Model-theoretic Semantics: Names and verbs

Models

A model M for predicate logic is an ordered pair $\langle U, [\![\cdot]\!]\rangle$, in which:

- U is a a nonempty set of individuals;
- $[\![\cdot]\!]$ is an interpretation function.

Ling-exp $\dashrightarrow$ $[\![\cdot]\!]$ $\dashrightarrow$ some object built from U

A sample model

$U = \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$

a:	a girl named *Ada*
b:	a girl named *Becky*
c:	a girl named *Cindy*

Facts

- a, b and c are girls.
- a and b danced, but c didn't.
- a saw b, but b didn't see a.
- c saw a, and a saw c, too.
- a and c are Americans, but b is a British.

Names

$[\![\text{Ada}]\!] = \mathsf{a}$ $\quad [\![\text{Becky}]\!] = \mathsf{b}$ $\quad [\![\text{Cindy}]\!] = \mathsf{c}$

Type e

Intransitive verbs

$[\![\text{dance}]\!] = \{x \mid x \text{ dances}\} = \{\mathsf{a}, \mathsf{b}\} \subseteq U$

Characteristic functions

$$[\![\text{dance}]\!] = \begin{bmatrix} \mathsf{a} \dashrightarrow 1 \\ \mathsf{b} \dashrightarrow 1 \\ \mathsf{c} \dashrightarrow 0 \end{bmatrix}$$

Type $e \to t$

Transitive verbs

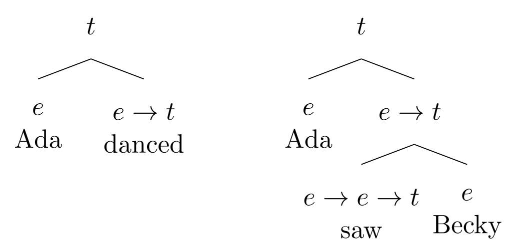
$$[\![\text{see}]\!] = \{\langle x, y\rangle \mid x \text{ saw } y\}$$
$$= \{\langle \mathsf{a}, \mathsf{b}\rangle,\ \langle \mathsf{c}, \mathsf{a}\rangle,\ \langle \mathsf{a}, \mathsf{c}\rangle\} \subseteq U \times U$$

Characteristic functions

$$[\![\text{see}]\!] = \begin{bmatrix} \langle \mathsf{a},\mathsf{b}\rangle \dashrightarrow 1 \\ \langle \mathsf{a},\mathsf{c}\rangle \dashrightarrow 1 \\ \langle \mathsf{c},\mathsf{a}\rangle \dashrightarrow 1 \\ \langle \mathsf{a},\mathsf{a}\rangle \dashrightarrow 0 \\ \langle \mathsf{c},\mathsf{b}\rangle \dashrightarrow 0 \\ \langle \mathsf{c},\mathsf{c}\rangle \dashrightarrow 0 \\ \langle \mathsf{b},\mathsf{a}\rangle \dashrightarrow 0 \\ \langle \mathsf{b},\mathsf{c}\rangle \dashrightarrow 0 \\ \ldots \end{bmatrix} = \begin{bmatrix} \mathsf{a} \dashrightarrow \begin{bmatrix} \mathsf{a} \dashrightarrow 0 \\ \mathsf{b} \dashrightarrow 0 \\ \mathsf{c} \dashrightarrow 1 \end{bmatrix} \\ \mathsf{b} \dashrightarrow \begin{bmatrix} \mathsf{a} \dashrightarrow 1 \\ \mathsf{b} \dashrightarrow 0 \\ \mathsf{c} \dashrightarrow 0 \end{bmatrix} \\ \mathsf{c} \dashrightarrow \begin{bmatrix} \mathsf{a} \dashrightarrow 1 \\ \mathsf{b} \dashrightarrow 0 \\ \mathsf{c} \dashrightarrow 0 \end{bmatrix} \end{bmatrix}$$

Type $e \to e \to t$

Combination



Functional Application

$$\frac{f :: a \to b \qquad x :: a}{f(x) :: b}$$

1. $[\![\text{Ada danced}]\!] = [\![\text{danced}]\!]([\![\text{Ada}]\!])$
2. $[\![\text{Ada saw Becky}]\!] = [\![\text{saw}]\!]([\![\text{Becky}]\!])([\![\text{Ada}]\!])$

Formal concepts

Set "a collection of distinct objects, considered as an object in its own right"
Defining sets

- Extensionally: $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$
- Intensionally: $\{x \mid x \text{ is a girl}\}$

Empty set: $\emptyset$ or $\{\}$

Relations between sets

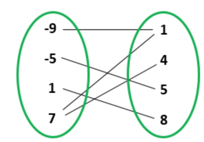
- Proper subset: $A \subset B$ iff $A \subseteq B$ and $B \not\subseteq A$
- Equivalence: $A = B$ iff $A \subseteq B$ and $B \subseteq A$
- Superset: $B \supseteq A$ iff $A \subseteq B$

Power sets $\wp(A) = \{B \mid B \subseteq A\}$

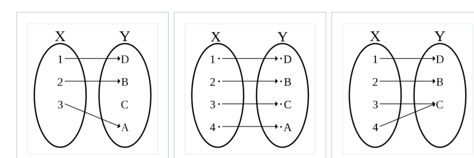
Pairs $\langle \mathsf{a}, \mathsf{b}\rangle \neq \langle \mathsf{b}, \mathsf{a}\rangle$ $\qquad$ n-**tuples** $\langle \mathsf{a}, \mathsf{b}, \mathsf{c}, \ldots\rangle$

Relations as sets of pairs (Cartesian products)

$$A \times B := \{\langle x, y\rangle \mid x \in A \text{ and} y \in B\}$$



Function any relation where each input is paired with at most one output



injection $\qquad$ bijection $\qquad$ surjection

Type a 'name tag' of a set (D_e, D_t)
Basic types: e (individuals); t (truth values)
Function types: $a \to b$