UBC Virtual Physics Circle

A Hacker's Guide to Brownian motion

David Wakeham

June 25, 2020

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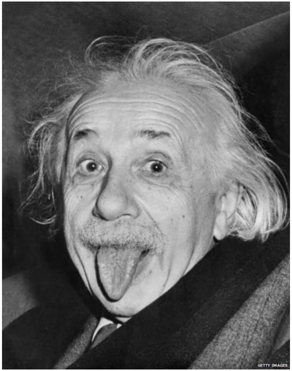

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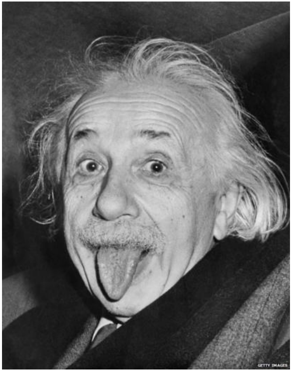

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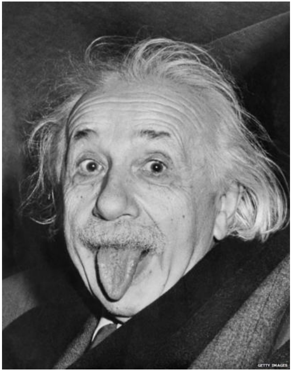

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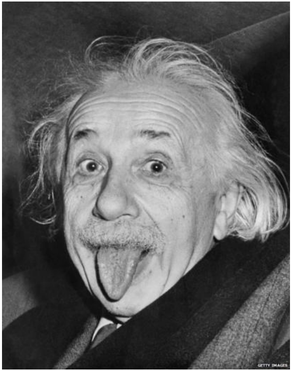

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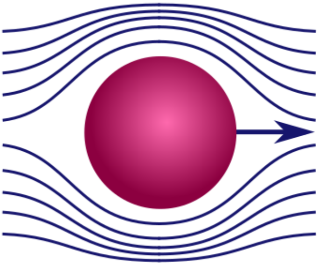

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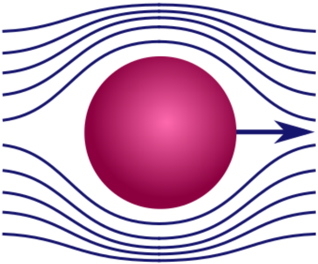

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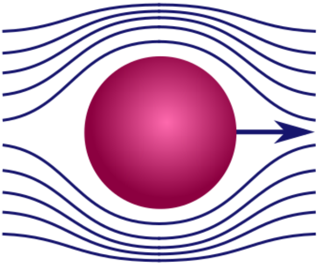

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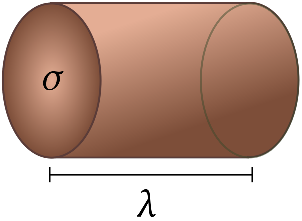

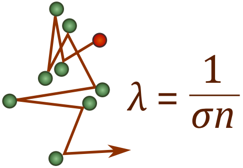

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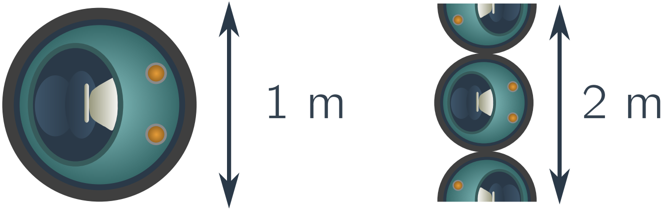

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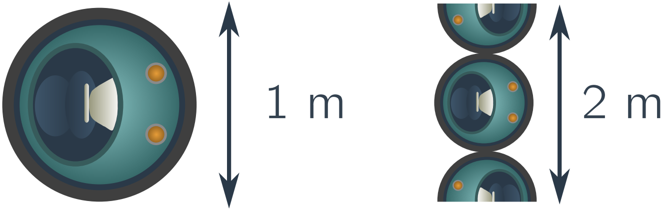

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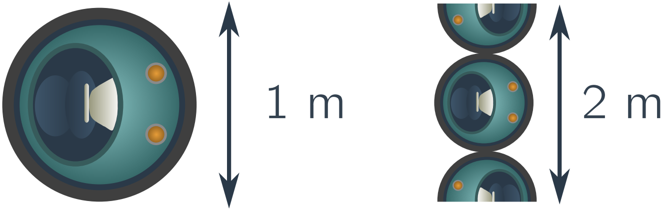

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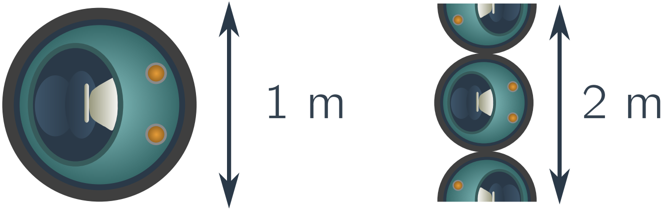

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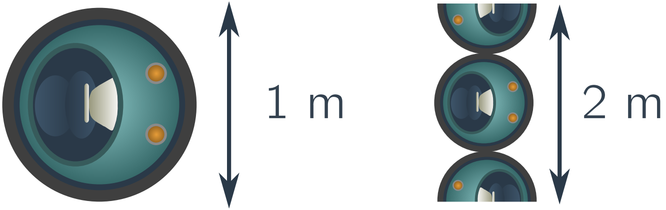

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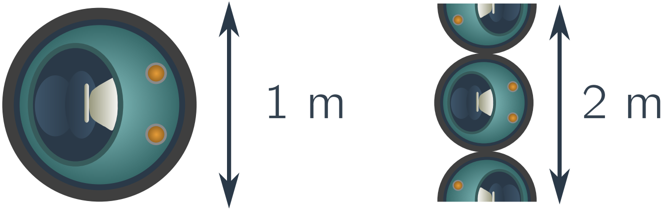

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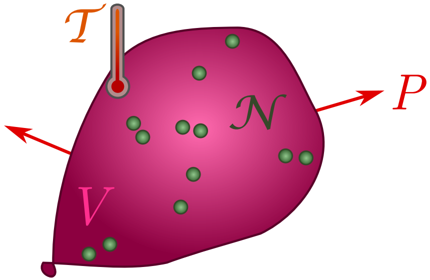

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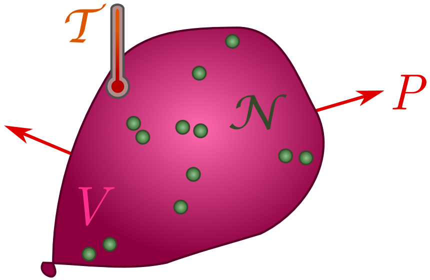

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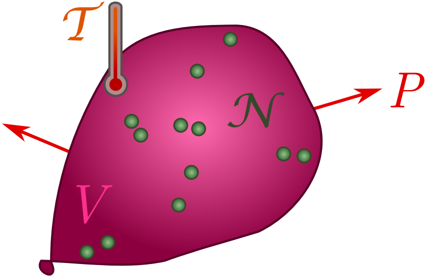

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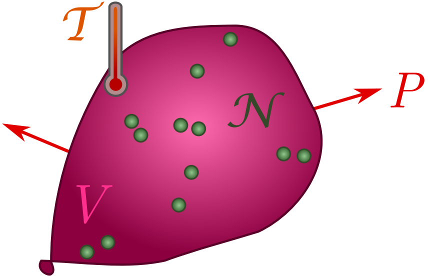

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(c) From part (a), $[PV] = (\Xi L^3)(\mathcal{E}/L^3) = \Xi\mathcal{E}$.

Exercise 2: ideal gas (solution)

(a) Usually, V has units $[V] = L^3$. But the volume of a gas grows with particle number, so $[V] = \Xi L^3$.

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Lucretius, Brown, Einstein

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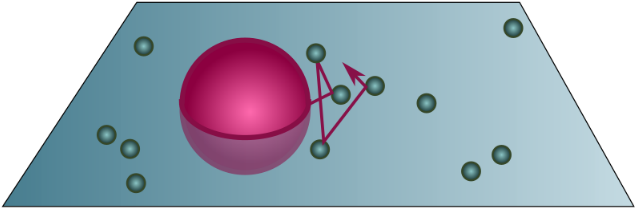

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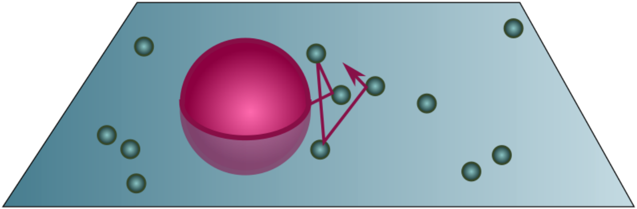

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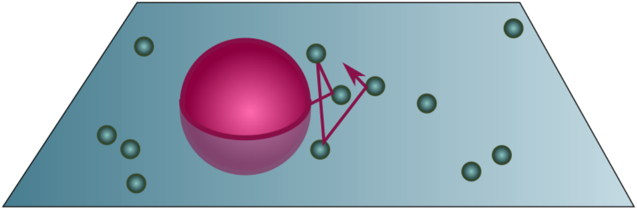

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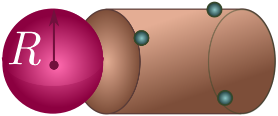

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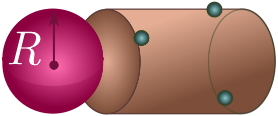

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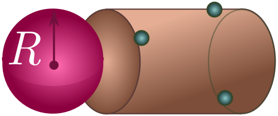

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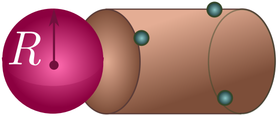

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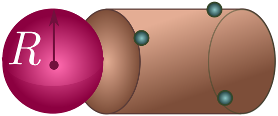

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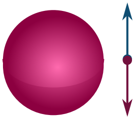

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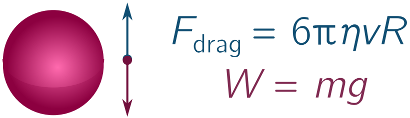

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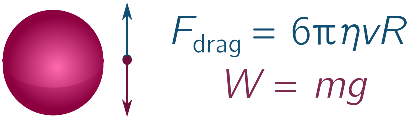

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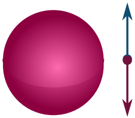

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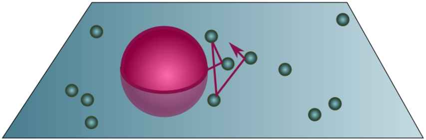

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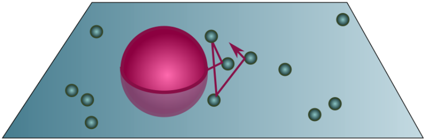

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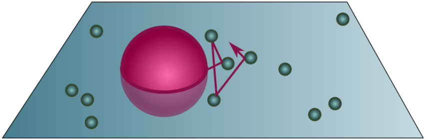

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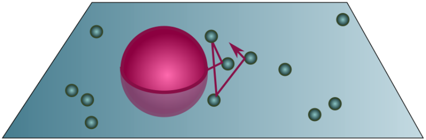

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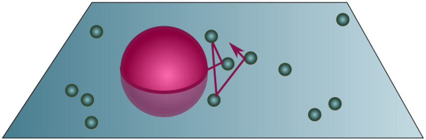

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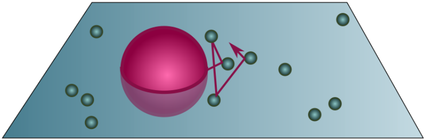

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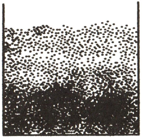

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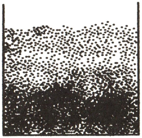

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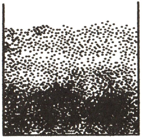

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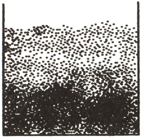

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This was one of no fewer than five methods Einstein proposed for measuring Avogadro's constant!

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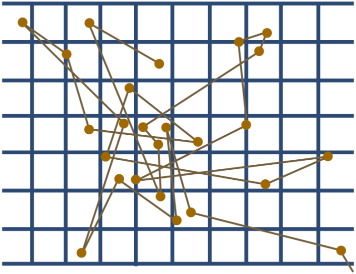

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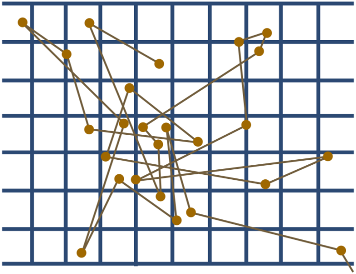

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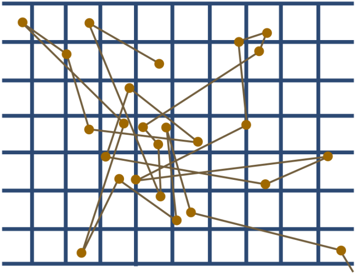

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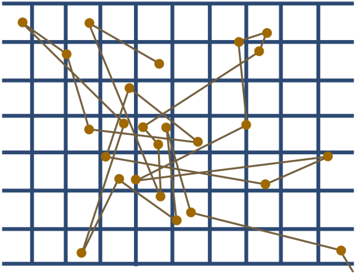

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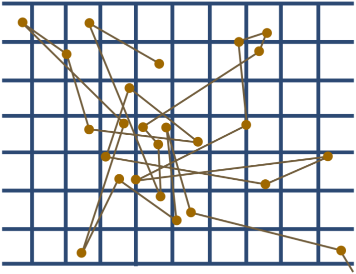

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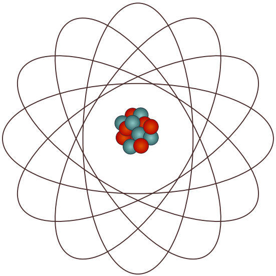

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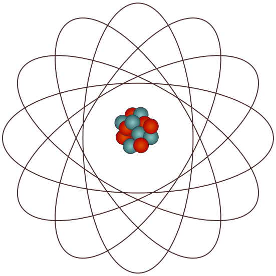

(e) What is the approximate mass of a nucleon?

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