

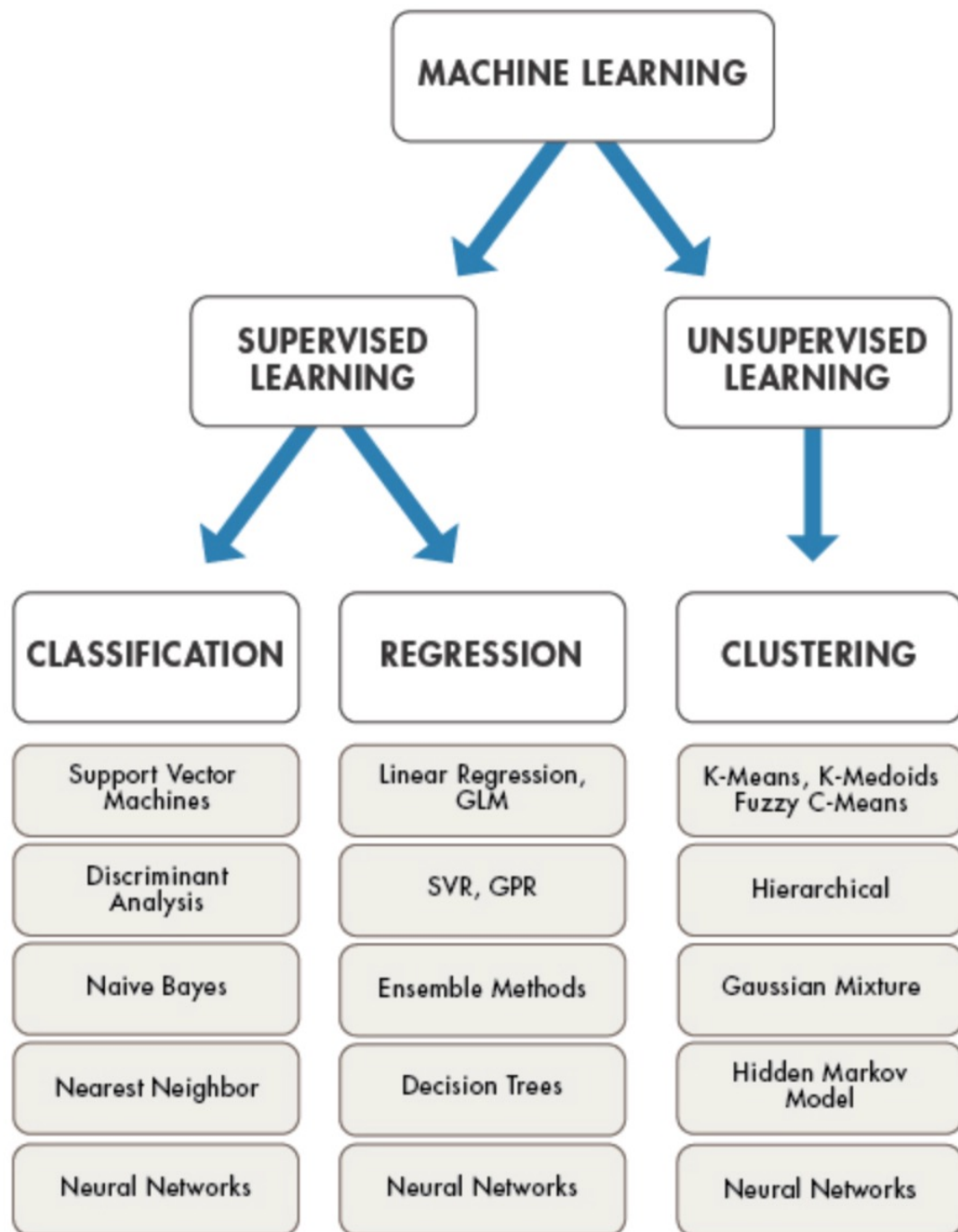
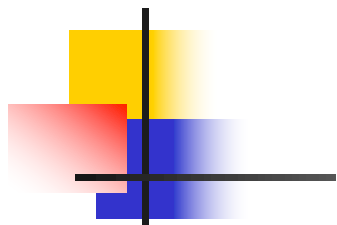
Clustering

- **Outline:**

1. Introduction

2. K-means clustering algorithm

3. Gaussian mixture model clustering algorithm



Clustering

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- 1. Introduction**

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What is clustering?

- Unsupervised learning
- Input: an unlabeled dataset
- Output: **groups** (**clusters**)
- Principle: dividing the examples into a number of **groups** (**clusters**) such that examples in the same group are more similar to other examples in the same group than those in other groups.
- **Goal:** to find distinct groups or “clusters” within a data set.

Clustering

- **Duration:** 2 hrs

- **Outline:**

1. Introduction

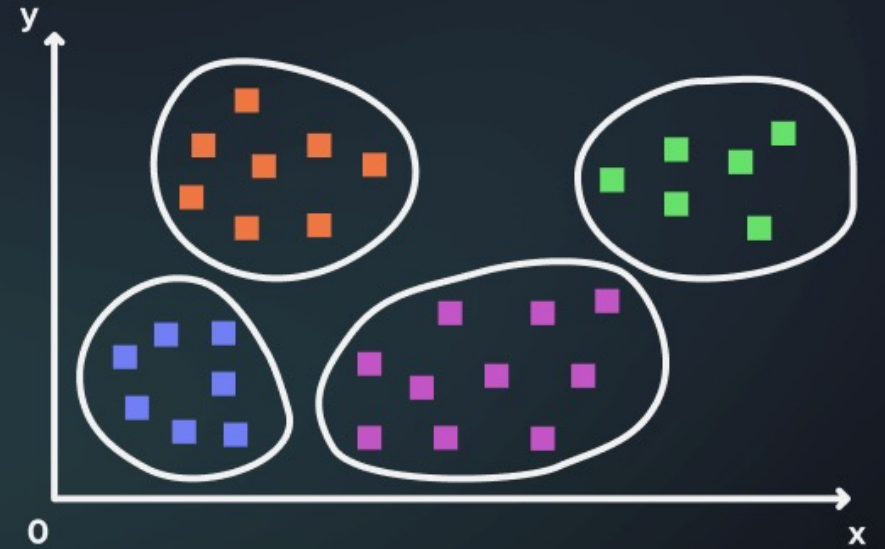
- 2. K-means clustering algorithm**

3. Gaussian mixture model clustering algorithm

Before K-Means



After K-Means



General

- K-means clustering is the most commonly used clustering algorithm.
- K-means clustering is a distance-based algorithm.
- K-means tries to group the closest points to form a cluster (K-means tries to minimize the variance of data points within a cluster).
- K-means is best used on small data sets because it iterates over *all* of the data points → it'll take more time to classify data points in the large data set.

K-means clustering implementation

- Step 1: initialization
 - Partition the data points into K clusters randomly. Find the centroids of each cluster
- Step 2: data clustering

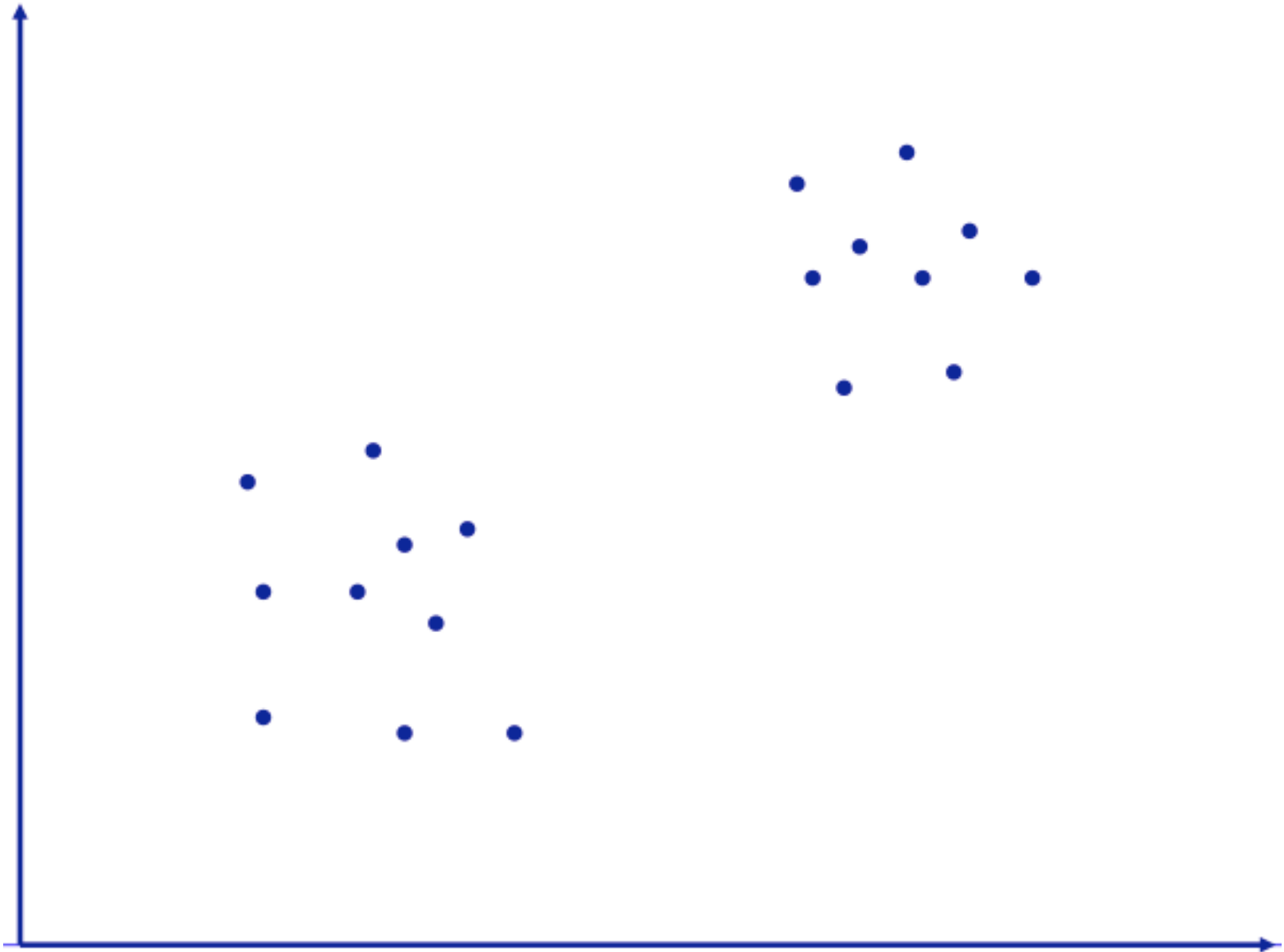
For each data point:

 - Calculate the distance from the data point to each cluster
 - Assign the data point to the closest cluster

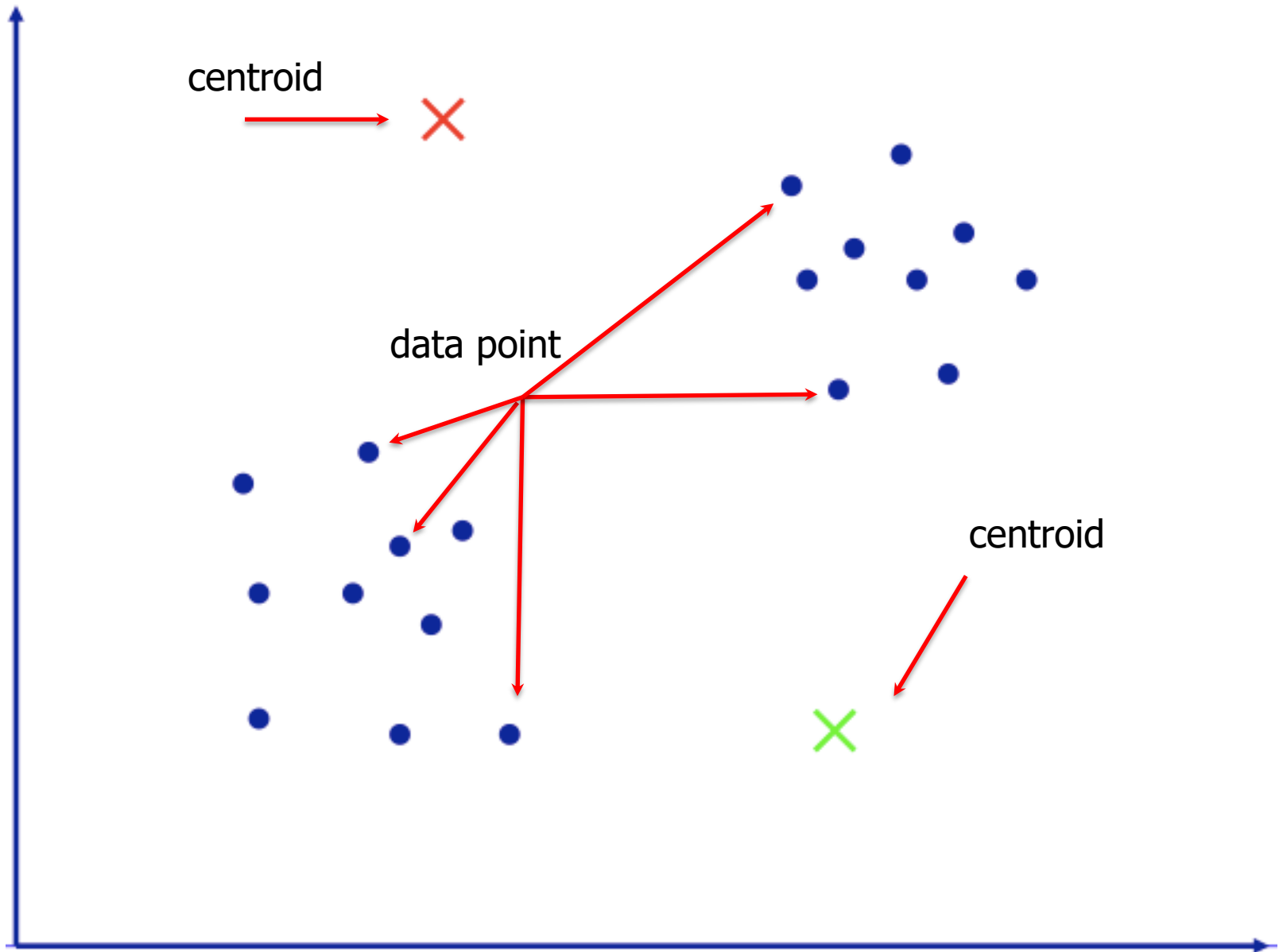
K-means clustering implementation

- Step 3: centroid determination
 - Re-compute the centroid of each cluster
- Step 4: iteration
 - Repeat step 2 and step 3 until terminated

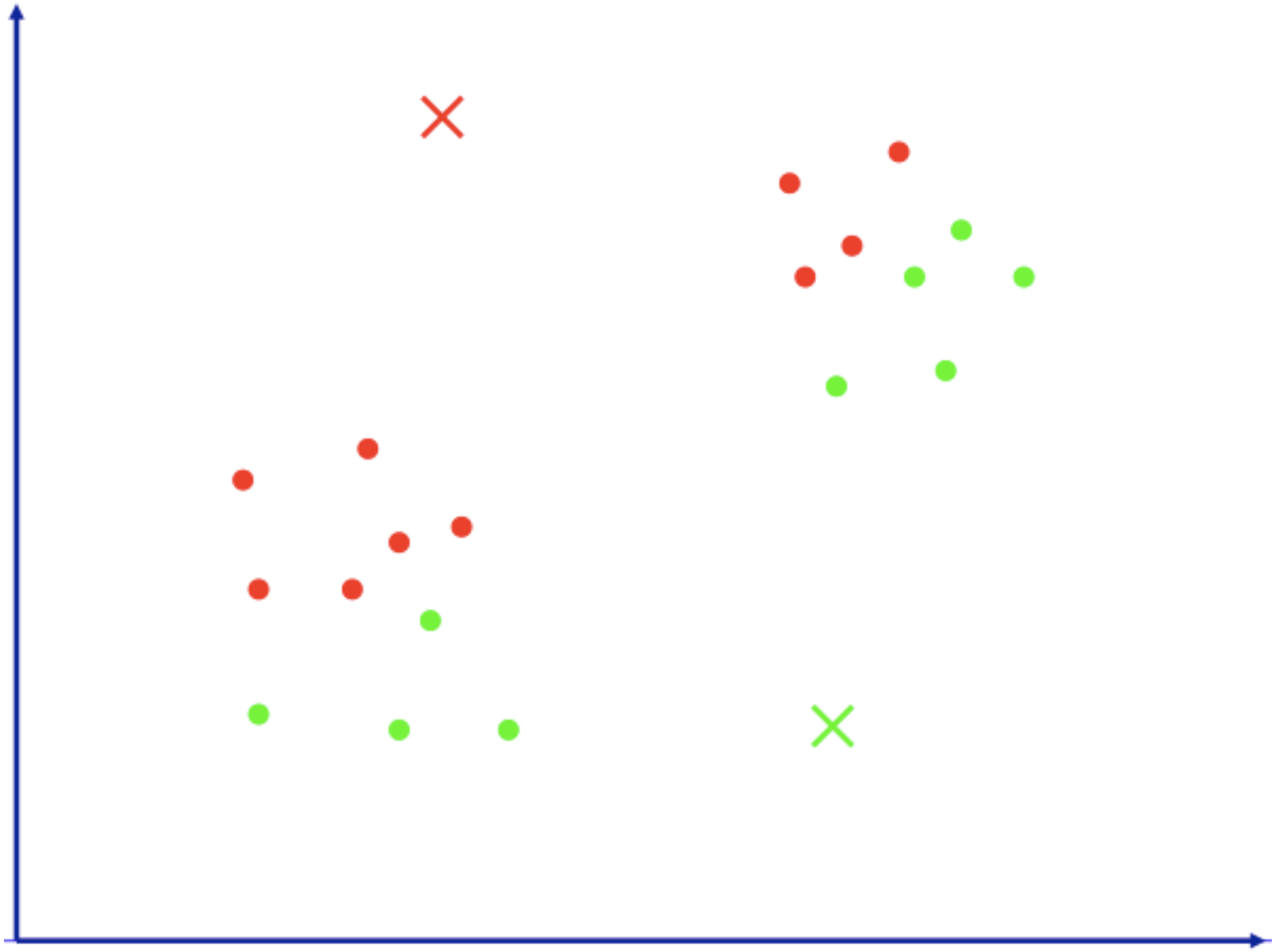
K-means clustering - illustration



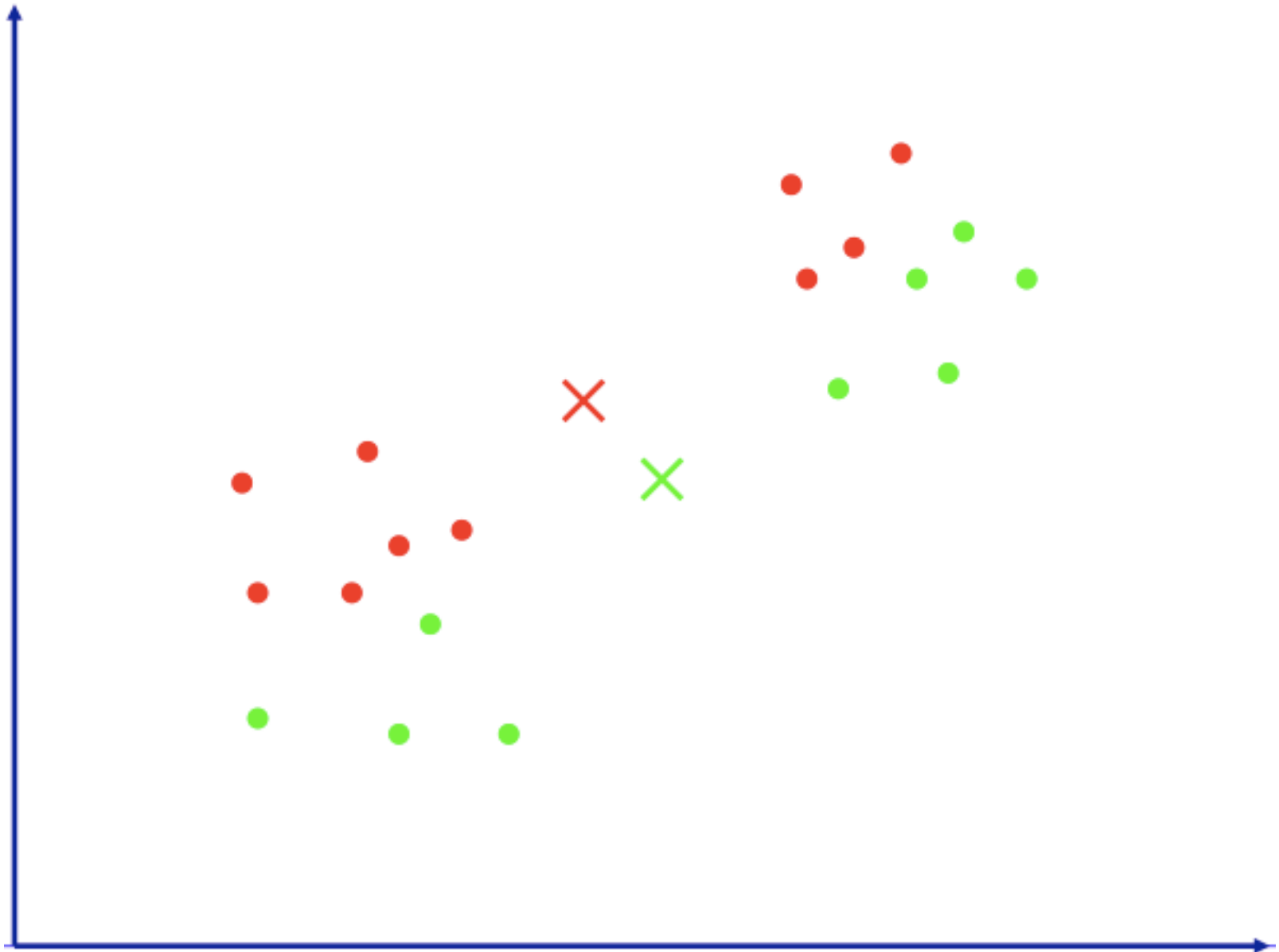
K-means clustering - illustration



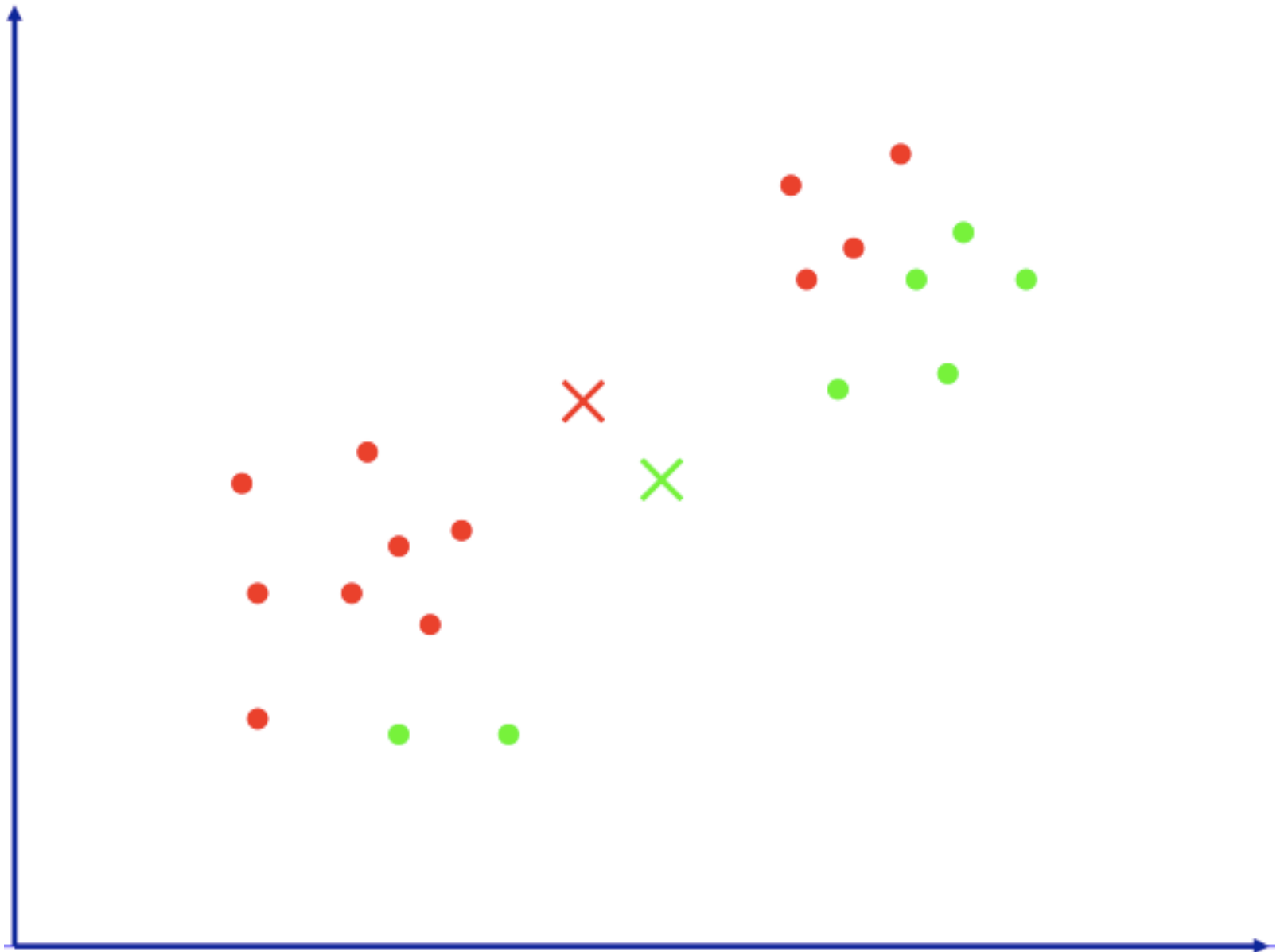
K-means clustering - illustration



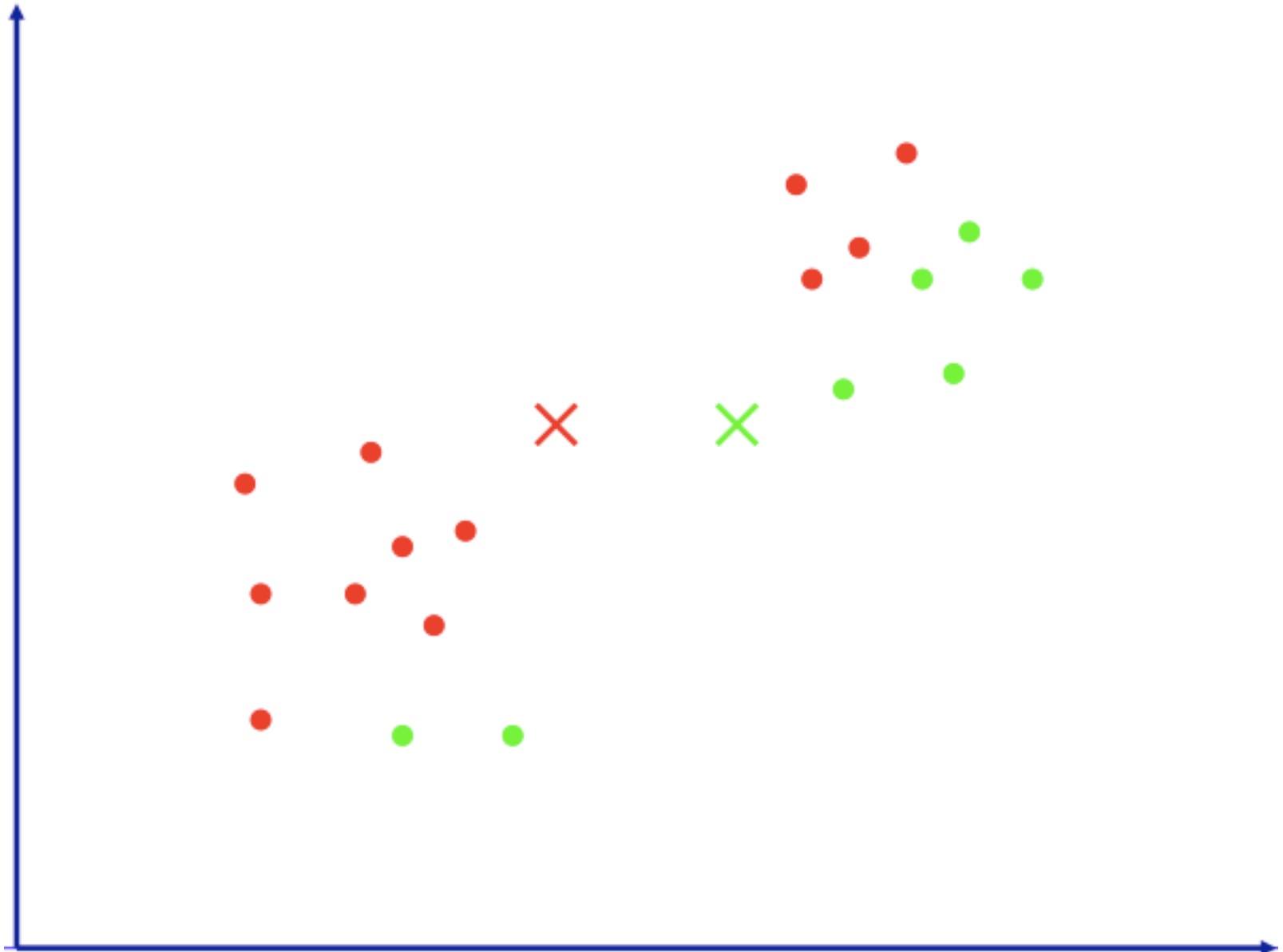
K-means clustering - illustration



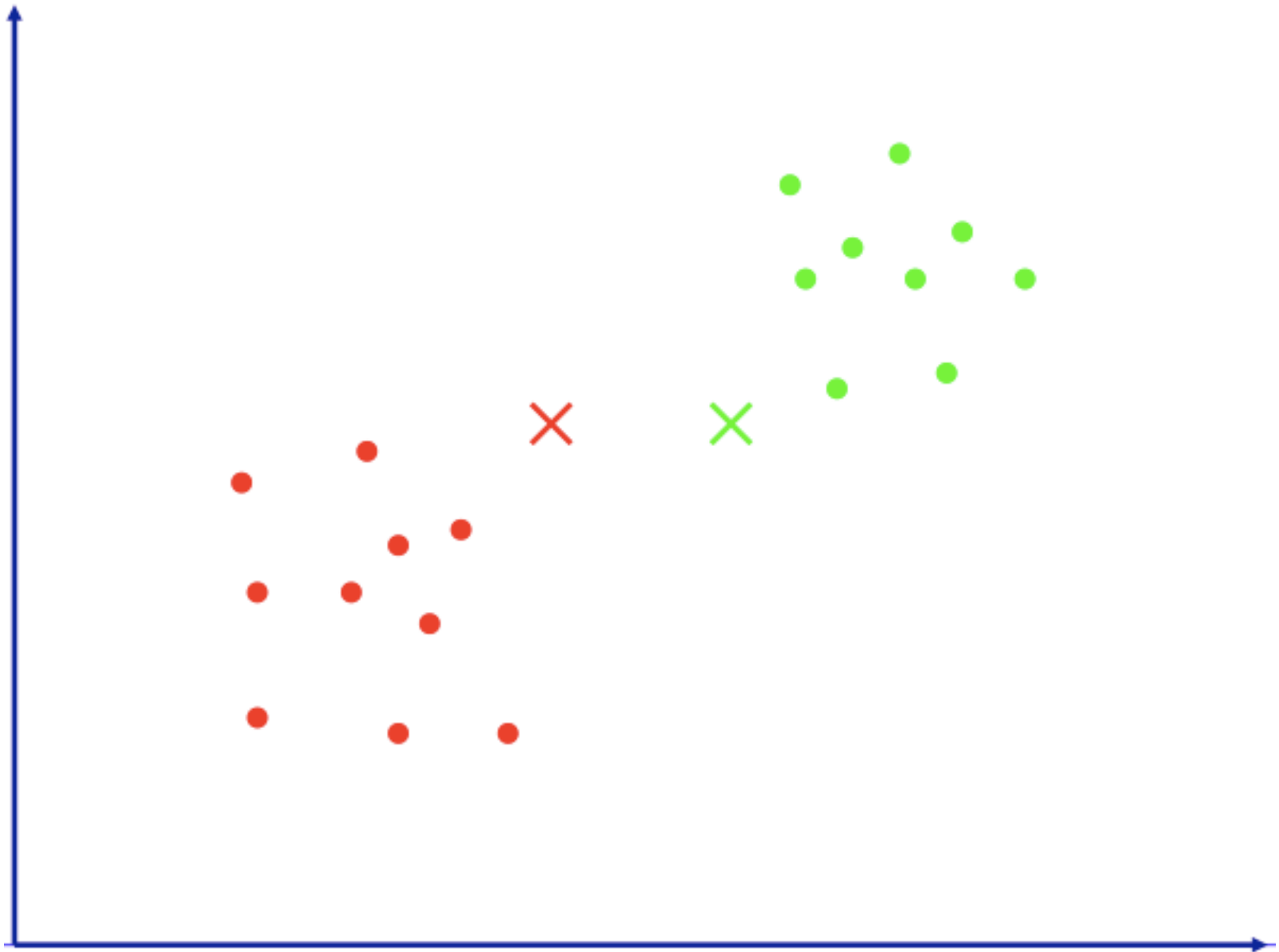
K-means clustering - illustration



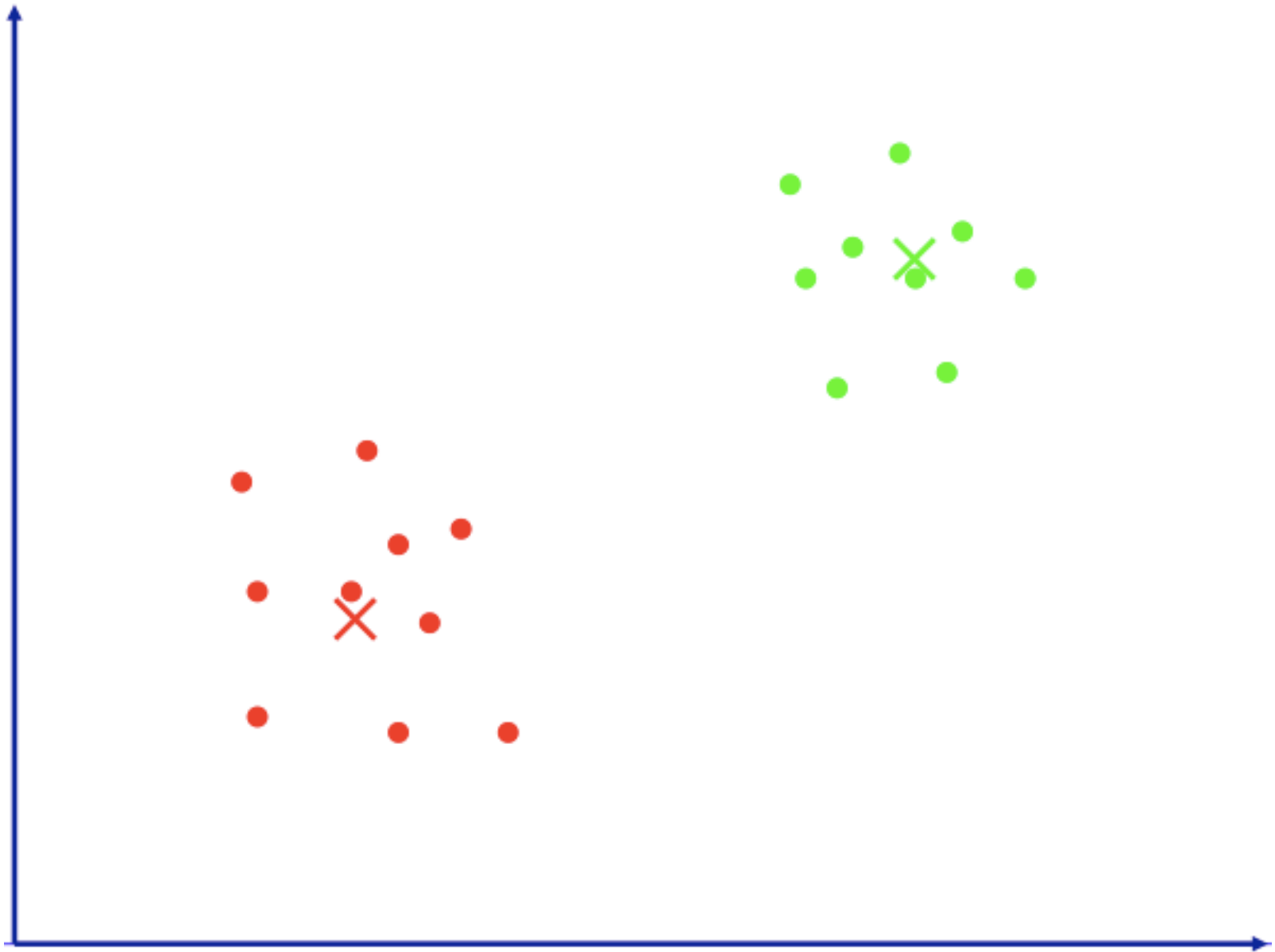
K-means clustering - illustration



K-means clustering - illustration

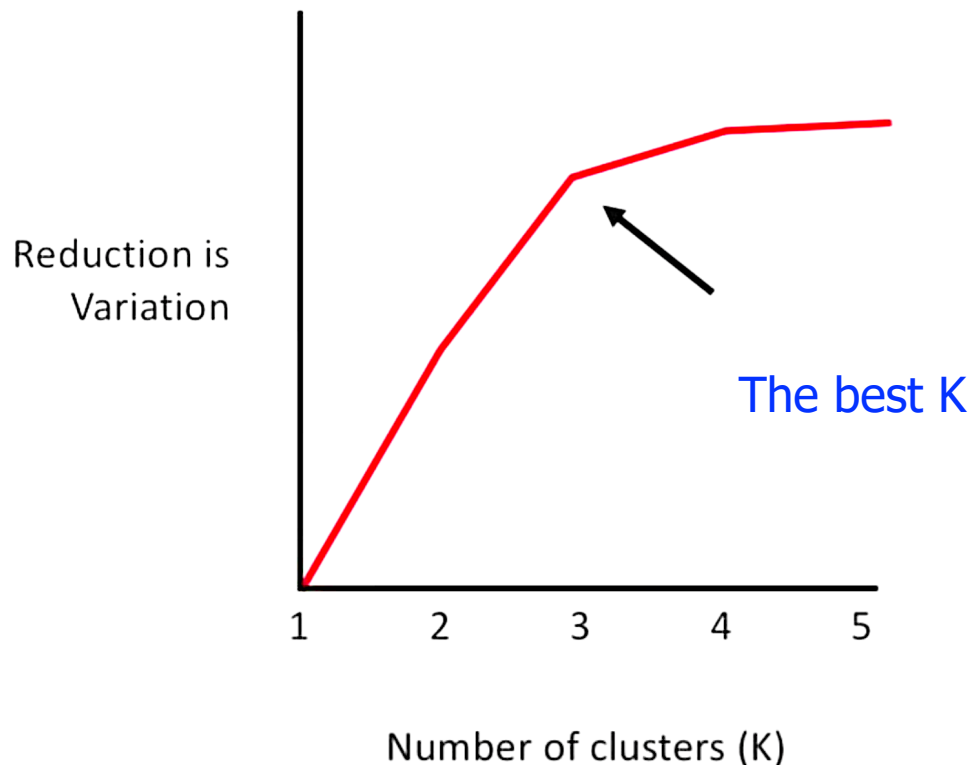


K-means clustering - illustration



How to figure out the best value of K?

- Just try the different value of K
- Check the total variation within each cluster



Bài tập áp dụng 1

- Cho 2 trọng tâm của 2 cụm (cluster) của dữ liệu 2D như sau:
 - Centroid của cụm 1: $(1,5)$
 - Centroid của cụm 2: $(4,1)$
- Giả sử có 3 mẫu dữ liệu A, B, C có các vector đặc trưng lần lượt là: $(1.1,1.2)$, $(2.0,3.0)$ và $(6.3,1.5)$
- Cho biết các mẫu dữ liệu này thuộc về cụm nào?

Bài tập áp dụng 2

- Cho ảnh sau:



- Bằng phương pháp K-means clustering với $K = 3$, hãy trích ra bông hoa trên nền đen như ảnh sau:



Bài tập về nhà

- Ứng dụng phương pháp Kmeans clustering phát hiện quả chín trên cây.



Bài tập về nhà (tt)

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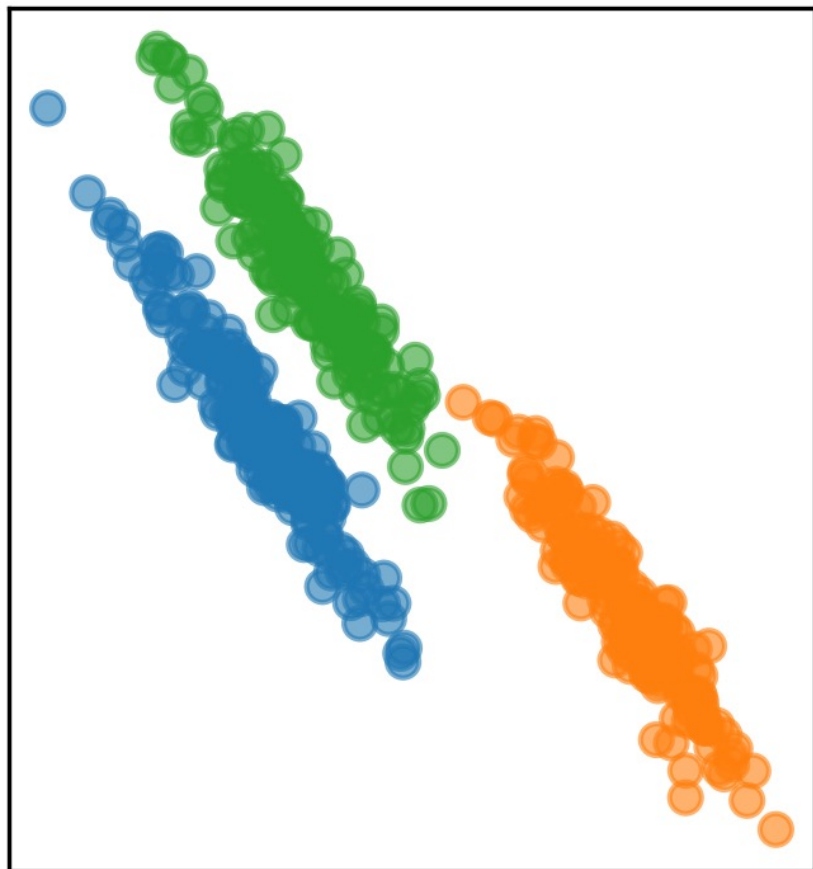


Clustering

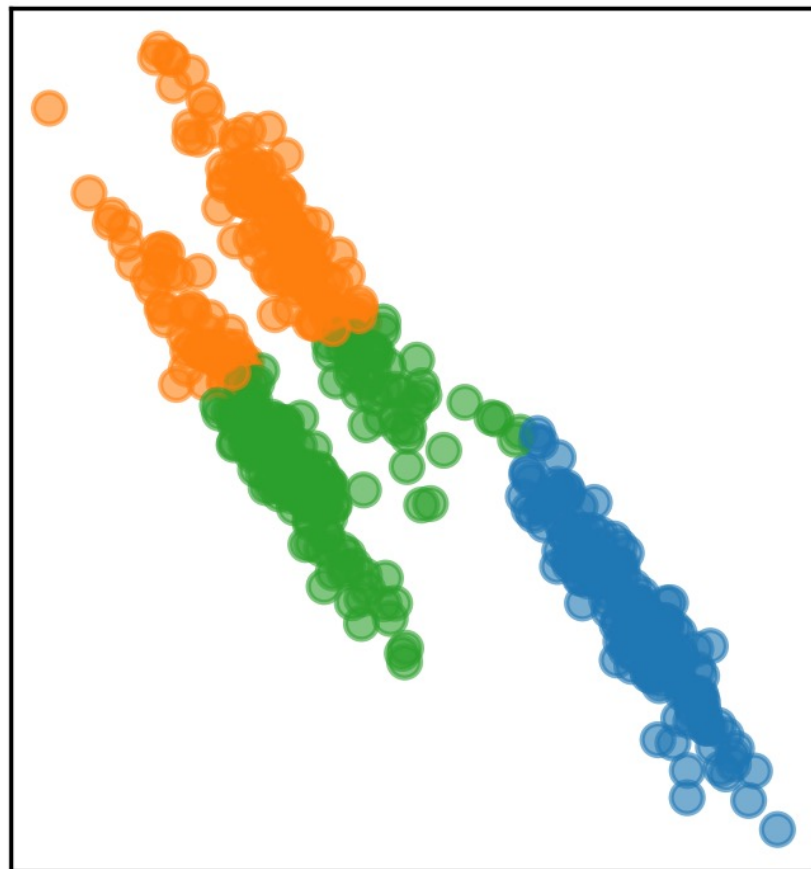
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Drawback of K-means

GaussianMixture



KMeans



GMM clustering

- GMM clustering is a powerful clustering algorithm.
- GMM clustering is distribution-based.

Gaussian distribution

- Gaussian distribution \equiv Normal distribution
- Gaussian distribution has a bell-shaped curve.
- The data points symmetrically distributed around the mean value.

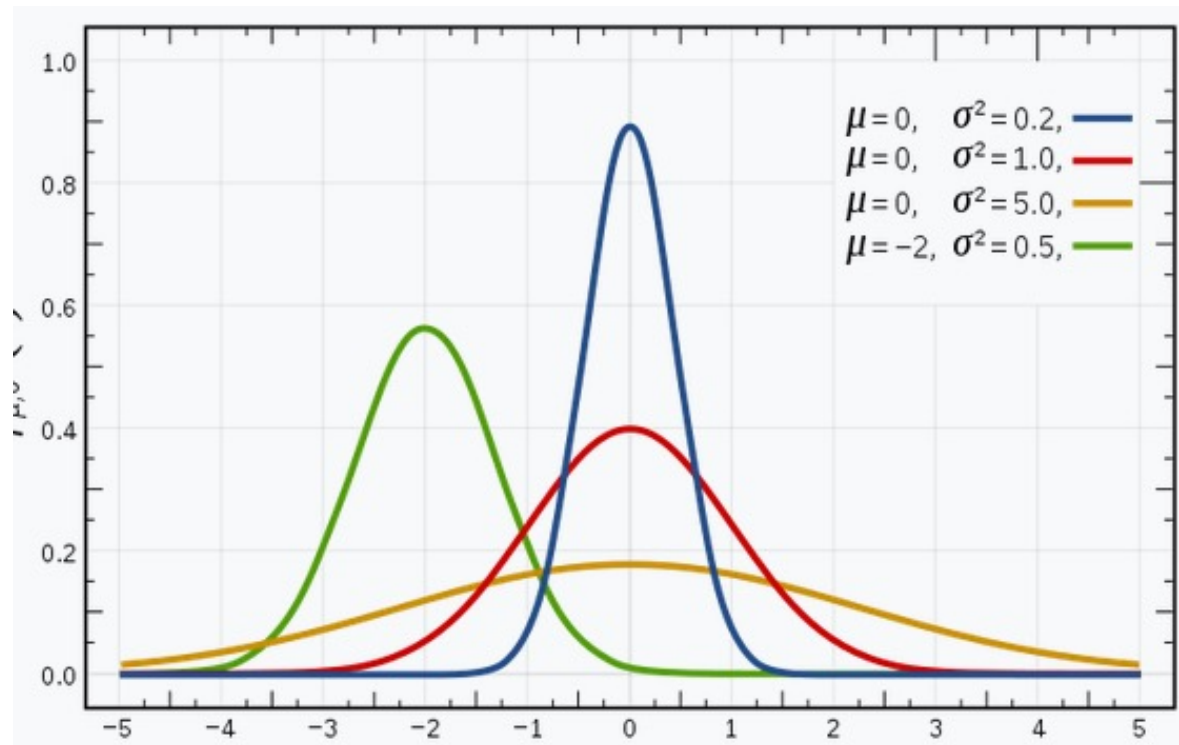
1D Gaussian pdf

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

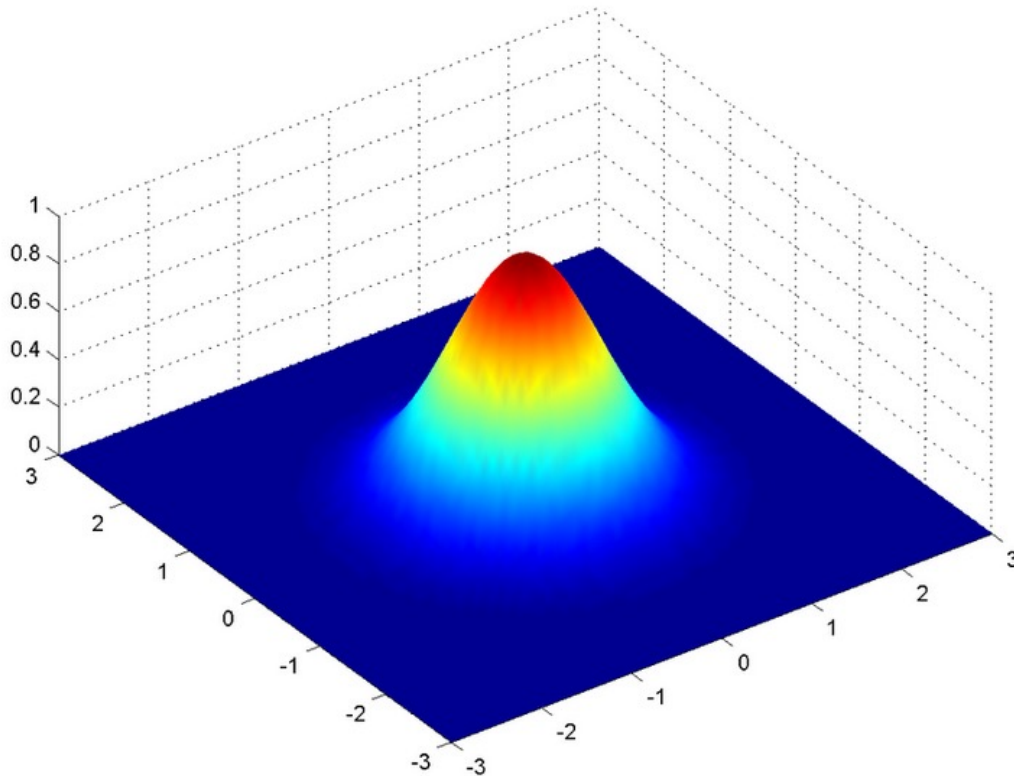
x : input data

μ : mean

σ^2 : variance.



2D Gaussian pdf



\mathbf{x} : input vector (length = 2)

$\boldsymbol{\mu}$: mean vector (length = 2)

$\boldsymbol{\Sigma}$: 2×2 covariance matrix

$$f(\mathbf{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{\sqrt{2\pi|\boldsymbol{\Sigma}|}} \exp \left[-\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right]$$

Gaussian mixture model

- Linear combination of M Gaussian distributions
- pdf of GMM:

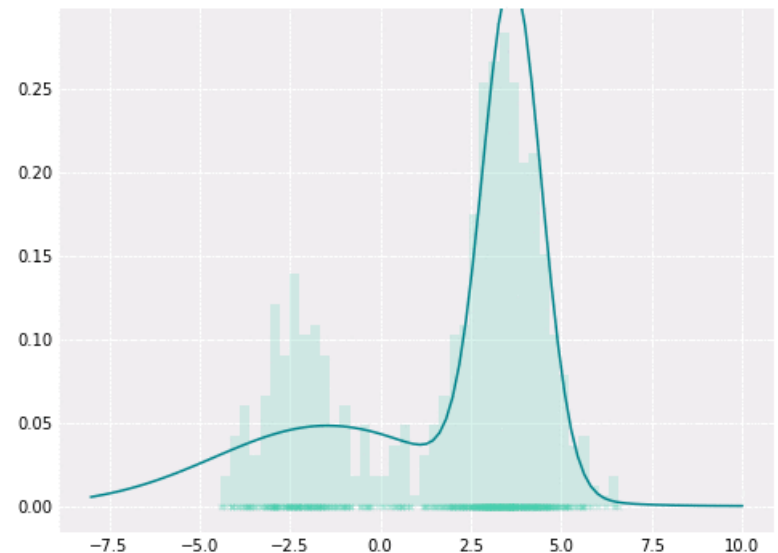
$$p(\mathbf{x}|\lambda) = \sum_{i=1}^M w_i g(\mathbf{x}|\mu_i, \Sigma_i)$$

Ex: $M = 2$

- \mathbf{x} : D-dimension data
- w_i : *mixing coefficients*,
- $1 \leq w_i \leq M$ for all $i = 1, \dots, M$

and $\sum_{i=1}^M w_i = 1$

- g : *Gaussian density components*



GMM clustering algorithms

- GMM parameters:
 - number of Gaussian components (M)
 - weights (ω_i)
 - Gaussian components (mean μ , covariance Σ)
- GMM assumes that all the data points are generated from a mixture of a finite number of Gaussian distributions, and each of these distributions represent a cluster → tends to group the data points belonging to a single distribution together.

GMM clustering algorithm

- GMM training input:
 - number of Gaussian components (M) \equiv number of clusters
 - training data points (\mathbf{x})
- Goal: to model this data using GMM
 - Mixing coefficients $\omega_1, \omega_2, \dots, \omega_M$
 - Mean $\mu_1, \mu_2, \dots, \mu_M$
 - Covariance $\Sigma_1, \Sigma_2, \dots, \Sigma_M$
- Solution: EM algorithm

Expectation-Maximization (EM) algorithm

- EM is a statistical algorithm for finding the right model parameters.
- EM is used when the data has missing values (latent variables).
- EM tries to use the existing data → determine the optimum latent variables → find the model parameters → go back and update the latent variable, and so on.
- **E-step:** the available data is used to estimate (guess) the values of the missing variables
- **M-step:** based on the estimated values generated in the E-step, the complete data is used to update the parameters

GMM-based motion detection

<https://www.youtube.com/watch?v=0nz8JMyFF14&t=844s>