Management of Power Systems With In-line Renewable Energy Sources

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May 14, 2018

Abstract

Power flowing along a feeder line may modeled by an ordinary differential equations (ODE). The feeder line in our model starts with a main generator and has a second source injecting power in the midpoint of the line (such as a renewable energy source). Solving the ODE backwards as an initial value problem (IVP) initial values of real power, reactive power and voltage may be obtained for the formulation of forward problem. The graph for real power, reactive power, and voltage of the ODE solution characterises quality reliability and efficiency of the energy transmission process. By comparing spatial profiles of these characteristics with and without the injection of the midpoint (renewable) energy source, the desired solution for better line performance may be determined. By solving the ODE under such optimal conditions, conditions are found. In particular the optimal value for angle of the phase shift is determined and the minimal input voltage from the main generator is found, under condition that the voltage at the endpoint to be 90% of a standard value.

Acknowledgments

In performing our final assignments, we took the help and the guideline from some respected people with our greatest gratitude. The completion of this final report gives us much pleasure. We would like to give pleased thanks to our mentor Jared McBride and our course instructor Ilder Gabitov from the University of Arizona for giving us a good guideline for this final report throughout numerous consultations. We would also like to expand our deepest gratitude to all those who have directly and indirectly guided us in writing this report. We thank all the people for their grateful contributions to our report

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Contents

1 Background and previous result

1.1 Background

Traditional electric power networks consist of several key network components including generators, power loads and power transmission lines that remains unchanged during network operation. With the development of renewable energy sources, the situation has changed dramatically. Over time, with the integration of renewable energy sources to the power network, not only additional sources of energy may appear, but there is also a presence of dynamical transformation of power loads into energy sources and vise versa. This dynamic reconfiguration of the network allows a reduction in the power of the main generators and, as a result, optimizes fuel consumption by generating stations. To make it possible, two main tasks need to be solved: 1) how to reduce, in an optimal way, the power of the main generators and 2) how to adjust the parameters of renewable energy sources so that the network parameters remain within acceptable operational limits.

This project is devoted to the analysis of the optimization problem given in the case of a power line with distributed loads together with a renewable energy source g. The line is fed by a generator G as shown in Fig. 1. In accordance with existing regulations acceptable

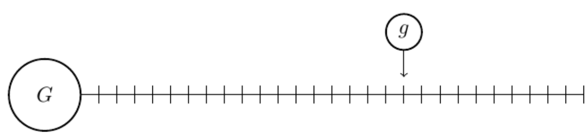

Figure 1: Illustration of the model considered in the project. A power line with distributed loads and renewable source of energy g fed by a generator G.

voltage variations along the power line must be within the interval of $90\% - 105\%$, where 100% corresponds to the value of standard voltage [1]. The power of the renewable source g depends on weather conditions and varying with time. The presence of an additional power source g allow a reduction in the power of the generator G, which will lead to a decrease in the voltage of the generator G. Electric energy generated by the renewable source g enters the power line in the form of alternating current. In this case, the phase shift between current and voltage can be controlled by hardware. This means that the ratio of active and reactive power being injected into the network can be controlled on the hardware level. This further means that the ratio of the active and reactive power components is an additional parameter to control the network parameters that allows an optimal reduction of the power of the main generator G. In order to solve the optimization problem, we investigate the introduction of feedback, in terms of a linear function of reactive power injected to the power line, on the voltage at the renewable energy source g [2].

1.2 Capacitor bank

It is important to understand how it is possible to achieve less power loss within the high-voltage transmission process. The fundamental basis of the application of the renewable energy is to consider its real-life characteristics, such as the weak sunlight during daytime and nights for the solar energy supply, or small/no wind for the wind power supply. In our model, these occasions are treated as the shutdown moment for the renewable energy generators. The occasional turning-on and shut-off would lead to the instability of the voltage. Therefore, it is necessary to put the capacitor bank into the power grid after regular intervals of the line in order to increase the power factor of the electricity transmission within the line. With the influence of the inductance of the line, the reactive power will increase and let the current lead the voltage. These characteristics will let the power grid have a bad performance. Besides, a capacitor bank added to the line will draw the current ahead of the voltage, which could compensate the real power and reduce the reactive power. Therefore, as the power factor is increased, the voltage will be more stable. For example, in the high-voltage transmission of electricity, electricity is generated in the factories, then the voltage load tap changer boosts the voltage up to around $110kV$ for long-distance transmissions. Then the voltage regulator is the device helping the line to stabilize the voltage at a high value. Later, since the power is converted multiple times with devices with large inductance, capacitor banks help to draw the current ahead of the voltage in order to compensate the real power and reduce the reactive power. In this way, the power factor for the power transmission is increased and the effective transmission efficiency is increased accordingly. The overhead recloser is the switch of the overhead power transmission status controlled by the control station, and this switch is only for the station to control based on the amount of the energy needed to supply the feeder line. Lastly, the transformer is the device decreasing the voltage to $120V$ for loads daily usage.

1.3 Previous result

The primary goal of the first part of this analysis is the management of the renewable energy to maximize the utility of real power and minimize the consumption of reactive power. To reach this goal, the foundational formula to facilitate this endeavor is the distribution flow ODE function of real power, reactive power, and voltage from [3], [4]:

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\frac{P^2+Q^2}{v^2} \\ q - x\frac{P^2+Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix} \tag{1}$$

where $v(0) = 1.05,\ P(L) = 0,\ Q(L) = 0,\ L = 0.5$ (here L is a dimensionless quantity corresponding to the length of the line). When trying to solve this ODE, setting voltage to 1.05 is the first trail and it gives a good result. Hence, since the engineering voltage margin, $[0.9, 1.05]$ (see [2]), accepts this value, the boundary condition of voltage leaves as 1.05.

1.4 Nose curve

Due to the impossibility for computers to catch the bad performance of voltage drops, drawing a nose curve is one way to show this situation, which is derived from the rescaled formulas from the DistFlow function (1). From (1), the rescaled formula of the DistFlow ODE functions (1) is shown below:

$$-\frac{d}{ds}\begin{pmatrix}\rho\\ \tau\\ \nu\end{pmatrix} = \begin{pmatrix}\operatorname{sgn}(p) - \frac{\rho^2+\tau^2}{\nu^2}\\ A - B\frac{\rho^2+\tau^2}{\nu^2}\\ -\frac{\rho+B\tau}{\nu}\end{pmatrix} \tag{2}$$

where $\nu(0) = 1$, $\rho(0) = 0$, $\tau(0) = 0$, which is a Cauchy problem. We normalized the initial value of voltage to be 1 as a $s = 0$, and draw the graph of voltage with respect to the distance s. When voltage drops dramatically, the electricity equipment will not work due to voltage not flowing from low-voltage position to high-voltage position. To solve this problem, we need to solve the Cauchy problem (2) in the rescaled time: $s : 0 \to s_*$, the length of the feeder line and the voltage with respect to voltage are:

$$L = \frac{s_*}{\sqrt{|p|r}}v(L), \qquad v(L) = \frac{1}{v(s_*)} \tag{3}$$

A fact to understand from the two formulas (3) is that there will be two stable solutions in this system. This represents two possible voltages throughout the transmission process. As the distance goes further, the differences between those two voltages will be less until the power is eliminated eventually at the end point and there will be only one solution of voltage. Use the nondimensionalized system (3) to draw the nose curve of Figure 2 in [1]. By [3],we get $p = -1$, $q = -0.5$, $r = 1$. This graph shows that the amount of renewable energy power injected into the feeder line cannot exceed the threshold dependent when it eventually reaches one end of the s as the distance goes further.

1.5 Motivation

With the continuous growing demand of power and unavoidable issues in traditional energy generation such as heavy pollution and high requirement of transmission route, using renewable energy is extremely urgent. Renewable energy can solve environmental concerns by saving traditional energy. In the normal situation, if voltage drops below 0.9 in the transmission, it will be an excessive loss of power. Hence, by guaranteeing voltage at end point to be at least 0.9, adding a renewable energy device with management can control fluctuation

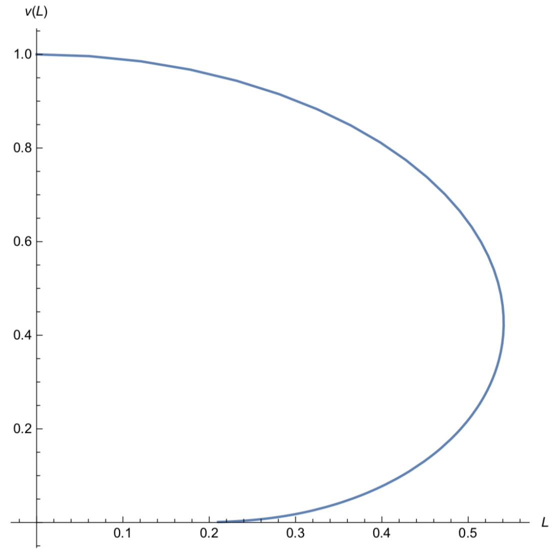

Figure 2: The voltage change throughout the transmission process

of voltage and increase the working efficiency of the main generator in order to save the maximum value of traditional energy and decrease the environmental pollution.

1.6 Goal

In order to save power from the main generator, we need to add a device to control the voltage fluctuation along the feeder line to make sure that voltage at end point is 0.9. Then we will adjust this device with phase shift between alternating current and voltage to find the minimal value of angle of phase shift required to minimize the power from the main generator in order to save energy.

2 Our model and solution

2.1 Norm Situation

This graph shows the normal situation in the power transmission. Voltage, real power and reactive power are all decreased straightly in the feeder line with the usage of traditional energy generator. Initial values of these three variables are given by the reference paper [3]. In the ideal situation, voltage drops to 0.8 and real and reactive power changes to 0 in the final values. Thus, we consider that adding renewable energy into the feeder line would change this linear decreased situation.

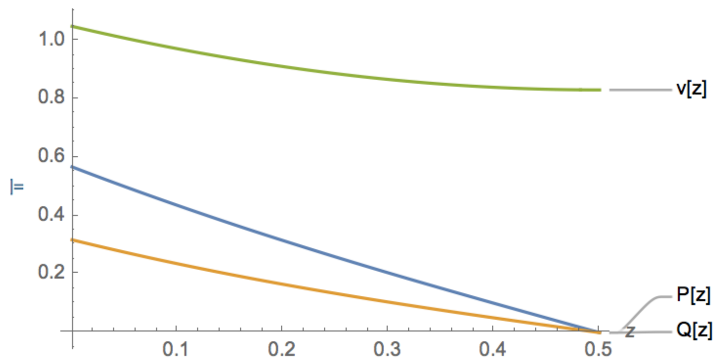

Figure 3: Norm graph of the real and reaction power and voltage

2.2 Our Model

In the figure 1, G refers to the traditional power generator for the total transmission that we put at the start of the feeder line. Each small vertical line represents either the production loads or the consumption loads along the feeder line. For convenience, we set up the value of these loads (resistances and inductances) to be equal to 1. g indicates the renewable energy we need to add into one point of the feeder line. It should be noted that the injected renewable energy power needs to keep the end voltage floats between 0.9 to 1.05, which is the engineering voltage margin [2].

In the previous work, we derived the DistFlow ODE function which showed that voltage, real power and reactive power drop straightly in the transmission. Then we came up with the model of feeder line to increase the usage of voltage in the transmission. It is hard to find the minimum value intuitively of injected renewable energy power which can help us achieve increasing the utilization of voltage. Thus, we use the approximation of Diracs δ- function to figure out the minimum value of injected power and stimulate the practical application. Specifically, we add the renewable energy both into the real power and reactive power. This method avoids energy heat releases instead of doing work, increases usage of voltage and reaches the engineering voltage margin.

2.3 The Approximation of Diracs Delta Function

The Approximation of Dirac δ- function represents renewable energy power injected into the feeder line:

$$g(z) = -1 + \frac{\beta}{\varepsilon\sqrt{\pi}} \exp\left(-\left(\frac{z-0.5L}{\epsilon}\right)^2\right) \tag{4}$$

where the arbitrary width of area $\varepsilon = 0.015$, $L = 0.5$, $z \in [0, 0.5]$. Since the renewable energy cannot be stored, the width can be as small as possible until it reaches the smallest value. Hence, from the approximation of the Dirac δ-function (4) we conclude that β can represent

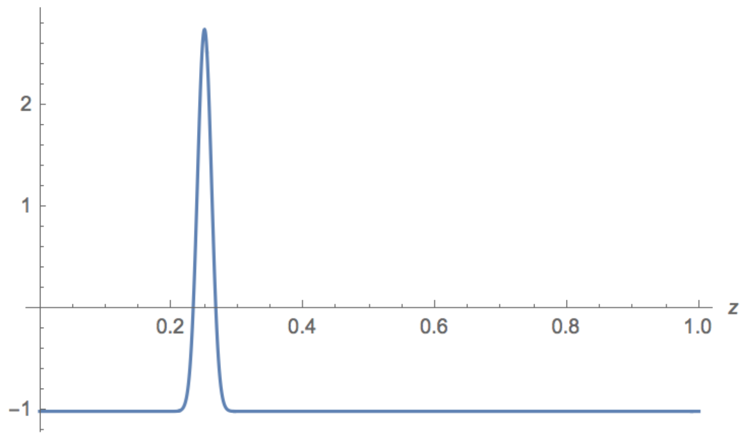

Figure 4: The approximation of Dirac's δ-function

the total power injected, which will eventually represent the minimum value of power that can control the voltage to be with in the margin. Considering of this circumstance, knowing how much power injected will help us find the angle of phase shift, which will be mentioned later.

2.4 Renewable energy added to the real power

Since the real power is the effective power consumed on loads that cannot be transformed back and the reactive power is the ineffective power creating the electromagnetic field that can be converted back to electricity, the first trial was to inject the renewable energy into the real power only to see the performance of power grid. Using the ODE functions with the Dirac δ-function in real power consumption equation, the solution was solved by Mathematica for the graph of P, Q, v for the comparison with the normal P, Q, v graphs without the renewable energy generator. It is shown in the graph that there is a part of the line where the reactive power is greater than the real power, and the voltage still have a major drop at the end of the line. In practical case, it means that the electricity is producing more heat or electromagnetic field rather than doing effective work. And the electricity supplying efficiency are decreasing as the goes further in the line. Hence, we need to come up with another method for best performance of every load along the feeder line.

2.5 Renewable energy added to the real and reactive power

The second method tried is to add the renewable energy in both real and reactive power. The Dirac δ-function was put into both real and reactive power to simulate the impact of a renewable energy generator. Similar to the previous analysis process, the ODE was solved by Mathematica to compute the graph of P, Q, v. Only at this time, differential equations of real power and of reactive power are solved with the addition of the Dirac δ-function. Therefore,

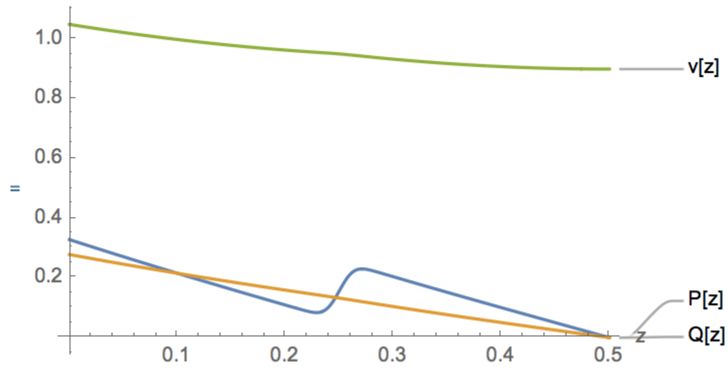

Figure 5: When the renewable power adds into real power

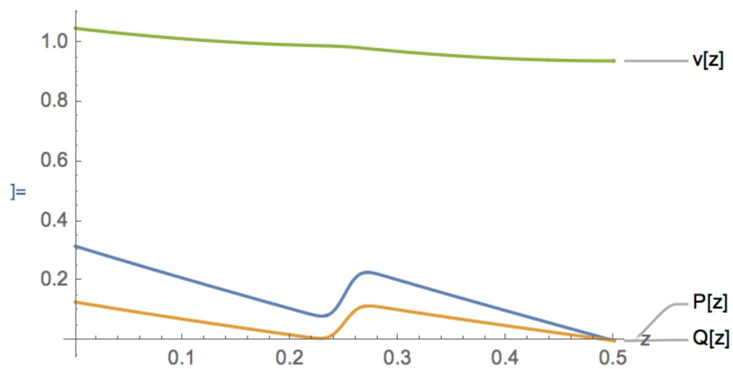

Figure 6: When the renewable power adds into real and reaction power

the real power P is consistently greater than the reactive power Q, which indicates a very effective feeder line transmission in most of the cases. Practically, it means that the loads in the power line are consistently consuming power in the line. It is important to note that this model is set in a theoretical ideal case; in practical cases, the reactive power should never be zero, and for some of the electricity consuming devices, such as ovens, the consumption of reactive power should be greater than that of real power.

2.6 Phase difference between real and reactive power

Having the function of alternating current and voltage can get the integral function for real power and reactive power:

$$V = u_0 \sin(\omega t) \tag{5}$$

$$I = I_m \sin(\omega t + \phi) \tag{6}$$

According to function (5) and (6), we can get the P and Q:

$$P = u_0 I_m \sin(\omega t)\sin(\omega t + \phi)$$

$$P = u_0 I_m \int_0^T \sin(\omega t)\sin(\omega t + \phi)dt$$

$$P = \frac{u_0 I_m}{2} \int_0^T \cos(\phi) - \cos(2(\omega + \phi))dt$$

Solve for real power by setting the function to be approached to 0 :

$$\int_0^T cos(2(\omega + \phi))dt \to 0$$

$$P {\to} cos(\phi)$$

$$Q = I^2 x$$

$$Q = I_m^2 sin^2(\omega t + \phi)x$$

$$Q = I_m^2 x \int_0^T 1 - cos(2(\omega + \phi))dt$$

$$Q {\to} 1$$

$$cos(\phi) = \frac{P}{\sqrt{P^2 + Q^2}}$$

In Figure (1), Eventually, real power approaches cosine phi and reactive power approaches to 1. This shows that the phase difference is determined by the value of phi. In this case, we need to find the value of ϕ within the margin $(0, \frac{\pi}{2})$ and use the phase difference to find out how much power we can save from the main generator to guarantee that the voltage at the end will be at least 0.9.

The function calculating the real and reactive power with respect to the alternating current and voltage from the three-phase circuit is:

$$p = \sqrt{3} I u \cos(\phi) \tag{7}$$

$$q = \sqrt{3} I u \sin(\phi) \tag{8}$$

By reference paper [3], $p = -1, q = -0.5$, which are used to solve the function (7) and (8). The solution is that the value of phase shift between current and voltage is 26.57.

3 Voltage control in presence of renewable energy source

Incorporation of renewable energy sources allows for reduction of the load on the main generator, however, it leads to the problem of the management of the parameters of the power line. The optimal reduction of the input power must be implemented in such a way as to ensure that the voltage $v(L)$ remains at the power line output within the required limits. In addition, variations of the voltage along the power line should be minimized. It should be also taken into account that the power of renewable energy sources in most cases depends on weather conditions and time of day and can vary randomly. An essential component of the renewable power source is convertor DC current to AC current. One of the controllable parameter in this case is the phase shift between generated current and voltage. This phase shift can be used as an active input parameter for controlling power line characteristics. On the other hand it was shown in [2] that voltage value v at the location of the renewable source of energy $L/2$, $v = v(L/2)$ can be used as a feedback information for optimal adjustment of the phase shift or optimal management of the input into the power line of the reactive and active power components.

We consider two cases: a) undepleted regime of the renewable power source operation and b) taking account of saturation effects. System of the equations corresponding to the first case reads as:

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p + q_{rn} - r\frac{P^2+Q^2}{v^2} \\ q + q_{rn} - x\frac{P^2+Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix} \tag{9}$$

Here the term in the right hand side of the first equation:

$$p_{rn} = \frac{\beta}{\varepsilon\sqrt{\pi}} \exp\left[-\left(\frac{z-0.5L}{\varepsilon}\right)^2\right] \equiv \beta\delta_\varepsilon(z - \frac{L}{2}) \tag{10}$$

corresponds to the active power injected to the power line by renewable source, here β - injected total power, δ_ε is an approximation of Dirac's δ-function (idealization of point source of energy as the result of continuous limit model approximation). The term in the right hand side of the second equation:

$$q_{rn} = \frac{\beta}{\varepsilon\sqrt{\pi}} \exp\left[-\left(\frac{z-0.5L}{\varepsilon}\right)^2\right] \theta(v(z) - \nu) \tag{11}$$

corresponds to reactive power injected to the power line by the renewable source, which is a linear function of the voltage v [2]. Here θ and ν are optimization parameters. It was shown above that injection of the reactive power is allows to reduce input voltage and improve overall voltage profile along the power line. This improvement is illustrated in Fig. 5 and Fig. 6. However, power limitations of the renewable source of energy and power depletion effects in this case in not taken into account. System of equations taking into account power

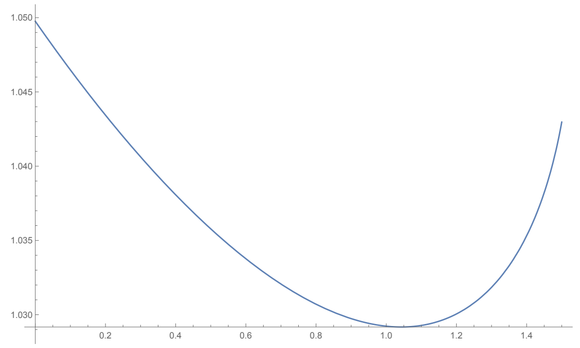

Figure 7: Input voltage $v(0)$ as function of θ (see equations (12))., here $L = 0.5,\ \beta = 0.2,\ \varepsilon = 0.015,\ r = 1,\ x = 1, p = -1,\ q = -0.5\ , \nu = 0.3,\ v(L) = 0.9$.Vertival axis corresponds to $v(0)$ and horizontal axis to θ.

saturation effects has the following form:

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p + \frac{\beta\sqrt{1-\theta(v-\nu)^2}}{\varepsilon\sqrt{\pi}} \exp\left[-\left(\frac{z-L/2}{\varepsilon}\right)^2\right] - r\frac{P^2+Q^2}{v^2} \\ q + \frac{\beta\theta}{\varepsilon\sqrt{\pi}} \exp\left[-\left(\frac{z-L/2}{\varepsilon}\right)^2\right](v-\nu) - x\frac{P^2+Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix} \tag{12}$$

This system of equations allows to optimize system performance. To be more precise, at the given value of the power of renewable source β and at the fixed value of the voltage at the end of the power line $v(L) = 0.9$ which corresponds to the lower level acceptable voltage value, find minimal value of the input voltage $v(0)$. Let us consider as an example the case corresponding to $L = 0.5,\ \beta = 0.2,\ \varepsilon = 0.015,\ r = 1,\ x = 1, p = -1,\ q = -0.5\ , \nu = 0.3,\ v(L) = 0.9$. The functional dependence of $v(0)$ on θ is shown in Fig. 7. The optimal value of θ is $\theta \approx 1.05$ and corresponding input voltage value is $v(0) \approx 1.029$.

However, Fig. 7 represents only one slice on two parametric space (θ, ν). To understand general behavior of $v(0)$ and to find the optimal regime it is necessary to analyze functional dependence on both $(\theta\ \nu)$ parameters. The behavior of the input voltage $v(0)$ as the function of θ and ν can be characterized by contour plot and 3D plot shown in Fig. 8 and in Fig. 9.

Both figures Fig. 8 and Fig. 9 show that for this particular choice of parameters optimal point conversions to $(\theta, \nu) = (1.05, 0)$. The input voltage in this case is $v(0) = 1.029$.

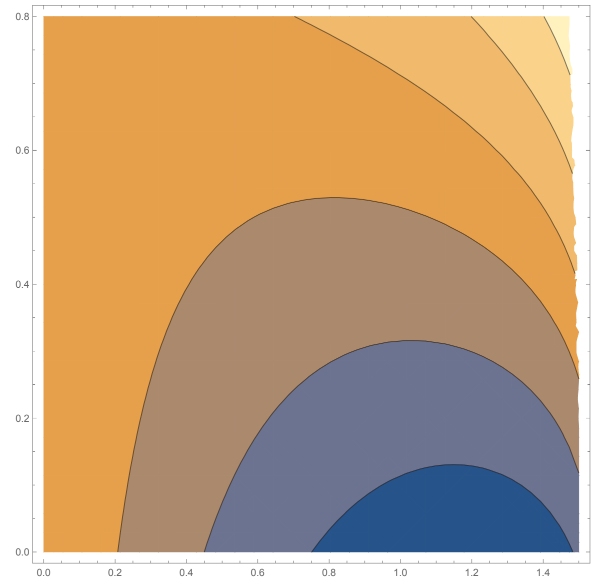

Figure 8: Contour plot of $v(0)$ as function of (θ and ν) (see equations (12)), here $L = 0.5$, $\beta = 0.2$, $\varepsilon = 0.015$, $r = 1$, $x = 1, p = -1$, $q = -0.5$, $v(L) = 0.9$. Vertical axis corresponds to ν and θ changes along horizontal axis.

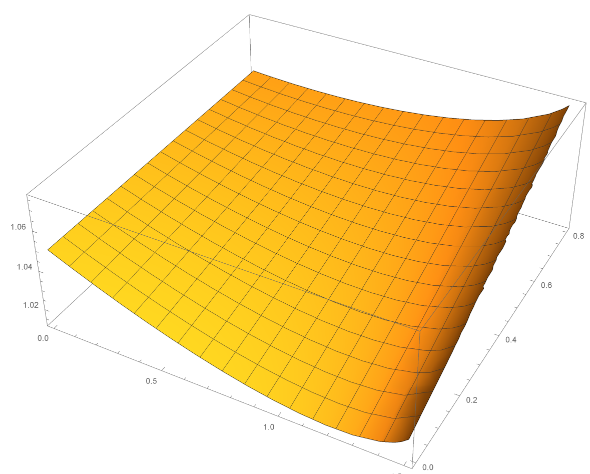

Figure 9: 3D surface plot of $v(0)$ as function of (θ and ν) (see equations (12)), here $L = 0.5$, $\beta = 0.2$, $\varepsilon = 0.015$, $r = 1$, $x = 1, p = -1$, $q = -0.5$, $v(L) = 0.9$. Vertical axis corresponds to $v(0)$, ν and θ are changing in the intervals $0 \le \nu \le 0.8$, $0 \le \theta \le 1.5$.

4 Conclusion

Based on the model set up, we need to add the renewable energy injection power into the feeder line to make a better voltage by controlling its fluctuations. According to the figure (5) and (6), we summarize that after adding the renewable energy, we need to control and adjust the main generator to compensate the reactive power in order to keep the voltage at the end point to be 0.9 after injecting the renewable energy. From a particular case study, we conclude that controlling the value of phase shift angle can let voltage at the end reach 0.9 when optimization parameters is 0. Based on figure (6) and (7), we can get the minimum value of the input voltage when the voltage at the end reaches 0.9. Overall, we compute the minimum value of input voltage to be 1.029 when the phase shift value is 1.05.

5 Potential application

In a map constructed used information gathered by NASA and National Renewable Energy Lab of Department of Energy, USA, it is easy to find that the city of Tucson is located in the region that could use solar energy for the renewable energy substitution. For cities in the solar circles, this model can be used for the constant high-voltage transmissions from the energy generator to the loads. Also, if the size of the renewable energy farms increases, and the electricity generated becomes more than the needs of the local area, it can be used to support more than one city. Due to the constant high-voltage transmission and limited power loss during the transmission process, the cities in the demand circles could also get benefits from the usage of the renewable energy.

6 Future work

Based on the model and the results we have, future work should be focused on solving the relation of the apparent power (total power) of the renewable energy with the phase shift angle and the relation of the apparent power of the renewable energy with the time coefficients. The relation between the apparent power with the phase shift angle shows that for the angle we should draw the current ahead of the voltage for a minimum production at the main generator. This gives us the controlling parameter function in the engineering field when the total power produced is limited at a certain level. The relation between the apparent power and the time coefficient takes the instability of the renewable power into consideration. Practically, it shows that the controlling scheme of the actions needs to be taken when the renewable energy sources injected are not sufficient due to their own characteristics. With these two relation equations, this model could be partially modified under particular circumstances and applied to real life scenarios.

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