Modular inclusions and wormholes

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Abstract

A summary of "Comments on black hole interiors and modular inclusions" [1] for the strings group meeting at the University of British Columbia. A criminally brief overview of algebraic quantum field theory and traversable wormholes is included.

Contents

1 AQFT background

1.1 Reeh-Schlieder

We'll start with a brief overview of some aspects of *algebraic quantum field theory* (AQFT), including the *Tomita-Takesaki theorems*. The basic objects of study in AQFT are *(von Neumann) algebras*, which are bounded operators on Hilbert space closed under various operations: you can add them, you can multiply them, you can take daggers, you can take limits. Finally, they contain the identity operator. For the moment, we will consider Type III von Neumann algebras which do not possess a well-defined trace, and return to finite-dimensional Type I algebras later.[1]

Given an open region of spacetime U, there is an algebra $\mathcal{A}$ of bounded operators on U. By evolving forward and backward in time, the operators naturally have support on the *domain of the dependence* $\hat{U} = D[U]$, which we'll draw as a diamond (Fig. 1). This is called a *local*

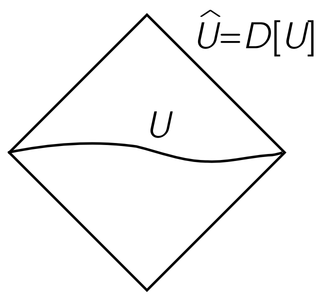

Figure 1: The domain of dependence of an open set in spacetime, U.

algebra. The set of all bounded operators which commute with everything in $\mathcal{A}$ is called the *commutant*, and is denoted by $\mathcal{A}'$. Schematically,

$$[\mathcal{A}, \mathcal{A}'] = 0. \tag{1}$$

It's usually true that the commutant of a local algebra is just the local algebra associated to U', the *causal complement* of U, i.e. the largest set spacelike separated from U. The relation $\mathcal{A}'_U = \mathcal{A}_{U'}$ is called *Haag duality* [2].

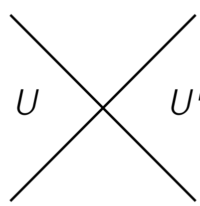

Figure 2: (Haag-dual notions: causal complements and commutant algebras.

[1]We can classify Type III algebras using the spectrum of the modular operator Δ_Ψ for a cyclic, separating vector $|\Psi\rangle$ (see below for definitions). It is believe that in quantum field theories, for any such Ψ, the spectrum of Δ_Ψ (including accumulation points) is $\mathbb{R}^+$ [2]. Equivalently, the spectrum of the modular Hamiltonian is $\mathbb{R}$. This is called a Type III_1 algebra.

Local algebras have some surprising and deep properties. The first is the *Reeh-Schlieder theorem*. Roughly speaking, this says that, given some local algebra $\mathcal{A}$, we can create *any state* in the Hilbert space by acting on the vacuum with operators in the algebra. By applying operators here, we can create the moon instantaneously! More precisely, the image of the vacuum state $|\Omega\rangle$ is dense in Hilbert space:

$$\overline{\mathcal{A}|\Omega\rangle} = \mathcal{H}, \tag{2}$$

and we say that the vacuum is *cyclic* with respect to $\mathcal{A}$. This result appears to violate causality and so on; the operators which create the moon must somehow be unphysical. Indeed, we will show in §2.1 that locality-violating operators are non-unitary.

The moral of Reeh-Schlieder is not that we can violate locality, but rather, that unitarity and locality are closely related. A second moral is that the vacuum is highly entangled, with *short distance divergences* preventing me from drawing a line between the regions and splitting the Hilbert into factors on either side [2]. Mathematically speaking, one can exploit a "chain" of locally entangled Bell pairs to act with operators *here*, and do stuff over *there*. Reeh-Schlieder is a fancy, field-theoretic iteration of the old EPR paradox of spooky action at a distance.

Applying Reeh-Schlieder to $\mathcal{A}'$, we learn that the vacuum is also cyclic with respect to the commutant. This implies that the only operator in $\mathcal{A}$ that annihilates the vacuum is the $\mathbf{0}$ operator itself. To see why, suppose that $\mathcal{O} \in \mathcal{A}$ annihilates the vacuum. If $\mathcal{O}' \in \mathcal{A}'$ is any operator in the commutant, then

$$0 = \mathcal{O}|\Omega\rangle = \mathcal{O}'\mathcal{O}|\Omega\rangle = \mathcal{O}\mathcal{O}'|\Omega\rangle. \tag{3}$$

But since the vacuum is cyclic for $\mathcal{A}'$, as we vary $\mathcal{O}'$ we span a dense subset of Hilbert space. It follows from the closure properties of $\mathcal{A}$ that $\mathcal{O}$ annihilates the whole Hilbert space! This is only possible if $\mathcal{O} = \mathbf{0}$, the null operator. We say that the vacuum is *separating* with respect to $\mathcal{A}$ because only the $\mathbf{0}$ operator annihilates it; it's never in the kernel of a non-trivial operator.

1.2 Tomita-Takesaki theory

Cyclic and separating vectors are dense in Hilbert space, since (for instance) it's easy to see that $A|\Omega\rangle$ is cyclic, separating for invertible $A \in \mathcal{A}$. Since most states exhibit a short-distance divergence in entanglement, it is *universal* and associated to the algebra rather than the state. But the state serves as a "witness", and we can use it to probe the algebraic structure of entanglement.

Tomita-Takesaki theory is a tool which makes this precise, and allows us to (in principle) classify Type III algebras. Given a cyclic, separating state $|\Omega\rangle$ for an algebra $\mathcal{A}$, there is a antilinear operator S which maps the action of an operator to the action of its dagger:

$$S : \mathcal{O}|\Omega\rangle \mapsto \mathcal{O}^\dagger|\Omega\rangle. \tag{4}$$

We can perform a polar decomposition of S, splitting it into an antiunitary operator J, and a positive operator $\Delta^{1/2}$:

$$S = J\Delta^{1/2}. \tag{5}$$

Prosaically, this is just a rotation (with a flip) followed by a scaling. The operators J and Δ are called *modular operators*, and they satisfy

$$J^2 = I, \quad JS^z J = S^{-z}. \tag{6}$$

The two *fundamental theorems* of Tomita-Takesaki theory are as follows:

1. Conjugation by J maps an algebra to its commutant,

$$J\mathcal{A}J = \mathcal{A}'. \tag{7}$$

2. Algebras are closed under conjugation by Δ^{is} for real s, also called *modular flow*:

$$\Delta^{is}\mathcal{A}\Delta^{-is} = \mathcal{A}, \quad \forall s \in \mathbb{R}. \tag{8}$$

We can think of the modular flow as evolution in modular time, with a corresponding *modular Hamiltonian*:

$$H = -\log \Delta. \tag{9}$$

Another example is the *thermofield double* model of entangled AdS black holes [3]. The cyclic separating vector is just the thermofield double state, which can be described as a thermally entangled state on two CFTs:

$$|\mathrm{TFD}\rangle = \frac{1}{Z[\beta]^{1/2}} \sum_E e^{-\beta E/2} |E_L\rangle |E_R\rangle = \rho_\beta^{1/2} |\mathrm{EPR}\rangle, \tag{10}$$

where $|\mathrm{EPR}\rangle$ is the (non-normalisable) maximally entangled state on the two CFTs. It follows that the modular operator is the thermal density matrix:

$$\Delta = \frac{1}{Z[\beta]} e^{-\beta H} = \rho_\beta.$$

2 Modular inclusions and mirrors

2.1 Localised state and mirror operators

Conjugation by J maps an algebra to its commutant. But what about *localised states* rather than operators? To create a localised state, we act on the vacuum $|\Omega\rangle$ (or any cyclic, separating vector) with a unitary operator $\mathcal{O} \in \mathcal{A}$:

$$|\psi\rangle = \mathcal{O}|\Omega\rangle. \tag{11}$$

An observer in U' is confined to probing the state with operators $\mathcal{O}' \in \mathcal{A}'$. These operators have the same expectation value in $|\psi\rangle$ as the vacuum state, since the operators commute:

$$\langle \mathcal{O}' \rangle_\psi = \langle \Omega | \mathcal{O}^\dagger \mathcal{O}' \mathcal{O} | \Omega \rangle = \langle \Omega | \mathcal{O}' \mathcal{O}^\dagger \mathcal{O} | \Omega \rangle = \langle \mathcal{O}' \rangle_\Omega. \tag{12}$$

It follows that a state localised to U looks like the vacuum to an observer in U'. Incidentally, this proves our earlier claim that Reeh-Schlieder connects unitarity and locality: if $\mathcal{O}$ is unitary, the state it creates really is localised to U.

We can map a localised state on U to a localised state on U' using the modular Hamiltonian, provided we evolve in *Euclidean (modular) time*. Formally, we write

$$|\psi'\rangle = \Delta^{1/2}|\psi\rangle = \Delta^{1/2}\mathcal{O}|\Omega\rangle. \tag{13}$$

An intuition for the factor of $\Delta^{1/2}$ comes from the Rindler wedge: the Wick-rotated geometry has a $U(1)$ symmetry where boosts become rotations, so $\Delta^{1/2}$ simply rotates our state by π.

Let's check this works. We need a couple of properties from earlier, and the fact the vacuum is invariant under CPT:

$$J^2 = I, \quad S = J\Delta^{1/2}, \quad J|\Omega\rangle = |\Omega\rangle. \tag{14}$$

It follows that

$$|\psi'\rangle = J^2\Delta^{1/2}\mathcal{O}|\Omega\rangle = JS\mathcal{O}|\Omega\rangle = J\mathcal{O}^\dagger|\Omega\rangle = J\mathcal{O}^\dagger J|\Omega\rangle = \mathcal{O}'|\Omega\rangle, \tag{15}$$

where $\mathcal{O}' = J\mathcal{O}^\dagger J \in \mathcal{A}'$ is some operator in U' by (7).[2] The operator $\mathcal{O}' \in \mathcal{A}'$ is totally different from $\Delta^{1/2}\mathcal{O}$, which is not in either algebra, but they act on states in the same way. In other words, with access to the modular Hamiltonian, we also obtain a *state-dependent* map for mapping between operators in U and U'. We call $\Delta^{1/2}\mathcal{O}$ a *mirror operator*.

An observer in the right exterior, who has access to the full density matrix, can use the state-dependent mirror operators to describe physics in the left exterior. The *Papadodimas-Raju proposal*, in brief, is that a typical state in a one-sided black hole has enough thermal entanglement to mimic the construction of TFD mirror operators [4].

2.2 Modular inclusions

We can fine-grain the notion of localised state by considering the *modular inclusion* of algebras. The gist of modular inclusions is simple: they are subalgebras preserved under modular flow in one direction.[3] Formally, suppose that $\mathcal{N}$ is a subalgebra of $\mathcal{M}$, with a common cyclic separating vector $|\Omega\rangle$, and modular operators $\Delta_\mathcal{N}, \Delta_\mathcal{M}$. The inclusion $\mathcal{N} \subseteq \mathcal{M}$ is a *half-sided* $\pm$ *modular inclusion* if flowing backwards (forwards) in the modular time of $\mathcal{M}$ preserves $\mathcal{N}$:

$$\Delta_\mathcal{M}^{\mp is}\mathcal{N}\Delta_\mathcal{M}^{\pm is} \subseteq \mathcal{N}, \quad \forall s \geq 0. \tag{16}$$

We restrict to half-sided inclusions since "two-sided" inclusion implies $\mathcal{M} = \mathcal{N}$.

Given a modular inclusion, the difference in modular Hamiltonians

$$p = \frac{1}{2\pi}\left(H_\mathcal{M} - H_\mathcal{N}\right) \tag{17}$$

[2] Of course, a much simpler way to map to an excitation in U' is to apply J, $|\psi'_J\rangle = J\mathcal{O}|\Omega\rangle = J\mathcal{O}J|\Omega\rangle = \mathcal{O}'_J|\Omega\rangle$. The problem is that this conjugates $\mathcal{O}$, rather than $\mathcal{O}^\dagger$, and after applying the T in CPT, this gives an operator with the correct sense in time.

[3] This is analogous to a normal subgroups, or any other stable subobject.

is a positive operator.[4] Exponentiating[5] p gives a one-parameter family

$$U^a = e^{iap}, \tag{18}$$

which controls the algebraic relations between $\mathcal{M}$ and $\mathcal{N}$. It satisfies the identities [1]

$$\mathcal{N} = U^{-1}\mathcal{M}U \tag{19}$$
$$\Delta_{\mathcal{N}}^{is} = U\Delta_{\mathcal{M}}^{is}U^{-1} \tag{20}$$
$$J_{\mathcal{M}}J_{\mathcal{N}} = U^2 \tag{21}$$
$$J_{\mathcal{M}}U^aJ_{\mathcal{M}} = J_{\mathcal{N}}U^aJ_{\mathcal{N}} = U^{-a}. \tag{22}$$

For the moment, we will be consider only the special case where the two wedges share a null boundary. This means that p acts in a manifestly geometric way [6], with U^{-a} translating in the null direction a times the translation needed to take $\mathcal{M}$ to $\mathcal{N}$. This is also called a *modular translation* (Fig. 3).

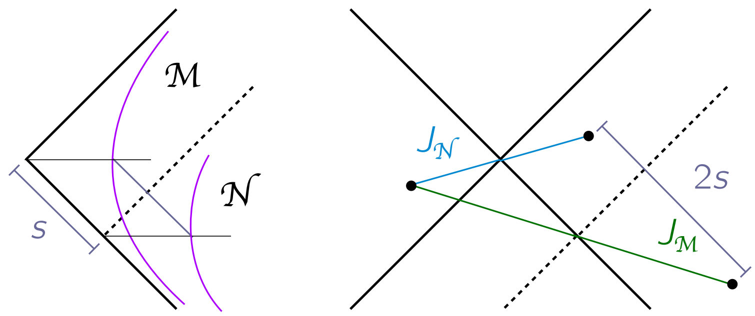

Figure 3: *Left.* Modular translating $\mathcal{M}$ to obtain the subalgebra $\mathcal{N}$. *Right.* Proof-by-picture of (22) for modular translations.

The geometric action of U allows us to "prove" (19–22) using pictures. The first two are obvious, since conjugation by U (or U^{-1}) just modular translates the algebra or modular flow operators. The J identities are more interesting. For instance, to prove (22), we CPT flip around the point of intersection using either $\mathcal{J}_{\mathcal{M}}$ or $\mathcal{J}_{\mathcal{N}}$, modular translate by U^a, then flip back to obtain evolution by U^{-a} (Fig. 3). You can use similar arguments to derive (21).

3 Traversable wormholes

Modular inclusions fine-grain the notion of localised state, and hence, the state-dependent construction of mirror operators in §2.1. We will use this to probe deformations of the TFD. We start by considering the *traversable wormhole* of Gao, Jafferis and Wall (GJW) [7].

[4] Loosely speaking, the modular spectrum of the subalgebra $\mathcal{N}$ is included in the spectrum of $\mathcal{M}$. For a simple proof, see [5].

[5] Since these are operators, we technically define exponentiation by

$$U^a = \lim_{n\to\infty}(\Delta_{\mathcal{M}}^{ia/2\pi n}\Delta_{\mathcal{N}}^{-ia/2\pi n})^n.$$

3.1 Background

We can perturb the TFD (10) by briefly coupling the CFTs, via a *double-trace deformation*:

$$\delta H(t) = \int dx^{d-1}\, h(t)\mathcal{O}_L(t,x)\mathcal{O}_R(-t,x).$$

Here, $\mathcal{O}_L$ and $\mathcal{O}_R$ are *relevant* operators on the left and right CFT, with scaling dimension $\Delta < d/2$. This requires the alternative boundary conditions of [8]. Relevant deformations affect the IR of the CFT and leave the UV alone. By the UV/IR correspondence, in the gravity dual the low-energy EFT is unaffected but the deep bulk will be deformed. The envelope function $h(t)$ is switched on briefly at $t = 0$, with

$$h(t) = h\left(\frac{2\pi}{\beta}\right)^{2-2\Delta}\mathbb{I}(t_0 < t < t_f), \tag{23}$$

for some small, dimensionless parameter $h > 0$, and $\mathbb{I}$ is an indicator function so that the coupling is only switched on from $t = t_0$ to $t = t_f$. The corresponding Kruskal variables for $V = 0$ are $t_0, t_f \to U_0, U_f$.

On the quantum side, coupling the CFTs allows us to send signals between them. This is an instance of a general phenomenon called *regenesis* [9]. We can also view the coupling as providing a *quantum channel* for sending information between the left and right CFT [10, 11]. In the bulk, this is dual to shooting in *negative energy shockwaves* from both boundaries. These shockwaves decrease the ADM mass and shrink the future horizon, effectively opening up the throat of the wormhole. More carefully, we demonstrate this by finding null geodesics (tangent k_μ, affine parameter λ) which violate the average null energy condition (ANEC):

$$\int T^{\mu\nu}\, k_\mu k_\nu\, d\lambda < 0. \tag{24}$$

These are light rays which pass through the wormhole.

We very briefly outline the calculation for the case of the BTZ black hole with AdS radius ℓ and horizon radius r_h, but refer the reader to [7] for details. The alternative boundary conditions modify the bulk 2-point function for ϕ, the dual scalar to $\mathcal{O}_{L,R}$. Going to Kruskal coordinates $x = (U, V)$, and setting $V = V' = 0$ so that we are evaluate along a horizon, the change is

$$\Delta G(U, U') = hC_0\int_{U_0}^{U}\frac{dU_1}{U_1}\int_1^{U/U_1}\frac{2\,dy}{\sqrt{y^2-1}}\left(\frac{U_1}{U - U_1 y}\right)^{\Delta}\left(\frac{1}{U'U_1 + y}\right)^{\Delta} + (U \leftrightarrow U'), \tag{25}$$

for a constant $C_0 \propto r_\mathrm{H}^{2-2\Delta}(2\pi/\beta)^{2-2\Delta}$. In turn, this can be used to find the expectation for the stress-energy via the method of *point-splitting*:

$$T_{\mu\nu}(x') = \lim_{x\to x'}\left[\partial_\mu\partial'_\nu G(x,x') - \frac{1}{2}g_{\mu\nu}\partial_\sigma\partial^{\sigma\prime}G(x,x') - \frac{1}{2}g_{\mu\nu}M^2 G(x,x')\right], \tag{26}$$

where $M^2 = \Delta(\Delta - 2)$ is the mass squared of the dual scalar field. Since $T_{\mu\nu} = 0$ in the unperturbed BTZ geometry, we can think of this as a 1-loop correction to the stress energy.

The UU component T_{UU} can be numerically evaluated. One finds that T_{UU} is *negative* when the coupling is switched on, positive when it is switched off, and decays to 0^+ at late times as

$$T_{UU} \sim \frac{4h\Delta^2 C_0}{U^{2\Delta+2}} \log U \log\left(\frac{U_f}{U_0}\right). \tag{27}$$

This negative decrement in T_{UU} is precisely the negative energy shockwave. To prove traversability, the ANEC integral (24) can be explicitly performed along the horizon:

$$\int_{U_0}^{\infty} dU\, T_{UU} = -\frac{h\Gamma(2\Delta+1)^2}{2^{4\Delta}(2\Delta+1)\Gamma(\Delta)^2\Gamma(\Delta+1)^2\ell} \frac{{}_2F_1(\frac{1}{2}+\Delta, \frac{1}{2}-\Delta; \frac{3}{2}+\Delta; (1+U_0^2)^{-1})}{(1+U_0)^{\Delta+1/2}}. \tag{28}$$

Plotting numerically, one finds this is negative for all $0 < \Delta < 1$, so the wormhole is traversable as claimed.

An important point is that (27) is telling us how the black hole thermalises at late times. Soon after U_f, when we switch the coupling off and (27) becomes valid, the polynomial denominator oustrips the logarithmic numerator and $T_{UU} \approx 0$. This means that the geometry to first order is just the usual Hartle-Hawking state, but with a smaller horizon. We will not write the expression for the change δE in the ADM mass, but it is indeed negative [7].

3.2 Naive modular inclusion

We call the algebras for left and right exterior $\mathcal{M}_L$ and $\mathcal{M}_R$. Due to the perturbation, these are no longer analogous to Rindler wedges, but we ignore this complication for the moment. The exterior algebras no longer commute due to the double-cone overlap or *centre* $\mathcal{M}_L \cap \mathcal{M}_R$. But we can modular translate to subalgebras $\mathcal{N}_{L,R} \subseteq \mathcal{M}_{L,R}$ which do commute:

$$\mathcal{N}_L' = \mathcal{N}_R. \tag{29}$$

These are the local algebras associated with local algebras in the Hartle-Hawking state prior to the double-trace perturbation.

We now focus on right-hand side of the Penrose diagram, and define $\mathcal{D}_R$ (Fig. 5) as the difference of small and large wedges:

$$\mathcal{D}_R = \mathcal{M}_R - \mathcal{N}_R. \tag{30}$$

This strip corresponds to the opened throat of the wormhole, when the perturbation is still thermalising. Again, we will ignore this and use the simpler approach of [1] for the moment.

Our results in §2 will allow us localise excitations in the “interior” strip $\mathcal{D}_R$ using operators in the “exterior" $\mathcal{N}_R$. Geometrically, the idea is straightforward:

1. Start with an operator $D \in \mathcal{D}_R$ and perform CPT with respect to $\mathcal{M}$. This gives an operator $D' \in \mathcal{M}_R'$.
2. Since $\mathcal{M}_R' \subseteq \mathcal{N}_R'$ is also a modular inclusion, we can map D' to an operator $N' \in \mathcal{N}_R'$.
3. Conjugate N' with respect to $\mathcal{N}$, yielding a local operator in the “exterior” $\mathcal{N}_R$.

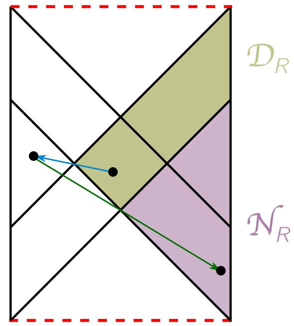

Figure 4: Constructing a state-dependent operator using local algebras in the traversable wormhole.

Algebraically, this is easy to implement:

$$D \to D' = J_{\mathcal{M}} D J_{\mathcal{M}} \to N' = U^{-1} D' U \to N = J_{\mathcal{N}} N' J_{\mathcal{N}}. \tag{31}$$

More explicitly, using (21) and (22), we have

$$N = J_{\mathcal{N}} U^{-1} J_{\mathcal{M}} D J_{\mathcal{M}} U J_{\mathcal{N}} = U^3 D U^{-3}. \tag{32}$$

It's easy to check that N and D have the same effect on the state, since the vacuum is invariant under U:

$$N|\Omega\rangle = U^3 D U^{-3}|\Omega\rangle = U^3 D|\Omega\rangle = SD^\dagger U^{-3}|\Omega\rangle = SD^\dagger|\Omega\rangle = D|\Omega\rangle. \tag{33}$$

One can double-check algebraically [1] that $N \in \mathcal{N}_R$.

3.3 Algebraically emergent spacetime?

I'll finish with Jefferson's qualitative remarks on emergent spacetime. Algebraically, negative-energy shockwaves "push" the exterior algebras together and create a centre. Similarly, a positive-energy *Shenker-Stanford* shockwave, or an anti-time ordered series of such shockwaves [12], "pulls" the algebras apart. The exterior algebras will commute, but no longer be commutant. This invites us to imagine a family of spacetimes obtained by perturbing the TFD with anti-time-ordered shockwaves of positive or negative energy. These spacetimes are not dynamically related, but there is presumably some natural embedding of "nearby" spacetimes in the sequence.[6]

There's two limits we can take. The first is to add enough negative-energy shockwaves to cause the black hole to evaporate completely. If this process has a holographic description, it should terminate at the Hawking-Page transition, where the canonically dominant geometry shifts from black hole to thermal AdS. This is defined on a single copy of AdS. Geometrically, one naively expects an infinitely tall Penrose diagram, which is consisten with thermal AdS; on the CFT side, two coupled CFTs could perhaps, in some circumstances, become a single CFT.

A second limit is to add lots of positive-energy shockwaves until we disentangle the CFTs altogether. with an infinitely long wormhole between the two CFTs. The claim is that you can

[6] For instance, the traversable wormhole can be viewed as a perturbation of the TFD.

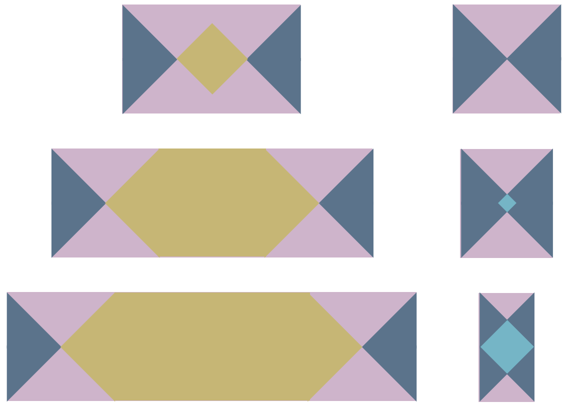

Figure 5: A family of spacetimes obtained by anti-time-ordered shocks. *Left.* Shenker-Stanford shocks. *Right.* GJW shocks.

interpolate between these two pictures by taking the unentangled CFTs and stitching them together with negative-energy shockwaves; this should be algebraically dual to a family of modular inclusions. This procedure seems unlikely to work from the boundary perspective, since entangling and coupling CFTs are different (though a coupling can generate entanglement). A nice proof-of-concept would be a calculation checking the picture is valid for a positive-energy shockwave followed by a negative-energy shockwave, and vice versa.

My general concern is that this picture relies on the validity of effective field theory in the bulk. Spamming your spacetime with shockwaves is a recipe for leaving a perturbatively well-defined code subspace, and entering a realm where we either cannot make calculations, or do not expect a holographic dual at all. Before the emergence of spacetime can be understood using modular inclusions, we need to know when modular inclusions are a valid description of spacetime!

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