Einstein's Singular Embarrassment

David Wakeham

Albert Einstein is best known for his theory of *special relativity*: a Dalí-esque realm where clocks melt, long poles fit into short barns, and mass equals energy. But his best was yet to come. In 1915, Einstein formulated *general relativity*, succeeding Newton's theory of gravity in the same way special relativity succeeds Newtonian mechanics. Elegantly connecting physics and geometry, it is, by many accounts, the most beautiful scientific theory ever devised. In John Wheeler's maxim,

Spacetime tells matter how to move.
Matter tells spacetime how to curve.

According to this theory, a large mass like the sun curves spacetime like a bowling ball deforms a rubber sheet. Planets execute their circular orbits not because a mysterious force pulls them through space, but because they roll like marbles along a curved geometry (Fig. 1).

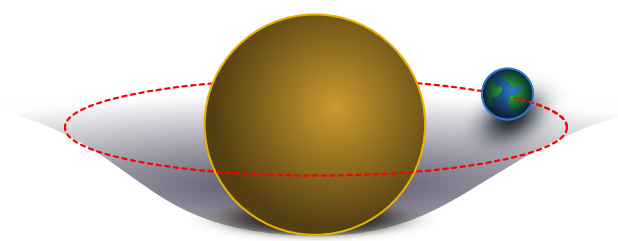

Figure 1. The sun bends spacetime like a bowling ball on a rubber sheet.

A few weeks after Einstein presented his results, something on the Russian frontlines. Karl Schwarzschild, a German astrophysicist and artillery officer, solved Einstein's equations exactly to determine how a sphere of matter dimples the spacetime around it. The answer was sensible, giving the circular orbits we see in the sky. But for a very dense sphere, two surprises emerged: first, there is a *light-trapping region* around the sphere; second, if the sphere is contracted to a point, spacetime in the vicinity curves so dramatically that general relativity itself breaks down (Fig. 2). This is called a *singularity*, mathematical parlance for "disaster". Schwarzschild died of pemphigus a year later, but bequeathed to the world the enigma of black holes.

Einstein admired the simplicity of Schwarzschild's solution, but was uncomfortable with its implication that *time itself* ended inside the black hole. He would later argue that the singularity was an artefact, a bug coming from the assumption that the geometry was perfectly spherical. It would disappear, he concluded, "in the real world", where symmetries are harder to come by and matter would strenuously object to collapse. The maestro had spoken, and for many physicists, that settled the matter.

In 1964, almost 50 years after Schwarzschild's discovery and 10 years after Einstein's death, a professor of mathematics crossed a busy London street. An impression flashed into his mind—a skin of light, pulsing outwards and *getting smaller*—but he lost it again in the hustle and

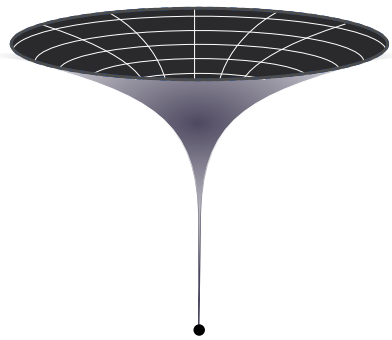

Figure 2. A Schwarzschild black hole and its singularity.

bustle of midday traffic. The course of 20th century physics might be very different if he had not, by luck, recalled it the following day. The professor was Roger Penrose, and his mental image would inspire the most important technical development since Schwarzschild: the *Penrose singularity theorem*, a mathematical result showing that singularities are not mere bugs arising from symmetry, but inevitable features of a curvy universe. It is perhaps a mercy Einstein did not live to see the most elegant theory ever devised, the apotheosis of his genius, robustly predict its own demise.

Penrose shared the 2020 Nobel Prize in Physics with Andrea Ghez and Reinhard Genzel, astronomers who discovered black holes in our very own Milky Way. Penrose laid the groundwork for these observations, since if Einstein's prejudice still held sway, no one would search for them! Let's unpack Penrose's proof. The starting point is the idea that *gravity is attractive*, focusing light like a magnifying glass. If light rays are already converging, gravity cannot pry them apart and they will collide (Fig. 3). Penrose's flash of inspiration, as he crossed the street, was the notion of a *closed trapped surface*: a complete surface like a sphere, with an inside and an outside, such that light heading *outwards* from the sphere converges. We call this a "light trap". It may sound impossible for outgoing light rays to get closer, but this is exactly what happens in the light-trapping region of a black hole![1]

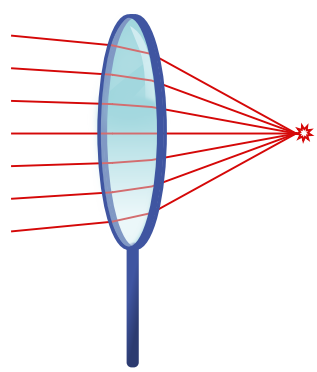

Figure 3. If rays are already converging, gravity focuses them like a magnifying glass.

Remember that gravity is a magnifying glass. This means rays going in either direction from the light trap are focused and collide with each other. That's why we call it a light trap!

[1]To be clear, the event horizon of a black hole is almost but not quite a light trap! The light that heads outwards *hovers* on the horizon, rather than getting closer. But any surface inside is a light trap!

We can picture this scenario in two dimensions (Fig. 4), with space on the horizontal axis and time in the vertical direction. The light trap is represented by two points—the equivalent of a sphere in one dimension—with an inside (dotted black line) and an outside (solid black line). The dark red rays head outwards and lighter red head inwards, converging at the dark and light red stars.

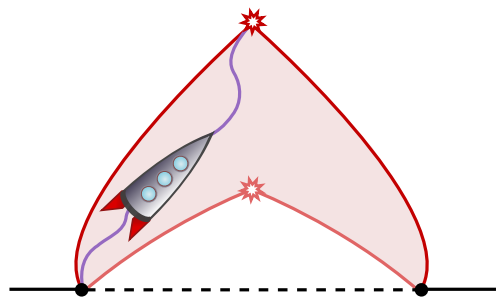

Figure 4. Any light ray leaving the light trap collides with another ray.

Penrose showed carefully that the *future* of the light trap—the points in spacetime that can be reached by a spaceship starting at one of the black dots—is the pink blob in between. It is finite, and cannot be extended, so time ends at the top of the blob. To see why, imagine that a light ray is allowed to pass through a collision, represented by a star in Fig. 4. There will be a point on the extended ray which can be reached by a "zig-zag" light ray path which swaps between rays. The kink in this zig-zag path can be smoothed out, like pulling on a crimped thread, to form a slower path that a rocket ship could use to arrive at the same point. And because rocket ships can always be a little slower, or a little faster, this path can be varied to reach a small ball of points in the vicinity. This zig-zag argument, depicted in Fig. 5, shows that a rocket can visit any point on an extended light ray.

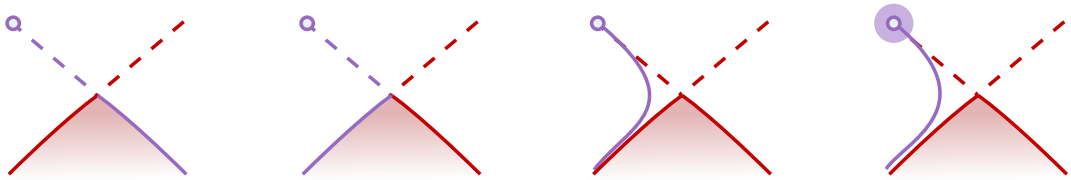

Figure 5. A point on an extended ray can also be reached by a zig-zag path which switches rays at the collision. This zig-zag can be smoothed into a shorter path a rocket could take, and varied slightly to visit nearby points.

Penrose realized that this was impossible. Since nothing can travel faster than light, and rockets are strictly slower, the *edge* of the future consists of points reached only by light ray. Let's expand on why. For a point on the edge, any small ball we draw around it will partly fall outside the future (Fig. 6). Otherwise, it's not on the edge, but in the middle! If a rocket could reach such a point, we could vary the rocket's speed a little to reach a ball of nearby points, completely contained in the future; since this cannot be true for a point on the edge, only light rays, whose speed cannot be varied, are allowed to get there. The zig-zag argument told us that any point on an extended light ray can be reached by rocket. It follows that *no extended ray is on the edge of the future*. But since the future must have an edge (rockets can't go

everywhere!), and there are no other light rays to provide it, Penrose concluded that the future must end in a singularity at the top of the blob.

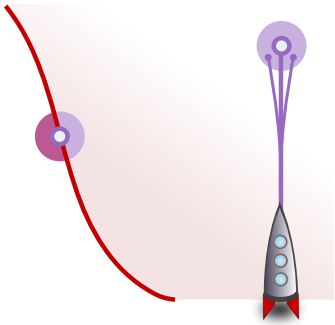

FIGURE 6. On the left, a point on the edge of the future. Any ball around it lies partly outside the accessible region. On the right, a point in the interior of the future, surrounded by a ball of rocket-accessible points.

In contrast to Schwarzschild's perfect sphere, a light trapping surface can be puckered, deformed, and filled with matter; the singularity will form regardless. Evidently, the bugs are not in in Schwarzchild's mathematics, but Einstein's beautiful theory. Whether singularities are truly bugs in Nature, or are somehow patched up by new physics, remains to be seen. But as Penrose's fellow laureates discovered, black holes—Schwarzschild's disquieting legacy and Einstein's embarrassment—can be found in our galactic backyard.