Model-theoretic Semantics: Names and verbs

Models

A model M for predicate logic is an ordered pair $\langle U, \llbracket\cdot\rrbracket\rangle$, in which:

- U is a a nonempty set of individuals;
- $\llbracket\cdot\rrbracket$ is an interpretation function.

Ling-exp ⇢ $\llbracket\cdot\rrbracket$ ⇢ some object built from U

A sample model

$U = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$	a:	a girl named *Ada*
	b:	a girl named *Becky*
	c:	a girl named *Cindy*

Facts

- a, b and c are girls.
- a and b danced, but c didn't.
- a saw b, but b didn't see a.
- c saw a, and a saw c, too.
- a and c are Americans, but b is a British.

Names

$\llbracket\text{Ada}\rrbracket = \mathsf{a}$ $\quad\llbracket\text{Becky}\rrbracket = \mathsf{b}$ $\quad\llbracket\text{Cindy}\rrbracket = \mathsf{c}$

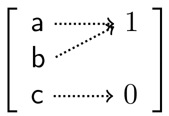
Type e

Intransitive verbs

$\llbracket\text{dance}\rrbracket = \{x \mid x \text{ dances}\} = \{\mathsf{a}, \mathsf{b}\} \subseteq U$

Characteristic functions

$\llbracket\text{dance}\rrbracket =$



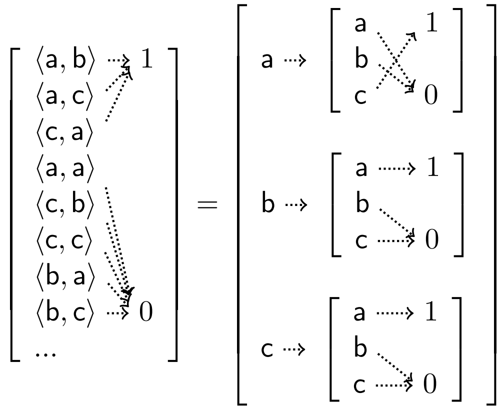
Type $e \to t$

Transitive verbs

$$\llbracket\text{see}\rrbracket = \{\langle x, y\rangle \mid x \text{ saw } y\}$$
$$= \{\langle \mathsf{a}, \mathsf{b}\rangle,\ \langle \mathsf{c}, \mathsf{a}\rangle,\ \langle \mathsf{a}, \mathsf{c}\rangle\} \subseteq U \times U$$

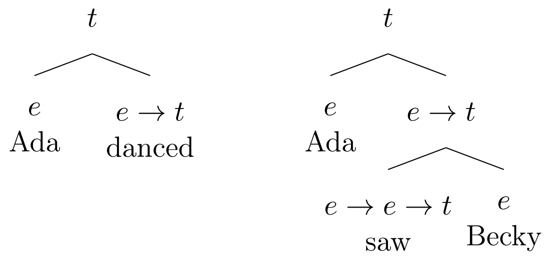
Characteristic functions

$\llbracket\text{see}\rrbracket =$



Type $e \to e \to t$

Combination



Functional Application

$$\frac{f :: a \to b \qquad x :: a}{f(x) :: b}$$

1. $\llbracket\text{Ada danced}\rrbracket = \llbracket\text{danced}\rrbracket(\llbracket\text{Ada}\rrbracket)$
2. $\llbracket\text{Ada saw Becky}\rrbracket = \llbracket\text{saw}\rrbracket(\llbracket\text{Becky}\rrbracket)(\llbracket\text{Ada}\rrbracket)$

Formal concepts

Set "a collection of distinct objects, considered as an object in its own right" (thanks wiki)
Defining sets

- Extensionally: $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$
- Intensionally: $\{x \mid x \text{ is a girl}\}$

Empty set: $\emptyset$ or $\{\}$

Relations between sets

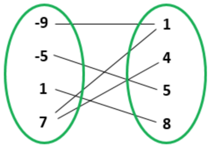
- Proper subset: $A \subset B$ iff $A \subseteq B$ and $B \not\subseteq A$
- Equivalence: $A = B$ iff $A \subseteq B$ and $B \subseteq A$
- Superset: $B \supseteq A$ iff $A \subseteq B$

Power sets $\wp(A) = \{B \mid B \subseteq A\}$

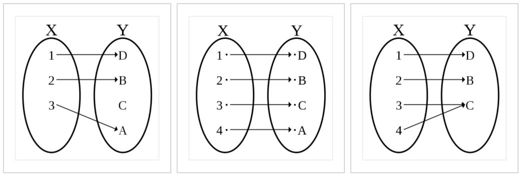
Pairs $\langle \mathsf{a}, \mathsf{b}\rangle \neq \langle \mathsf{b}, \mathsf{a}\rangle$ $\qquad$ n**-tuples** $\langle \mathsf{a}, \mathsf{b}, \mathsf{c}, \ldots\rangle$

Relations as sets of pairs (Cartesian products)

$$A \times B := \{\langle x, y\rangle \mid x \in A \text{ and} y \in B\}$$



Function any relation where each input is paired with at most one output



injection $\qquad$ bijection $\qquad$ surjection

Type a 'name tag' of a set (D_e, D_t)
Basic types: e (individuals); t (truth values)
Function types: $a \to b$

Currying

‘In mathematics and computer science, **currying** is the technique of converting a function that takes multiple arguments into a sequence of functions that each take a single argument.’ (thanks wiki)

Given a function $f : (X \times Y) \to Z$,

currying constructs a new function: $h : X \to Y \to Z$

NB: The operation **currying** is named after Haskell Curry, an American mathematician and logician,

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Exponential notation of functions $f \in Y^X$

Given $X = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$ and $Y = \{1, 0\}$,

1. For each x in X, there are two mappings from $\{x\}$ to Y

   $S_{\mathsf{a}} = \{[\mathsf{a} \mapsto 1], [\mathsf{a} \mapsto 0]\}$; $S_{\mathsf{b}} = \{[\mathsf{b} \mapsto 1], [\mathsf{b} \mapsto 0]\}$; $S_{\mathsf{c}} = \{[\mathsf{c} \mapsto 1], [\mathsf{c} \mapsto 0]\}$

2. Possible functions from X to Y:

$$\begin{bmatrix} \mathsf{a} \mapsto 1, \\ \mathsf{b} \mapsto 1, \\ \mathsf{c} \mapsto 0 \end{bmatrix} \begin{bmatrix} \mathsf{a} \mapsto 1, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 0 \end{bmatrix} \begin{bmatrix} \mathsf{a} \mapsto 0, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 0 \end{bmatrix} \begin{bmatrix} \mathsf{a} \mapsto 1, \\ \mathsf{b} \mapsto 1, \\ \mathsf{c} \mapsto 1 \end{bmatrix}$$

$$\begin{bmatrix} \mathsf{a} \mapsto 1, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 1 \end{bmatrix} \begin{bmatrix} \mathsf{a} \mapsto 0, \\ \mathsf{b} \mapsto 1, \\ \mathsf{c} \mapsto 1 \end{bmatrix} \begin{bmatrix} \mathsf{a} \mapsto 0, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 1 \end{bmatrix} \begin{bmatrix} \mathsf{a} \mapsto 0, \\ \mathsf{b} \mapsto 1, \\ \mathsf{c} \mapsto 0 \end{bmatrix}$$

3. The total number of the functions $X \mapsto Y$ is: $2 \times 2 \times 2$

4. $\left|Y^X\right| = |Y|^{|X|} = 2^3$

Proof of Currying

$$\begin{aligned} |Z^{(X \times Y)}| &= |Z|^{|X \times Y|} \\ &= |Z|^{|X| \times |Y|} \\ &= (|Z|^{|Y|})^{|X|} \\ &= |(Z^Y)^X| \end{aligned} \qquad \begin{aligned} |Z^{(X \times Y)}| &= |Z|^{|X \times Y|} \\ &= |Z|^{|X| \times |Y|} \\ &= (|Z|^{|X|})^{|Y|} \\ &= |(Z^X)^Y| \end{aligned}$$

Currying n-ary relations

$$\llbracket \text{see} \rrbracket = \begin{bmatrix} \langle \mathsf{a}, \mathsf{b} \rangle \mapsto 1, \\ \langle \mathsf{a}, \mathsf{c} \rangle \mapsto 1, \\ \langle \mathsf{a}, \mathsf{a} \rangle \mapsto 0, \\ \langle \mathsf{b}, \mathsf{a} \rangle \mapsto 0, \\ \langle \mathsf{b}, \mathsf{c} \rangle \mapsto 0, \\ \langle \mathsf{b}, \mathsf{b} \rangle \mapsto 0, \\ \langle \mathsf{c}, \mathsf{a} \rangle \mapsto 1, \\ \langle \mathsf{c}, \mathsf{b} \rangle \mapsto 0, \\ \langle \mathsf{c}, \mathsf{c} \rangle \mapsto 0 \end{bmatrix}$$

Turning n-ary relations to n-place functions

$$\begin{bmatrix} \mathsf{a} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 0, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 1 \end{bmatrix} \\ \mathsf{b} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 1, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 0 \end{bmatrix} \\ \mathsf{c} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 1, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 0 \end{bmatrix} \end{bmatrix} \qquad \begin{bmatrix} \mathsf{a} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 0, \\ \mathsf{b} \mapsto 1, \\ \mathsf{c} \mapsto 1 \end{bmatrix} \\ \mathsf{b} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 0, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 0 \end{bmatrix} \\ \mathsf{c} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 1, \\ \mathsf{b} \mapsto 0, \\ \mathsf{c} \mapsto 0 \end{bmatrix} \end{bmatrix}$$

Right to Left Left to Right

Model-theoretical Semantics: Connectives

Connectives

Negation: Tom didn't leave.
Conjunction: Tom danced and Andy sang.
Disjunction: Tom danced or Andy sang.
Conditional: If Andy sang, then Tom danced.

Truth table

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

p	$\neg p$
1	0
0	1

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

Meaning

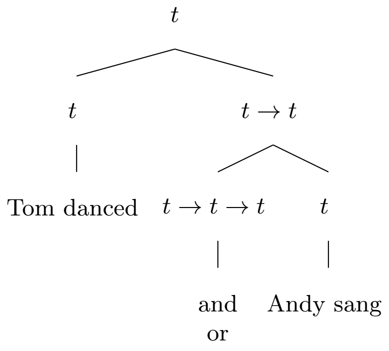
$$[\![\text{and}]\!] = \begin{bmatrix} 1 \mapsto \begin{bmatrix} 1 \mapsto 1 \\ 0 \mapsto 0 \end{bmatrix} \\ \\ 0 \mapsto \begin{bmatrix} 1 \mapsto 0 \\ 0 \mapsto 0 \end{bmatrix} \end{bmatrix} \qquad [\![\text{not}]\!] = \begin{bmatrix} 1 \mapsto 0 \\ 0 \mapsto 1 \end{bmatrix}$$

$$[\![\text{or}]\!] = \begin{bmatrix} 1 \mapsto \begin{bmatrix} 1 \mapsto 1 \\ 0 \mapsto 1 \end{bmatrix} \\ \\ 0 \mapsto \begin{bmatrix} 1 \mapsto 1 \\ 0 \mapsto 0 \end{bmatrix} \end{bmatrix}$$

Type

$$\begin{aligned} &[\![\text{not}]\!] :: t \to t \\ &[\![\text{and}]\!] :: t \to t \to t \\ &[\![\text{or}]\!] \;\; :: t \to t \to t \end{aligned}$$

Composition



Logical equivalences

1. Double negation elimination

- $\neg\neg p = p$

2. Distributive laws

- $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
- $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

3. Associative laws

- $(p \wedge q) \wedge r = p \wedge (q \wedge r)$
- $(p \vee q) \vee r = p \vee (q \vee r)$

4. DeMorgan's laws

- $\neg(p \wedge q) = \neg p \vee \neg q$
- $\neg(p \vee q) = \neg p \wedge \neg q$

5. Eliminability of the material conditional

- $p \to q = \neg p \vee q$

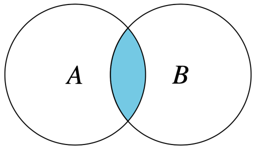
A sample proof

$(p \wedge q) \to r = p \to (q \to r)$

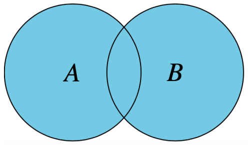
Proof:
$$\begin{aligned} (p \wedge q) \to r &= \neg(p \wedge q) \vee r && (5) \\ &= (\neg p \vee \neg q) \vee r && (4) \\ &= \neg p \vee (\neg q \vee r) && (3) \\ &= p \to (\neg q \vee r) && (5) \\ &= p \to (q \to r) && (5) \quad \square \end{aligned}$$

Operations on sets

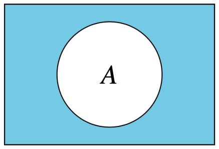
Intersection $A \cap B := \{x \mid x \in A \text{ and } x \in B\}$



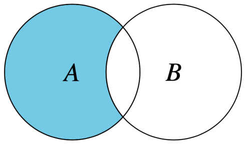
Union $A \cup B := \{x \mid x \in A \text{ or } x \in B\}$



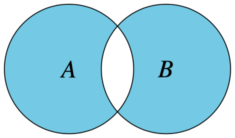
Complementation $\overline{A} := \{x \mid x \notin A\}$



Difference $A - B := \{x \mid x \in A \text{ and } x \notin B\}$



Exclusive union $A \underline{\cup} B := (A \cup B) - (A \cap B)$



Different kinds of meaning: Presupposition

What is taken for granted

Possessives

(1) **Emma's son** is smart.
⇝ Emma has a son.

Factive verbs

(2) Becky **knows** Alex is tall.
⇝ Alex is tall.

Additive adverbs

(3) Kelly wore a hat, **too**.
⇝ Someone else wore a hat.

Definites

(4) **The** student is smart.
⇝ There is a unique student.

Gender feature

(5) **She** is knowledgable.
⇝ The person referred to by *she* is female.

Definition

Presuppositions are inferences **backgrounded** and **taken for granted** (Redundancy).

1. A sentence can be felicitously uttered only in contexts where its presupposition is true.
2. Expressions triggering presuppositions are presupposition triggers.

Three value logic (weak Kleene)

A sentence is neither true (1) nor false (0), but undefined (#) iff its presupposition is false.

ϕ	ψ	$\neg\phi$	$\phi \wedge \psi$	$\phi \vee \psi$	$\phi \rightarrow \psi$
1	1	0	1	1	1
1	0	0	0	1	0
1	#	0	#	#	#
0	1	1	0	1	1
0	0	1	0	0	1
0	#	1	#	#	#
#	1	#	#	#	#
#	0	#	#	#	#
#	#	#	#	#	#

What are you protesting

(6) A: Is Emma's son smart?
B: No, he isn't.
B': $^{\#}$No, Emma doesn't have a son.
B": Hey wait a minute. I didn't know Emma has a son.

Projection

(7) Emma's son is **not** smart. (negation)
⇝ Emma has a son.

(8) **Is** Emma's son smart? (question)
⇝ Emma has a son.

(9) **Maybe** Emma's son is smart. (modal)
⇝ Emma has a son.

(10) **If** Emma's son is smart, **then** he can resolve this puzzle. (conditional)
⇝ Emma has a son.

Filtering

In a sentence consisting of multiple sub-clauses,

1. the presupposition of a sub-clause may be satisfied locally;
2. the whole sentence does not have the presupposition.

Conjunction

(11) Emma has a son and <u>her son</u> is very smart.
$\not\rightsquigarrow$ Emma has a son.

Conditional

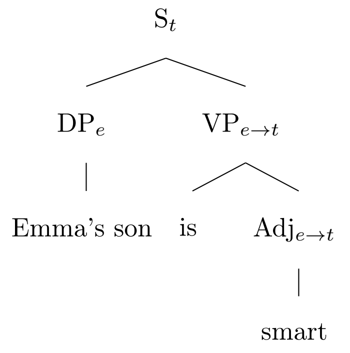
(12) If Emma has a son, she would send <u>her son</u> to that school.
$\not\rightsquigarrow$ Emma has a son.

Disjunction

(13) Either Emma doesn't have a son, or <u>her son</u> doesn't live with her.
$\not\rightsquigarrow$ Emma has a son.

Definedness condition

$[\![\text{Emma's son}]\!] = \mathsf{a}$ Type: e
defined only if Emma has a son



- $[\![\text{smart}]\!] = x \mapsto x \in \{y \mid y \text{ is smart}\}$ Type: $e \rightarrow t$
- $[\![\text{Emma's son is smart}]\!] = [\![\text{smart}]\!]([\![\text{Emma's son}]\!])$
$= \mathsf{a} \in \{y \mid y \text{ is smart}\}$
defined only if
Emma has a son

At issue meaning: $\mathsf{a} \in \{y \mid y \text{ is smart}\}$

Non-at-issue meaning: the definedness condition

Other types of non-at-issue meanings

Scalar implicature (Cancelable)

(14) Emma ate three apples.
⇝ Emma ate **only** three apples.

(15) Emma ate three apples. In fact, she ate four.

Supplement (non-at-issue but new)

(16) Did Alex, who you mistreated, press charges?

(17) A: Alex is incompetent.
B: Does Alex knows he is?
B': $^{\#}$Is Alex, who is incompetent, aware of this?

Variables and assignments: pronouns and quantifiers

Pronouns and quantifiers

Pronoun: She is smart.

Quantification: Every girl is smart.

Some boy is silly.

Multiple quantifiers: Every girl saw a boy.

Quantifier+pronoun: No kid likes their school.

What does a pronoun or a quantifier denote?

Variables

A pronoun does not have a deterministic meaning. Its value varies depending on the context.

(1) Ada solved the puzzle. She is smart.
(2) Becky got an A. She is smart.

Formally, variables do not have deterministic values, either.

Assignment (function)

$$g = \begin{bmatrix} x \mapsto \mathsf{a} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \qquad g' = \begin{bmatrix} x \mapsto \mathsf{a} \\ y \mapsto \mathsf{a} \\ z \mapsto \mathsf{c} \end{bmatrix}$$

$\llbracket \text{she}_x \rrbracket^{M,g} = g(x)$

Which g is picked is **contextually** determined.

- For (1), $g(x) = \mathsf{a}$

$$\begin{aligned} \llbracket \text{she}_x \text{ is smart} \rrbracket^{M,g} &= \llbracket \text{smart} \rrbracket^{M,g}(\llbracket \text{she}_x \rrbracket^{M,g}) \\ &= \mathsf{smart}(g(x)) \\ &= \mathsf{smart}(\mathsf{a}) \end{aligned}$$

- For (2), $g'(x) = \mathsf{b}$

$$\begin{aligned} \llbracket \text{she}_x \text{ is smart} \rrbracket^{M,g'} &= \llbracket \text{smart} \rrbracket^{M,g'}(\llbracket \text{she}_x \rrbracket^{M,g'}) \\ &= \mathsf{smart}(g'(x)) = \mathsf{smart}(\mathsf{b}) \end{aligned}$$

Universal quantification

$\llbracket \text{everyone is smart} \rrbracket^{M,g} = \forall x.\mathsf{smart}(x)$

$$\begin{tikzcd} & g^{x\mapsto \mathsf{a}} = \begin{bmatrix} x \mapsto \mathsf{a} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \\ g = \begin{bmatrix} x \mapsto \mathsf{a} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \arrow[ur, dotted] \arrow[r, dotted] \arrow[dr, dotted] & g^{x\mapsto \mathsf{b}} = \begin{bmatrix} x \mapsto \mathsf{b} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \\ & g^{x\mapsto \mathsf{c}} = \begin{bmatrix} x \mapsto \mathsf{c} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \end{tikzcd}$$

'$\forall x.\mathsf{smart}(x)$' is equivalent to:

$\mathsf{smart}(g^{x\mapsto \mathsf{a}}(x)) \wedge \mathsf{smart}(g^{x\mapsto \mathsf{b}}(x)) \wedge \mathsf{smart}(g^{x\mapsto \mathsf{c}}(x))$

Existential quantification

$\llbracket \text{someone is silly} \rrbracket^{M,g} = \exists x.\mathsf{silly}(x)$

$$\begin{tikzcd} & g^{x\mapsto \mathsf{a}} = \begin{bmatrix} x \mapsto \mathsf{a} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \\ g = \begin{bmatrix} x \mapsto \mathsf{a} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \arrow[ur, dotted] \arrow[r, dotted] \arrow[dr, dotted] & g^{x\mapsto \mathsf{b}} = \begin{bmatrix} x \mapsto \mathsf{b} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \\ & g^{x\mapsto \mathsf{c}} = \begin{bmatrix} x \mapsto \mathsf{c} \\ y \mapsto \mathsf{b} \\ z \mapsto \mathsf{c} \end{bmatrix} \end{tikzcd}$$

'$\exists x.\mathsf{silly}(x)$' is equivalent to:

$\mathsf{silly}(g^{x\mapsto \mathsf{a}}(x)) \vee \mathsf{silly}(g^{x\mapsto \mathsf{b}}(x)) \vee \mathsf{silly}(g^{x\mapsto \mathsf{c}}(x))$

Multiple quantifiers

$\llbracket \text{everyone saw someone} \rrbracket^{M,g} = \forall x \exists y.\mathsf{see}(y)(x)$

$$\begin{tikzcd} & & g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{a}} \\ & g^{x\mapsto \mathsf{a}} \arrow[ur, dotted] \arrow[r, dotted] \arrow[dr, dotted] & g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{b}} \\ & & g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{c}} \\ & & g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{a}} \\ g \arrow[uuur, dotted] \arrow[r, dotted] \arrow[dddr, dotted] & g^{x\mapsto \mathsf{b}} \arrow[ur, dotted] \arrow[r, dotted] \arrow[dr, dotted] & g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{b}} \\ & & g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{c}} \\ & & g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{a}} \\ & g^{x\mapsto \mathsf{c}} \arrow[ur, dotted] \arrow[r, dotted] \arrow[dr, dotted] & g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{b}} \\ & & g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{c}} \end{tikzcd}$$

'$\forall x \exists y.\mathsf{see}(y)(x)$' means:

$$\bigwedge \left\{ \begin{array}{l} \bigvee \left\{ \begin{array}{l} \mathsf{see}(g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{a}}(y))(g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{a}}(x) \\ \mathsf{see}(g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{b}}(y))(g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{b}}(x)) \\ \mathsf{see}(g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{c}}(y))(g^{x\mapsto \mathsf{a}, y\mapsto \mathsf{c}}(x)) \end{array} \right\} \\ \\ \bigvee \left\{ \begin{array}{l} \mathsf{see}(g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{a}}(y))(g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{a}}(x) \\ \mathsf{see}(g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{b}}(y))(g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{b}}(x)) \\ \mathsf{see}(g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{c}}(y))(g^{x\mapsto \mathsf{b}, y\mapsto \mathsf{c}}(x)) \end{array} \right\} \\ \\ \bigvee \left\{ \begin{array}{l} \mathsf{see}(g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{a}}(y))(g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{a}}(x) \\ \mathsf{see}(g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{b}}(y))(g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{b}}(x)) \\ \mathsf{see}(g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{c}}(y))(g^{x\mapsto \mathsf{c}, y\mapsto \mathsf{c}}(x)) \end{array} \right\} \end{array} \right\}$$

How about the other meaning: $\exists y \forall x.\mathsf{see}(y)(x)$

Bound pronouns

$\llbracket \text{no one likes their}_x \text{ school} \rrbracket^{M,g} = \neg \exists x.\mathsf{like}(\mathsf{school\text{-}of\text{-}}x)(x)$

The value of *their*$_x$ **co-varies** with $\exists x$

Lambda calculus: Relative clauses

λ-notation

Consider a function:

$$f : \mathbb{N} \mapsto \mathbb{N} \text{ for every } x \in \mathbb{N}.\ f(x) = x + 1$$

The function can be expressed as follows:

$$f = \lambda x : x \in \mathbb{N}.x + 1$$

The (smallest) function which maps every x such that $x \in \mathbb{N}$ to $x + 1$

Semantic types of λ-terms

If x is type of a and E is type of b, then $\lambda x.E$ is type of $a \to b$.

Intransitive verbs

$$[\![\text{dance}]\!]^{M,g} = \begin{bmatrix} \mathsf{a} \dashrightarrow 1 \\ \mathsf{b} \nearrow \\ \mathsf{c} \dashrightarrow 0 \end{bmatrix} \quad \text{i.e.,} \quad \begin{array}{l} f : D_e \mapsto D_t \\ \text{for every } x, f(x) = 1 \\ \text{iff } x \text{ dance} \end{array}$$

$$[\![\text{dance}]\!]^{M,g} = \overbrace{\lambda x}^{e} :\in D_e.\underbrace{\mathsf{dance}(x)}_{t}$$

Transitive verbs

$$\begin{bmatrix} \mathsf{a} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 0 \\ \mathsf{b} \mapsto 0 \\ \mathsf{c} \mapsto 1 \end{bmatrix}, \mathsf{b} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 1 \\ \mathsf{b} \mapsto 0 \\ \mathsf{c} \mapsto 0 \end{bmatrix}, \mathsf{c} \mapsto \begin{bmatrix} \mathsf{a} \mapsto 1 \\ \mathsf{b} \mapsto 0 \\ \mathsf{c} \mapsto 0 \end{bmatrix} \end{bmatrix}$$

$$\begin{aligned} [\![\text{see}]\!]^{M,g} = & f : D_e \mapsto D_{e\to t} \\ & \text{for every } x,\ f(x) = f' : D_e \mapsto D_t \\ & \qquad \text{for every } y,\ f'(y) = 1 \\ & \qquad \text{iff } y \text{ see } x \end{aligned}$$

$$[\![\text{see}]\!]^{M,g} = \overbrace{\lambda x}^{e} : x \in D_e \overbrace{\lambda y}^{e} : y \in D_e.\underbrace{\mathsf{see}(x)(y)}_{t}$$

λ-reduction

η-equivalence

$$f = \lambda x.f(x)$$

Two functions are equivalent iff they return the same values for every argument

$$[\![\text{dance}]\!]^{M,g} = \mathsf{dance} = \lambda x.\mathsf{dance}(x)$$

β-reduction

$$(\lambda x.E_1)(E_2) = E_1[E_2/x]$$

"$E_1[E_2/x]$" is the expression just like E_1, but where every free occurrence of x has been replaced by E_2.

$$\begin{aligned} [\![\text{dance}]\!]^{M,g}([\![\text{Ada}]\!]^{M,g}) &= [\lambda x.\mathsf{dance}(x)](\mathsf{a}) \\ &= \mathsf{dance}(\mathsf{a}) \end{aligned}$$

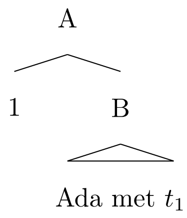
α-equivalence

$$\lambda x.E = \lambda y.E[y/x]$$

A specific choice of a bound variable doesn't matter.

$$[\![\text{dance}]\!]^{M,g} = \lambda x.\mathsf{dance}(x) = \lambda y.\mathsf{dance}(y)$$

λ-abstraction

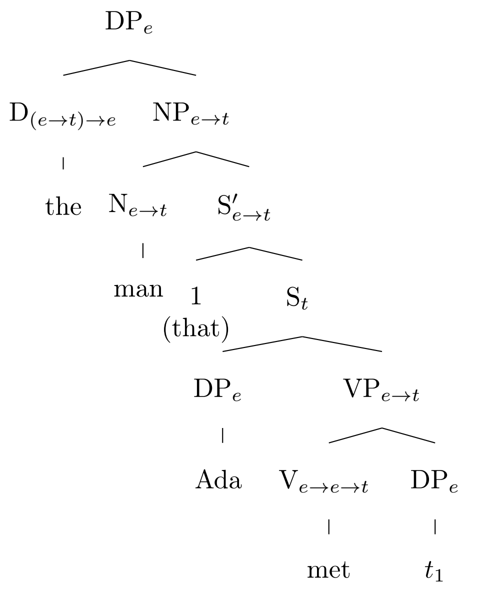


If A is branching node with daughters $n \in \mathbb{N}$ and B of type b, then A has type $a \to b$ and for any assignment g, $[\![\text{A}]\!]^{M,g} = \lambda x.[\![\text{B}]\!]^{g^{n\mapsto x}}$

$$\begin{aligned} & [\![1\ [\text{Ada met } t_1]]\!]^{M,g} \\ & = \lambda x.[\![\text{Ada met } t_1]\!]^{M,g^{1\mapsto x}} \\ & = \lambda x.[\![\text{met}]\!]^{M,g^{1\mapsto x}}([\![t_1]\!]^{M,g^{1\mapsto x}})([\![\text{Ada}]\!]^{M,g^{1\mapsto x}}) \\ & = \lambda x.\mathsf{met}(g(1))(\mathsf{a}) = \lambda x.\mathsf{met}(x)(\mathsf{a}) \end{aligned}$$

Relative clauses

(1) the man that Ada met



---

1. $[\![t_1]\!]^{M,g} = g(1)$ (the same as a pronoun)
2. $[\![\text{S}']\!]^{M,g} = \lambda x.[\![\text{S}]\!]^{M,g^{1\mapsto x}} = \lambda x.\mathsf{met}(x)(\mathsf{a})$
3. $[\![\text{the}]\!]^{M.g} = \lambda P.\text{the entity } y \text{ s.t. } P(y)$
   defined only if
   $\exists y.[P(y) \wedge \forall x.P(x) \to x = y]$

Predicate Modification

$$\frac{P :: e \to t \qquad\qquad Q :: e \to t}{\lambda x.P(x) \wedge Q(x) :: e \to t}$$

---

1. $[\![\text{NP}]\!]^{M,g} = \lambda x.[\![\text{man}]\!]^{M,g}(x) \wedge [\![\text{S}']\!]^{M,g}(x)$
   $= \lambda x.\mathsf{man}(x) \wedge \mathsf{met}(x)(\mathsf{a})$
2. $[\![\text{DP}]\!]^{M,g} = [\![\text{the}]\!]^{M,g}([\![\text{NP}]\!]^{M,g})$
   $=$

Generalized quantifiers: Basics

Quantification in FOL

Suppose Ada and Becky are girls and Carl is a boy.

- Ada danced but Becky didn't.
- Carl also danced.

$$\begin{tikzcd}
 & g^{x\mapsto \mathsf{a}} \ \mathsf{girl}(g^{x\mapsto \mathsf{a}}(x)) = 1;\ \mathsf{dance}(g^{x\mapsto \mathsf{a}}(x)) = 1 \\
g \arrow[ur, dotted] \arrow[r, dotted] \arrow[dr, dotted] & g^{x\mapsto \mathsf{b}} \ \mathsf{girl}(g^{x\mapsto \mathsf{b}}(x)) = 1;\ \mathsf{dance}(g^{x\mapsto \mathsf{b}}(x)) = 0 \\
 & g^{x\mapsto \mathsf{c}} \ \mathsf{girl}(g^{x\mapsto \mathsf{c}}(x)) = 0;\ \mathsf{dance}(g^{x\mapsto \mathsf{c}}(x)) = 1
\end{tikzcd}$$

(1) Every girl danced $\rightsquigarrow \forall x.\mathsf{girl}(x) \to \mathsf{dance}(x)$

(2) A girl danced $\rightsquigarrow \exists x.\mathsf{girl}(x) \wedge \mathsf{dance}(x)$

Since it is **not** the case that $\mathsf{girl}(x) \to \mathsf{dance}(x)$ is true for all individuals, $\forall x.\mathsf{girl}(x) \to \mathsf{dance}(x)$ is false.

What about (2)?

Problem

(3) Most girls danced.

Suppose there're three girls, a boy and a cat:

Scenario I: (3) is true

- Ada and Becky danced, but Cindy didn't
- Donald didn't danced.
- Kitty didn't dance.

Scenario II: (3) is false

- Ada danced, but Becky and Cindy didn't
- Donald danced.
- Kitty didn't dance.

Consider the following formulas:

1. $\mathsf{most}\ x.\mathsf{girl}(x) \to \mathsf{dance}(x)$ — 1 in I; $\boxed{1}$ in II
2. $\mathsf{most}\ x.\mathsf{girl}(x) \wedge \mathsf{dance}(x)$ — $\boxed{0}$ in I; 0 in II
3. $\mathsf{most}\ x.\mathsf{girl}(x) \vee \mathsf{dance}(x)$ — 1 in I; $\boxed{1}$ in II

Relations between sets

$[\![\text{every girl danced}]\!] = \{x \mid \mathsf{girl}(x)\} \subseteq \{x \mid \mathsf{dance}(x)\}$

$[\![\text{a cat danced}]\!] = \{x \mid \mathsf{cat}(x)\} \cap \{x \mid \mathsf{dance}(x)\} \neq \emptyset$

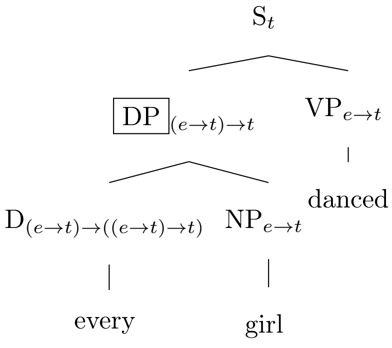
$[\![\text{most girls danced}]\!] = \dfrac{|\mathsf{girl} \cap \mathsf{dance}|}{|\mathsf{girl}|} > \dfrac{1}{2}$

$[\![\text{more than two girls danced}]\!] = |\mathsf{girl} \cap \mathsf{dance}| > 2$

$[\![\text{at most two girls danced}]\!] = |\mathsf{girl} \cap \mathsf{dance}| \leqslant 2$

$[\![\text{no girl danced}]\!] = |\mathsf{girl} \cap \mathsf{dance}| = \emptyset$

What are quantifiers



Suppose:

- Ada, Becky, Cindy and Donald danced
- Ada, Becky, Cindy and Eric sang

$$\mathsf{girl} = \{\mathsf{a,b,c}\} \subseteq \begin{array}{l} \mathsf{dance} = \{\mathsf{a,b,c,d}\} \\ \mathsf{girl} = \{\mathsf{a,b,c}\} \\ \mathsf{sing} = \{\mathsf{a,b,c,e}\} \end{array}$$

$[\![\text{every girl}]\!] = \{\{\mathsf{a,b,c,d}\}, \{\mathsf{a,b,c}\}, \{\mathsf{a,b,c,e}\}\}$

1. $[\![\text{every girl is a girl}]\!] = [\![\text{girl}]\!] \in [\![\text{every girl}]\!]$
2. $[\![\text{every girl danced}]\!] = [\![\text{dance}]\!] \in [\![\text{every girl}]\!]$
3. $[\![\text{every girl sang}]\!] = [\![\text{sang}]\!] \in [\![\text{every girl}]\!]$

$[\![\text{every girl}]\!] = \lambda P.\mathsf{girl} \subseteq P$ Type: $(e \to t) \to t$

Boolean compounds

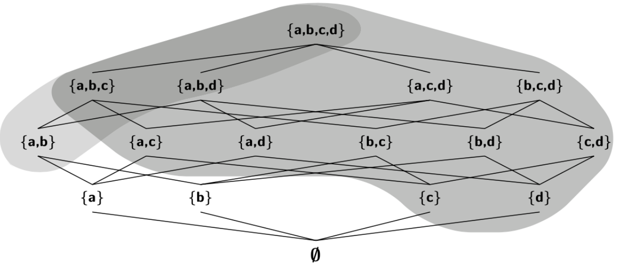
(4) Every girl **and** at least one boy danced.
(5) Every girl **or** at least one boy danced.
(6) Every girl **but** not every boy danced.

Suppose:

- Ada and Becky danced and sang.
- Carl danced but did not sing.
- Donald sang but did not dance.
- Ada and Carl wore a hat.
- Becky and Carl are tall.
- Ada and Donald likes swimming.
- Becky and Donald wore a scarf.

$$[\![\text{every girl}]\!] = \left\{ \begin{array}{l} \mathsf{girl} = \{\mathsf{a,b}\} \\ \mathsf{dance} = \{\mathsf{a,b,c}\} \\ \mathsf{sing} = \{\mathsf{a,b,d}\} \\ \mathsf{human} = \{\mathsf{a,b,c,d}\} \end{array} \right\}$$

$$[\![\text{at least one boy}]\!] = \left\{ \begin{array}{l} \mathsf{boy\ who\ danced} = \{\mathsf{c}\} \\ \mathsf{boy\ who\ sang} = \{\mathsf{d}\} \\ \mathsf{boy} = \{\mathsf{c,d}\} \\ \mathsf{wear.hat} = \{\mathsf{a,c}\} \\ \mathsf{wear.scarf} = \{\mathsf{b,d}\} \\ \mathsf{tall} = \{\mathsf{b,c}\} \\ \mathsf{like.swim} = \{\mathsf{a,d}\} \\ \mathsf{dance} = \{\mathsf{a,b,c}\} \\ \mathsf{sing} = \{\mathsf{a,b,d}\} \\ \mathsf{human} = \{\mathsf{a,b,c,d}\} \end{array} \right\}$$



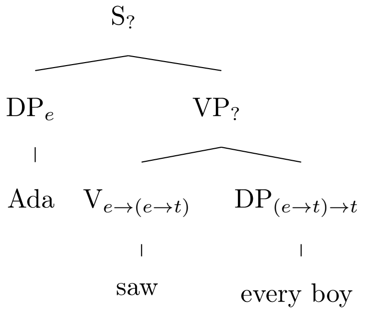
(picture credited to Szabolcsi 2010)

$[\![\text{every girl } \textbf{and} \text{ at least one boy}]\!]$

Logical Form

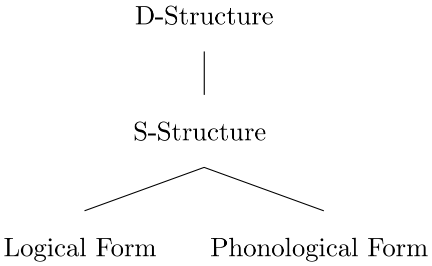
Object quantifiers

(1) Ada saw every boy.



Logical Form (LF)

Y-model of grammar:



1. LF is a **SYNTACTIC** component.
2. An LF product serves as the input of $[\![\cdot]\!]$.

Movement

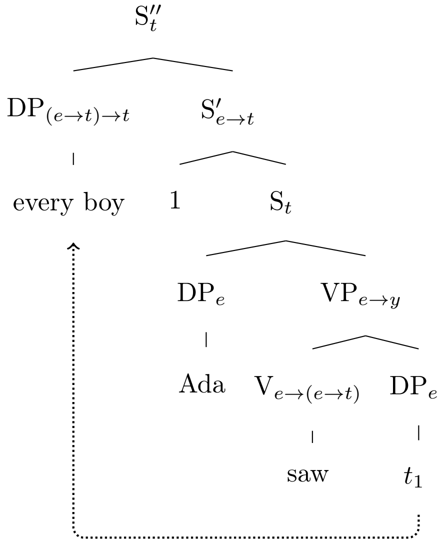
(2) Who did Ada see?

D-Structure: $[_{\text{S}}$ Ada $[_{\text{VP}}$ see who$]]$

S-Structure: $[_{\text{S}''}$ Who $[_{\text{S}'}$ did $[_{\text{S}}$ Ada see t $]]]$

A deep structure is transformed to a surface structure via movement.

Quantifier Raising (QR)



1. $[\![\text{S}]\!]^{M,g} = \mathsf{see}(g(1))(\mathsf{a})$
2. $[\![\text{S}']\!]^{M,g} = \lambda x.\mathsf{see}(x)(\mathsf{a})$
3. $[\![\text{S}'']\!]^{M,g} = [\![\text{every boy}]\!]^{M,g}(\lambda x.\mathsf{see}(x)(\mathsf{a}))$
   $= \{x \mid \mathsf{boy}(x)\} \subseteq \{x \mid \mathsf{see}(x)(\mathsf{a})\}$

Two quantifiers

(2) A boy saw every girl.

LF: [[a boy] [1 [every girl] [2 [t_1 saw t_2]]]]

$[\![\text{a boy}]\!]^{M,g}$ $(\lambda x.[\![\text{every girl}]\!]^{M,g}(\lambda y.\mathsf{see}(y)(x)))$

$= [\![\text{a boy}]\!]^{M,g}$ $(\lambda x.\{y \mid \mathsf{girl}(y)\} \subseteq \{y \mid \mathsf{see}(y)(x)\})$

$= \{x \mid \mathsf{boy}(x)\} \cap \{x \mid \{y \mid \mathsf{girl}(y)\} \subseteq \{y \mid \mathsf{see}(y)(x)\}\} \neq \emptyset$

LF: [[every girl] [2 [a boy] [1 [t_1 saw t_2]]]]

$[\![\text{every girl}]\!]^{M,g}$ $(\lambda y.[\![\text{a boy}]\!]^{M,g}(\lambda x.\mathsf{see}(y)(x)))$

$= [\![\text{every girl}]\!]^{M,g}$ $(\lambda y.\{x \mid \mathsf{boy}(x)\} \cap \{x \mid \mathsf{see}(y)(x)\} \neq \emptyset)$

$= \{y \mid \mathsf{girl}(y)\} \subseteq \{y \mid \{x \mid \mathsf{boy}(x)\} \cap \{x \mid \mathsf{see}(y)(x)\} \neq \emptyset\}$

Scope ambiguity

Assuming three boys, Peter, Tom, and Carl, and two ladies, Ada and Becky:

(3) Ada **didn't** see **every** boy.

- not > every: a saw t, but not p and c.
- every > not: a didn't see t, p and c.

(4) **At most two** boys saw **every** girl.

- at most two > every: p and t both saw a and b.
- every > at most two: p and t saw a, and t saw b.

Scope islands

It is not always the case that a quantifier can take wide scope.

(3) **No** boy thought that [**every** girl could dance].

- no > every: there were some boys who thought some girls could dance.
- #every > no: for every girl x, there was no boy who thought x could dance.

(4) John dated **a** woman [who loves **every** man].

- a > every: there is one girl such that she loves every man.
- #every > a: for any man x, there is a (different) woman who loves x.

QR is clause bounded.

Exceptional scope

(5) **Each** boy believes that [**a** paper of mine contains an error].

- each > a: for any boy x, there's a different paper of mine y s.t. x believes y contains an error.
- a > every: there is a paper of mine y s.t. for any boy x, x believes y contains an error.

Empirical properties of quantifiers: Monotonicity

Any

1. *Ann ate **any** fish last night.
2. *A girl ate **any** fish last night.
3. Ann didn't eat **any** fish last night.
4. No girl ate **any** fish last night.
5. *Every girl who read a book talked to **any** professor.
6. Every girl who read **any** book talked to a professor.

Monotonic increasing

Suppose we have three ladies, Ann, Becky, and Cindy, and one body, Donald,

- Ann, Cindy, and Donald walked.
- Ann, Becky, and Cindy talked.

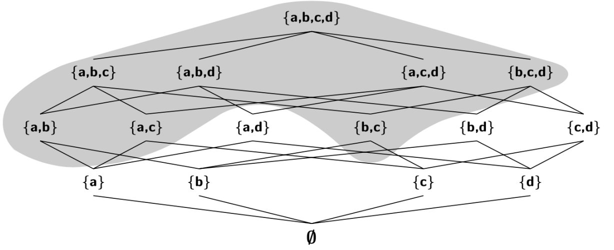
$$\boxed{(\mathsf{walk} \cap \mathsf{talk}) \subseteq \mathsf{walk} \subseteq (\mathsf{walk} \cup \mathsf{talk})}$$

---

1. More than one girl walked and talked.
2. More than one girl walked.
3. More than one girl walked or talked.

$$\boxed{\text{1 entails 2, 2 entails 3, and 1 entails 3}}$$

$[\![$more than one girl$]\!]$ =



The generalized quantifier GQ is **monotonically increasing** iff whenever X is an element of GQ, all supersets of X are elements of GQ.

Monotonic decreasing

Suppose we have two ladies, Ann and Becky, and two boys, Carl and Donald

- Ann and Becky jogged.
- Carl walked.
- Carl and Donald talked.

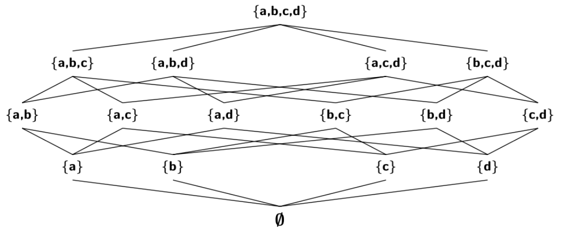
$$\boxed{(\mathsf{walk} \cap \mathsf{talk}) \subseteq \mathsf{walk} \subseteq (\mathsf{walk} \cup \mathsf{talk})}$$

---

1. No girl walked and talked.
2. No girl walked.
3. No girl walked or talked.

$$\boxed{\text{3 entails 2, 2 entails 1, and 3 entails 1}}$$

$[\![$no girl$]\!]$ =

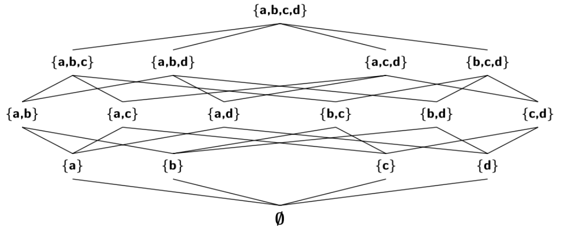


The generalized quantifier GQ is **monotonically decreasing** iff whenever X is an element of GQ, all subsets of X are elements of GQ.

Non-monotonic

1. Exactly two girls jogged.

$[\![$exactly two ladies$]\!]$ =



Downward and upward entailing

1. Every girl who read a book talked to a professor.
2. Every girl who read a novel talked to a professor.

- $\mathsf{novel} \subseteq \mathsf{book}$;
- $\{x \mid x \text{ read a novel}\} \subseteq \{x \mid x \text{ read a book}\}$
- $[\![1]\!] = \{x \mid x \text{ read a novel}\} \subseteq \{x \mid x \text{ talk to prof}\}$;
- $[\![2]\!] = \{x \mid x \text{ read a book}\} \subseteq \{x \mid x \text{ talk to prof}\}$;
- $[\![2]\!]$ entails $[\![1]\!]$

---

1. Every girl who read a book talked to a teacher.
2. Every girl who read a book talked to a professor.

- $\mathsf{professor} \subseteq \mathsf{teacher}$;
- $\{x \mid x \text{ talk to prof}\} \subseteq \{x \mid x \text{ talk to teacher}\}$
- $[\![2]\!] = \{x \mid x \text{ read book}\} \subseteq \{x \mid x \text{ talk to prof}\}$;
- $[\![1]\!] = \{x \mid x \text{ read book}\} \subseteq \{x \mid x \text{ talk to teacher}\}$;
- $[\![1]\!]$ entails $[\![2]\!]$

The maximality problem

1. At least two girls walked. = There is a set of men with cardinality at least two such that all its elements walk.
2. At most two girls walked. ≠ There is a set of men with cardinality at most two such that all its elements walk.
3. Exactly two girls talked. ≠ There is a set of men with cardinality exactly two such that all its elements walk.

Sentence 1, but not 2 and 3, is true in a situation in which more than two girlsgirls walked.

The diagrams in this handout are credited to Szabolcsi (2010).

Empirical properties of quantifiers: Witnesses and conservativity

Topic set

Suppose that we have three ladies, Annie, Becky, and Cindy, and one boys, Donald:

$$[\![\text{more than one girl}]\!] = \lambda P.|\mathsf{girl} \cap P| > 1$$
$$= \{P \mid |\mathsf{girl} \cap P| > 1\}$$

$$\left\{ \begin{array}{ccc} \{\mathsf{a, b, c, d}\}, & \{\mathsf{a, b, c}\}, & \{\mathsf{a, c, d}\}, \\ \{\mathsf{a, b, d}\}, & \{\mathsf{b, c, d}\}, & \{\mathsf{a, b}\}, \\ \{\mathsf{a, c}\}, & \{\mathsf{b, c}\} & \end{array} \right\}$$

Topic set = the smallest live-on set:
A generalized quantifier $\mathcal{Q}$ lives on a set of entities L if, for any set of entities X, $X \in \mathcal{Q}$ iff $(X \cap L) \in \mathcal{Q}$.

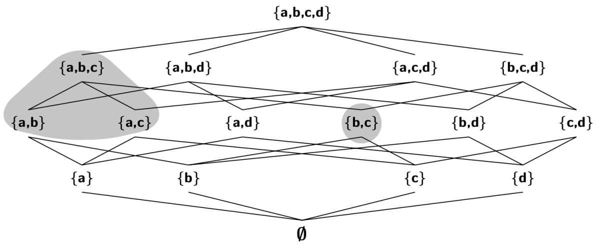
What is the topic set of *more than one girl*?

Which syntactic phrase corresponds to the topic set of *more than one girl*?

Witness set

W is a witness set of a generalized quantifier $\mathcal{Q}$ iff (a) $W \in \mathcal{Q}$; (b) $W \subseteq L$ and L is $\mathcal{Q}$'s topic set.

The witness sets of *more than one girl*:

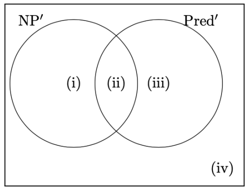


(the picture credited to Szabolcsi 2010)

- If a generalized quantifier $\mathcal{Q}$ is monotonic increasing, for any X, $X \in \mathcal{Q}$ iff $\exists W.W \subseteq X$;
- If a generalized quantifier $\mathcal{Q}$ is monotonic decreasing, for any X, $X \in \mathcal{Q}$ iff $\exists W.(X \cap L) \subseteq W$.

Conservativity

Consider four subsets of the universe of discourse:



(the picture credited to Szabolcsi 2010)

1. $[\![\text{some}]\!]$: (ii) is not empty
2. $[\![\text{more than one}]\!]$: (ii) has more than one element
3. $[\![\text{every}]\!]$: (i) is empty
4. $[\![\text{most}]\!]$: $\dfrac{(ii)}{(i) \cup (ii)} = \dfrac{1}{2}$

A quantificational determiner does not make reference to (iii) and (iv).

A quantificational determiner $\mathcal{D}$ is conservative iff $\mathcal{D}(A)(B) = \mathcal{D}(A)(A \cap B)$

1. Every girl is smart ↔ Every girl is a smart girl
2. Some boy is smart ↔ Some boy is a smart boy
3. Most girlsgot an A ↔ Most girlsare girlswho got an A

Symmetry

1. Some girlsare smart ↔ some smart people are ladies
2. At least two girlsgot an A ↔ at least two people who got an A are girls
3. No girlsare silly ↔ No silly people are girls
4. Most girlsare smart $\not\leftrightarrow$ most smart people are girls

A quantificational determiner is symmetric iff $\mathcal{D}(A)(B) = \mathcal{D}(B)(A)$

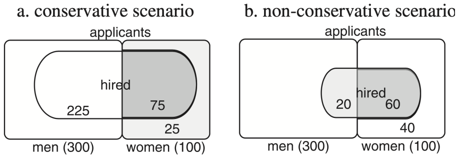
Pair-list answers

1. Which book did **every boy** read?
   Pair-list answer: Max read *Moby Dick*; Kyle *The Great Gatsby*; Sam *Oliver Twister*.
2. Which book did **no boy** read?
   #Pair-list answer: Max didn't read *Moby Dick*; Kyle didn't read *The Great Gatsby*; Sam didn't read *Oliver Twister*.
3. Which book did **at most two boys** read?
   #Pair-list answer: Max read *Moby Dick*; Kyle *The Great Gatsby*.
4. Which book did **two of the boys** read?
   ?(?)Pair-list answer: Max read *Moby Dick*; Kyle *The Great Gatsby*.

Quantifier	Minimal witness set
every boy	$\{\mathsf{m, k, s}\}$
two of the boys	$\{\mathsf{m, k}\}$, $\{\mathsf{k, s}\}$, $\{\mathsf{m, s}\}$
no boy	$\emptyset$
at most two boys	$\emptyset$

Is every determiner conservative

1. Huáwéi gù-le 75% de nǚshēng.
   Huawei hire-ASP 30% woman
   a. 'Huawei hired 30% women.'
   b. 'Huawei hired 30% of the women.'



(the picture credited to Ahn & Sauerland 2018)

Conservative: $[\![75\%]\!](\mathsf{woman})(\lambda x.\mathsf{hire}(x)(\mathsf{h}))$
$= [\![75\%]\!](\mathsf{woman})(\lambda x.\mathsf{woman}(x) \wedge \mathsf{hire}(x)(\mathsf{h}))$
= 75% of the women are the ones hired by Huawei.

Non-conservative: $[\![75\%]\!](\lambda x.\mathsf{hire}(x)(\mathsf{h}))(\mathsf{woman})$
$= [\![75\%]\!](\lambda x.\mathsf{hire}(x)(\mathsf{h}))(\lambda x.\mathsf{woman}(x) \wedge \mathsf{hire}(x)(\mathsf{h}))$
= 75% of the people hired by Huawei are women.

Event semantics

What are events

(1) Brutus stabbed Caesar yesterday.

The event described by (1): $\begin{bmatrix} \text{Agent} : \mathsf{b} \\ \text{Patient} : \mathsf{c} \\ \text{Nature} : \mathsf{stabbing} \\ \text{Runtime} : \mathsf{yesterday} \end{bmatrix}$

Truth condition:

- (1) is true in M if and only if $\langle \mathsf{b}, \mathsf{c} \rangle \in [\![\text{stab}]\!]^M$
- (1) is true if and only if (1) describes an event e s.t.
  - the agent of e is Brutus,
  - the patient of e is Caesar,
  - e is stabbing,
  - the runtime of e is yesterday.

Adding events to a standard model: $\langle D_e, D_v, [\![\cdot]\!] \rangle$

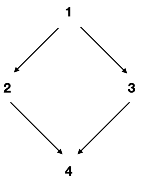
A sentence expresses quantification over eventualities, which describes eventualities.

(1) $\rightsquigarrow \exists e.\mathsf{stab}(e) \wedge \mathsf{ag}(e, \mathsf{b}) \wedge \mathsf{pa}(e, \mathsf{c}) \wedge \tau(e, \mathsf{yesterday})$

Diamond entailment

1. Brutus stabbed Caesar in the back with a knife.
2. Brutus stabbed Caesar in the back.
3. Brutus stabbed Caesar with a knife.
4. Brutus stabbed Caesar.

1. I've got big black cars.
2. I've got big cars.
3. I've got black cars.
4. I've got cars.



- $\exists x.\mathsf{cars}(x) \wedge \mathsf{big}(x) \wedge \mathsf{black}(x) \wedge \mathsf{get}(x)(\mathsf{spk})$
- $\exists e.\mathsf{stab}(e) \wedge \mathsf{ag}(e, \mathsf{b}) \wedge \mathsf{c} \wedge \mathsf{in}(e, \mathsf{bk}) \wedge \mathsf{inst}(e, \mathsf{k})$

Different types of eventualities

process	**state**	**accomplishment**	**achievement**
run	*know*	*make a chair*	*recognize*
walk	*believe*	*paint a picture*	*reach*
swim	*have*	*build a house*	*find*
⋮	⋮	⋮	⋮

Telicity [$\pm$ natural ending point]

- **Achievement** and **accomplishment** are telic.
  - Sue made a chair in/*for an hour.
  - Kate reached the peak in/*for an hour.
- **Process** and **state** are atelic.
  - Max ran/walked/swam for/*in an hour.
  - Owen believed the story for/*in his entire life.

Describing eventualities

1. Caesar knows Brutus.
   $\rightsquigarrow \exists s.\mathsf{know}(s) \wedge \mathsf{ho}(s, \mathsf{c}) \wedge \mathsf{th}(s, \mathsf{b})$
2. Peter made Karl cry.
   $\rightsquigarrow \exists e \exists e'.\mathsf{ag}(e, \mathsf{p}) \wedge \mathsf{cry}(e') \wedge \mathsf{ag}(e', \mathsf{k}) \wedge \mathsf{cause}(e, e')$
3. In every burning, oxygen is consumed.
   $\rightsquigarrow \forall e.\mathsf{burn}(e) \rightarrow \exists e'.\mathsf{consume}(e') \wedge \mathsf{th}(e', \mathsf{O_2}) \wedge \mathsf{in}(e', e)$

Tense

A simple existential treatment of tense (anchoring the runtime of an event):

1. John saw Mary. (Past)
   $\rightsquigarrow \exists e \exists t.t < \mathsf{now} \wedge \tau(e) \subseteq t \wedge \mathsf{see}(e) \wedge \mathsf{ag}(e, \mathsf{j}) \wedge \mathsf{th}(e, \mathsf{m})$
2. Owen reaches the peak. (Present)
   $\rightsquigarrow \exists e \exists t.t = \mathsf{now} \wedge \tau(e) \subseteq t \wedge \mathsf{reach}(e) \wedge \mathsf{ag}(e, \mathsf{o}) \wedge \mathsf{th}(e, \mathsf{p})$
3. Kyle will leave. (Future)
   $\rightsquigarrow \exists e \exists t.t > \mathsf{now} \wedge \tau(e) \subseteq t \wedge \mathsf{leave}(e) \wedge \mathsf{ag}(e, \mathsf{k})$

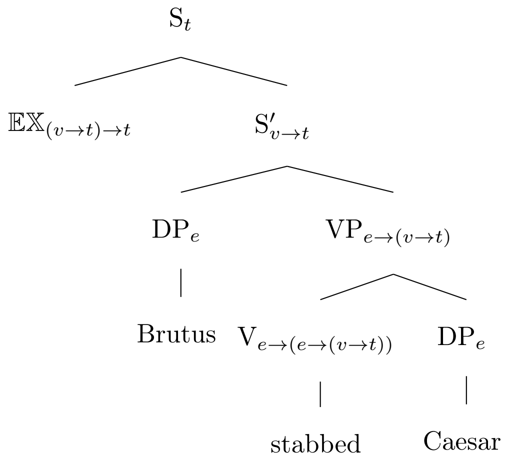
The narrowest scope

1. Each boy hugged Sue.
   - $\forall x.\mathsf{boy}(x) \rightarrow \exists e.\mathsf{hug}(e) \wedge \mathsf{ag}(e, x) \wedge \mathsf{th}(e, \mathsf{s})$ ✓
   - $\exists e.\forall x.\mathsf{boy}(x) \rightarrow \mathsf{hug}(e) \wedge \mathsf{ag}(e, x) \wedge \mathsf{th}(e, \mathsf{s})$ ✗
2. Max didn't love Sue.
   - $\neg(\exists e.\mathsf{love}(e) \wedge \mathsf{ag}(e, \mathsf{m}) \wedge \mathsf{th}(e, \mathsf{s}))$ ✓
   - $\exists e.\neg(\mathsf{love}(e) \wedge \mathsf{ag}(e, \mathsf{m}) \wedge \mathsf{th}(e, \mathsf{s}))$ ✗
3. Exactly one boy hugged Mary.
   - $|\mathsf{boy} \cap \{x : \exists e.\mathsf{hug}(e) \wedge \mathsf{ag}(e, x) \wedge e, \mathsf{m}\}| = 1$ ✓
   - $\exists e.|\mathsf{boy} \cap \{x : \mathsf{hug}(e) \wedge \mathsf{ag}(e, x) \wedge e, \mathsf{m}\}| = 1$ ✗
4. Brutus stabbed Caesar and Leo stabbed Hamlet.
   - $(\exists e.\mathsf{ag}(e, \mathsf{b}) \wedge \mathsf{stab}(e) \wedge \mathsf{pt}(e, \mathsf{c})) \wedge (\exists e'.\mathsf{ag}(e', \mathsf{l}) \wedge \mathsf{stab}(e') \wedge \mathsf{pt}(e', \mathsf{h}))$ ✓
   - $\exists e.\mathsf{ag}(e, \mathsf{b}) \wedge \mathsf{stab}(e) \wedge \mathsf{pt}(e, \mathsf{c}) \wedge \mathsf{ag}(e, \mathsf{l}) \wedge \mathsf{stab}(e) \wedge \mathsf{pt}(e, \mathsf{h})$ ✗

Composition

$[\![\text{stab}]\!] = \lambda x \lambda y \lambda e.\mathsf{stab}(e) \wedge \mathsf{ag}(e, x) \wedge \mathsf{pt}(e, y)$

$[\![\mathbb{EX}]\!] = \lambda V \exists e.V(e)$


S_t
$\mathbb{EX}_{(v \to t) \to t}$
$\mathrm{S}'_{v \to t}$
DP_e
$\mathrm{VP}_{e \to (v \to t)}$
Brutus
$\mathrm{V}_{e \to (e \to (v \to t))}$
DP_e
stabbed
Caesar