3 entails 1

⟦no lady⟧ =

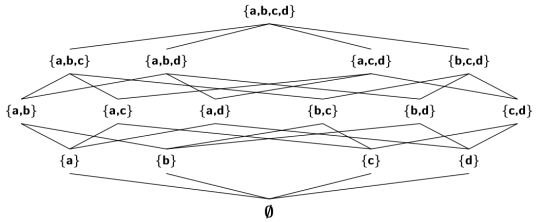


The generalized quantifier GQ is **monotonically decreasing** iff whenever X is an element of GQ, all subsets of X are elements of GQ.

Non-monotonic

1. Exactly two ladies jogged.

⟦exactly two ladies⟧ =



Downward and upward entailing

1. Every lady who read a book talked to a professor.
2. Every lady who read a novel talked to a professor.

- $\mathsf{novel} \subseteq \mathsf{book}$;
- $\{x \mid x \text{ read a novel}\} \subseteq \{x \mid x \text{ read a book}\}$
- $[\![1]\!] = \{x \mid x \text{ read a novel}\} \subseteq \{x \mid x \text{ talk to prof}\}$;
- $[\![2]\!] = \{x \mid x \text{ read a book}\} \subseteq \{x \mid x \text{ talk to prof}\}$;
- $[\![2]\!]$ entails $[\![1]\!]$

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1. Every lady who read a book talked to a professor.
2. Every lady who read a book talked to a teacher.

- $\mathsf{professor} \subseteq \mathsf{teacher}$;
- $\{x \mid x \text{ talk to prof}\} \subseteq \{x \mid x \text{ talk to teacher}\}$
- $[\![2]\!] = \{x \mid x \text{ read book}\} \subseteq \{x \mid x \text{ talk to prof}\}$;
- $[\![1]\!] = \{x \mid x \text{ read book}\} \subseteq \{x \mid x \text{ talk to teacher}\}$;
- $[\![1]\!]$ entails $[\![2]\!]$

The maximality problem

1. At least two ladies walked. = There is a set of men with cardinality at least two such that all its elements walk.
2. At most two ladies walked. ≠ There is a set of men with cardinality at most two such that all its elements walk.
3. Exactly two ladies talked. ≠ There is a set of men with cardinality exactly two such that all its elements walk.

The sentence 1, but not 2 and 3, is true in a situation in which more than two ladies walked.