Peeking behind black hole horizons with boundary states

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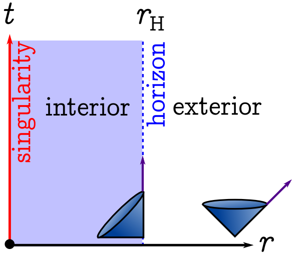
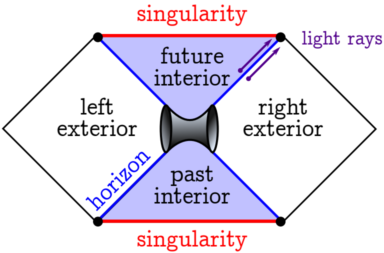
I. Black holes

Classical black holes

- Focus on Schwarzschild black hole (BH)[†] for simplicity.
- In coordinates of far-away observer, light cone of infalling observer tips over. No signals can be sent to infinity past horizon at $r = r_{\mathrm{H}}$.



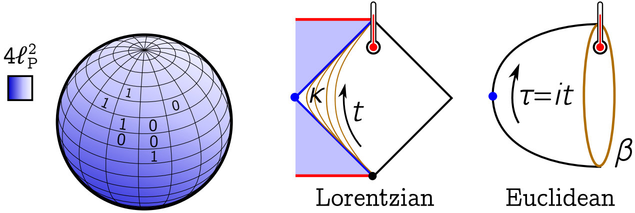


- Horizon appears singular from far away. Go to conformally compact coordinates: light rays on $\pm 45^\circ$ lines and finite spacetime diagram.[‡]
- For Schwarzschild BH, we get two exteriors joined by wormhole.

[†] Schwarzschild 1916. [‡] Penrose 1964; Carter 1966.

Black hole thermodynamics

- Remarkable fact: black holes are thermodynamic systems.[†]
- The entropy is proportional to the horizon area, $S = A/4G$.[‡] Since $G = \ell_P^2$, think of horizon as screen with Planck length-sized pixels!

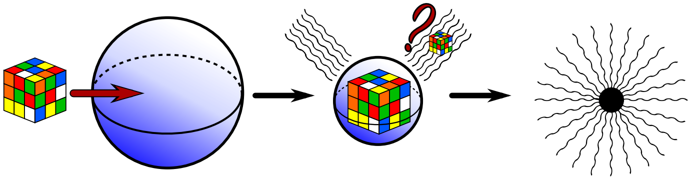


- BHs emit Hawking radiation at $T = \kappa/2\pi$, where κ is proper acceleration for observer fixed at horizon[§] (tension in fishing rod).
- Elegant derivation: $\beta = 1/T$ is period of BH in imaginary time.[♭]

[†] Bardeen, Carter and Hawking 1973. [‡] Hawking 1971; Bekenstein 1972. [§] Hawking 1974. [♭] Gibbons and Hawking 1977.

The information problem and complementarity

- Hawking realised that radiation leads to a paradox: BHs evaporate into thermal noise. They destroy information about what fell in![†]

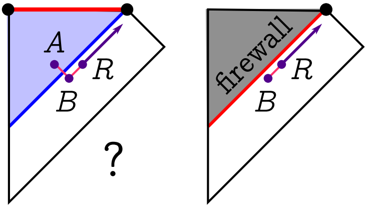


- Process is irreversible. But fundamental laws are reversible, with monotonicity of second law arising from coarse-graining.
- BH complementarity: horizon is quantum xerox machine, copying infalling state into Hawking radiation, but in scrambled form. Looks thermal to a casual observer![‡]
- But: scrambling mechanism unclear and in tension with no-cloning.[§]

[†] Hawking 1975. [‡] Susskind, Thorlacius and Uglum 1993. [§] Susskind and Thorlacius 1993.

Firewalls

- Even with BH complementarity, paradoxes for infalling observer.
- Scrambling maximally entangles near-horizon degrees of freedom B with Hawking radiation R.†
- To get smoothly varying quantum fields, need lots of short-distance entanglement, meaning B is entangled with mode A across horizon.‡



- But B can have at most one maximally entangled partner!§
- One option: drop AB entanglement, creating massive energy spike at horizon called "firewall".♭ Vaporises anyone who tries to cross!

† Page 1993. ‡ Unruh and Wald 1984. § Coffman, Kundu, and Wootters 1999; Mathur 2009.
♭ Almheiri, Marolf, Polchinski and Sully 2012.

Simulating the interior

- Second option, more in line with complementarity: identify modes A inside BH with some modes outside.[†] Avoids monogamy issue.
- Simulate interior of BH using exterior degrees of freedom. To reproduce short-distance entanglement, use entanglement of exterior quantum state $|\Psi\rangle$, giving state-dependent[‡] recipe for interior.

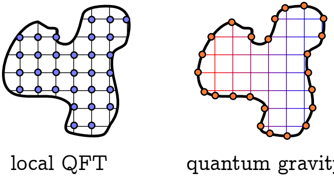
- Don't expect to recover all spacetime behind horizon, e.g. second Schwarzschild universe. Should instead get state-dependent amount of interior, though not clear where it breaks down.[§]
- We will see a precise realisation of these ideas in AdS/CFT!

[†] Papadodimas and Raju 2014; Maldacena and Susskind 2013. [‡] Papadodimas and Raju 2015.
[§] Shenker and Stanford 2013; de Boer, van Breukelen, Lokhande, Papadodimas and Verlinde 2018.

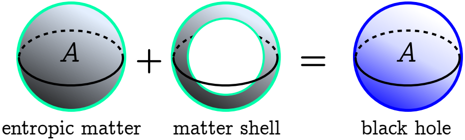
II. AdS/CFT

Gravity is holographic

- AdS/CFT is a theory of quantum gravity where we have some control. Natural place to explore physics of quantum black holes!
- First motivation: quantum gravity is holographic.† Unlike local QFT, gravity degrees of freedom scale with area rather than volume.



- Susskind's argument: BHs maximise entropy density, since we can collapse high entropy matter into BH and violate second law.

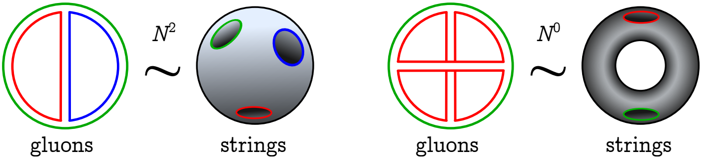


- Since $S \propto A$ in BH, area law scaling is the best you can do.

† 't Hooft 1993; Susskind 1995.

Gravity is colourful

- Second motivation is (less universal) statement that quantum gravity can be colourful. QCD with many colours looks like string theory!
- Consider adjoint fields Φ^i_j (e.g. gluons) of SU(N). When N is large, colour indices decouple and we record separately in diagrams.†



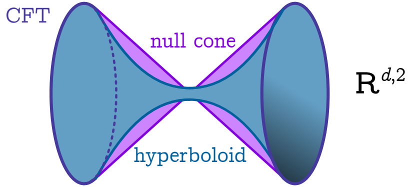
- Diagrams in one-to-one correspondence with simplest surface they can be embedded on. Like closed string scattering!
- After rescaling Φ, Feynman expansion becomes genus (hole) expansion. Like string perturbation theory with coupling $g_s \sim 1/N$.
- As $N \to \infty$, keep only tree-level strings. Suggests large-N Yang-Mills theory can encode classical gravity!

† 't Hooft 1974.

Matching symmetries

- In 1997, Maldacena[†] found holographic/colourful example: 5D string theory dual to large-N, conformally invariant Yang-Mills in 4D.
- Soon generalised to $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ correspondence:[‡] quantum gravity in $d+1$-dimensional anti-de Sitter space is dual to a large-N conformal field theory (CFT) in flat d-dimensional space.
- Check symmetries! AdS_{d+1} is $\mathrm{SO}(d,2)$-invariant surface in $\mathbb{R}^{d,2}$:

$$X_M X^M = X_1^2 + \cdots + X_d^2 - X_0^2 - X_{d+1}^2 = L^2.$$

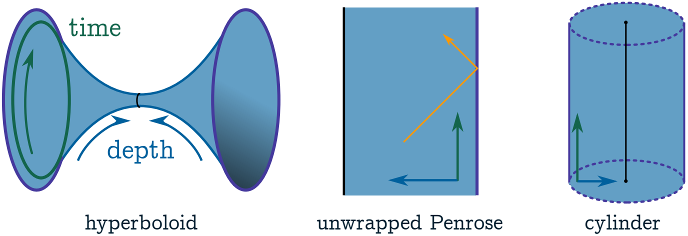


- CFT_d can be lifted to projective null cone[§]: $X^M X_M = 0$ (null), $X \sim \lambda X$ (projective). Symmetry group $\mathrm{SO}(d,2)$ acts linearly.

[†] Maldacena 1997. [‡] Gubser, Klebanov and Polyakov 1998; Witten 1998. [§] Dirac 1935.

Pictures of AdS/CFT

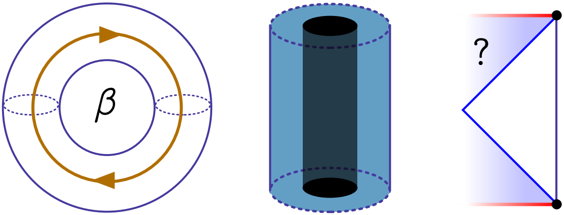
- The hyperboloid embedding of AdS_{d+1} is periodic in time! To get rid of closed timelike curves, unwrap periodic direction.
- The other important direction is depth: distance from the null cone in $\mathbb{R}^{d,2}$ embedding. The centre of space is represented by black line.
- From Penrose diagram, we can ping light rays off boundary. AdS/CFT is basically quantum gravity in a box.



- The CFT lives on $\mathbb{R}^{d-1} \times \mathbb{R}$, but we can compactify space so that it lives on a sphere $\mathbb{S}^{d-1}$. So boundary is higher-dimensional cylinder.

Thermal states and black holes

- AdS/CFT is a correspondence between Hilbert spaces.
- Pure AdS is vacuum state of CFT. Now consider thermal state of CFT (canonical ensemble), with mixed density matrix $\rho \propto e^{-\beta H}$.
- Correlators have period β in imaginary time,† which we geometrise by wrapping CFT cylinder into donut.



field theory black hole interior?

- Dual geometry is thermal cloud at low T, but at high T, thermal cloud collapses into BH.‡ In equilibrium with "pinged" radiation.
- Information problem: mixed state ρ doesn't tell us about interior.

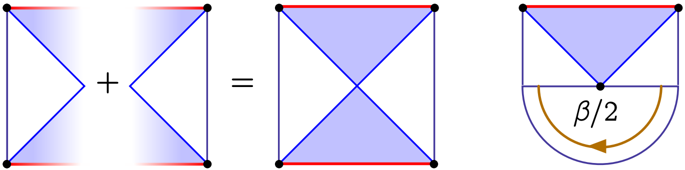
† Matsubara 1955. ‡ Hawking and Page 1983; Witten 1998.

Purification and wormholes

- If uncertainty comes from mixedness of ρ, what happens if we purify?
- The recipe for purifying mixed ρ is copy system, entangle, and apply $\sqrt{\rho}$. For density $\rho \sim e^{-\beta H}$, we get thermofield double (TFD):†

$$|\mathrm{EPR}\rangle \sim \sum_E |E\rangle_1 |E\rangle_2, \quad |\mathrm{TFD}\rangle \sim \sum_E e^{-\beta E/2} |E\rangle_1 |E\rangle_2.$$

- Each system separately has the exterior of a black hole, so natural to expect that TFD is dual to AdS wormhole.‡



- Note that $|\mathrm{TFD}\rangle = e^{-\beta H/2}|\mathrm{EPR}\rangle$ is a path integral recipe for a wormhole: evolve entangled state by $\beta/2$ in Euclidean time.§

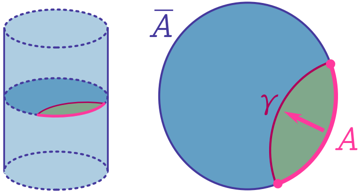
† Israel 1976. ‡ Maldacena 2001. § Hartle and Hawking 1976.

Entanglement and geometry

- TFD = wormhole suggests geometry/entanglement are connected.[†]
- Ryu-Takayanagi (RT) formula gives similar connection between geometry/entanglement for subsystems of CFT.[‡]
- Consider subregion A of CFT in state $|\Psi\rangle$. Entanglement with complementary system $\bar{A}$ measured by entanglement entropy S_A:

$$\rho_A = \mathrm{Tr}_{\bar{A}}|\Psi\rangle\langle\Psi|, \quad S_A = -\mathrm{Tr}_A[\rho_A \log \rho_A] \overset{\mathrm{RT}}{=} \frac{\mathrm{Area}(\gamma)}{4G},$$

where γ is minimal surface homologous to A.



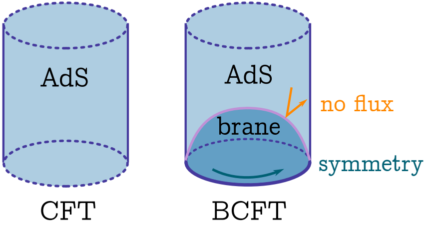
- The upshot is that gravity encodes subsystem entanglement.

[†] Van Raamsdonk 2009; Swingle 2009; Maldacena and Susskind 2013. [‡] Ryu and Takayanagi 2006; Hubeny, Rangamani, and Takayanagi 2007.

III. Boundary states

AdS/BCFT

- A boundary CFT (BCFT) is a CFT on a half-space.[†] This breaks $SO(d, 2) \to SO(d-1, 2)$ and implies no flux through boundary.
- We are led to conjecture that CFT boundary is dual to brane with $SO(d-1, 2)$ symmetry and Neumann (no flux) condition.[‡]



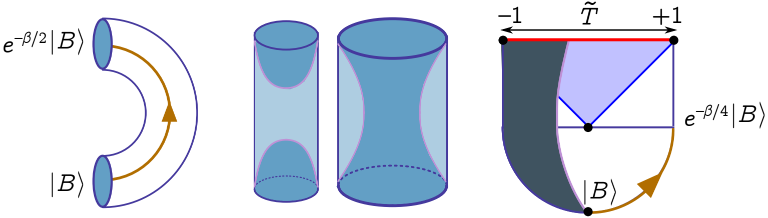
- Model different boundary conditions by varying brane tension T. "No flux" tells us T is proportional to extrinsic curvature K:

$$K = \frac{d}{(1-d)} 8\pi G T \equiv -d\tilde{T}.$$

[†] Cardy 1984. [‡] Karch and Randall 2001; Takayanagi 2011.

Boundary state black holes

- We can place boundaries in time. Boundary condition becomes boundary (B) state $|B\rangle^\dagger$ and $SO(d-1,2) \to SO(d,1)$.
- For thermal state, cut donut in half to get finite cylinder with two boundary components. Two brane topologies consistent with symmetry: disconnected (high β) and connected (low β).

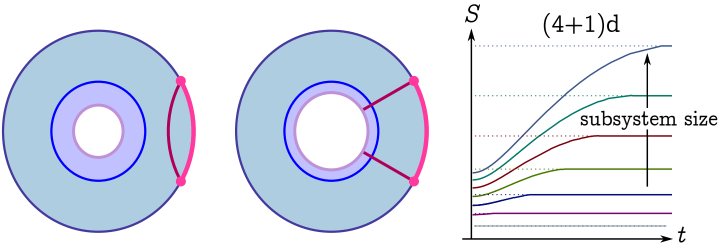


- Connected phase is a wormhole with a spherical brane. It hits singularity in finite proper time. $\tilde{T}$ determines where it hits.
- Path integral recipe: pick boundary state $|B\rangle$ and evolve by $e^{-\beta/4}$.
- Get state-dependent amount of interior, as advertised!

† Cardy 1989. ‡ Fujita, Takayanagi and Tonni 2011; Almheiri, Mousatov, and Shyani 2018.

Hawking radiation and subsystem entanglement

- Can we learn about how state $|B\rangle$ is encoded in radiation? Hard to do explicitly, but subsystem entanglement is good surrogate.
- Use RT! Minimal surface stays outside horizon or ends on brane.[†]



- $\tilde{T}$ is encoded in dependence of entanglement on time and subsystem size. Observer could check these in Hawking radiation!
- We found surfaces analytically in 3D, numerically in $\geq$ 4D.
- Excellent match with subsystem entanglement in SYK model[‡] (dual of AdS_2) for thermally evolved B states[§].

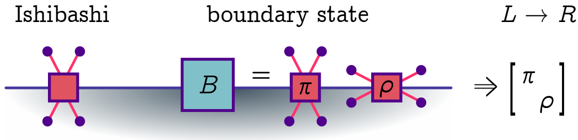
[†] Harlow 2016. [‡] Sachdev and Ye 1993; Kitaev 2015. [§] Maldacena and Kourkoulou 2017.

Boundary state simulation?

- Can we use entanglement resources of B state to simulate interior?
- For field h, no flux condition gives Ishibashi states[†] $|h\rangle\rangle$ entangling left- (L) and right-moving (R) modes n on either side of boundary.
- Take linear combinations, impose symmetry to get B states $|B_\alpha\rangle$:

$$|h\rangle\rangle = \sum_n |hn\rangle_L |hn\rangle_R, \quad |B_\alpha\rangle = \sum_h U^h_\alpha |h\rangle\rangle = \sum_{hn} U^h_\alpha |hn\rangle_L |hn\rangle_R.$$

- Neat fact: tension is proportional to overlap $\langle\langle \text{vac}|B_\alpha\rangle$.[‡]

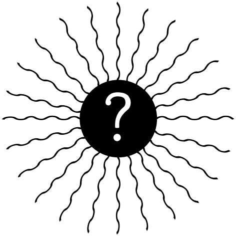


- Flipping $|hn\rangle_R \to \langle hn|_R$ shows that B_α is "twisted" map from L to R. Equivalently, B states "twisted" version of EPR state on LR.
- Todo: construct bulk interior operators from B state entanglement.

[†] Ishibashi 1989. [‡] Harvey, Kachru, Moore, and Silverstein 1999.

Loose threads

- Can we do cosmology on the brane?[†] Perhaps in charged BH!
- Enlarge AdS/BCFT dictionary so we can understand microscopic dynamics of brane, including backreaction and brane-localised fields.
- Compare apples with apples: check AdS_3 predictions against entanglement entropy in CFT_2.[‡]
- Finally, see if B states tell us anything about more general BHs.[§]



Thanks for listening! Questions?

[†] Randall and Sundrum 1999; Karch and Randall 2000; Hebecker and March-Russell 2001. [‡] Cardy and Calabrese 2009. [§] Almheiri 2018.