UBC Physics Olympics: Quzzics ideas

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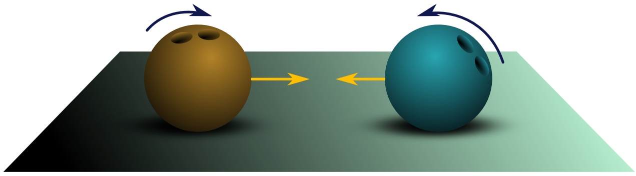
Preliminary questions

1 Collision course

Problem. Two bowling balls collide on a flat plane. They are subject to no external forces, and you can ignore the effect of rolling friction. The balls possess various physical properties:



I. Linear momentum

II. Angular momentum

III. Kinetic energy

IV. Total energy

After the collision, which quantities are guaranteed to be conserved?

A. None of the above.

B. I and III.

C. I, II, III and IV.

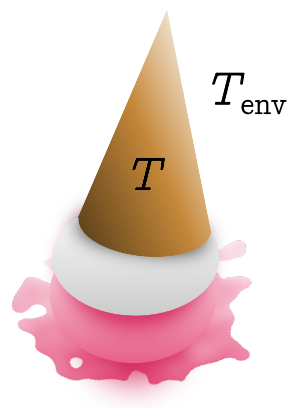
D. I, II, and III.

E. I, II, and IV.

Solution. *E. I, II and IV.* Since no external forces act on the system, total linear momentum is conserved. No external forces also means no external torque, so angular momentum is conserved. Finally, if the collision is inelastic, kinetic energy will *not* be conserved. But *total energy* must always be conserved! The kinetic energy lost in an inelastic collision is converted into some combination of heat, sound, and deformation of the colliding objects. This means E is correct.

2 Ice creams and X rays

Problem. A physics student buys ice cream on a hot day and goes to the beach. The ice cream begin to melt, so they briefly place it in a cool box. What effects does this have?



I. It *decreases* the rate the ice cream melts.

II. It *increases* the rate the ice cream melts.

III. It lowers the temperature.

IV. The ice cream emits fewer X rays.

Choose all options that apply:

A. III only.

B. I, II, III and IV.

C. I and III.

D. II and III.

E. II, III and IV.

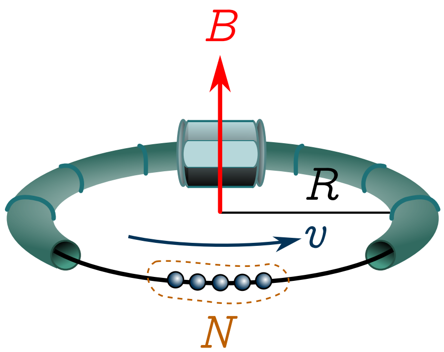
Solution. *E. II, III and IV.* Clearly, placing the ice cream in a cool box lowers the temperature. Since it increases the temperature difference between the ice cream and the hot air around it, it will *increase* the rate of heat transfer, according to Newton's law of cooling:

$$\frac{\Delta T}{\Delta t} = -k(T - T_{\text{env}}).$$

This means II and III hold, while I does not. Finally, hot objects emit radiation across the whole spectrum, with hotter objects emitting more energetic rays. Thus, a cooler ice cream will emit fewer high-energy rays like X rays and IV holds. The correct answer is therefore E.

3 Accelerating protons at the LHC

Problem. The LHC probes the subatomic structure of matter by smashing together protons close to the speed of light. The protons are accelerated with the help of a powerful magnetic field B, perpendicular to the plane of the accelerator. Suppose the beam line is a circle of radius R, and a packet of N protons, mass m_{p}, is accelerated to speed v. How much work does the magnetic field do?



A. $W = \frac{1}{2}Nm_{\mathrm{p}}v^2$.

B. $W = 2\pi RNBev$.

C. $W = \hbar eBN/m_{\mathrm{p}}$.

D. We need more information about the other forces on the protons.

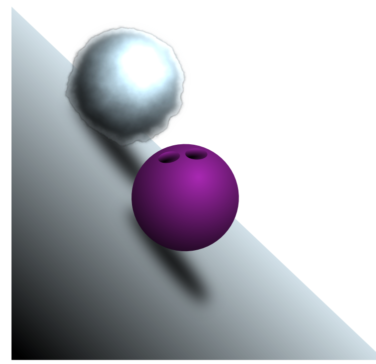
E. No work.

Solution. *E. No work.* Since the magnetic force is centripetal, it is always perpendicular to the direction of motion and hence the *magnetic field does no work*. Something else must be doing the accelerating! D is on the right track, but E is the best answer. The remaining options are red herrings:

- A is the total change in kinetic energy;
- B is the magnitude of the magnetic force times the circumference of the accelerator;
- C is the cyclotron frequency multiplied by $\hbar$, the energy of a photon at that frequency.

4 Snowballs and slow balls

Problem. A snowball and a bowling ball are released from the top of a snowy hill at the same time. As the snowball travels, it accumulates snow and its mass increases, while the mass of the bowling ball remains constant. Assuming both have constant density and roll without slipping, which arrives at the bottom of the hill first?



A. The snowball arrives first.

B. The bowling ball arrives first.

C. They arrive at the same time.

D. Whichever has greater density.

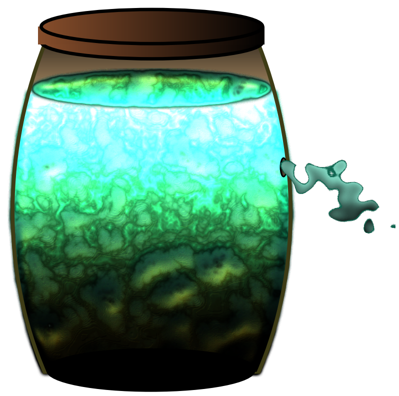
E. Not enough information to answer.

Solution. *B. The bowling ball arrives first.* We can think of the accumulation of snow as a collision at each time step. In time Δt, a snowball of mass m picks up some velocity $v = a\Delta t$, where a depends on the slope. It then *collides* completely inelastically with a patch of snow of mass Δm, and conservation of momentum implies that

$$p = mv = (m + \Delta m)v' \quad \Longrightarrow \quad v' = \frac{m}{m + \Delta m} < v.$$

Thus, the snowball is slowed down by "collisions" with the snow, and arrives later than the bowling ball.

5 High-pressure pickle



Problem. Picklemakers Phil and Lil are brewing up their next delicious batch of pickles. Cucumbers sit in a barrel of brine, density ρ, with air pressure P above the liquid. Usually $P = 0$ (vacuum), but Phil and Lil periodically open the barrel to add ingredients to their pickle solution. During this process, $P = P_{\text{atm}}$. After they finish, they restore the vacuum. The brine density ρ also increases.

Unbeknownst to Phil and Lil, there is a hole in the barrel below the waterline, slowly leaking at a rate f_0. When the barrel is opened to add ingredients, it leaks at rate f_1. Once it is closed and the vacuum restored, it leaks at rate f_2. What is the relation between leaks?

A. $f_0 < f_2 < f_1$

B. $f_0 < f_1 < f_2$

C. $f_1 < f_0 < f_2$

D. There is not enough information to answer.

E. The leak rates are all the same.

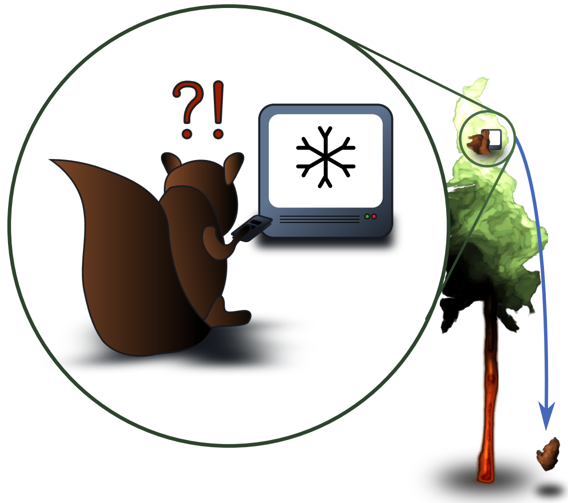
Solution. *A.* $f_0 < f_2 < f_1$. We have a column of brine of mass per unit area ρh. There is a weight force $\rho g h$ pulling it out of the hole, and pressure difference $\Delta P = P_{\text{atm}} - P$ pushing it back up. This leads to a downwards force and hence acceleration on the column

$$F = \rho g h - \Delta P \quad \Longrightarrow \quad a = gh - \frac{\Delta P}{\rho}.$$

Initially, $P = 0$ so $a_0 = gh - P_{\text{atm}}/\rho$. When the lid is removed, $P = P_{\text{atm}}$ so that $a_1 = gh > a_0$. Finally, the vacuum is reestablished and density increased, so $a_2 = gh - P_{\text{atm}}/\rho' < a_1$. Since $\rho' > \rho$, it also follows that $a_2 > a_0$. It's plausible that the ordering of flow rates should be the same as the ordering of acceleration on the column, so we get answer B.

6 Pinecone projectile

Problem. In Stanley Park, a squirrel lives at the top of a huge Douglas fir tree. When the weather channel reports more snow, the squirrel angrily throws the remote control, detaching a pinecone from the tree and sending it towards the ground. Assuming the tree is very tall, and the pinecone is released from rest, where does it land relative to the base of the tree?



A. It lands directly at the base of the tree.

B. North of the base.

C. South of the base.

D. East of the base.

E. West of the base.

Solution. *D. East of the base.* If the Douglas fir has height h, the pinecone has some angular momentum

$$L = mv_{\text{tree}}(R+h) = m\omega(R+h)^2,$$

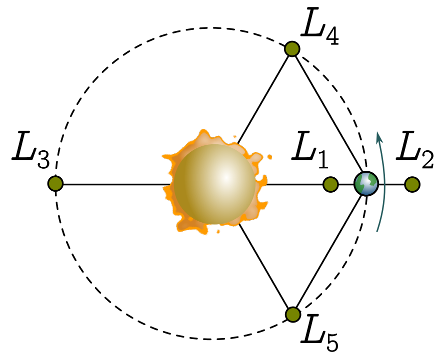
where ω is the angular velocity of the earth at the latitude of Stanley Park and $r = R + h$ is the radial distance from the centre of the earth. Since gravity is a central force, the angular momentum is conserved when the pinecone falls. When it reaches the surface of the earth at radius R, it will be moving at a velocity

$$v = \frac{L}{mR} = \omega\frac{(R+h)^2}{R} > \omega R = v_{\text{ground}}.$$

Since it is moving eastward relative to the ground, it lands to the east of the base.

7 Space junk

Problem. In the sun-earth system, there are five *Lagrange points*, where gravitational and centrifugul forces balance so that objects can orbit in a fixed position with respect to the rotating sun-earth system. They lie in the orbital plane and are pictured below. Space junk, cosmic dust, and other debris can get trapped at these points. At which points does most space junk accumulate?



A. L_1, L_2, L_3

B. L_4, L_5

C. L_1

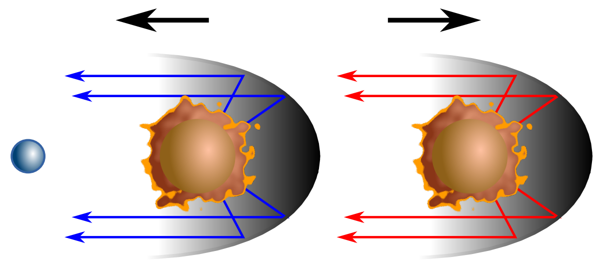
D. It accumulates at all points equally.

E. Junk doesn't accumulate; all orbits are unstable.

Solution. *B. L_4, L_5.* As E hints at, this is really a question about *stability*. The Lagrange point L_1 occurs because the gravitational attraction of the earth and sun balance out. Perturb a little towards either body and its gravity will dominate and pull the object in, so it is not stable. This means we can eliminate A and C. The question then becomes whether stable points exist; if they do, the answer must be B.

Some students may know this off the top of their heads, but it's possible to give a qualitative argument as follows. Suppose we push a small body at L_4 away from the sun. Its gravitational attraction to the sun decreases, but its speed decreases due to approximate conservation of angular momentum $L = mvr$. This reduces the angular velocity ω so it begins to fall towards the earth. But the gravity well of the earth causes it to speed up again, and move back towards the original point! In this way, a small perturbation leads to a stable "wobble" around L_4. A similar story holds for perturbations towards the sun or out the orbital plane, and for L_5. Thus, we expect both points to accumulate clumps of merrily wobbling space junk.

8 Solar ray gun



Problem. An advanced alien race surrounds their dying sun with a mirror, focusing the light and effectively turning it into a giant ray gun. They strap some rockets onto the ray gun, also powered by the sun, so that it can be moved around. The aliens are running out of water, and decide to melt a nearby ice planet to replenish their supplies.

The sun has a finite lifetime, so the aliens need to think about the best way to use its energy. One alien argues that, for best effect, they should move the ray gun towards the ice world so that the rays are blue-shifted and more energetic. Another argues that they should move away, so that they will be sending a longer wave train. A third argues that the energy delivered will be the same in either case. Which method is best?

A. Moving towards the planet.

B. Moving away from the planet.

C. Both yield the same result.

D. It depends on the wavelength of light.

E. It depends on the efficiency of the rockets.

Solution. *E. It depends on the efficiency of the rockets.* The energy from the sun is converted into three types of energy: (a) radiation delivered to the ice planet; (b) motion of the sun, due to the rockets and *recoil* from firing photons; and (c) any byproducts, like heat and light, from using inefficient rockets. If the rockets are perfectly efficient, we want to maximise item (a), so we should employ the rockets to ensure the sun is stationary (no kinetic energy) after the last photon is fired. This will correspond to blueshifting of photons while the sun moves towards the ice planet.

If the rockets are not completely efficient, we also have (c) to worry about it. In this case, we want to minimise the solar energy converted to categories (b) and (c), but this will depend in a complicated way on the efficiency of the rockets and the type of radiation. The best answer is therefore E.