Assigment 1

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1 Exercise 1

$$P(disease/+) = \frac{P(+/disease) \cdot P(disease)}{P(+)}$$

$$P(disease/+) = \frac{P(+/disease) \cdot P(disease)}{P(+/disease) \cdot P(disease) + P(+/nodisease) \cdot P(nodisease)}$$

$$P(disease/+) = \frac{0.98 \cdot 0.05}{0.98 \cdot 0.05 + 0.03 \cdot 0.95}$$

$$P(disease/+) = \frac{98}{155}$$

2 Exercise 2

1. Proof normalization

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-u}{\sigma})^2}$$

$$\int f(x)dx = \int \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-u}{\sigma})^2}$$

We choose: $z = \frac{x-u}{\sigma} => dx = \sigma dz$

$$\int f(x)dx = \int \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-z^2}{2}} \sigma dx$$

$$\int f(x)dx = \int \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dx$$

We choose: $y^2 = \frac{z^2}{2} => y = \frac{z}{\sqrt{2}} => dz = \sqrt{2}dy$

$$\int f(x)dx = \int \frac{1}{\sqrt{\pi}} e^{y^2} dy$$

We have $\int e^{y^2} dy = \sqrt{\pi}$ Gaussian Intergal

$$\int f(x)dx = 1$$

or

$$\int \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dx = 1(1)$$

2. Mean

$$E[x] = \int xf(x)dx = \int x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-u}{\sigma})^2} dx$$

We choose $y = x - u => dx = dy$

$$E[x] = \frac{1}{\sigma\sqrt{2\pi}} \int (y+u) \cdot e^{\frac{-1}{2}(\frac{y}{\sigma})^2} dx$$

$$E[x] = \frac{1}{\sigma\sqrt{2\pi}} \int y \cdot e^{\frac{-1}{2}(\frac{y}{\sigma})^2} dx + \frac{1}{\sigma\sqrt{2\pi}} \int u \cdot e^{\frac{-1}{2}(\frac{y}{\sigma})^2} dx$$

Because $y \cdot e^{-y^2}$is odd function so $\frac{1}{\sigma\sqrt{2\pi}} \int y \cdot e^{\frac{-1}{2}(\frac{y}{\sigma})^2} dx = 0$

$$E[x] = \frac{1}{\sigma\sqrt{2\pi}} \int u \cdot e^{\frac{-1}{2}(\frac{y}{\sigma})^2} dx$$

$$E[x] = u\frac{1}{\sigma\sqrt{2\pi}} \int \cdot e^{\frac{-1}{2}(\frac{y}{\sigma})^2} dx$$

From (1):

$$E(x) = u$$

3. Standard deviation

$$Var(x) = E((x-u)^2) = \int (x-u)^2 f(x)dx$$

$$=> Var(x) = \int (x-u)^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-u}{\sigma})^2} dx$$

We choose $y = x - u => dy = dx$

$$=> Var(x) = \frac{1}{\sigma\sqrt{2\pi}} \int y^2 e^{\frac{-1}{2}(\frac{y}{\sigma})^2} dx$$

We choose

$$u = y => du = dy$$

$$dv = ye^{\frac{-y^2}{2\sigma^2}} dy => v = \sigma^2 e^{\frac{-y^2}{2\sigma^2}}$$

$$=> Var(x) = \frac{1}{\sigma\sqrt{2\pi}} [ye^{\frac{-y^2}{2\sigma^2}} |_{\infty \to \infty} + \int \sigma^2 e^{\frac{-y^2}{2\sigma^2}} dy]$$

$$=> Var(x) = \frac{1}{\sigma\sqrt{2\pi}} \int \sigma^2 e^{\frac{-y^2}{2\sigma^2}} dy$$

$$=> Var(x) = \sigma^2 \frac{1}{\sigma\sqrt{2\pi}} \int e^{\frac{-y^2}{2\sigma^2}} dy$$

From (1)

$$=> Var(x) = \sigma^2$$

$$=> Std(x) = \sigma$$