

1 Fixed Functions in Graphic Pipeline

1.1 Rasterization

Addition Formulas:

$$\begin{aligned}\cos(x + y) &= \cos x \cos y - \sin x \sin y \\ \sin(x + y) &= \sin x \cos y + \cos x \sin y\end{aligned}$$

2D rotate matrix:

$$\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

2D matrix to rotate a vector 90 degrees counter-clockwise:

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

To judge vector v_1 is pointing to the right side of vector v_0 :

$$\left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{v}_0 \right) \cdot \vec{v}_1 > 0$$

In OpenGL, default visible triangles are counter-clockwise, thus left side of three edges form the triangle area.

1.2 Coordinate System Transform

Describe (u,v,w) space axis in (x,y,z) space:

$$\begin{bmatrix} \vec{u} & \vec{v} & \vec{w} \end{bmatrix} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \end{bmatrix} \times \begin{bmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{bmatrix} = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} \end{bmatrix} \times \mathbf{P}$$

2 Linear Algebra

2.1 Cross Product

Cross product definition differs in right-hand coordinate system and left-hand coordinate system ensuring that:

$$\begin{aligned}\vec{x} \times \vec{y} &= \vec{z} \\ \vec{y} \times \vec{z} &= \vec{x} \\ \vec{z} \times \vec{x} &= \vec{y}\end{aligned}$$

From this, it can be inferred that:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = (a_1 \vec{x} + a_2 \vec{y} + a_3 \vec{z}) \times (a_1 \vec{x} + a_2 \vec{y} + a_3 \vec{z}) = \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$