

# User Manual for BACH

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BACH implements Bausch's algorithm to evaluate the distribution function of a weighted sum of  $\chi_1^2$  random variables [1]; one important application is to compute the p-values associated with the Bayes factors [2]. There are two ways to run BACH : simple or in batch. Note in the following example, we use “./bach” as the program name; in the bach folder there are “bach-mac” (bach for Mac OS X) and “bach-linux” (bach for Linux). You may either rename the executable or modify the command lines below.

## 1 Simple run

Denoted by  $\chi_1^2$  a chi-squared random variable (r.v.) with 1 degree of freedom (d.f). Let  $Y = X_1 + 0.8X_2 + 0.6X_3 + 0.4X_4$ , where  $X_i \sim \chi_1^2$  independently. To compute  $P(Y > 10)$ , the command line is:

```
./bach 1 0.8 0.6 0.4 10
```

A p-value will be printed to the screen:

```
0.00914343
```

**Rules of input:** Numbers are separated by space; the last number is the statistic and the other numbers are coefficients.

Suppose we also want to compute  $P(Y > 20)$ . Of course we may repeat the above command line, substituting 10 with 20. But a more efficient way is to compute  $P(Y > 20)$  and  $P(Y > 10)$  simultaneously because we can reuse the distribution functions. The command line is:

```
./bach 1 0.8 0.6 0.4 10,20
```

Two p-values will be printed to the screen:

```
0.00914343 4.31004e-5
```

**Rules of input:** If we want to compute multiple p-values for the same coefficients, just append more test statistics in the end using ‘,’ as the delimiter.

## 2 Batch run

Suppose  $Z = 5X_5 + 0.2X_6$ , where  $X_i \sim \chi_1^2$ . If we want to compute  $P(Z > 34)$ ,  $P(Y > 10)$  and  $P(Y > 20)$  simultaneously in batch mode, we can create the following file (named “text.input”):

```
1 0.8 0.6 0.4 10,20
5 0.2 34
```

Each row in the input file is what you would type using command line save the “./bach”. To run BACH in batch mode:

```
./bach -i test.input
```

Three p-values will then be printed on the screen:

```
0.00914343 4.31004e-5
0.00932733
```

Within each row, multiple p-values are delimited by a tab ('\t'); each row in the output corresponds to the same row in the input file.

### 3 Output file

When ‘-o’ argument is invoked for batch mode, an output file will be generated.

```
./bach -i test.input -o test.output
```

This command will produce a ‘test.output’ which reads

p-value	error-bound	coefficients	statistic
0.00914343	3.02826e-15	0.4 0.6 0.8 1	10
4.31004e-05	5.37358e-11	0.4 0.6 0.8 1	20
0.00932733	3.55176e-08	0.2 5	34

The output file contains 4 columns: (1) p-value; (2) the upper bound of the absolute error; (3) coefficients of the  $\chi^2_1$  variables; (4) the value of the statistic. The delimiter between fields is the tab, and within the field of “coefficients” the delimiter is the blank space.

### 4 Options

- -i [string=]: the input filename. This is required for batch mode.
- -o [string=]: the output filename for the batch mode.
- -c [int=6]: the precision of all output (number of significant digits, default is 6).
- -h : stop the program and print the HELP.

There are additional options to control error bounds, some of which concern GMP library used in our implementation; one may send an email to inquire if on the off chance one is interested.

### References

- [1] Johannes Bausch. On the efficient calculation of a linear combination of chi-square random variables with an application in counting string vacua. *Journal of Physics A: Mathematical and Theoretical*, 46(50):505202, 2013.
- [2] Quan Zhou and Yongtao Guan. On the null distribution of Bayes factors in linear regressin. *JASA (Theory and Methods)*, 113(523): 1362 – 1371, 2018.