1 Fixed Functions in Graphic Pipeline

1.1 Rasterization

Addition Fomulars:

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

2D rotate matrix:

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

2D matrix to rotate a vector 90 degress counter-clockwise:

$$\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$$

To judge vector v1 is pointing to the right side of vector v0:

$$\left(\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \vec{v_0} \right) \cdot \vec{v_1} > 0$$

In OpenGL, default visiable triangles are counter-clockwise, thus left side of three edges form the triangle aera.

1.2 Coordinate System Transform

Describe (u,v,w) space axis in (x,y,z) space:

$$\begin{bmatrix} \vec{u} & \vec{v} & \vec{w} \end{bmatrix} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \end{bmatrix} \times \begin{bmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{bmatrix}$$
$$= \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \end{bmatrix} \times \mathbf{P}$$

$\mathbf{P}$ converts a vector from (u,v,w) space to (x,y,z) space; Inv($\mathbf{P}$) converts a vector from (x,y,z) space to (u,v,w) space:

$$\begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} = \begin{bmatrix} \vec{u} & \vec{v} & \vec{w} \end{bmatrix} \times \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix}$$

$$= \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \end{bmatrix} \times \mathbf{P} \times \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix}$$

$$= \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \end{bmatrix} \times \left(\mathbf{P} \times \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} \right)$$

For points transformation:

$$\begin{bmatrix} & \mathbf{P} & & \begin{matrix} u_{root} \\ v_{root} \\ w_{root} \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} & \mathbf{P}^{-1} & & -\mathbf{P}^{-1} \times \begin{matrix} u_{root} \\ v_{root} \\ w_{root} \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 Linear Algebra

2.1 Cross Product

Cross product definition differs in right-hand coordinate system and left-hand coordinate system ensuring that:

$$\vec{x} \times \vec{y} = \vec{z}$$
$$\vec{y} \times \vec{z} = \vec{x}$$
$$\vec{z} \times \vec{x} = \vec{y}$$

From this, it can be inferred that:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \left(a_1 \vec{x} + a_2 \vec{y} + a_3 \vec{z} \right) \times \left(a_1 \vec{x} + a_2 \vec{y} + a_3 \vec{z} \right)$$

$$= \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$