Inflation in scalar-tensor gravity

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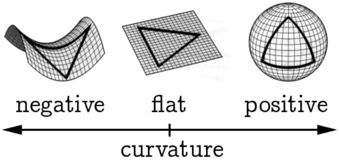
Nov 09, 2016

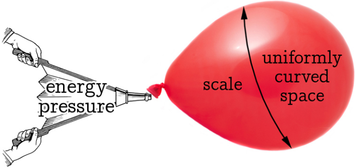
ARC Centre of Excellence for Particle Physics at the Terascale

Australian Government
Australian Research Council

Big Bang cosmology

- Cosmological Principle: at large scales, universe is the same everywhere and in all directions.
- Three possibilities: uniform sphere, saddle or plane.
- Geometries scale with time. Track with scale factor $a(t)$.

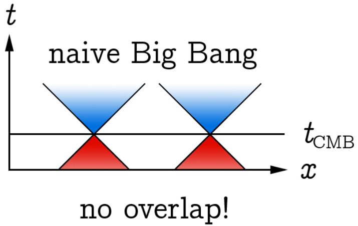




- Scaling is driven by energy and pressure of large-scale stuff.

Troubles with tuning

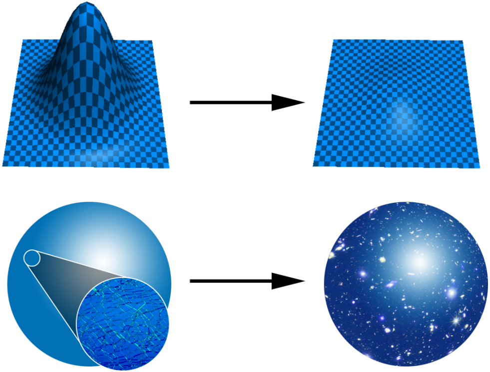
- Flatness problem: indications we live in flat space.
- This requires fine tuning of amount of matter in universe.
- Horizon problem: the CMB is highly uniform.



- But CMB splits into $\sim 10^4$ patches which never interacted! Necessitates fine tuning of patch conditions.

Inflation I

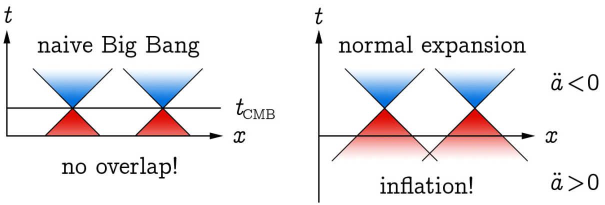
- Inflation is burst of accelerating expansion after Big Bang.
- Rapid growth smooths out lumps, creating flat, uniform space.



- Tiny quantum lumps later become large-scale structure.

Inflation II

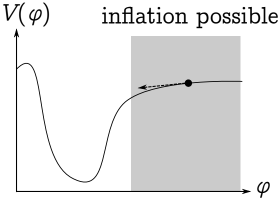
- To solve horizon problem, just grow light cones:



- Light cone width $\sim \int 1/\dot{a}$. Usually, $1/\dot{a}$ increases with time, and fixed by observation at time CMB forms.
- To grow cones, add big contribution early on. Simple way: increase $1/\dot{a}$ as we go back in time.
- Since $\partial_{-t}\dot{a} = \ddot{a}/a^2$, this is just accelerating expansion!

The scalar roller coaster

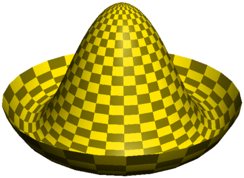
- A scalar ϕ with $V > T$ can cause inflation. Called inflaton.
- If $V \gg T$, have slow-roll inflation governed by shape of V.
- Slow roll on flat patches, ends on inclines.



- A successful model must
  - increase scale factor by $\sim e^{60}$ (60 e-folds);
  - predict right power law distribution for quantum lumps.

Higgs inflation

- Standard model has a unique scalar: Higgs boson.
- The Higgs gives mass to leptons, quarks, and weak force carriers $W^{\pm}$ and Z via spontaneous symmetry breaking.



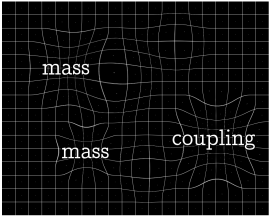
- Sadly, can't drive inflation! Mexican hat isn't flat enough.
- Bezrukov and Shaposhnikov [2008] coupled Higgs directly to gravity to get viable inflation. Called scalar-tensor gravity.

Scalar-tensor gravity

- Simplest approach to scalar-tensor gravity is action principle.
- Modify curvature term in Lagrangian for general relativity:

$$\mathcal{L}_{\mathsf{GR}} \supset \kappa \cdot \sqrt{-g}\mathcal{R} \quad \longrightarrow \quad \mathcal{L}_{\mathsf{ST}} \supset f(\phi) \cdot \sqrt{-g}\mathcal{R}.$$

- Can think of f as warping space on top of curvature.



- If $\phi \to v$, recover general relativity for $f(v) = \kappa$.

Rescaling and frames

- By locally rescaling metric, get new field χ governed by GR:

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega(f)^2 g_{\mu\nu}, \qquad \phi \to \chi.$$

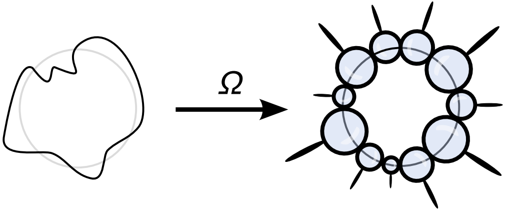


Figure: Rescaling metric = variable magnification.

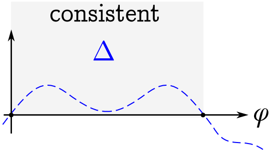
- Choosing f cleverly and rescaling allows χ to viably slow-roll.
- Metric $g_{\mu\nu}$ is called Jordan frame, $\tilde{g}_{\mu\nu}$ Einstein frame.
- Dictionary connects them but not physically equivalent.

de Sitter inflation

- If Jordan $\neq$ Einstein, motivates inflation in Jordan frame. Consider arbitrary coupling f and potential V.
- Simplest case is de Sitter evolution. Exponential scale factor:

$$a(t) = e^{Ht}, \quad H \equiv \dot{a}/a = \text{const}, \quad (\text{note } \ddot{a} > 0).$$

- Scalar-tensor equations imply quadratic for $\dot{\phi}$. Obtain consistent patches (real $\dot{\phi}$) for discriminant $\Delta \geq 0$.



- This constrains coupling f and potential V.

Perturbing de Sitter

- Earlier, inflation defined as accelerating expansion. Equivalently, slow change in Hubble parameter $H \equiv \dot{a}/a$.
- Suggests perturbation expansion around constant H:

$$
\begin{aligned}
H(t) &= H_0 + \lambda H_1(t) + \cdots \\
\phi(t) &= \phi_0(t) + \lambda \phi_1(t) + \cdots .
\end{aligned}
$$

  In other words, H_0, ϕ_0 terms represent de Sitter inflation.
- Plug expansion into scalar-tensor equations and get homogeneous matrix DE for first-order terms:

$$
\frac{d}{dt}\begin{bmatrix} \phi_1 \\ H_1 \end{bmatrix} = M(t)\begin{bmatrix} \phi_1 \\ H_1 \end{bmatrix}.
$$

- Still have consistency constraint at zeroth order.

Jordan frame slow-roll

- That is classical story, but what about quantum lumps? Requires QFT on curved, scalar-tensor background...
- For normal slow-roll, need parameters ϵ_1 and ϵ_2 to be small and approximately constant to get power-law behaviour:

$$\epsilon_1 = \frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}} .$$

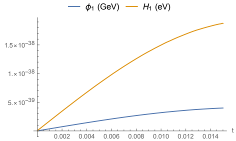
- In Jordan frame, two more slow-roll parameters ϵ_3 and ϵ_4:

$$\epsilon_3 = \frac{\dot{f}}{2Hf}, \quad \epsilon_4 = \frac{\dot{\alpha}}{2H\alpha}, \qquad \text{where } \alpha \equiv 1 + 3(f')^2/f .$$

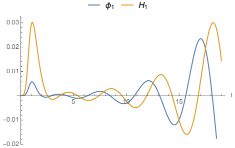
- Distribution determined by linear combination of ϵ_i.

Toy models

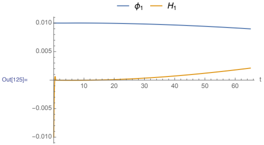
- Explored scalar-tensor inflation for various toy models:
  - Higgs inflation;
  - linearly coupled massive scalar;
  - coupling driven inflation.



(a) Higgs inflation.



(b) Massive scalar.



(c) Gaussian coupling.

- Massive scalar can oscillate resonantly, like reheating.
- By tuning parameters, usually possible to get 60 *e*-folds.
- "Bottom up" approach does not give Jordan frame slow-roll.

Top down approach

- Try imposing slow-roll constraints on ϵ_3, ϵ_4 directly:

$$\frac{\dot{f}}{2Hf} = \epsilon_3^{(0)} + \cdots , \quad \frac{\dot{\alpha}}{2H\alpha} = \epsilon_4^{(0)} + \cdots .$$

  $\epsilon_3^{(0)}$ and $\epsilon_4^{(0)}$ are small constants, "$\cdots$" higher order terms.

- Leads to explicit form $f(\phi_0(t))$, coupling as function of time.
- Use this to calculate ϵ_2. Find ϵ_2 is also small and roughly constant, with some suggestive cancellations.
- In principle, can work backwards to determine $f(\phi)$ and $V(\phi)$, and hence ϵ_1 and first-order dynamics. Work in progress!

Summary

- Inflation solves tuning problems with Big Bang model.
- Scalar-tensor gravity gives wider range of inflaton candidates.
- If Jordan $\neq$ Einstein, motivates inflation in Jordan frame. Perturbing around de Sitter inflation is one approach.
- Toy models had interesting dynamics, but realistic inflation requires top down approach.
- Ultimate goal: richer interplay of cosmology/particle physics.

Supplementary slides

FRW metric and Friedmann equations

- Cosmological principle implies FRW metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right].$$

Geometry given by $k \in \{0, \pm 1\}$: $k = 1$ is sphere, $k = -1$ is saddle, and $k = 0$ is flat space.

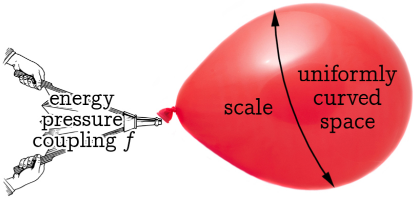
- Scale factor $a(t)$ controls expansion of space.
- Responds to energy and pressure via Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P).$$

Scalar-tensor cosmology

- Scalar-tensor action gives equivalent of Einstein's equation.
- Plug in FRW to get equivalent of Friedmann equations:

$$H^2 + H\dot{f} \propto \rho, \qquad H\dot{f} - 2\dot{H} - \ddot{f} \propto (\rho + P).$$



- Density and pressure related to ϕ, V by

$$\rho = T + V = \frac{1}{2}\dot{\phi}^2 + V, \quad P = \frac{1}{2}\dot{\phi}^2 - V.$$

Inflationary spectra

- Lumps are momentum modes for inhomogeneous corrections.
- Two types: scalar and tensor. Spectra obey power-law:

$$\Delta_s^2 = A_s k^{n_s - 1}, \qquad \Delta_t^2 = A_t k^{n_t}.$$

- Key observables are $n_s \simeq 0.97$ and $r \equiv A_t/A_s \lesssim 0.11$.

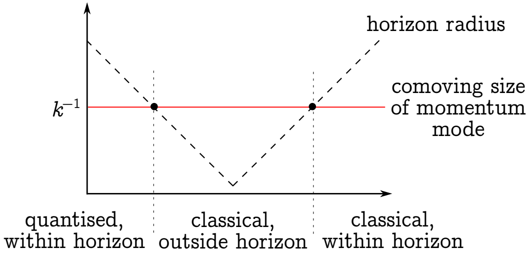


Figure: Life cycle of a momentum mode.

Jordan frame spectra

- Rough idea for spectral calculations:
  1. add inhomogeneous correction, $\phi(t,\mathbf{x}) = \bar{\phi}(t) + \delta\phi(t,\mathbf{x})$;
  2. expand inflaton $S[\phi,\dot{\phi}]$ action to second order, $S^{(2)}$;
  3. set corresponding variation to zero, $\delta S^{(2)} = 0$;
  4. quantise Mukhanov-Sasaki equation in Fourier space;
  5. evolve to horizon crossing.
- Calculation in Jordan frame uses rescaling dictionary:

$$\mathrm{J} \to \mathrm{E} \to \mathrm{J}.$$

- Jordan frame spectra:

$$\begin{aligned} n_s &\simeq 1 + 2(2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) \\ n_t &\simeq 2(\epsilon_1 - \epsilon_3) \\ r &\simeq 4\pi\left(-\frac{\epsilon_1}{16\pi f G} + 3\epsilon_3^2\right). \end{aligned}$$

Higgs inflation in Einstein frame

- Bezrukov and Shaposhnikov [2008] set $\phi =$ Higgs and

$$f(\phi) = \frac{1}{2}\left[M_{\rm pl}^2 + \xi(\phi^2 - v^2)\right].$$

- In Einstein frame, making $\xi \sim 50000\sqrt{\lambda}$ large flattens potential enough to satisfy constraints!
- Get $n_s \simeq 0.97$ and $r \simeq 0.003$, consistent with observation.

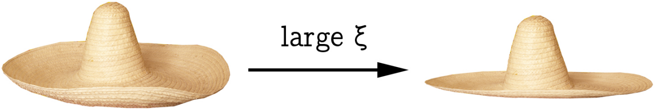


Figure: Flattening outer brim of Mexican hat.