Exercise Report-1

For

INF 4140/9140 -Numerical Analysis

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Exercise 4.1.1:

The interpolation problem in P_n leads to a linear system $V'c = f$, where V is a Vandermonde matrix. Write down the expression for the element v_{ij}:

Answer:

$$The\ Candermonde\ matrix: Vn = \begin{bmatrix} 1 & 1 & \ldots. & 1 \\ X1 & X2 & \ldots. & Xn \\ . & . & & \\ X1^{n-1} & X2^{n-1} & \ldots\ldots & Xn^{n-1} \end{bmatrix}$$

Exercise 4.1.3:

What is meant by a triangle family q1(x), q2(x), . . . , qn(x) of polynomials? Are all such families a basis for Pn?

Answer:

A triangle family of polynomials means a sequence of polynomials form a triangular matrix S.

q1(x) = S11,
q2(x) = S12 + S22x,
q3(x) = S13 + S23x + S33x^2
. . .
qn(x) = S1n + S2nx + S3n x^2+· · ·+Snn.x^n-1

for any j , Pj (x) = x^{j-1} can be expressed recursively and uniquely as
linear combinations of q1(x), . . . , qj (x) by inverse transformation. Thus every triangle family is a basis for Pm.

Exercise 4.1.5:

What good effects can be achieved by using over determination in polynomial interpolation?

Answer:

The over determination in polynomial interpolation can be used to (1) reduce the effect of random or irregular errors in the values of function. (2) give the polynomial a smoother behavior between the grid points.

Problems and Computer Exercises

Exercise 4.1.1 (a):

Plots Display:

Remarks:

For the display purpose, the 0 in difference has been replaced with the 1e-20 to avoid the 'infinite' in the logarithm display.

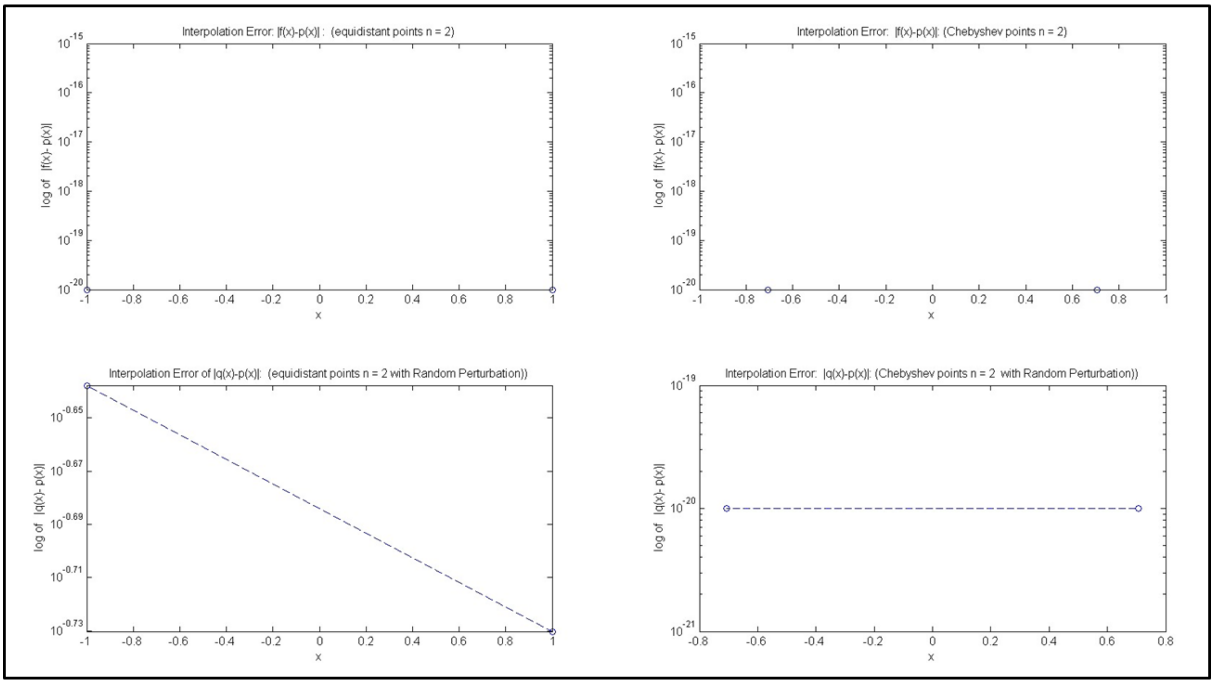

Figure1. Interpolation of function, with equidistant points and Chebyshev points, n =2

Top-Left: |f(x)-p(x)|: (equidistant points). **Top-right**: |f(x)-p(x)|: (Chebyshev points).

Bot-Left: |q(x)-p(x)|: (equidistant points). **Bot-right**: |q(x)-p(x)|: (Chebyshev points).

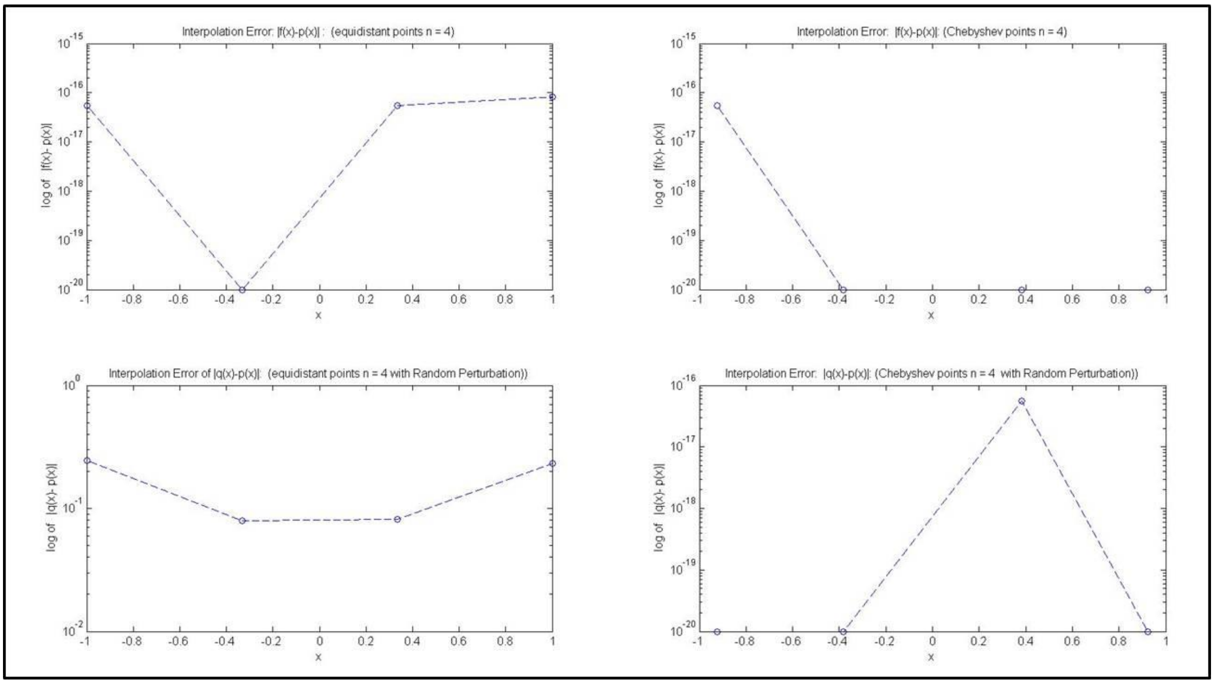

Figure2. Interpolation of function, with equidistant points and Chebyshev points, n =4

Top-Left: |f(x)-p(x)|: (equidistant points). **Top-right**: |f(x)-p(x)|: (Chebyshev points).

Bot-Left: |q(x)-p(x)|: (equidistant points). **Bot-right**: |q(x)-p(x)|: (Chebyshev points).

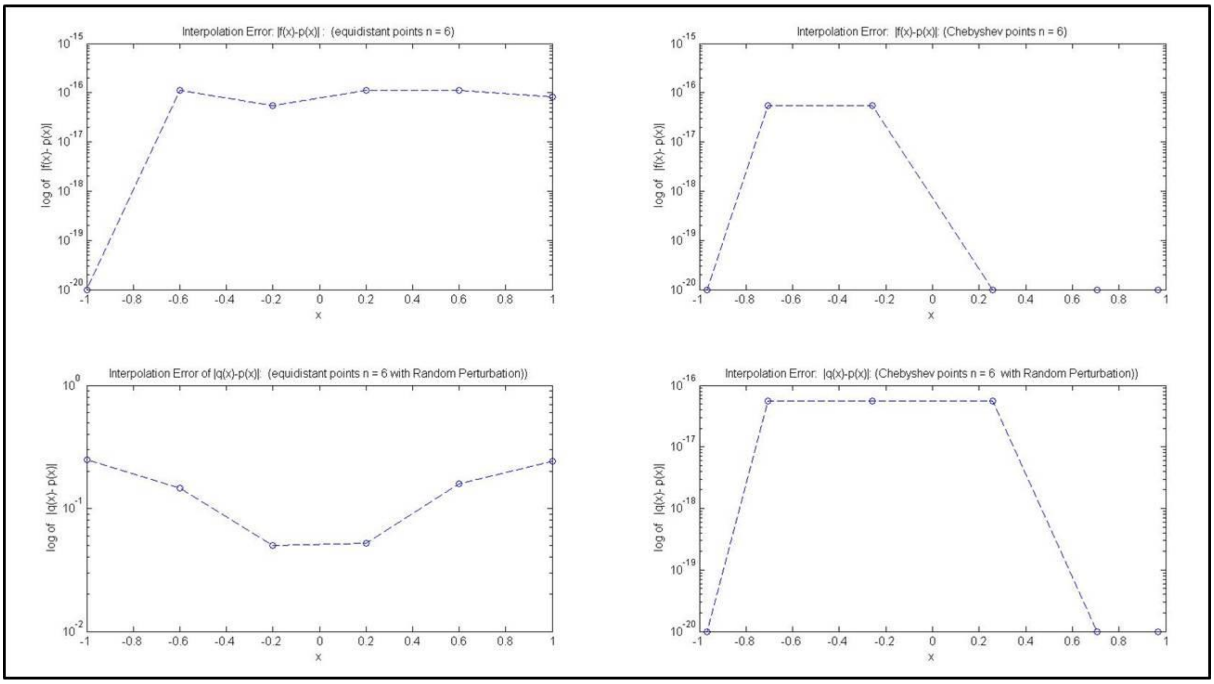

Figure3. Interpolation of function, with equidistant points and Chebyshev points, n =6

Top-Left: |f(x)-p(x)|: (equidistant points). **Top-right**: |f(x)-p(x)|: (Chebyshev points).

Bot-Left: |q(x)-p(x)|: (equidistant points). **Bot-right**: |q(x)-p(x)|: (Chebyshev points).

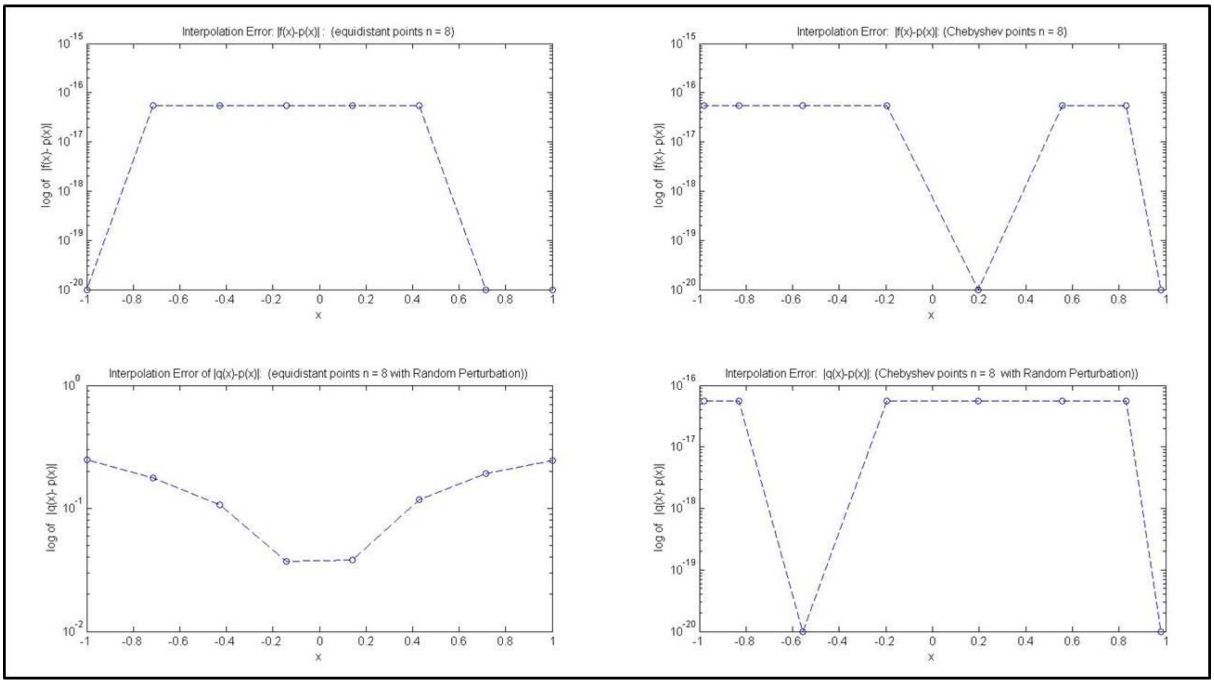

Figure4. Interpolation of function, with equidistant points and Chebyshev points, n =8

Top-Left: |f(x)-p(x)|: (equidistant points). **Top-right**: |f(x)-p(x)|: (Chebyshev points).

Bot-Left: |q(x)-p(x)|: (equidistant points). **Bot-right**: |q(x)-p(x)|: (Chebyshev points).

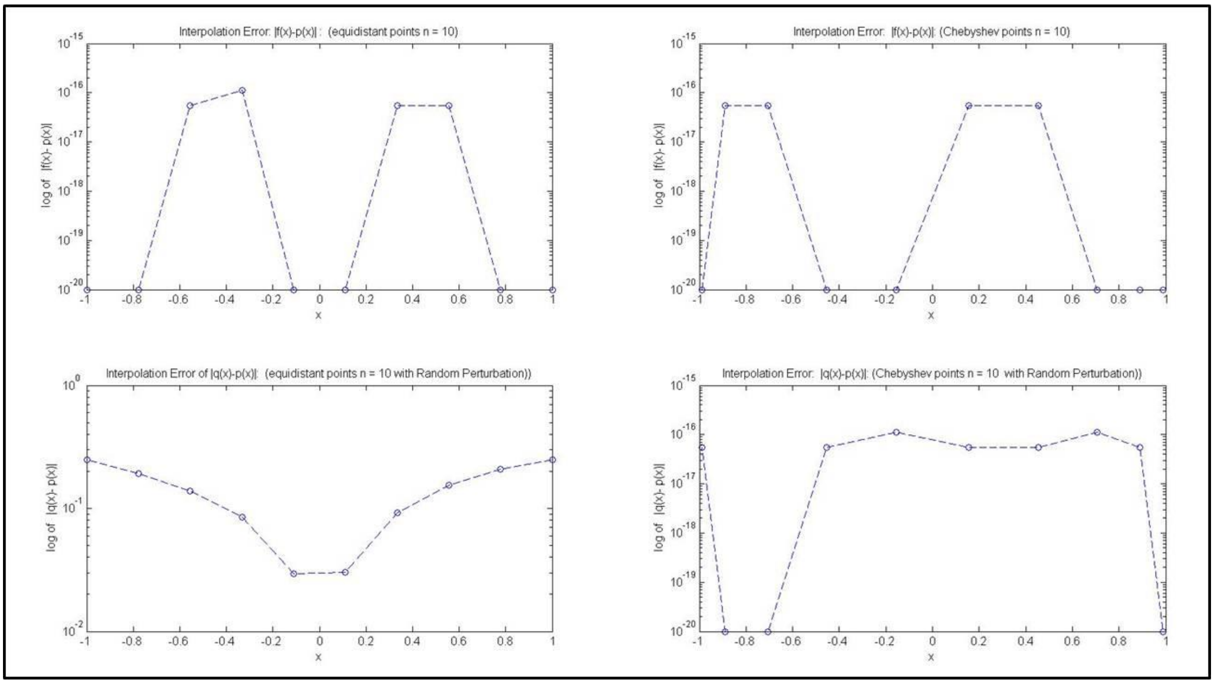

Figure5. Interpolation of function, with equidistant points and Chebyshev points, n =10

Top-Left: |f(x)-p(x)|: (equidistant points). **Top-right**: |f(x)-p(x)|: (Chebyshev points).

Bot-Left: |q(x)-p(x)|: (equidistant points). **Bot-right**: |q(x)-p(x)|: (Chebyshev points).

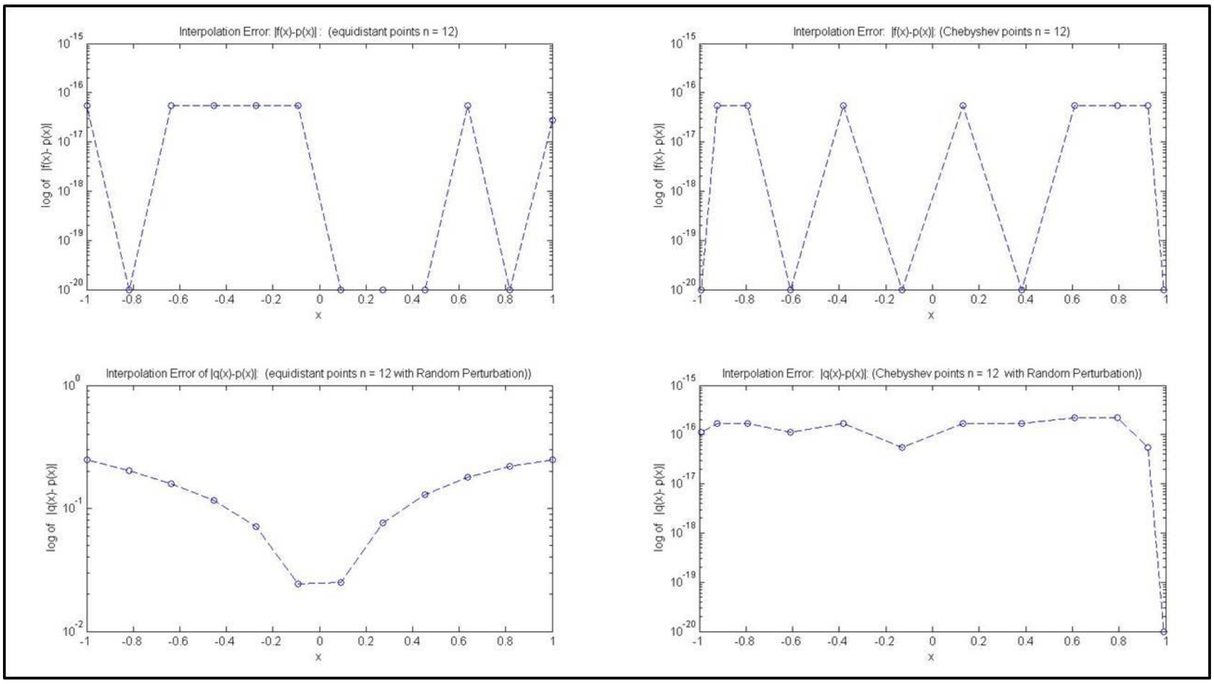

Figure6. Interpolation of function, with equidistant points and Chebyshev points, n =12

Top-Left: |f(x)-p(x)|: (equidistant points). **Top-right**: |f(x)-p(x)|: (Chebyshev points).

Bot-Left: |q(x)-p(x)|: (equidistant points). **Bot-right**: |q(x)-p(x)|: (Chebyshev points).

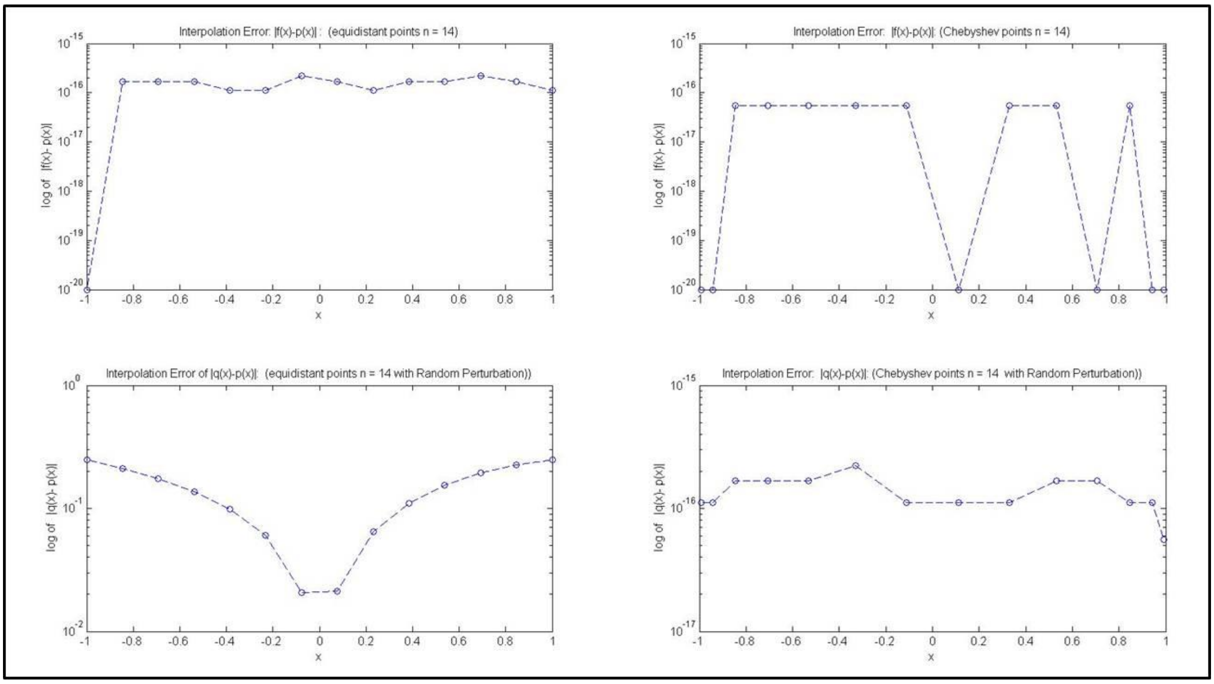

Figure7. Interpolation of function, with equidistant points and Chebyshev points, n =14

Top-Left: |f(x)-p(x)|: (equidistant points). **Top-right**: |f(x)-p(x)|: (Chebyshev points).

Bot-Left: |q(x)-p(x)|: (equidistant points). **Bot-right**: |q(x)-p(x)|: (Chebyshev points).

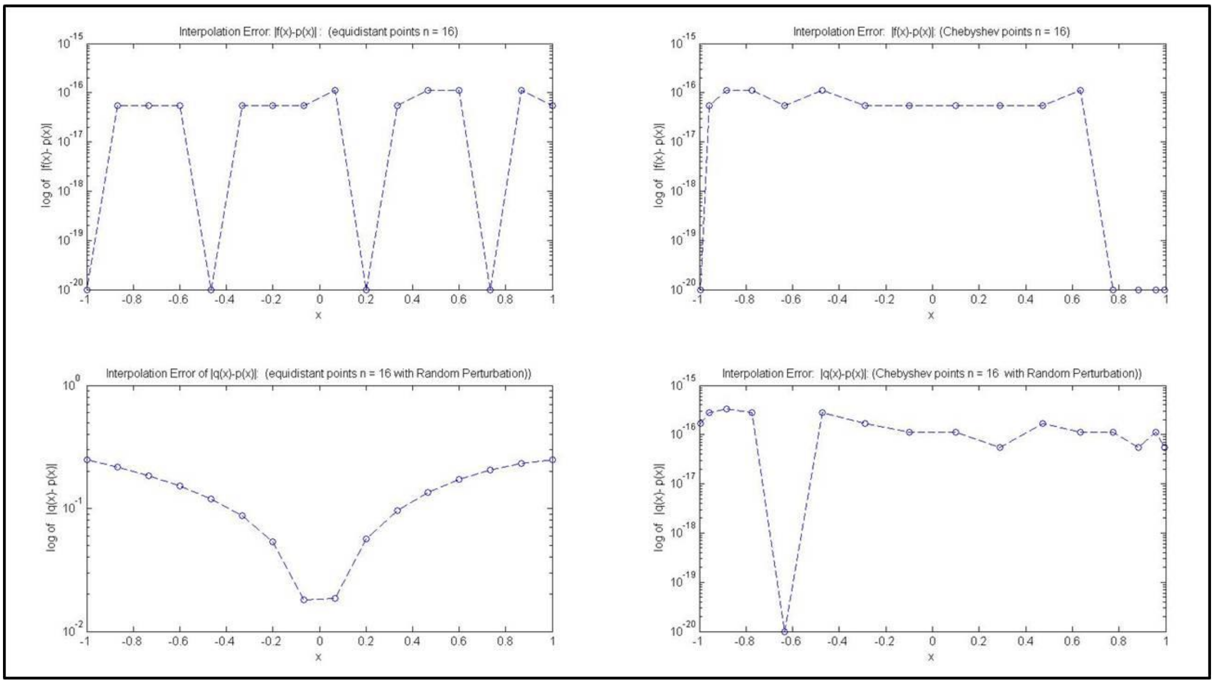

Figure8. Interpolation of function, with equidistant points and Chebyshev points, n =16

Top-Left: |f(x)-p(x)|: (equidistant points). **Top-right**: |f(x)-p(x)|: (Chebyshev points).

Bot-Left: |q(x)-p(x)|: (equidistant points). **Bot-right**: |q(x)-p(x)|: (Chebyshev points).

Exercise 4.1.1 (b):

Plots Display:

Remarks:

The random function y = sin(x*pi), x =[-1,1] is used for this test.

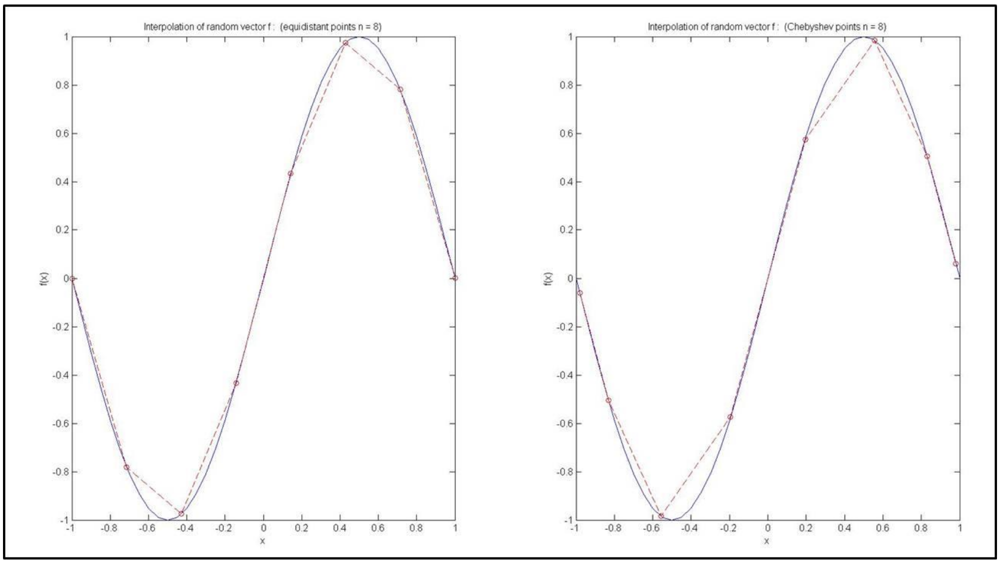

Figure9. Interpolation of random function, with equidistant points and Chebyshev points, n =8

Left: Display of f(x) and P(x) (equidistant points). **Right**: Display of f(x) and P(x) (Chebyshev points).

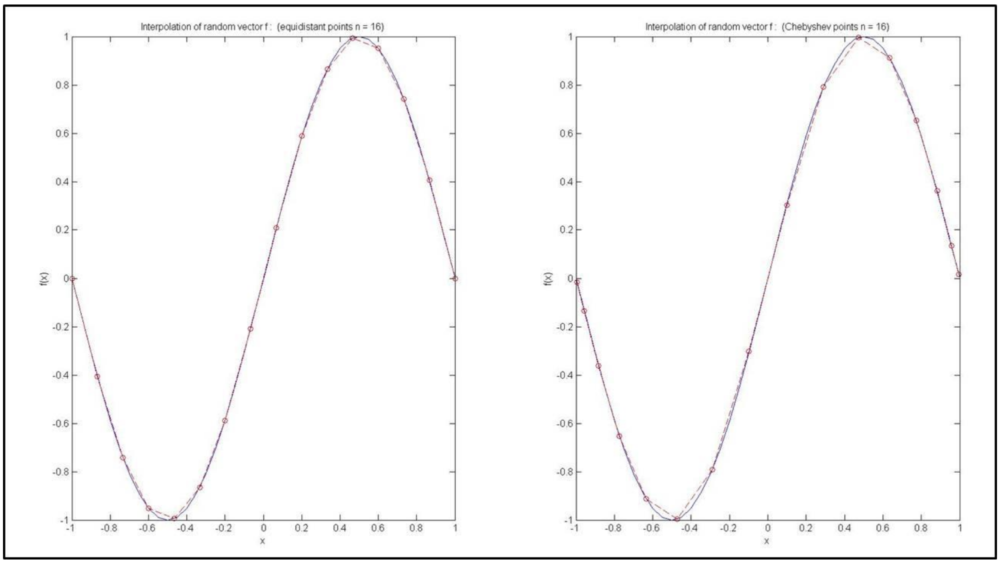

Figure10. Interpolation of random function, with equidistant points and Chebyshev points, n =16

Left: Display of f(x) and P(x) (equidistant points). **Right**: Display of f(x) and P(x) (Chebyshev points)